Instituto Galego de Física de Altas Enerxías

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Gluon fusion into HH and ZH at NLO QCD

Based on:
1) https://doi.org/10.1103/PhysRevLett.121.162003
2) https://doi.org/10.1007/JHEP07(2022)069
3) https://doi.org/10.1007/JHEP05(2021)168
4) https://doi.org/10.1007/JHEP08(2022)009

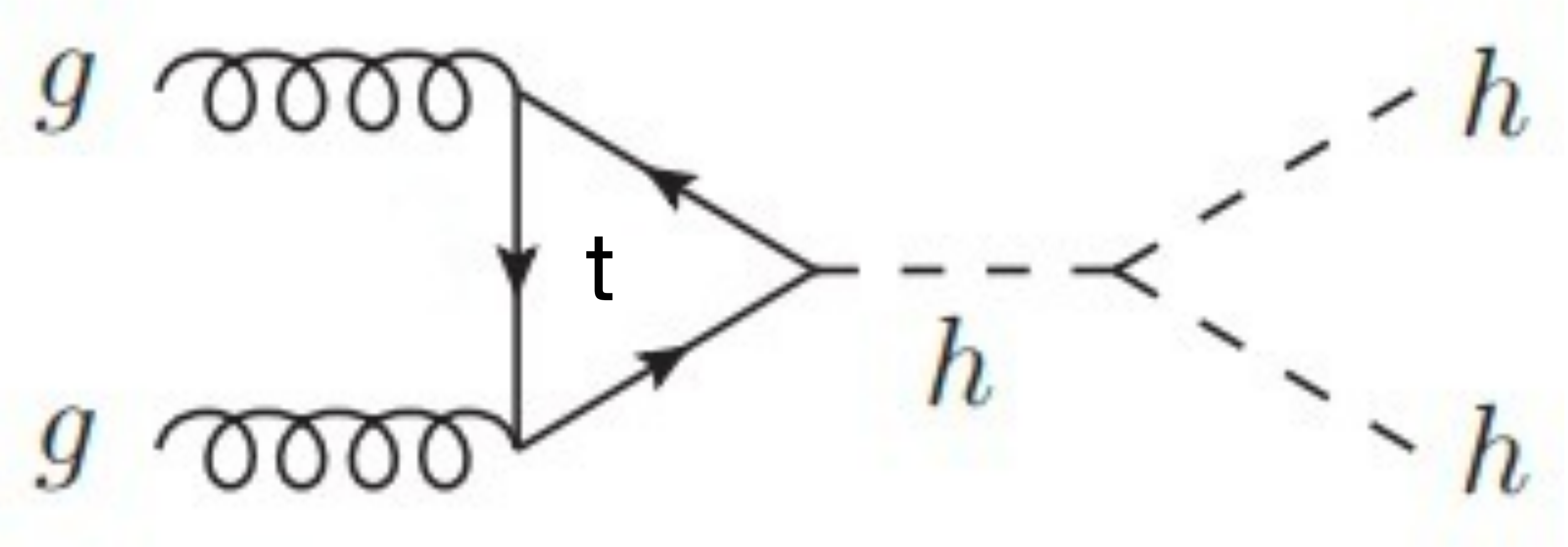
Double Higgs production

Luigi Bellafronte **Nov 10th, 2022**

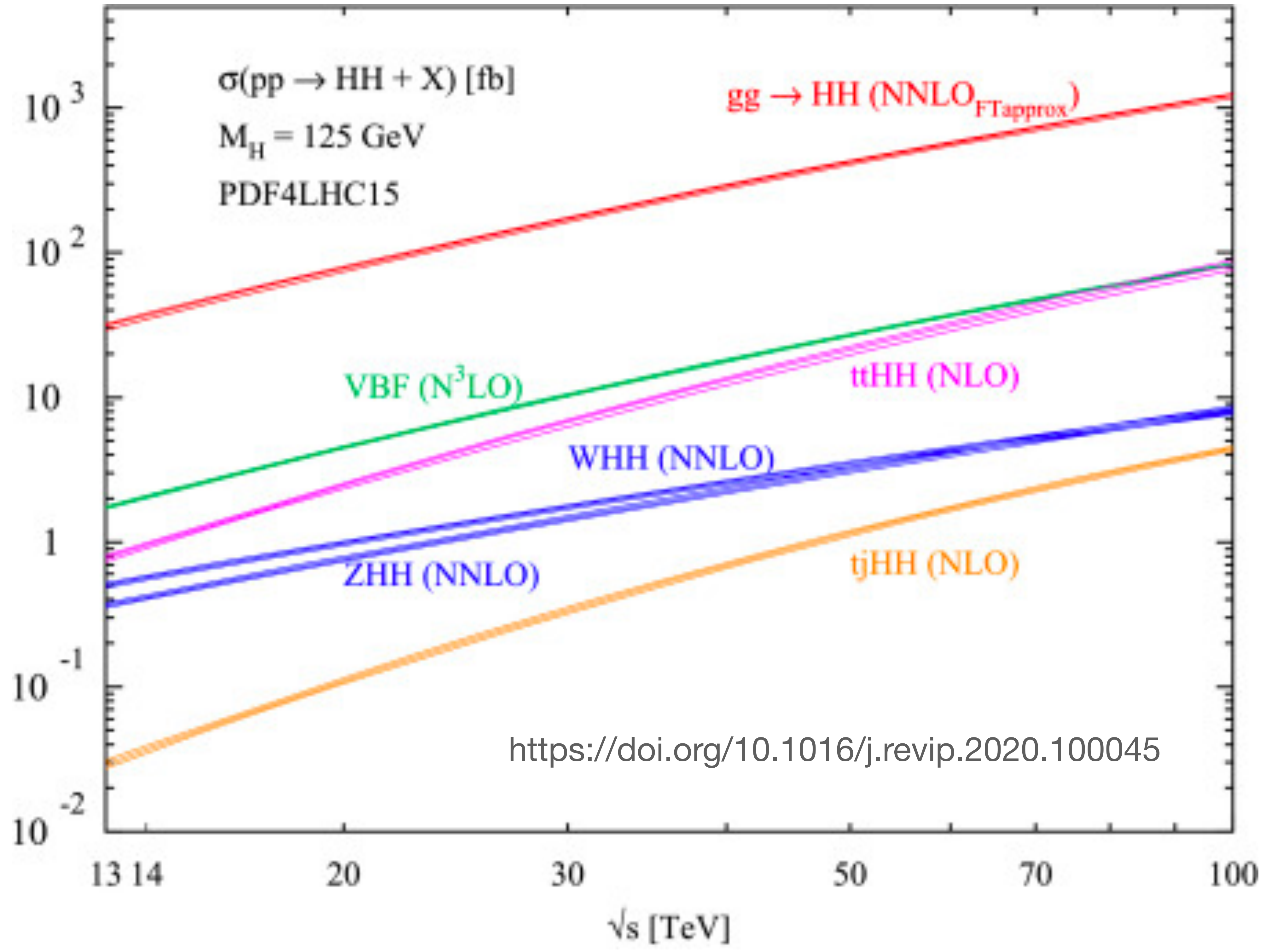
Motivation

gg->HH

$$V(h) = \frac{1}{2}m_h^2h^2 + \lambda v h^3 + \frac{1}{4}\lambda h^4$$



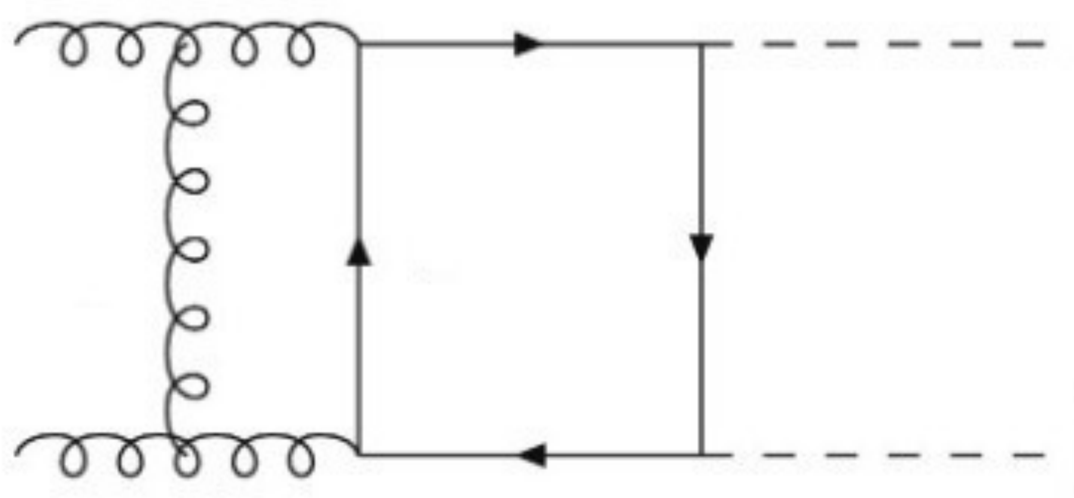
- The experimental exploration of the properties of the Higgs boson is one of the major targets of LHC.
- Trilinear Higgs self-coupling might be measurable from Higgs pair production.
- Sensitive to new physics.
- Higgs boson pairs are dominantly produced in the loop-induced gluon-fusion.



Definitions

$$\frac{d\sigma}{dt} \sim \left|F_1(s, p_t^2, m_t^2, m_h^2)\right|^2 + \left|F_2(s, p_t^2, m_t^2, m_h^2)\right|^2 \qquad p_t^2 = \frac{tu - m_h^4}{s} = \frac{s}{4}\sqrt{1 - \frac{4m_h^2}{s}}\sin^2\theta$$

- At the NLO, the F_i involve the calculation of very complicated integrals ($\gtrsim$ 10 k Integrals), that depend on 3 parameters.
- Exact analytic results for two-loop box diagrams with several energy scales cannot be derived with the present computational technology.



