QCD and leading Yukawa corrections to $gg \to HH$

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Karlsruher Institut für Technologie

Higgs 2022 - Pisa - Nov. 2022

Institute for Theoretical Particle Physics

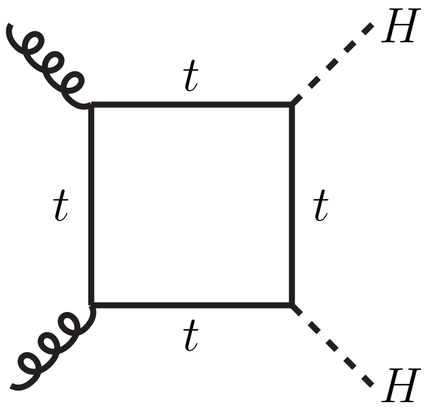
Motivation: Higgs self-coupling

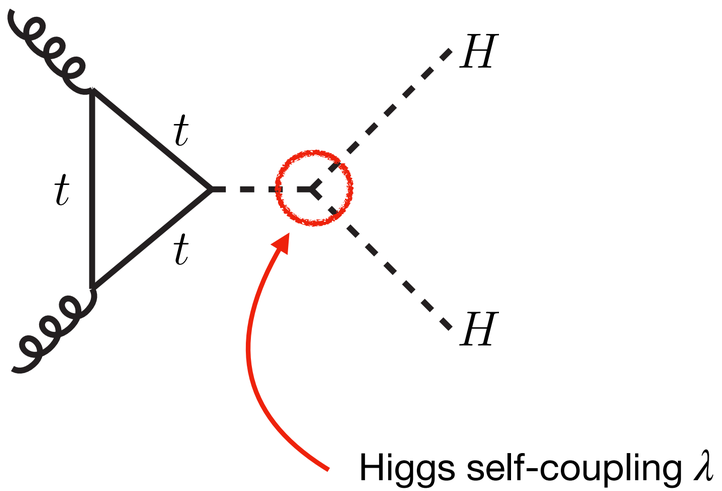
- Probe Higgs self-coupling in pair productions, and compare with the Standard Model value

  $\lambda = m_H^2/(2v^2) \approx 0.13$ in the Higgs potential

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

- Gluon-fusion channel dominates Higgs pair production at LHC





Overview of QCD and EW calculations

- **NLO QCD corrections with full m_t-dependence are known**
  - Numerical approach [Borowka, Geiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke, 16'], [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher, 18']
  - Expansions in different regions [Daoson, Dittmaier, Spira, 98'], [Grigo, Hoff, Melnikov, Steinhauser, 13'], [Degrassi, Giardino, Gröber, 16', and Bonciani, 18'], [Gröber, Maier, Raum, 17'], [Davies, Mishima, Steinhauser, Wellmann, 18', 19'], [Xu, Yang, 18', and Wang, Xu, 20']
  - Expansions + numerical approach [Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann, 19'], [Bellafronte, Degrassi, Giardino, Gröber, Vitti, 22']

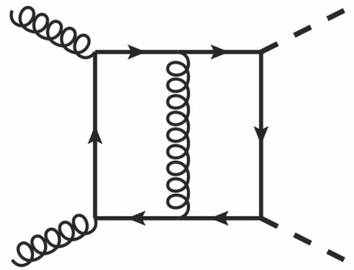
- **NNLO and N^{3}LO QCD corrections are available in large-m_t limit / expansion**
  - At NNLO [de Florian, Mazzitelli, 13'], [Grigo, Melnikov, Steinhauser, 14' and Hoff, 15'], [Davies, Herren, Mishima, Steinhauser, 19', 21'], [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, 18']
  - At N^{3}LO [Spira, 16'], [Gerlach, Herren, Steinhauser, 18'], [Banerjee, Borowka, Dhabi, Gehrmann, Ravindran, 18'], [Chen, Li, Chao, Wang, 19']

