$Hb\bar{b}$ production as an example of modern amplitudes calculations

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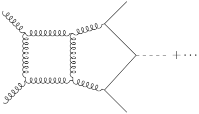
Typical workflow of loop amplitude computations

1. Draw all relevant Feynman diagrams:



2. Write down the integrand:

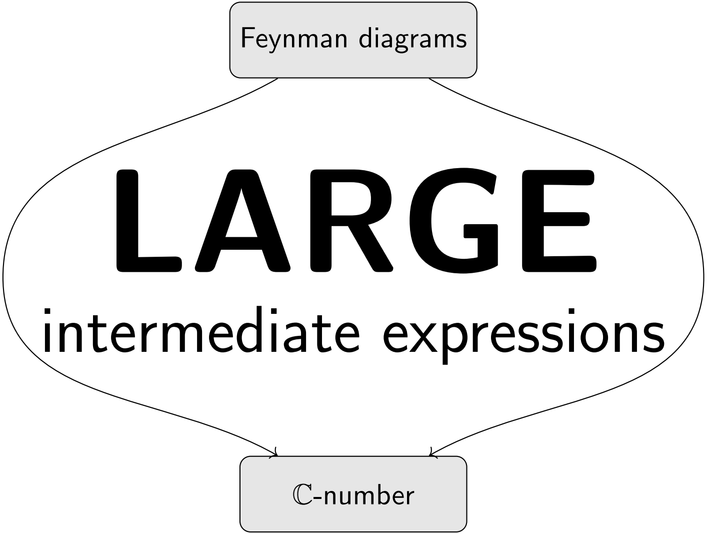
$$A = \sum_{T \in topologies} \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\sum_i c_i(\{p\}) m_i(\{k, p\})}{\prod_{j \in T} D_j(\{k, p\})}$$

3. Reduce the amplitude onto a set of master integrals:

$$A = \sum_j d_j\,(\epsilon, p) \times MI_j(\epsilon, p)$$

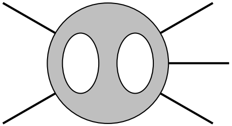
4. Evaluate the result at a chosen phase-space point

Complexity


Feynman diagrams
LARGE
intermediate expressions
$\mathbb{C}$-number

Complexity

- Complexity increases with **loop order** and **multiplicity**.
- Current QCD frontier: $2 \to 3$ scattering at NNLO.

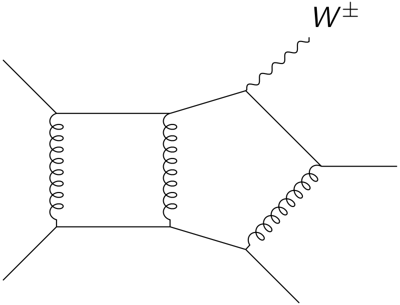


- Massless case: results for all relevant Feynman integrals available.
- One external mass: results for all planar + some non-planar integrals now available.

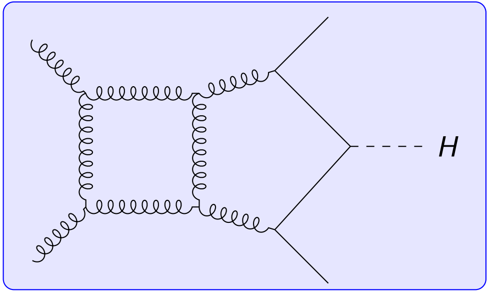
one-mass, planar:	Nov '15	[Papadopoulos, Tommasini, Wever]	(one penta-box, MPLs)
	May '20	[Abreu, Ita, Moriello, Page, Tschernow, Zeng]	(DEs+numerical sols)
	Sep '20	[Canko, Papadopoulos, Syrrakos]	(MPLs)
	Dec '20	[Syrrakos]	(1L pentagon, MPLs)
	Oct '21	[Chicherin, Sotnikov, Zoia]	(2L pentagon functions)
one-mass, non-planar	Oct '19	[Papadopoulos, Wever]	(one hexa-box, MPLs)
	July '21	[Abreu, Page, Ita, Tschernow]	(hexa-box, DEs+numerical sols)

Recent work

- $pp \to W/H + b\bar{b}$ at 2L (leading colour, massless b quarks)



[Badger, Hartanto, Zoia, Feb '21]



[Badger, Hartanto, **Kryś**, Zoia, July '21]

- $W(\to \ell\bar{\ell}') + 4$-partons at 2L (leading colour, massless quarks)
  [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, Oct '21]
- $pp \to W(\to l\nu)\gamma + j$ at 2L (leading colour, massless quarks)
  [Badger, Hartanto, **Kryś**, Zoia, Jan '22]