We have two strategies to deal with this problems:

A. A fully numerical exact evaluation S. Borowka et al., 1604.06447, J. Baglio et al., 2003.03227
B. An approximate analytic evaluation in order to reduce the number of scales in the problem

Method

Small pt expansion

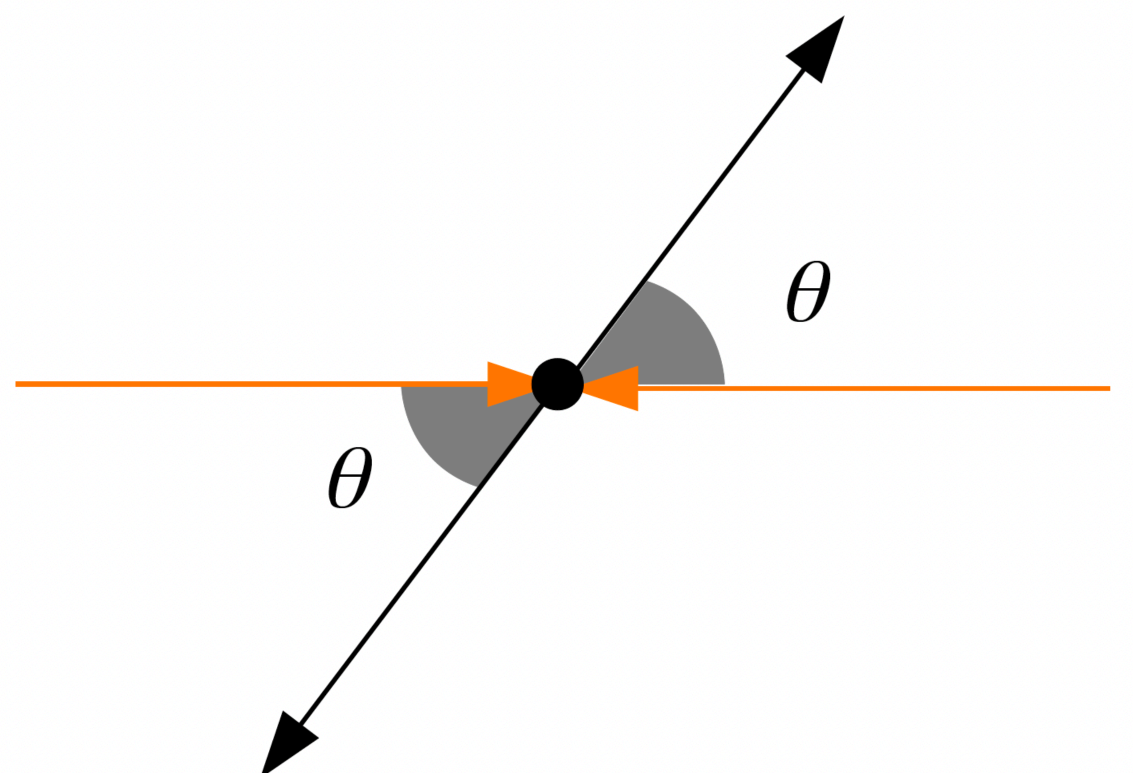
https://doi.org/10.1103/PhysRevLett.121.162003

- s and m_t are assumed to be the large energy scales while m_h and p_t are considered small.
- The validity of this approach is restricted to phase space regions where

$$\frac{|t|}{4m_t^2} \lesssim 1$$

- The expansion is equivalent to

$$\theta \sim 0$$



High-energy expansion

J.Davies et al., 1811.05489

- s and t are assumed to be the large energy scales while m_h and m_t are considered small.
- The validity of this approach is restricted to phase space regions where

$$\frac{|t|}{4m_t^2} \gtrsim 1$$

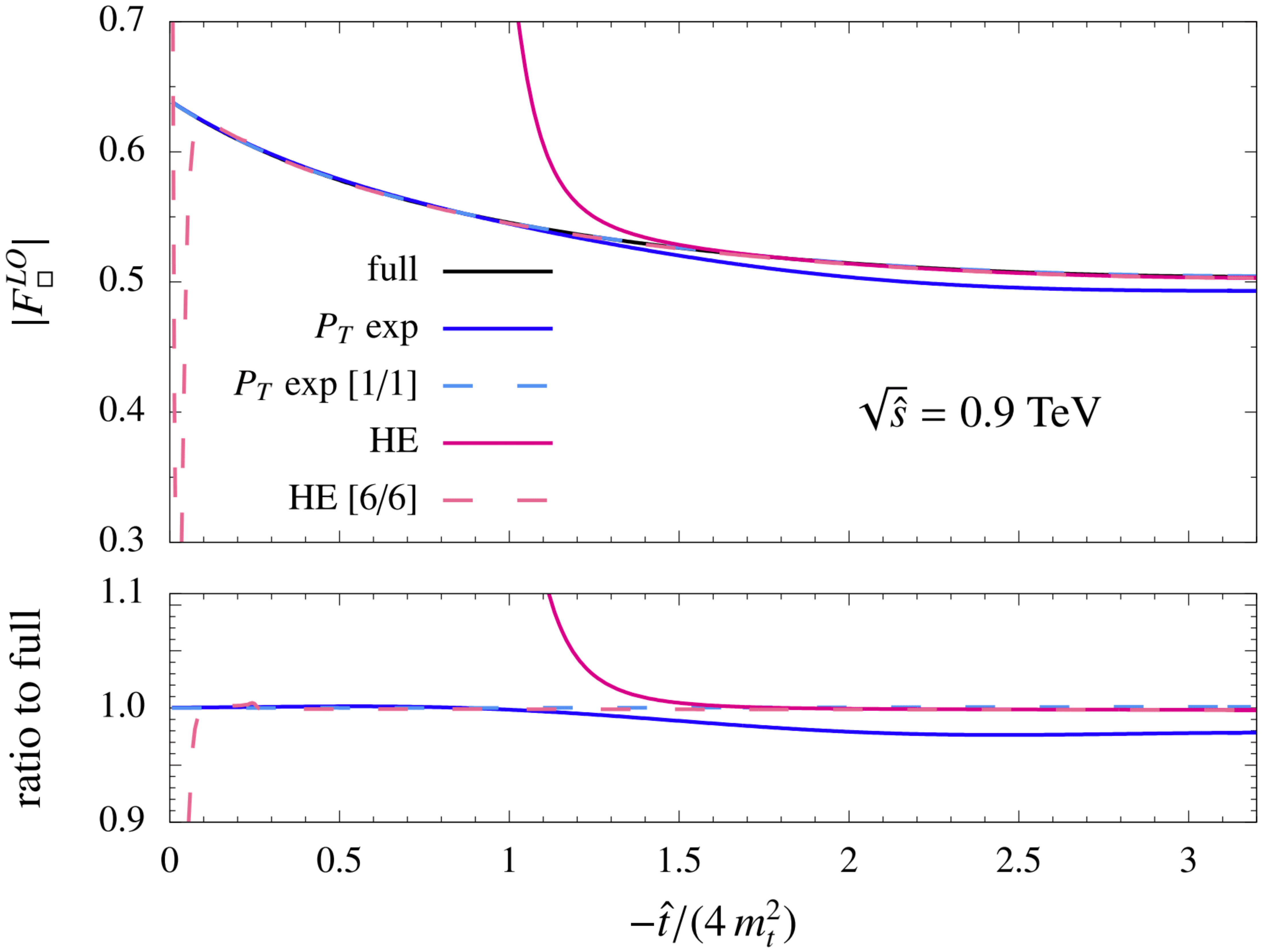
- The expansion is equivalent to

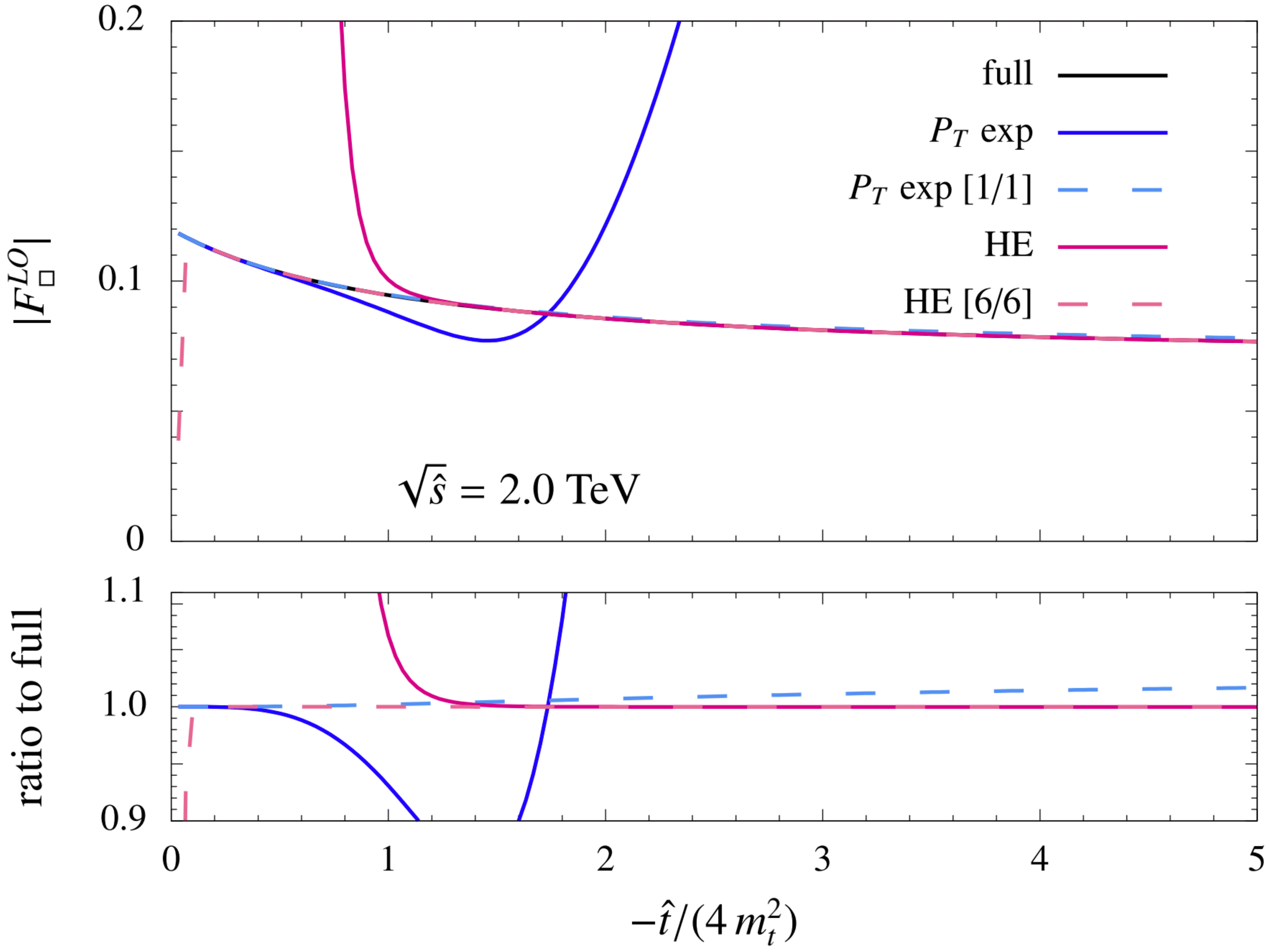
$$\theta \sim \frac{\pi}{2}$$

Padé Approximant

$$f(x) \simeq \sum_{k=0}^{r-1} c_k x^k \quad \Longrightarrow \quad [n,m]_f(x) = \frac{p_0 + p_1 x + \ldots + p_n x^n}{1 + q_1 x + \ldots + q_m x^m}$$