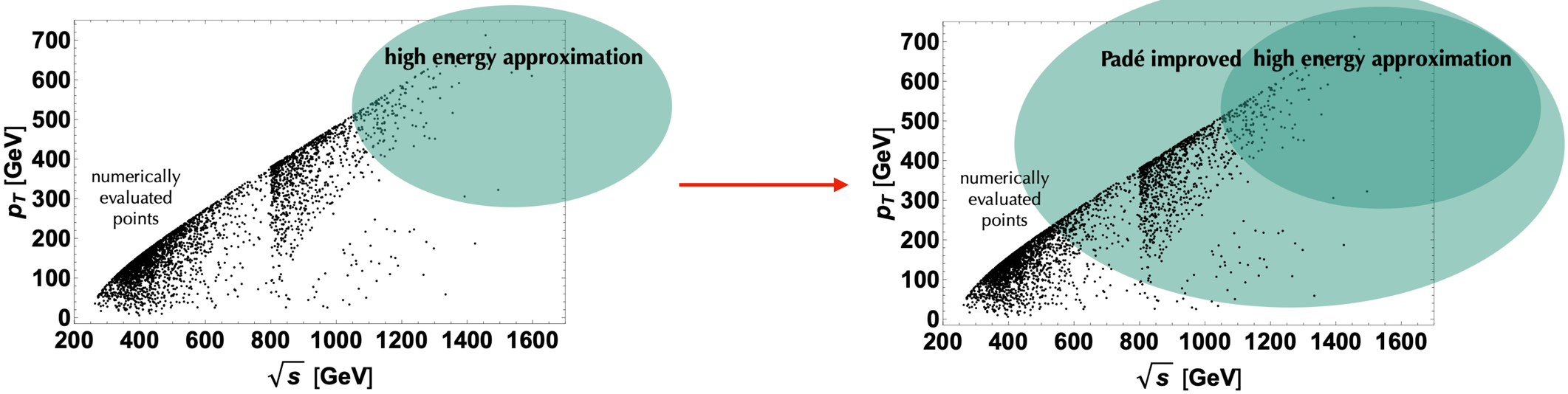
- **NLO EW corrections are partly available in small-m_t and large-m_t limits**
  - Yukawa-top correction [Davies, Mishima, Schönwald, Steinhauser, Zhang, 22'], [Mühlleitner, Schlenk, Spira, 22']

Recent QCD calculations @ NLO

[Davies, Mishima, Steinhauser, Wellmann, ***JHEP 03 (2018) 048, JHEP 01 (2019) 176***]

- **Virtual 2-loop** $gg \to HH$
  - Analytic high-energy expansion + Padé improvements to $\mathcal{O}(m_t^{32})$ in small-m_t limit



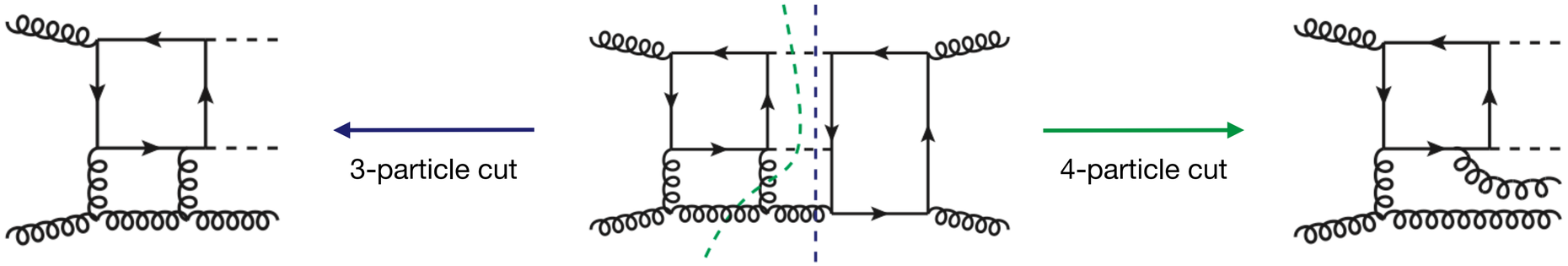


[From G. Mishima's talk at Radcor-Loopfest 2021]

Recent QCD calculations @ NNLO

[Davies, Herren, Mishima, Steinhauser, ***JHEP 05 (2019) 157***, ***JHEP 01 (2022) 049***]