Finite fields

- To avoid analytic complexity in intermediate steps, use numerical evaluations over **finite fields**
- We work with **rational numbers** modulo a large prime number:

$$q = \frac{a}{b} \longrightarrow q \bmod p \equiv \left(a \times \left(b^{-1} \bmod p\right)\right) \bmod p$$

$$\frac{3}{7} \equiv 2 \bmod 11$$

- One can reconstruct the analytic result from its many numerical evaluations
- `FiniteFlow` [Peraro, '19]

$pp \to b\bar{b}H$

- Three channels are relevant:
  - $0 \to \bar{b}(p_1) + b(p_2) + g(p_3) + g(p_4) + H(p_5)$
  - $0 \to \bar{b}(p_1) + b(p_2) + \bar{q}(p_3) + q(p_4) + H(p_5)$
  - $0 \to \bar{b}(p_1) + b(p_2) + \bar{b}(p_3) + b(p_4) + H(p_5)$
- After colour-decomposition, amplitude can be written as:

$$A = \sum_{T \in topologies} \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \frac{\sum_i c_i(\{p\}) mon_i(\{k,p\})}{\prod_{j \in T} D_j(\{k,p\})}$$

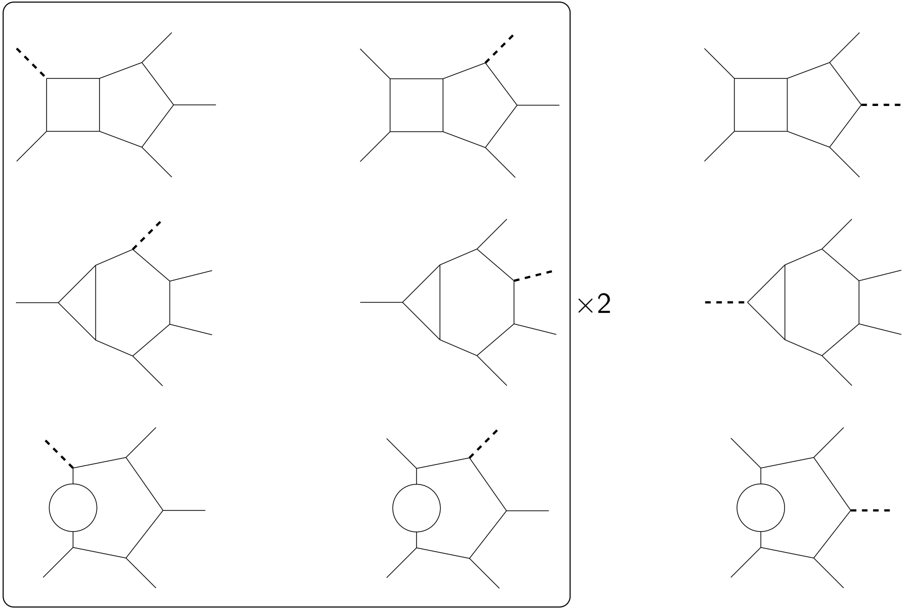
- Coefficients $c_i(p(x))$ are given a rational parametrisation using **momentum twistors** x

(Begin finite field sampling)

$pp \to b\bar{b}H$

- The amplitude is mapped onto scalar integrals within 15 **maximal topologies**

$pp \to b\bar{b}H$



$pp \to b\bar{b}H$

- The amplitude is mapped onto scalar integrals within 15 **maximal topologies**
- Scalar integrals are IBP-reduced onto a **master integral basis** [Laporta, '01], [Lee, '13]

$$A = \sum_i d_i\left(\epsilon, p(x)\right) \times MI_i(\epsilon, p)$$

- We work with MIs that satisfy canonical DEs [Henn, '13]:

$$\mathrm{d}\vec{MI} = \epsilon \left(\sum_{i=1}^{58} a_i \times \mathrm{d}\log w_i \right) \vec{MI}\,,$$

where the 'letters' w_i are algebraic functions of external kinematics [Abreu, Ita, Moriello, Page, Tschernow, Zeng, '20]

$pp \to b\bar{b}H$

- Map the MIs onto a basis of special functions $\{f\}$ related to the letters
- Subtract the poles to get the **finite remainder**:

$$F^{(L)} = \sum_i r_i\,(p(x))\,m_i(f)\,,$$

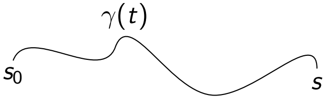
where $m_i(f)$ are monomials formed from elements of the **finite remainder function basis**