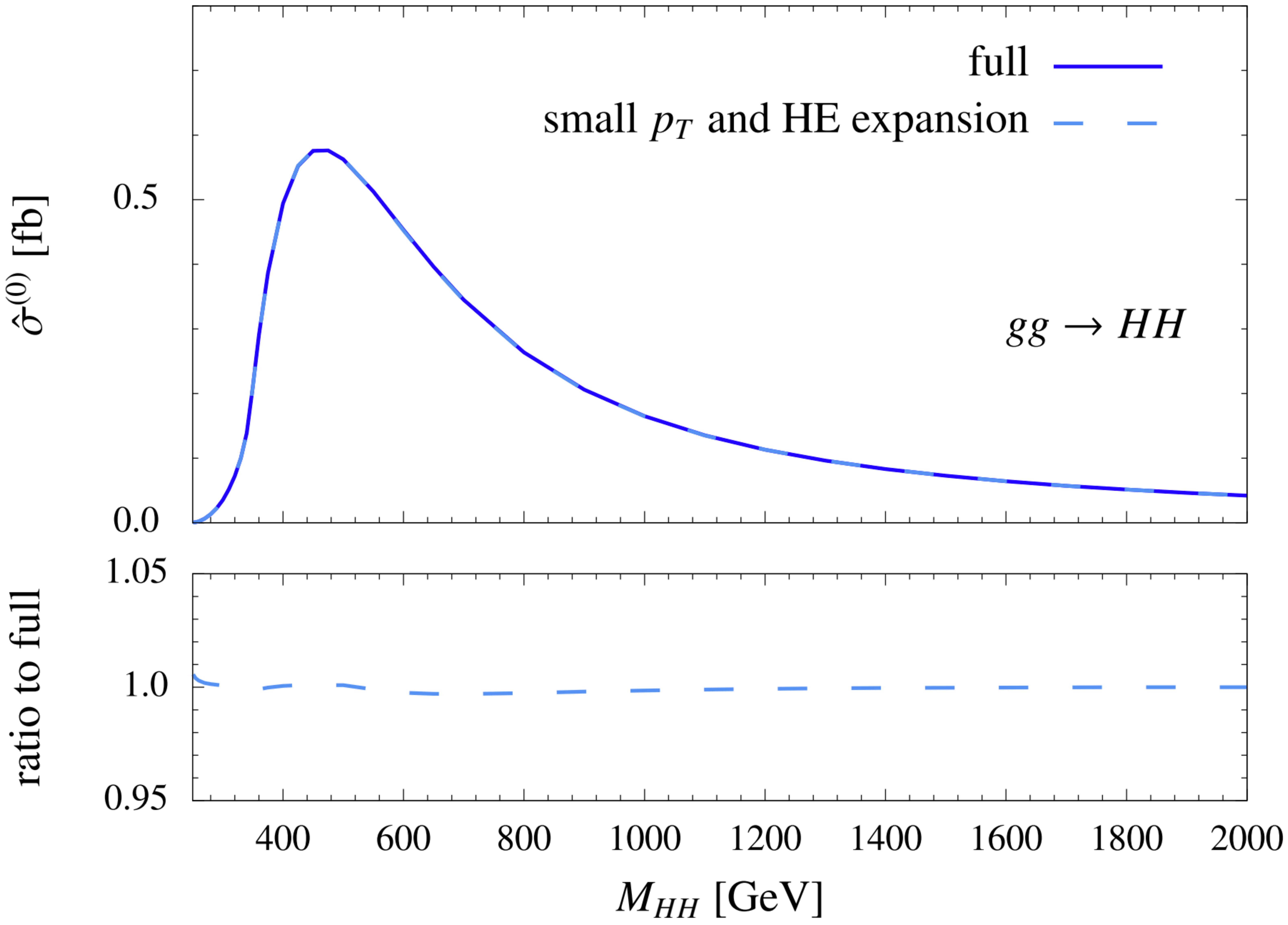
LO Form Factors





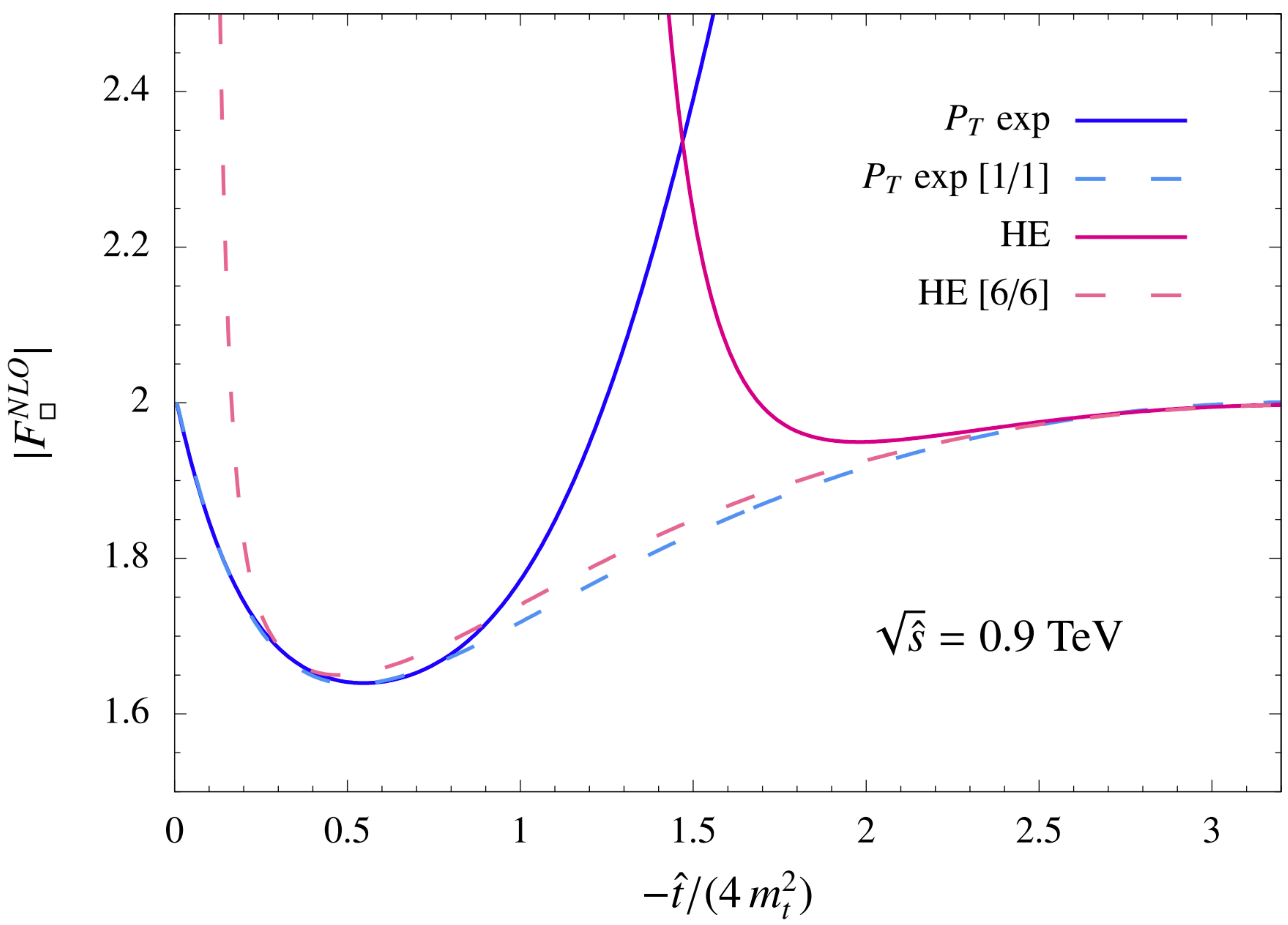
- **The differences between the two Padé near $|t|/4m_t^2 = 1$ are negligible.**
- **The Padé approximants allow us to reproduce the exact LO results with good accuracy.**

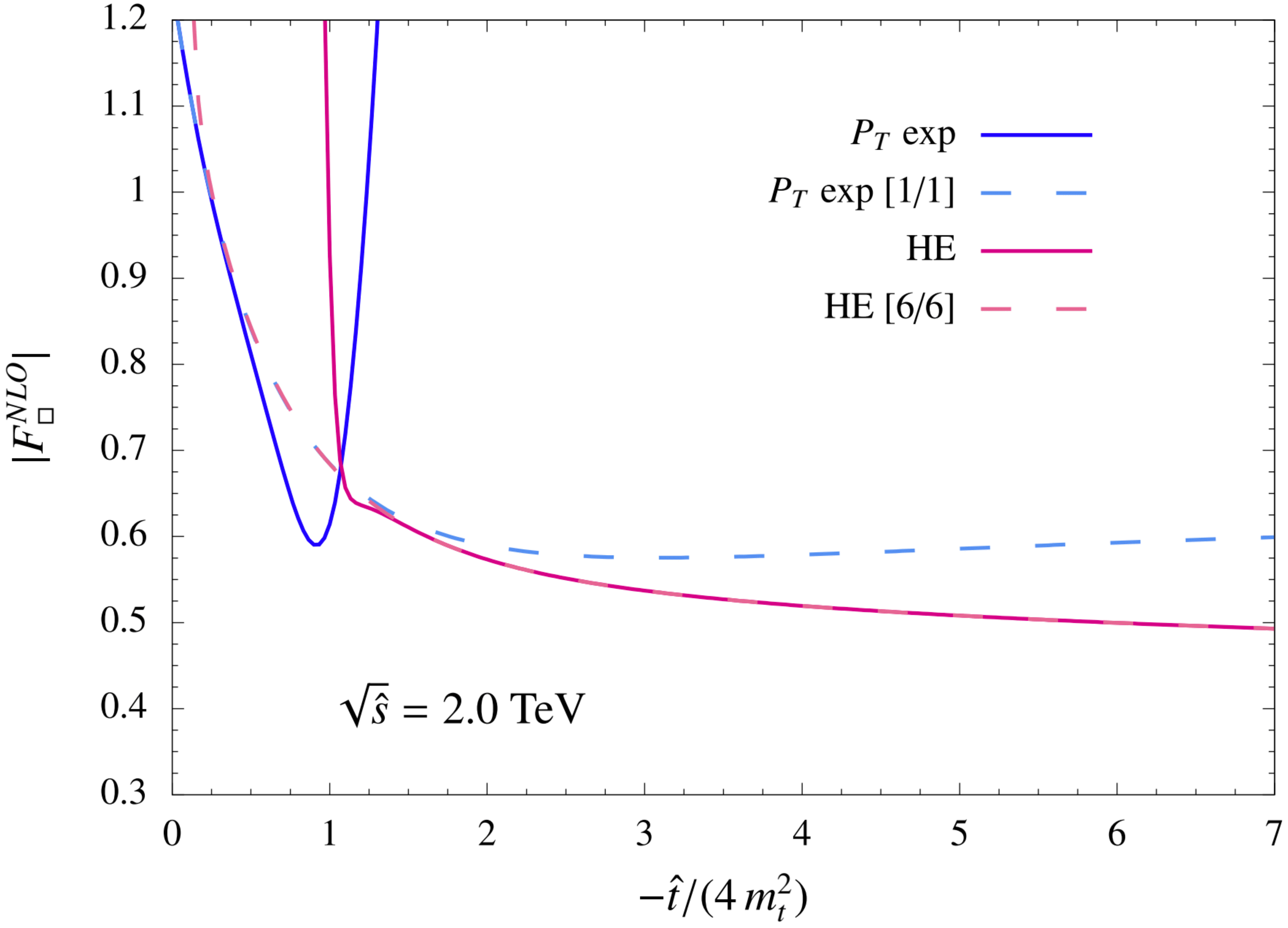
σ LO



- **The combination of the pT - and HE-Padé with respect to the exact prediction deviates by less than 1% in the cross section.**