- **Real-Virtual 2-loop** $gg \to HH + g$ **& Real-Real 1-loop** $gg \to HH + gg$
  - Compute via Optical theorem: convert phase-space integral to loop integrals
  - Analytic total cross section computed to $\mathcal{O}(1/m_t^8)$ in large-m_t limit (combined with virtual 3-loop corrections)



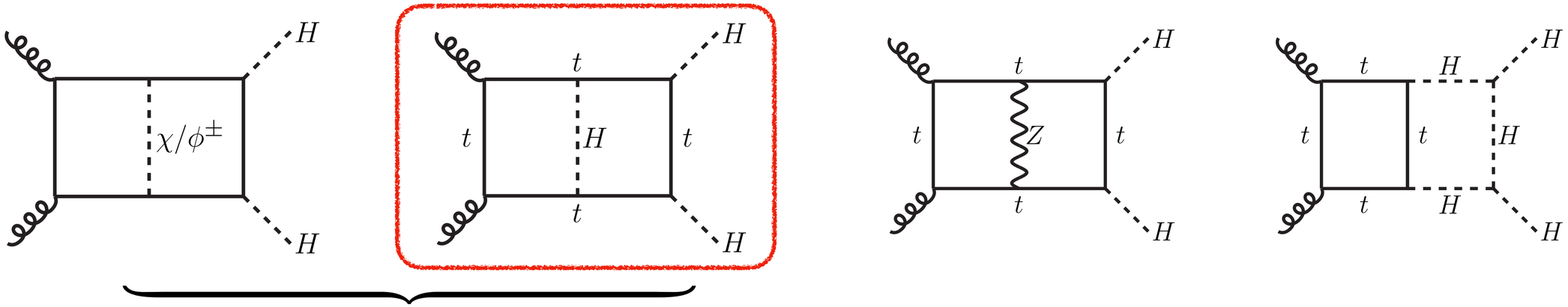
Real-Virtual

Real-Real

Towards NLO EW corrections: two-loop leading Yukawa

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]

- Sample two-loop diagrams contributing to NLO EW



sample diagrams of leading Yukawa correction $\sim \alpha_s y_t^4$

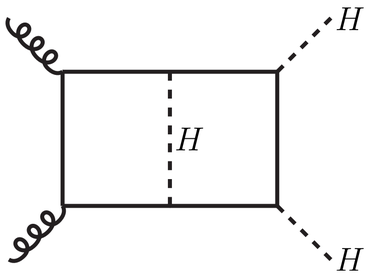
- Two-loop Yukawa (EW) corrections are not small, $y_t^2 = \dfrac{\alpha\, m_t^2}{2\sin^2\theta_w\, m_W{}^2} \sim \dfrac{1}{2}\alpha_s$

- **Aim:** analytic high energy expansion to leading Yukawa corrections with internal Higgs in $p_T^H \geq 120\,\mathrm{GeV}$ region

(See also Spira's talk)

Analytic high-energy expansion

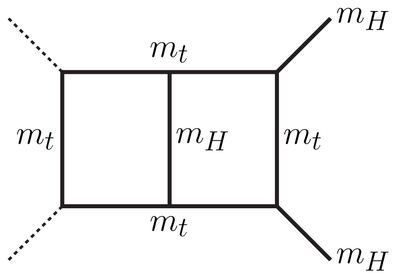
How?



kinematic invariants

$$s = (q_1 + q_2)^2$$
$$t = (q_1 + q_3)^2$$
$$u = (q_2 + q_3)^2$$

scalar master integral (MI)



solid line: massive
dashed line: massless

Analytic high-energy expansion

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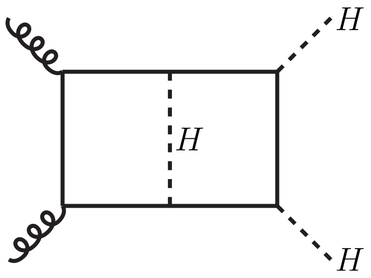

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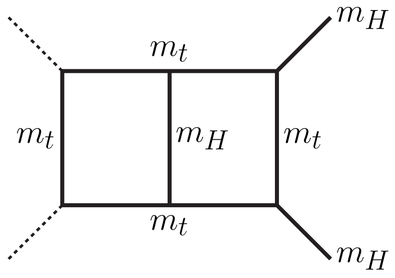