- Reconstruct the coefficients, now free of ϵ

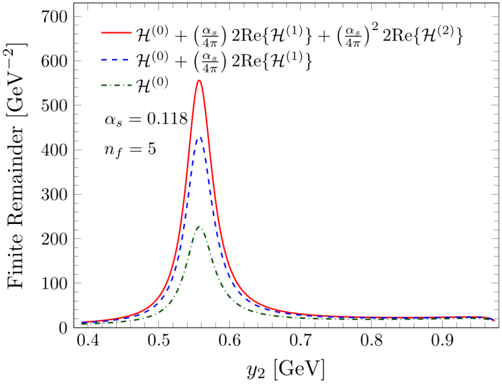
(End finite field sampling)

Evaluating special functions

- The finite remainder function basis is written in terms of **Chen's iterated integrals** [Chen, '77]
- The iterated integrals expose cancellations in finite remainders
- They satisfy differential equations as well, order-by-order up to $\mathcal{O}(\epsilon^4)$
- Solve them numerically in `DiffExp` using the method of **generalised series expansions** [Moriello, '19], [Hidding, '20]



Results



The finite remainders of the *gg* channel interfered with tree-level amplitudes, evaluated at a univariate phase-space slice.

$$p_1 = \frac{y_1\sqrt{s}}{2}(1\,,1\,,0\,,0) \qquad p_2 = \frac{y_2\sqrt{s}}{2}(1\,,\cos\theta\,,-\sin\theta\sin\phi\,,-\sin\theta\cos\phi)$$

$$p_3 = \frac{\sqrt{s}}{2}(-1\,,0\,,0\,,-1) \qquad p_4 = \frac{\sqrt{s}}{2}(-1\,,0\,,0\,,1)$$

Conclusion

- Calculated two-loop QCD amplitudes for $pp \to b\bar{b}H$
- Developed a `Mathematica` + `FORM` + `FiniteFlow` routine that can be adapted to other processes as needed
- Implemented several tools to overcome the complexity
- Integrals for non-planar topologies needed for $pp \to H + 2j$ and for $pp \to V + 2j$ beyond leading colour

Details of reconstruction

1. Linear relations between rational coefficients: $F^{(L)} = \sum_i r_i\,(p(x))\,m_i(f)$
   - Coefficients r_i are not independent
   - Find relations between them and choose the independent ones based on the lowest polynomial degree

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2. Factor matching:
   - Aid the reconstruction by providing an ansatz of factors related to the letters

$$\{\langle ij\rangle, [ij], \langle i|p_5|j], s_{ij}, s_{ij} - s_{kl}, s_{i5} - p_5^2, p_5^2, \mathrm{tr}_5, \Delta_1, \Delta_2,$$
$$s_{15}(s_{13} + s_{34}) - p_5^2 s_{34}, s_{25}(s_{24} + s_{34}) - p_5^2 s_{34}\}$$

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3. Univariate partial fractioning
   - Having guessed the denominator, construct a partial-fractioned ansatz

Details of reconstruction

$\bar{b}bggH$	helicity configurations	$r_i(x)$	independent $r_i(x)$	partial fraction in x_5	points
$F^{(2),1}$	$++++$	63/57	52/46	20/6	3361
	$+++-$	135/134	119/120	28/22	24901
	$++--$	105/111	105/111	22/12	4797
$F^{(2),n_f}$	$++++$	45/41	45/41	16/6	1381
	$+++-$	94/95	94/95	17/6	1853
	$++--$	89/95	62/69	18/3	2492
$F^{(2),n_f^2}$	$++++$	12/8	9/7	0/0	3
	$+++-$	11/16	11/16	3/0	22
	$++--$	12/20	8/16	8/0	242

Maximum numerator/denominator polynomial degrees and the sample points needed for the reconstruction of the finite remainder coefficients.

$$F^{(L)} = \sum_i r_i\left(p(x)\right) m_i(f)$$

Chen's iterated integrals

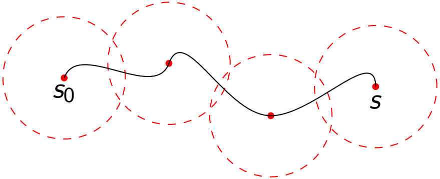
Defined as:

$$[w_{i_1}, \ldots, w_{i_n}]_{s_0}(s) = \int_0^1 \mathrm{d}t \frac{\mathrm{d}\log w_{i_n}(\gamma(t))}{\mathrm{d}t} \left[w_{i_1}, \ldots, w_{i_{n-1}}\right]_{s_0}(\gamma(t)), \quad [\,]_{s_0} := 1$$

The number of integration kernels w_i is known as **transcendental weight**.

They have several advantages:

1. Automatically implement functional relations
2. Singularities or branch points only where one of the letters vanishes or diverges
3. Simplify the finite remainder function basis through analytic cancellations



Our workflow