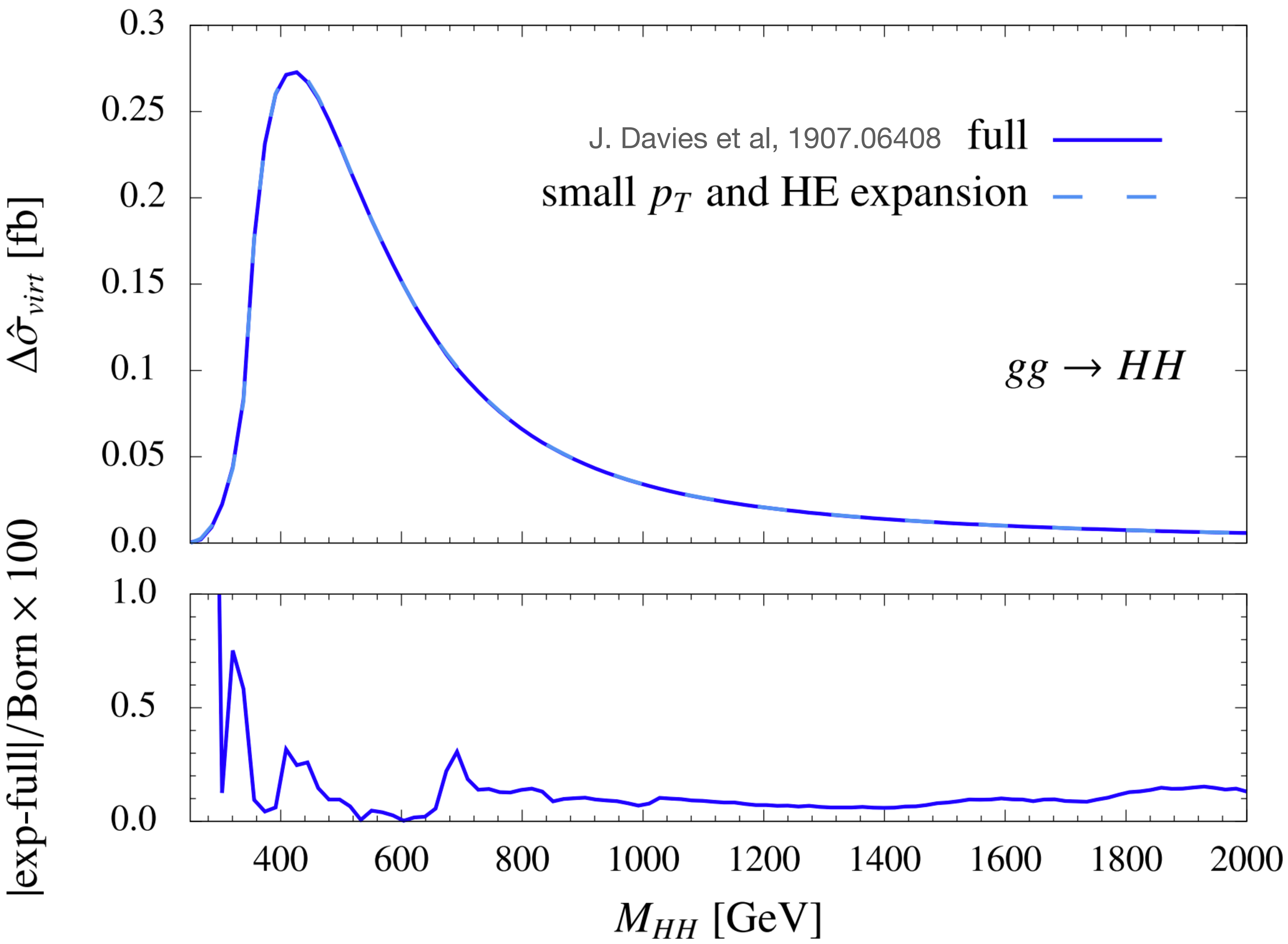
NLO Form Factors





- **The relative behaviour of the various approximations is analogous to what we observed at LO.**

σ NLO



- **The lower panel shows the absolute value of the difference between the two results, normalized to the partonic LO cross section.**
- **The full numerical result shows very good agreement with our results at every invariant mass, except for the first few bins at low MHH.**

NLO QCD Corrections to $gg \to ZH$

Marco Vitti - Padova University & INFN, Padova

Higgs 2022, Pisa
Nov 10 2022

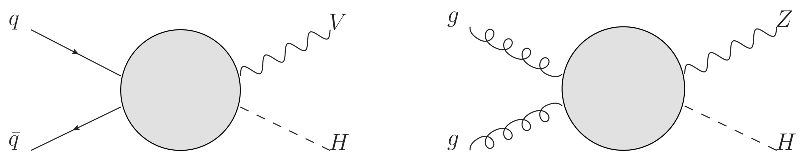
Why $gg \to ZH$? VH Production at the LHC

$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Two partonic channels in *pp->ZH*:

  $q\bar{q} \to ZH$ - dominant contribution

  $gg \to ZH$ - about 10% wrt to $q\bar{q} \to ZH$



- Theory prediction in MC codes:

  $q\bar{q} \to ZH$: NNLO accuracy [Han, Willenbrock- '91] [Brein, Djouadi, Harlander- 0307206]

  $gg \to ZH$: LO accuracy → Large scale uncertainties

Production mode	$\Delta_y^{<VH>}$
WH	$\pm 0.7\%$
$q\bar{q} \to ZH$	$\pm 0.6\%$
$gg \to ZH$	$\pm 25\%$

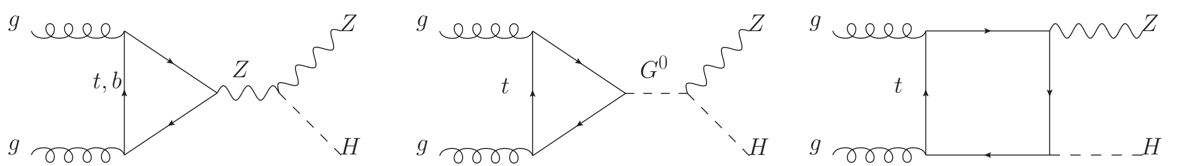
(No *gg*-channel for WH)

[CERN Yellow Report 4]

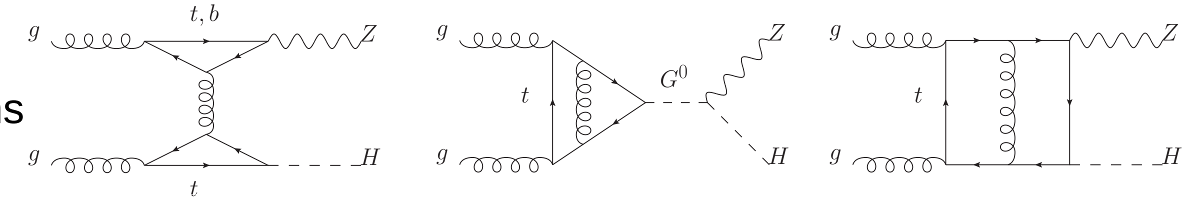
If we really want to improve the theory prediction **we need to go beyond LO in $gg \to ZH$**

SM Prediction for $gg \to ZH$

- **LO**: one-loop diagrams
  [Kniehl ('90) - Dicus, Kao ('88)]
- Top quark loops give dominant contribution (as in $gg \to HH$)



- **NLO**
  Virtual corrections: two-loop diagrams
- Two-loop boxes: full analytic result not available (as in $gg \to HH$)
- Alternative approaches:



OK: standard one-loop techniques

OK: [Spira et al. - 9504378] [Aglietti et al. - 0611266]

Hardest part in the calculation

- Numeric evaluation [Chen, et al. – 2011.12325] (combined with 2011.12314 in 2204.05225, see M. Kerner's talk)

- Analytic approximations:

Limit $mt \to \infty$
[Altenkamp et al. - 1211.50]

Large (but finite) top mass
[Hasselhuhn, Luthe, Steinhauser -1611.05881]

Small-mass expansion
[Wang et al. - 2107.08206]

High-energy expansion
[Davies, Mishima, Steinhauser - 2011.12314]

pt expansion
[Alasfar, Degrassi, Giardino, Groeber, MV - 2103.06225]

They can be combined as in $gg \to HH$
[Bellafronte et al. - 2202.12157]

Merging pt and HE expansions at NLO

- Lorentz decomposition in terms of 6 form factors

$$\hat{\mathcal{A}}^{\mu\nu\rho}(p_1,p_2,p_3) = \sum_{i=1}^{6} \mathcal{P}_i^{\mu\nu\rho}(p_1,p_2,p_3)\mathcal{A}_i(\hat{s},\hat{t},\hat{u},m_t,m_H,m_Z)$$

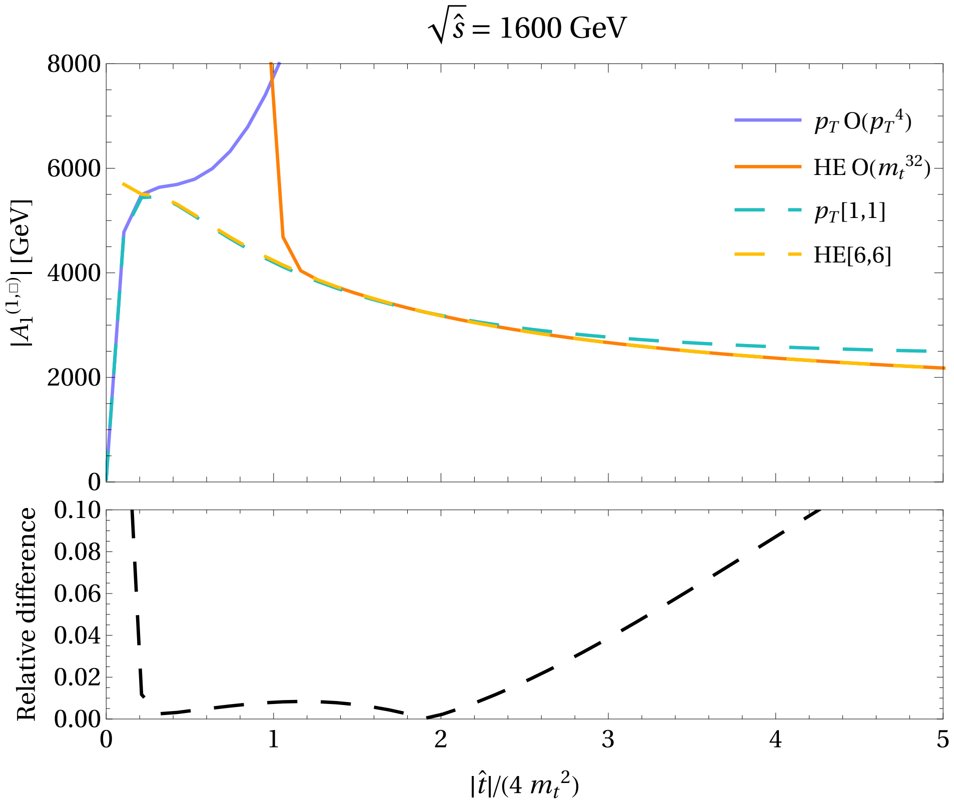
- For each FF we merged the following results