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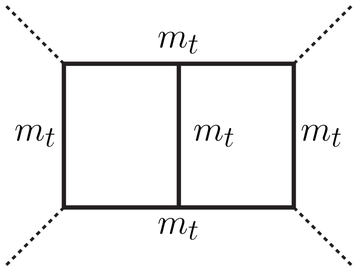
two expansions



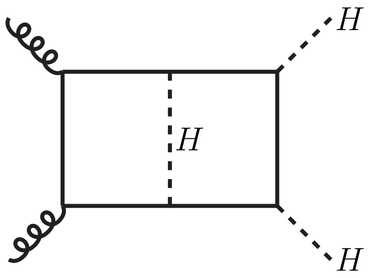
$$s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2$$

general to full EW corrections

need entire new family of MIs

Analytic high-energy expansion

How?



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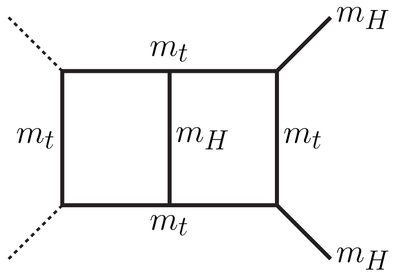

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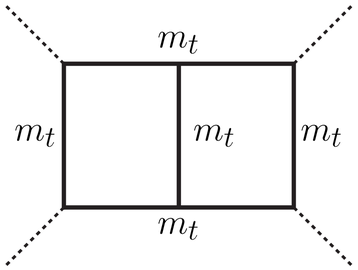

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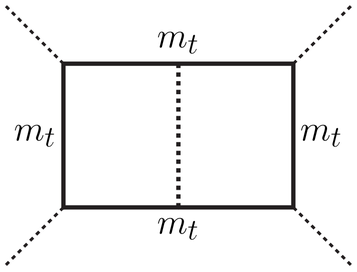

$s, t \gg m_t^2 \gg \left(m_H^{\text{int}}\right)^2, \left(m_H^{\text{ext}}\right)^2$

reduce to known QCD MIs

non-trivial to full EW corrections

Higher-order $(m_t^2/s)^n$ power corrections

Differential equations, asymptotic expansions and Mellin-Barnes integrals

1. Amplitudes are generated by **qgraf** [Nogueira], **q2e&exp** [Harlander, Seidensticker, Steinhauser], **FORM** [Vermaseren], and integration-by-parts (IBP) reduction yields 140 MIs to $\mathcal{O}(\epsilon^0)$ by **FIRE6 and ImproveMasters.m** [Chukharev, Smirnov[2]]
2. Linear system of coupled differential equations for 140 MIs derived with **LiteRed** [Lee]

$$\frac{\partial}{\partial\left(m_t^2\right)}\mathbf{I} = M(s,t,m_t^2,\epsilon)\,\mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{J}_1,\ldots,\mathcal{J}_{140}\right)^T$$

3. Higher-order $(m_t^2/s)^n$ corrections can be solved by inserting **power-log ansatz** for each MI

$$\mathcal{J}_n = \sum_{i=-2}^{0}\sum_{j=-1}^{40}\sum_{k=0}^{i+4} C_{(n)}^{ijk}(s,t)\,\epsilon^i\left(m_t^2\right)^j \log^k(m_t^2)$$

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challenging & require improved mathematical method

4. Boundary conditions are calculated in **asymptotic limit** $m_t \to 0$ by Mellin-Barnes method and analytic summations with help of **MB.m** [Czakon], **HarmonicSums.m** [Ablinger], **Sigma.m and EvaluateMultiSums.m** [Schneider]

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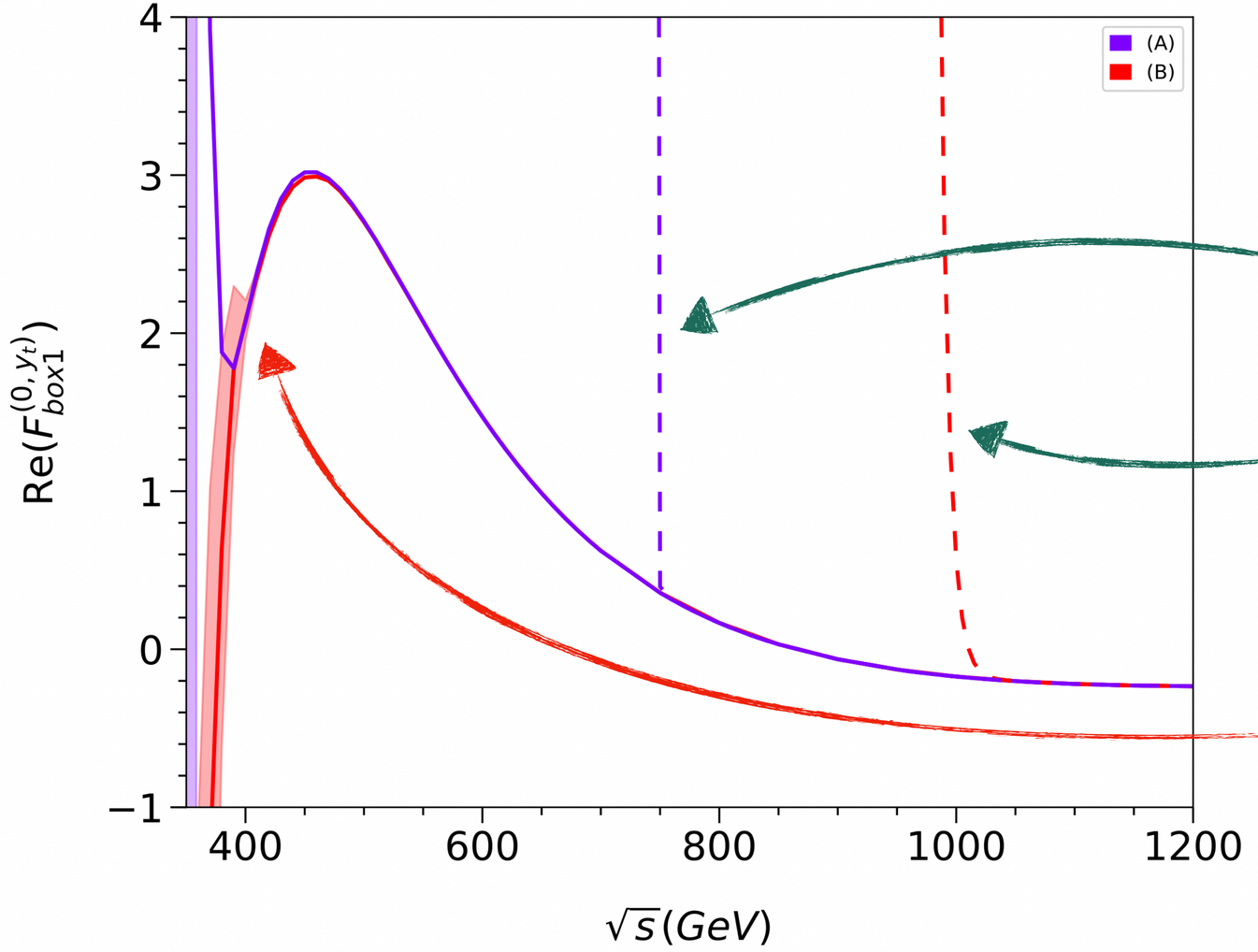
5. We solve all MIs to $\mathcal{O}\left(m_t^{120}\right)$ and apply Padé approximations at the level of Form factors

$$\mathcal{F}^N = \lim_{x\to 1}\sum_{i=0}^{N} f_i\,(m_t^2)^i\,x^i \qquad \Rightarrow \qquad \mathcal{F}^N = \lim_{x\to 1}\frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} = \lim_{x\to 1}[n/m](x)$$

Form factors for $gg \to HH$ with fixed scattering angle

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathcal{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathcal{F}_{\mathrm{box1}} + T_2^{\mu\nu}\,\mathcal{F}^{\mu\nu}_{\mathrm{box2}}$$