  - pt exp [2103.06225] improved by [1/1] Padé

  - HE exp [2011.12314] improved by [6/6] Padé

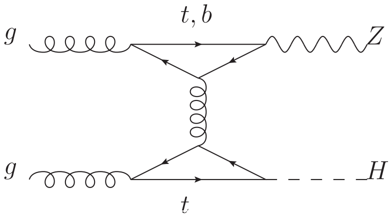
- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2$ $\rightarrow$ can switch without loss of accuracy

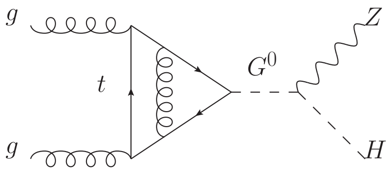
- Evaluation time for a phase space point below 0.1 *s*

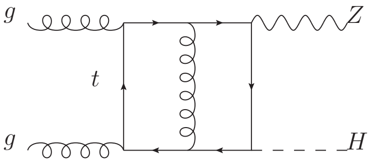


$gg \rightarrow ZH$ @ NLO in QCD: all ingredients

Virtual corrections (2→2, two loops): merging pt+HE expansions



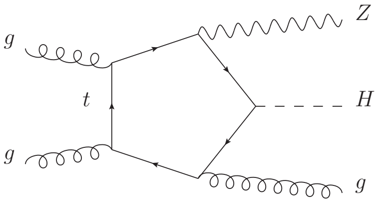




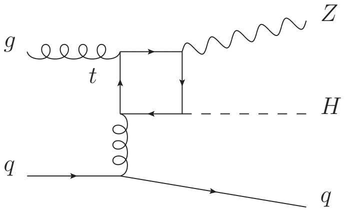
Real emission (2→3, one loop): automated evaluation (RECOLA2, MadGraph5)

We included all diagrams that:

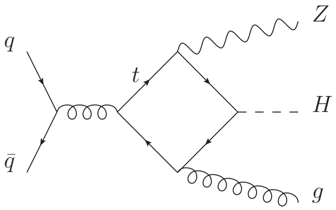
- give O(α_S^3) contribution to the cross section *pp->ZH*
- feature a closed fermion loop



$gg \rightarrow ZHg$



$qg \rightarrow ZHq$



$q\bar{q} \rightarrow ZHg$

Full NLO QCD Results

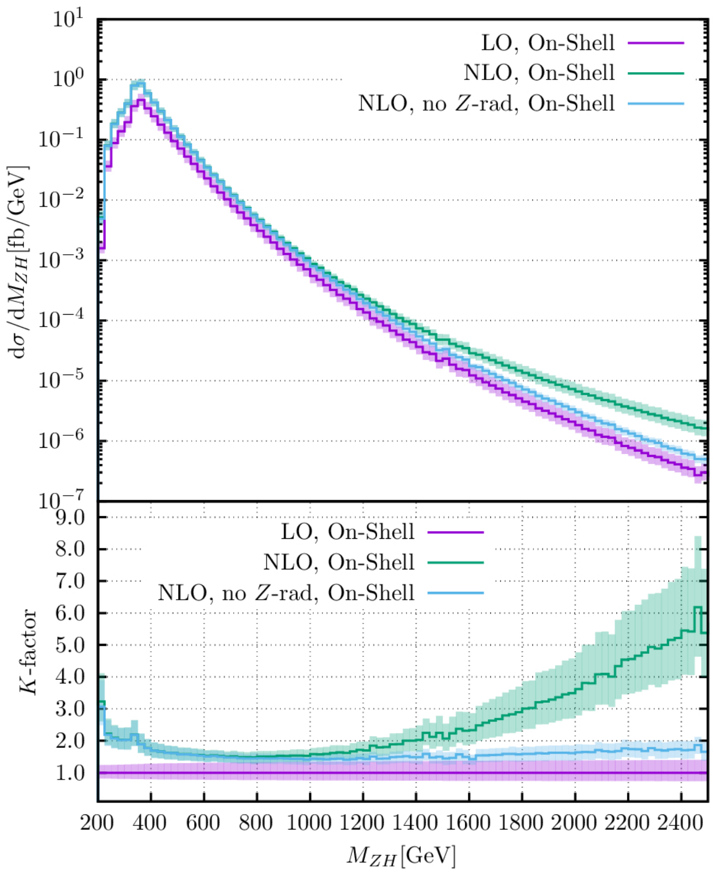
Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- Top mass renormalized both in OS and $\overline{\text{MS}}$ scheme
- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 2/3 wrt LO
- Agreement with independent calculation using small-mass expansion
  [Wang, Xu, Xu, Yang - 2107.08206]

M_{ZH} distribution

- K-factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail (M_{ZH}>1 TeV)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

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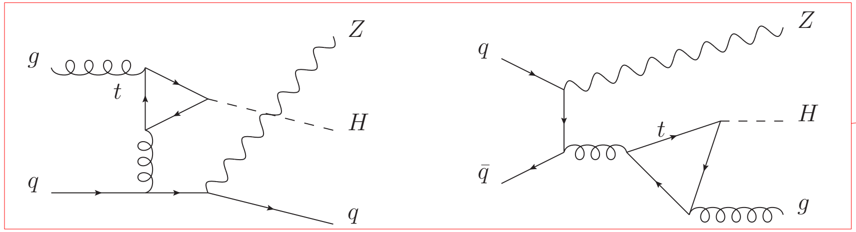
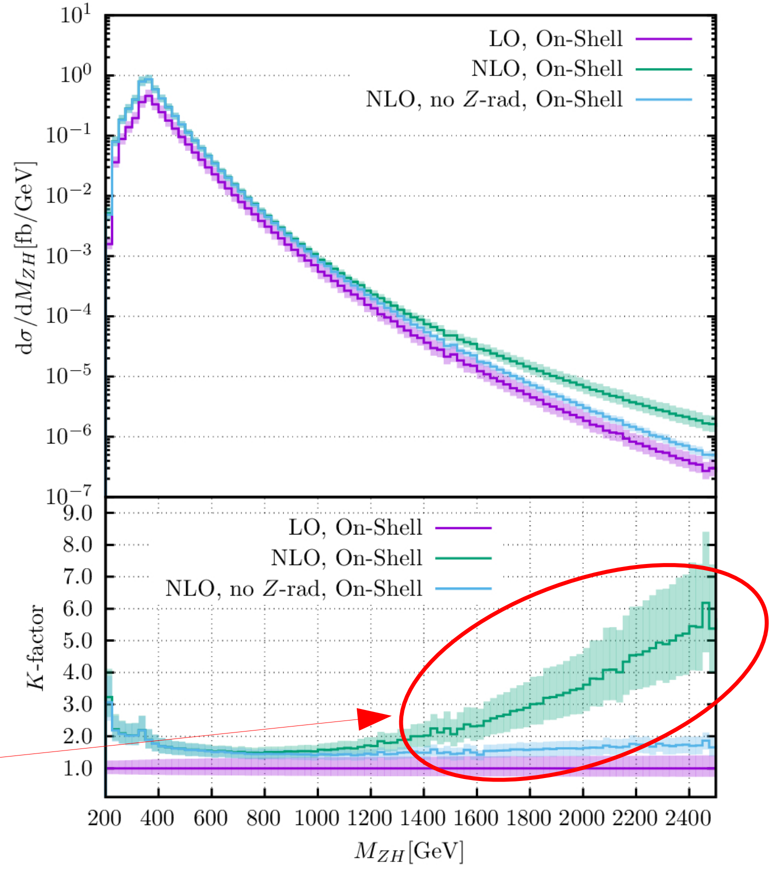
  [Wang, Xu, Xu, Yang - 2107.08206]

Z-radiated diagrams

- Large EW Logs

$$\log(M_Z/M_{ZH})$$





[Degrassi, Gröber, MV, Zhao - 2205.02769]

Top mass scheme uncertainty

- Take deviations of $\overline{\text{MS}}$ scheme wrt OS result as top mass scheme uncertainty

- Analytic results → change of top mass scheme is straightforward

$$F_i^{NLO,\overline{\text{MS}}} = F_i^{NLO,\text{OS}} - \frac{1}{4}\frac{\partial F_i^{LO}}{\partial m_t^2}\Delta_{m_t^2}$$

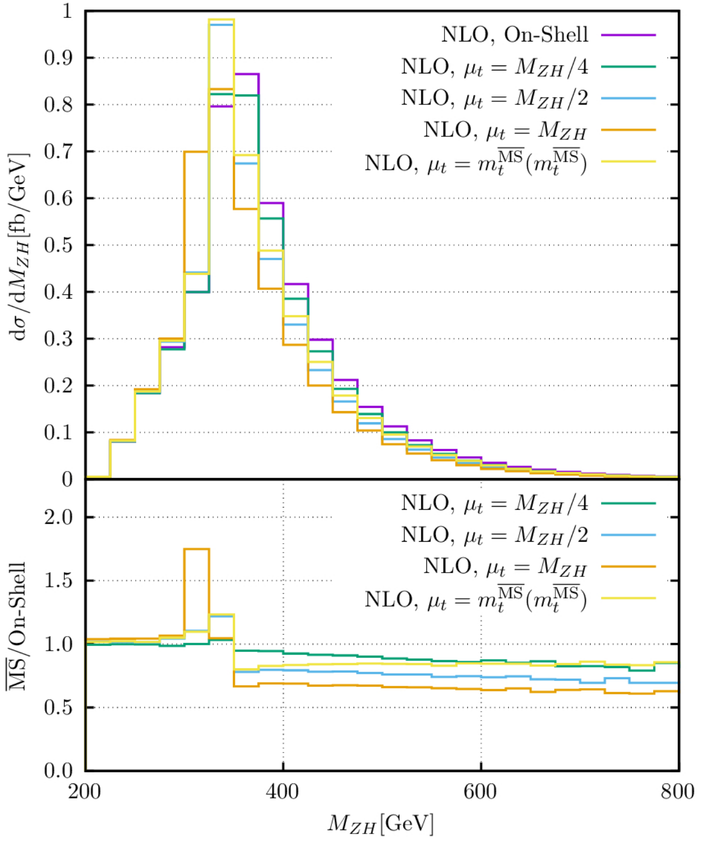
$$m_t^{OS} \rightarrow m_t^{\overline{MS}}(\mu_t)$$

$$\Delta_{m_t^2} = 2m_t^2 C_F\left[-4 + 3\log\left(\frac{m_t^2}{\mu^2}\right)\right]$$

- Same method already used for HH production
  [Baglio et al. - 1811.05692, 2003.03227]

Bin Width [GeV]	LO	NLO
1	$64.01^{+15.6\%}_{-35.9\%}$	$118.6^{+17.2\%}_{-27.0\%}$
5	$64.01^{+15.3\%}_{-35.6\%}$	$118.6^{+14.7\%}_{-24.9\%}$
25	$64.01^{+14.0\%}_{-33.1\%}$	$118.6^{+10.9\%}_{-20.8\%}$
100	$64.01^{+2.0\%}_{-25.3\%}$	$118.6^{+0.6\%}_{-13.7\%}$
∞	$64.01^{+0\%}_{-23.1\%}$	$118.6^{+0\%}_{-12.9\%}$