Scattering angle: $\theta = \dfrac{\pi}{2}$

Naive high-energy expansions starts to diverge at $\sqrt{s} \approx 700$ & 1000 GeV

$$\mathcal{F}^{58} = \lim_{x\to 1} \sum_{i=0}^{58} f_i\,(m_t^2)^i\,x^i$$

Padé improved high-energy expansions converge even at $\sqrt{s} \approx 400$ GeV

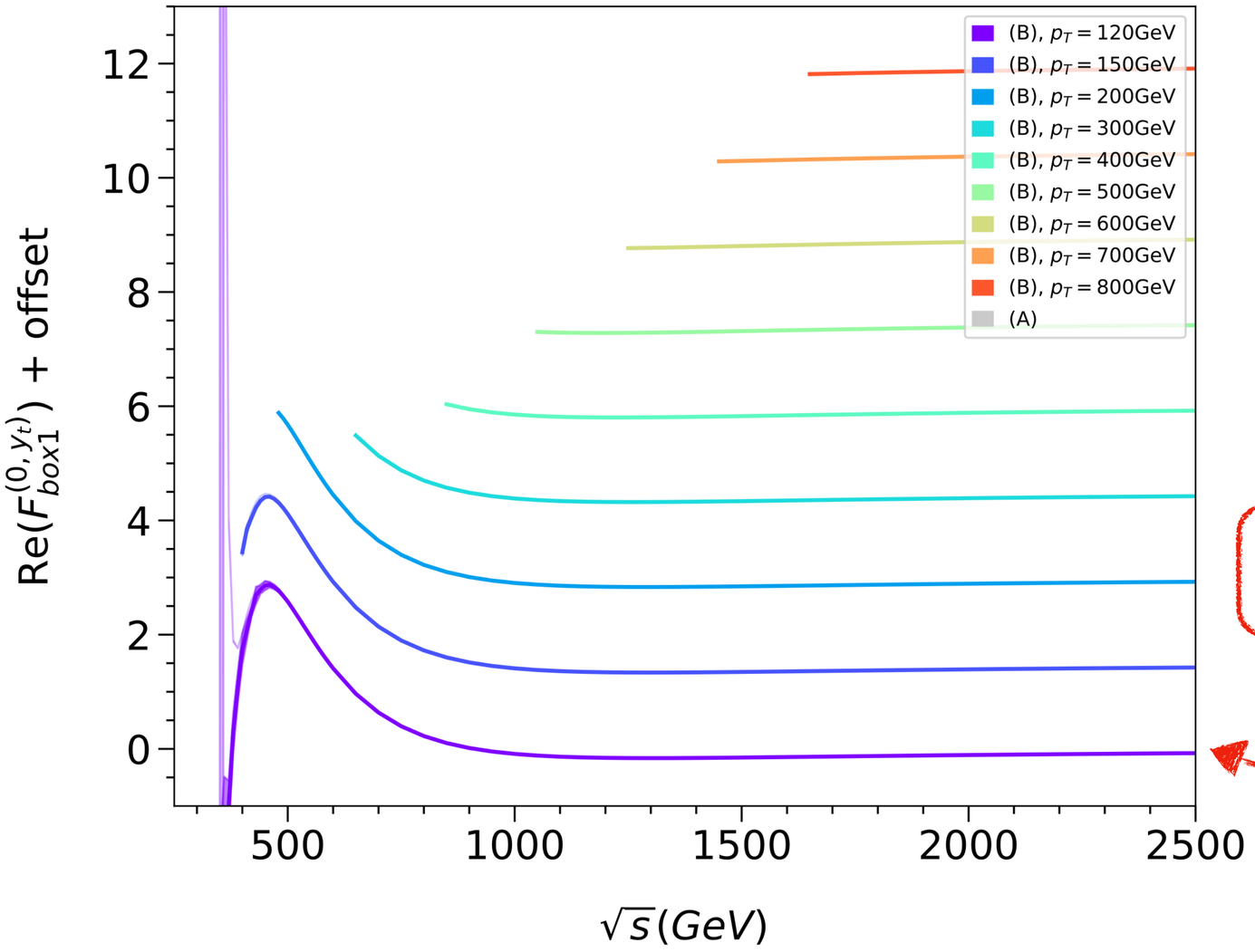
$$\mathcal{F}^{58} = \lim_{x\to 1} \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} = \lim_{x\to 1} [n/m](x)$$

Solid color lines: Padé improved results using MIs from $\mathcal{O}(m_t^{116})$ in two expansion approaches

Dashed color lines: Naive expansions at high energies to $\mathcal{O}(m_t^{116})$

Form factors for $gg \to HH$ with various fixed p_T^H

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathcal{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathcal{F}_{\text{box1}} + T_2^{\mu\nu}\,\mathcal{F}^{\mu\nu}_{\text{box2}}$$

$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

Padé improved high energy expansions converge even at $p_T^H = 120$ GeV

Color lines: Padé improved equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\text{int}}\right)^2$ using MIs from $\mathcal{O}(m_t^{116})$

Grey lines: Coincide with colourful lines (two approaches agree)

Conclusions and outlook

- We analytically compute NLO and NNLO QCD corrections in different regions to $gg \to HH$

  [JHEP: 03 (2018) 048, 11 (2018) 141, 01 (2019) 176, 05 (2019) 157, 01 (2022) 049]

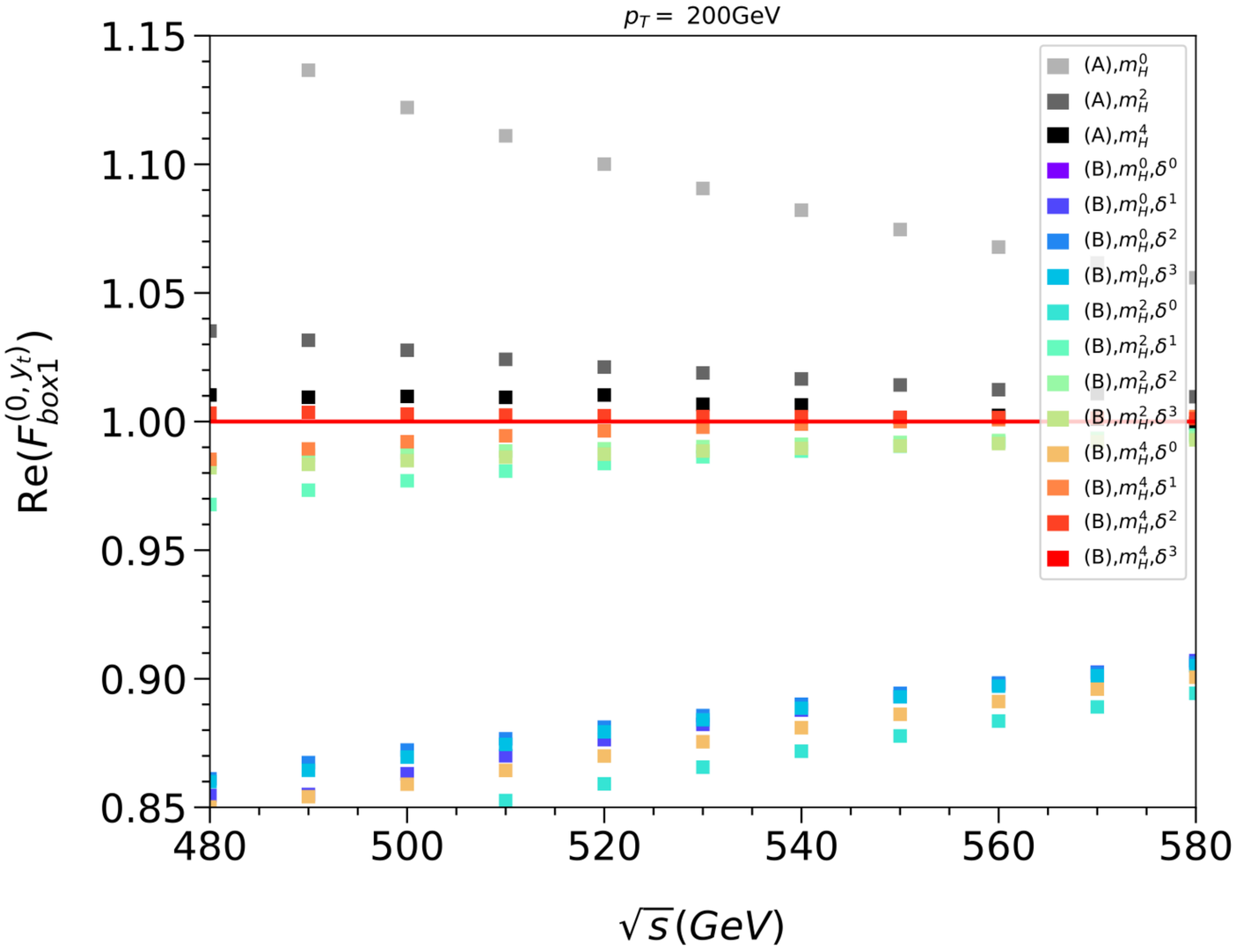
- As a first step towards analytic NLO EW corrections, we compute leading 2-loop Yukawa corrections to in high energy limit *[JHEP 08 (2022) 259]*
  - We propose two-expansion approaches for the challenging massive two-loop integral
  - Padé improved approximations for both expansions show good convergent results for $p_T^H > 120$ GeV with analytical master integrals up to $\mathcal{O}(m_t^{120})$