Avoid overestimate of mt uncertainty



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Conclusions

- Virtual QCD corrections to $gg \to HH$ and $gg \to ZH$ approximated analytically
- The pt and HE expansions are complementary
- Accuracy of 1% or below at the level of cross section
- Fast and flexible results for any point in phase space (can be included in MC)
- Full NLO (virtual+real) calculated for $gg \to ZH$:

  - large NLO corrections (see also pt distributions from M. Kerner's talk)

  - scale uncertainties reduced to the level of PDF+α_S (may be not enough for Hi-Lumi)

  - studied the impact of top mass scheme uncertainty

Thank you for your attention

Backup

Backward-Forward

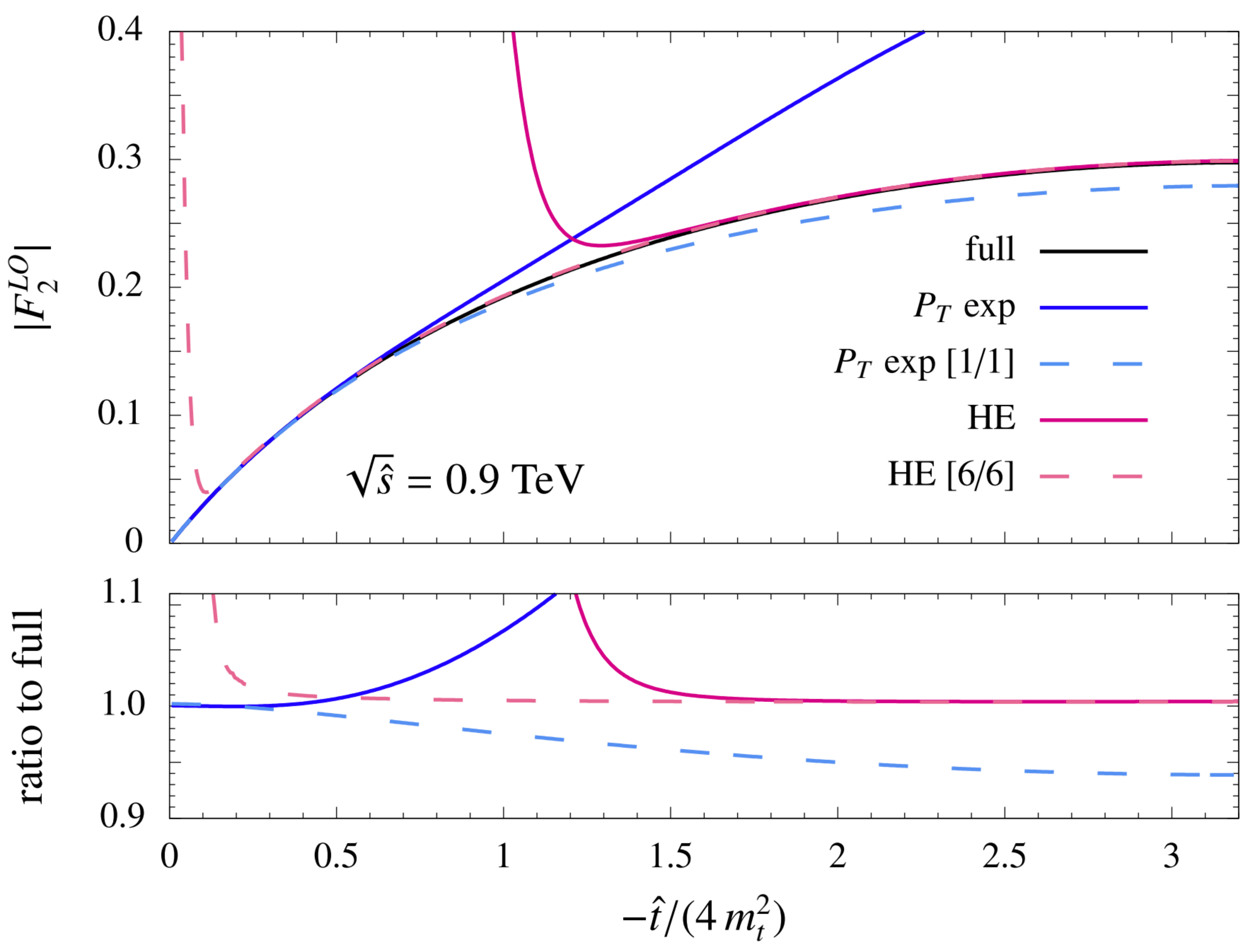
$$\sigma \propto \int_{t_i}^{t_f} dt' \mathcal{F}(t',u') = \int_{t_i}^{t_m} dt' \mathcal{F}(t',u') + \int_{t_m}^{t_f} dt' \mathcal{F}(t',u')$$
$$\sim \int_{t_i}^{t_m} dt' \mathcal{F}(t' \sim 0, u' \sim -s') + \int_{t_m}^{t_f} dt' \mathcal{F}(t' \sim -s', u' \sim 0)$$

If the amplitude is symmetric under $t' \leftrightarrow u'$ exchange then

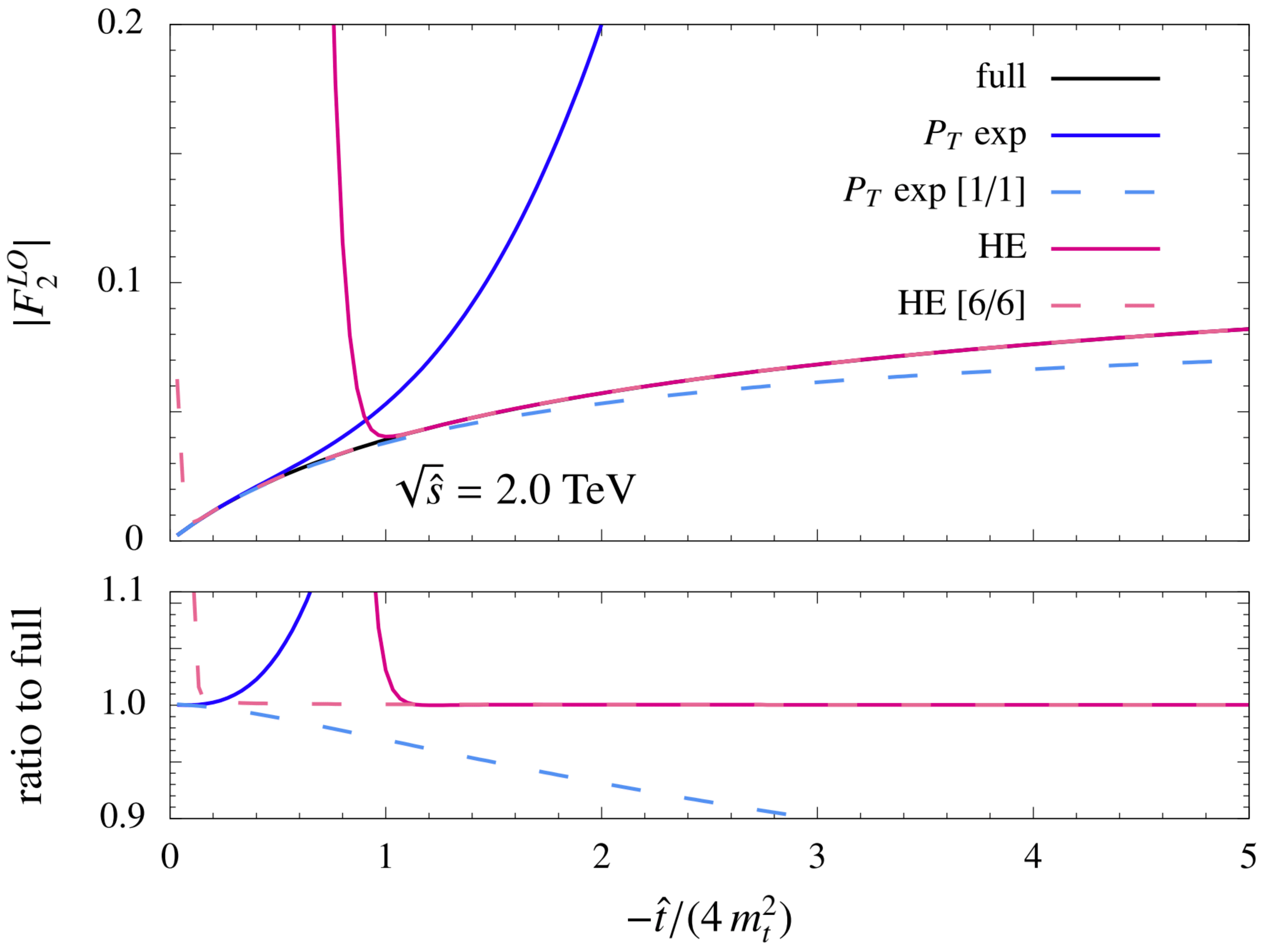
$$\sigma \propto \int_{t_i}^{t_m} dt' \mathcal{F}(0,-s') + \int_{t_m}^{t_f} dt' \mathcal{F}(-s',0)$$
$$= \int_{t_i}^{t_m} dt' \mathcal{F}(0,-s') + \int_{t_m}^{t_f} dt' \mathcal{F}(0,-s') = \int_{t_i}^{t_f} dt' \mathcal{F}(0,-s')$$

so that the expansion in the forward kinematics actually covers the entire phase space.

LO F2

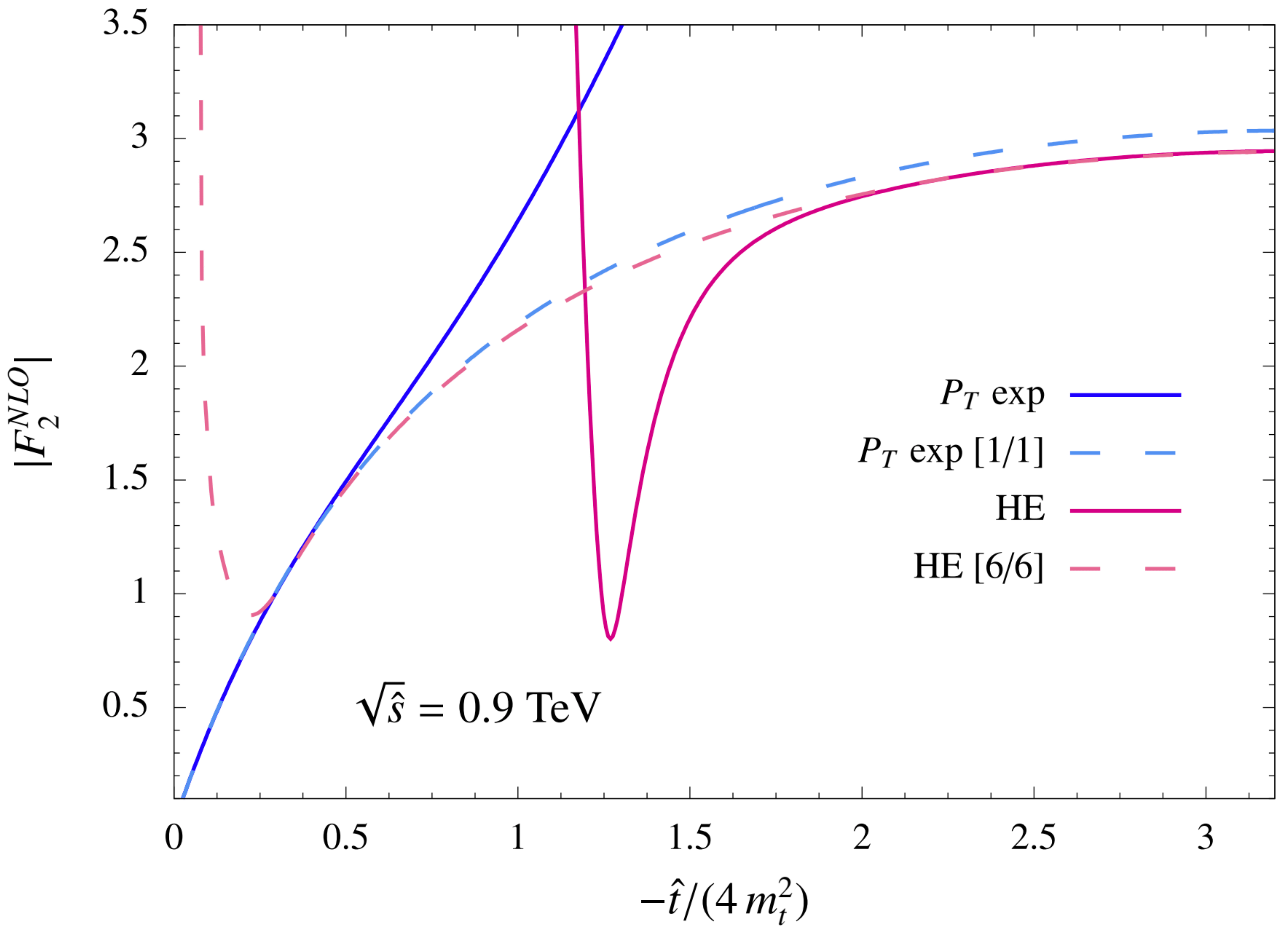


$|F_2^{LO}|$
full
P_T exp
P_T exp [1/1]
HE
HE [6/6]
$\sqrt{\hat{s}}$ = 0.9 TeV
ratio to full
$-\hat{t}/(4\,m_t^2)$

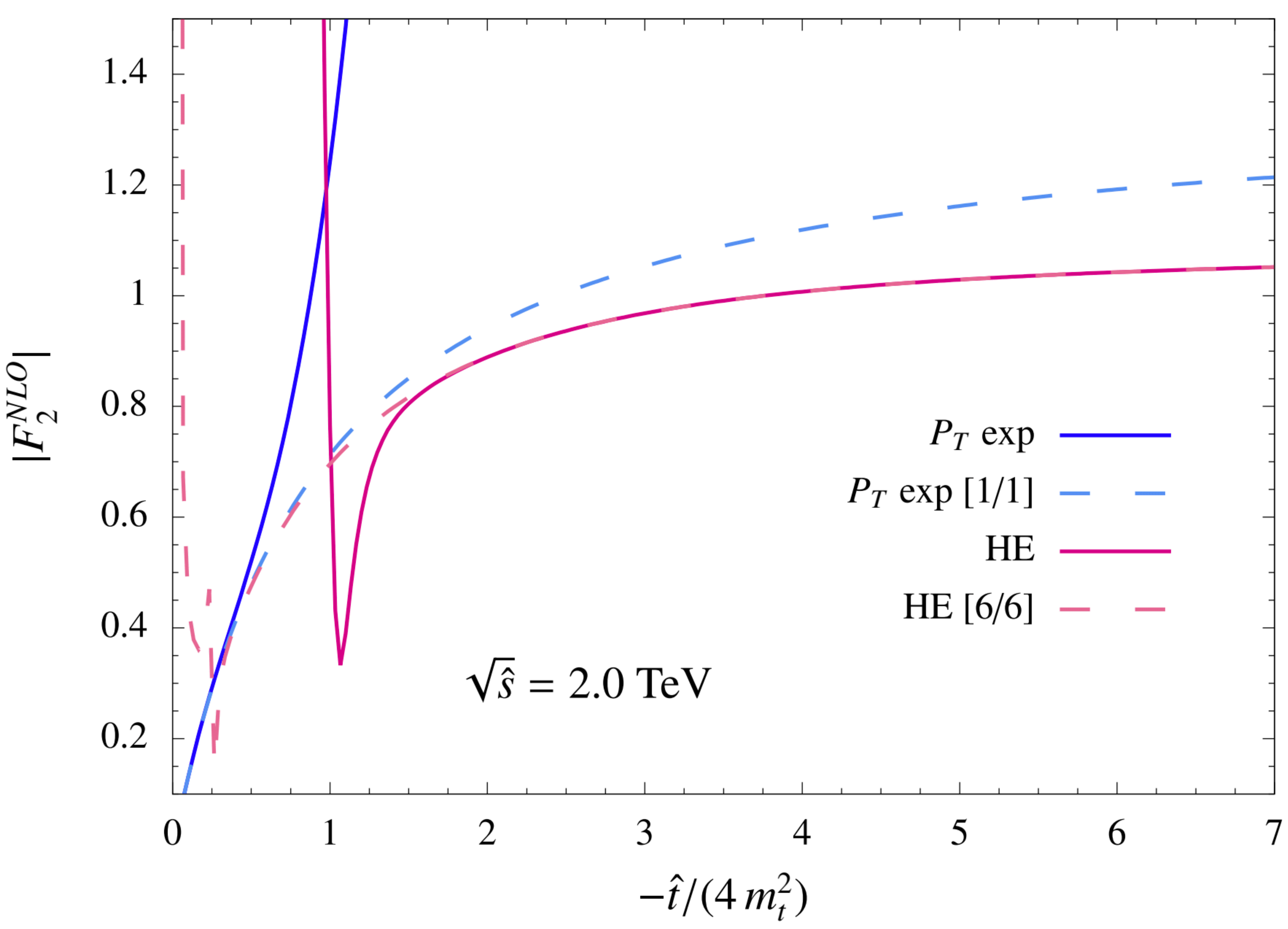


$|F_2^{LO}|$
full
P_T exp
P_T exp [1/1]
HE
HE [6/6]
$\sqrt{\hat{s}}$ = 2.0 TeV
ratio to full
$-\hat{t}/(4\,m_t^2)$

NLO F2



$|F_2^{NLO}|$
P_T exp
P_T exp [1/1]
HE
HE [6/6]
$\sqrt{\hat{s}}$ = 0.9 TeV
$-\hat{t}/(4\,m_t^2)$

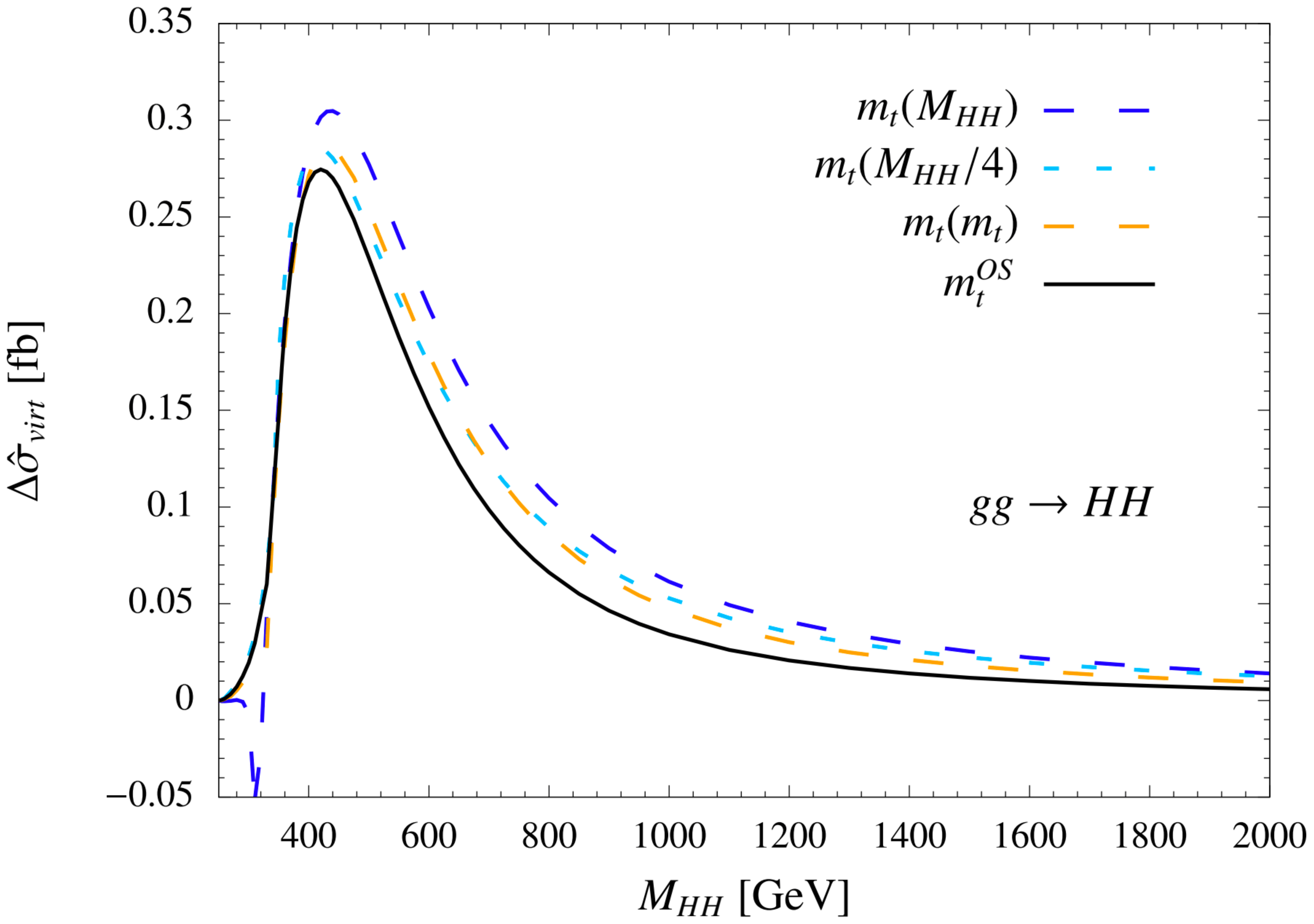


$|F_2^{NLO}|$
P_T exp
P_T exp [1/1]
HE
HE [6/6]
$\sqrt{\hat{s}}$ = 2.0 TeV
$-\hat{t}/(4\,m_t^2)$