- Future work, complete full Yukawa corrections and extend to full NLO EW corrections

Backup Slides

Convergence of expansions for $gg \to HH$ form factors

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathcal{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathcal{F}_{\text{box1}} + T_2^{\mu\nu}\,\mathcal{F}^{\mu\nu}_{\text{box2}}$$

The benchmark is expansion at $\mathcal{O}\left(m^4_{H(\text{ext})}, \delta^3, m_t^{116}\right)$.

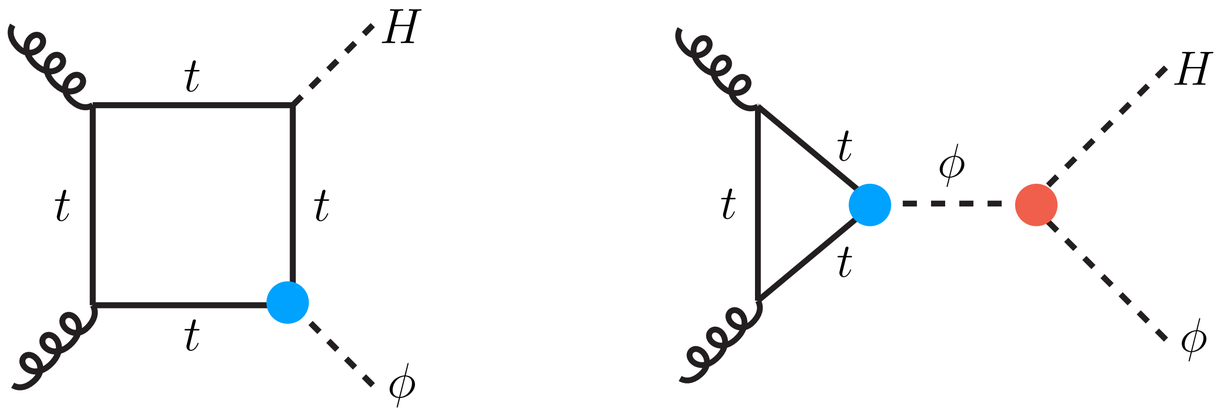
$$\delta = 1 - \frac{m_H^{(\text{int})}}{m_t}$$

Color points: Convergence plot of different expansion orders by ratios to the benchmark at fixed $p_T^H = 200$ GeV.

Chasing two-Higgs doublet model (2HDM) in $H\phi$ production

[Iguro, Kitahara, Omura, **Zhang**, ***ArXiv:2211.00011***]

- Additional EW scale scalars are interesting for baryogenesis, muon g-2, 95 GeV di-τ and di-photon excesses
- Light neutral CP-even scalar $\phi = H_2$ or CP-odd scalar $\phi = A$ ~ 100 GeV from 2HDM are good candidates



We consider the alignment limit, and constrains from flavor physics ($B_{(s)}$-meson mixing, $b \to s\gamma$), direct searches, vacuum stability and oblique corrections

- The cross section of $H\phi$ production can be large in the parameter region where excesses can be explained, e.g. $\sigma(pp \to HA \to b\bar{b}\tau\bar{\tau}) \approx K_{NLO} \times 5.6\,\text{fb}$