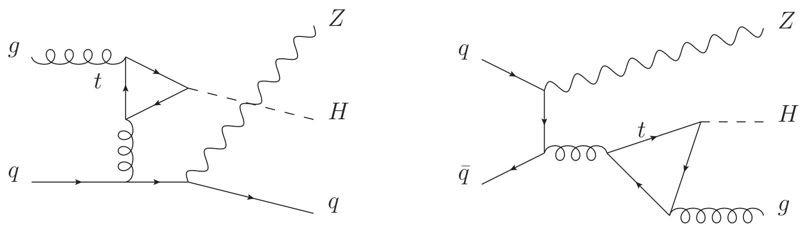
Comparison

M_{HH} [GeV]	$\hat{t}$ [GeV^2]	$\mathcal{V}_{\text{fin}}^{\text{Pade}}$	$\mathcal{V}_{\text{fin}}^{\text{grid}}$
280.9	$-7.783 \cdot 10^{3}$	$9.548 \cdot 10^{-6}$	$9.410 \cdot 10^{-6}$
411.4	$-6.627 \cdot 10^{4}$	$4.520 \cdot 10^{-4}$	$4.510 \cdot 10^{-4}$
586.96	$-6.925 \cdot 10^{4}$	$4.930 \cdot 10^{-4}$	$4.943 \cdot 10^{-4}$
716.55	$-1.816 \cdot 10^{5}$	$4.430 \cdot 10^{-4}$	$4.298 \cdot 10^{-4}$
1048.93	$-2.133 \cdot 10^{5}$	$2.952 \cdot 10^{-4}$	$3.104 \cdot 10^{-4}$
1855.32	$-1.678 \cdot 10^{6}$	$2.497 \cdot 10^{-4}$	$2.498 \cdot 10^{-4}$

Top mass dependence

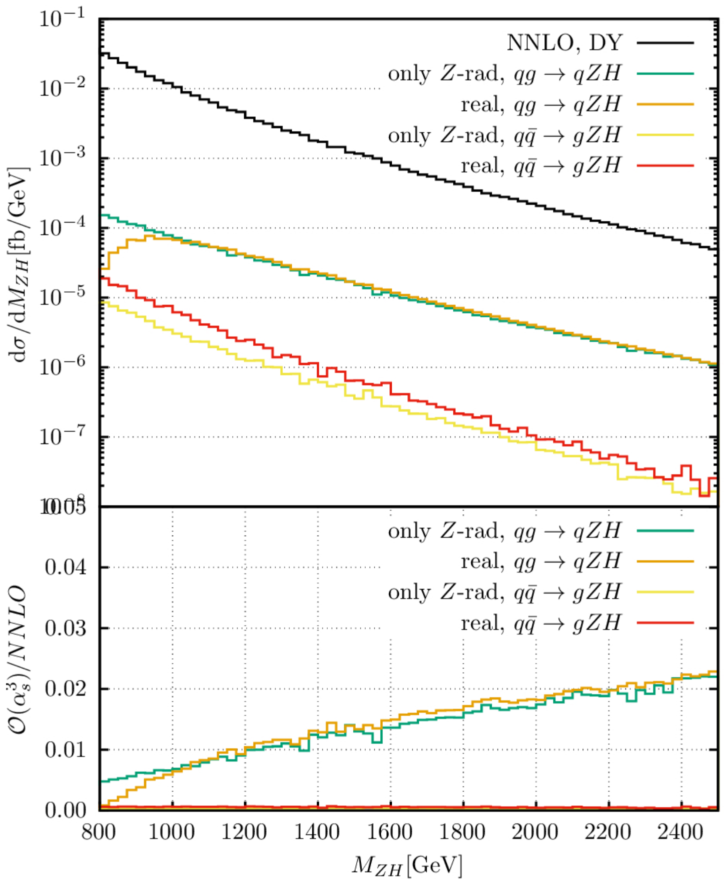


The effect of Z-radiated diagrams



In the high-energy tail (M_{ZH} > 1 TeV)

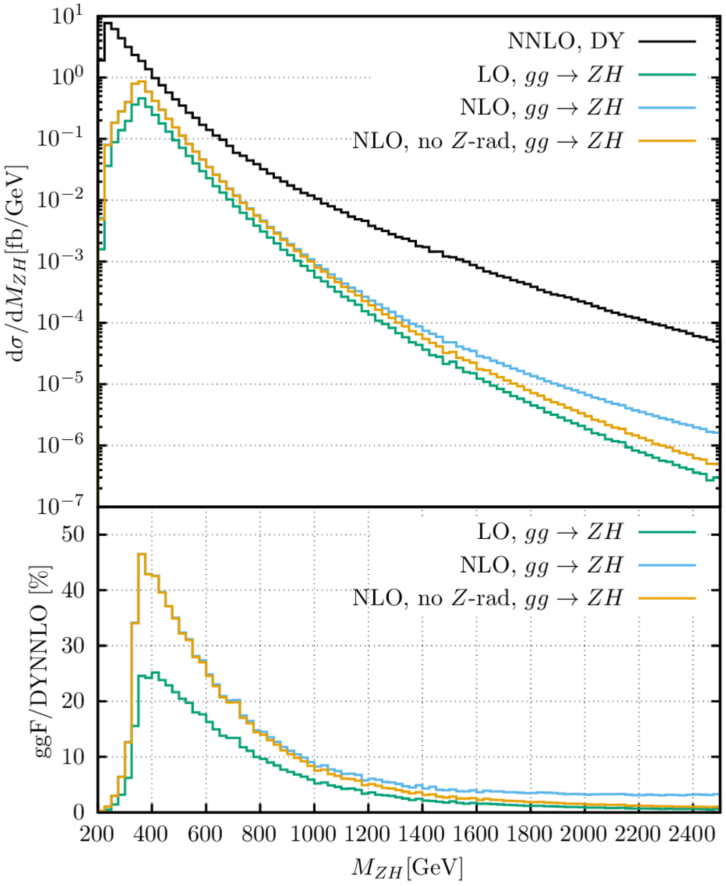
- **qg → ZHq channel**
  - Z-radiated diagrams dominate
  - Non-negligible contribution (up to 2% wrt DY)
- **$q\bar{q}$ → ZHg channel**
  - Z-radiated diagrams dominate
  - Negligible (PDF suppression)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

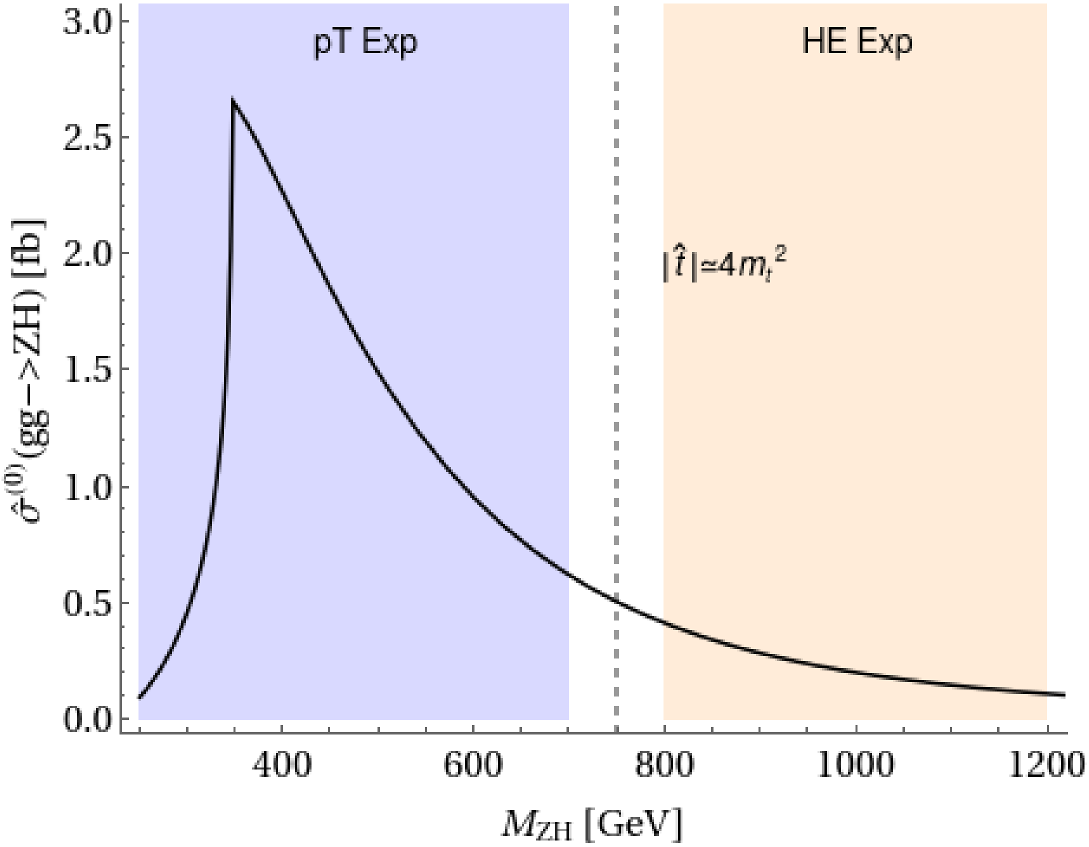
$gg \to ZH$ @ NLO: comparing with Drell-Yan contribution

- $\boldsymbol{gg \to ZH}$ is almost 50% of DY near $M_{ZH} \sim 2\, m_t$
- Because of Z -radiated diagrams the gg contribution falls off as rapidly as the DY one (ratio constant at ~ 2%)
- DY obtained using **vh@nnlo**
  [Harlander et al - 1802.04817]



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Comparing validity ranges



$gg \to ZH$ vs $gg \to HH$: pt expansion

- The application of the pt expansion to $gg \to ZH$ is technically more involved than $gg \to HH$:

  - 6 form factors (anti)symmetric under t<->u

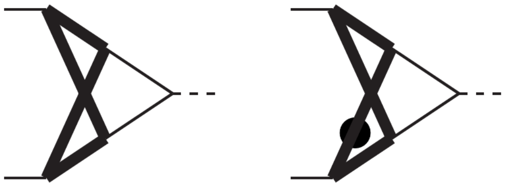
  - Box integrals depend on 5 energy scales

  - Treatment of γ^5 in D dimensions (Larin)

- However, the final result can be expressed in terms of the same 52 Master Integrals found in $gg \to HH$

  - 50 MIs expressed in terms of Generalized Harmonic Polylogarithms
    [Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

  - Two elliptic integrals [von Manteuffel, Tancredi ('17)]
    Semi-analytic evaluation implemented in Fortran routine
    [Bonciani, Degrassi, Giardino, Gröber ('18)]



Pt expansion: example

1) Consider a **one-loop** box integral

$$\int d^D q_1 \; \frac{(q_1^2)^{n_1}(q_1\cdot p_1)^{n_2}(q_1\cdot p_2)^{n_3}(q_1\cdot p_3)^{n_4}}{(q_1^2-m_t^2)[(q_1+p_2)^2-m_t^2][(q_1-p_1-p_3)^2-m_t^2][(q_1-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; make transverse momentum explicit

$$\frac{(q_1\cdot p_3)^{n_4}}{[(q_1-p_1-p_3)^2-m_t^2]}$$

$$\begin{aligned} p_3^\mu &= \frac{u'}{s'}p_1^\mu + \frac{t'}{s'}p_2^\mu + r_\perp^\mu \\ &= -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + \frac{\Delta_m}{s'}p_1^\mu + r_\perp^\mu \end{aligned}$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

$$\int d^D q_1 \; \frac{(q_1^2)^{n_1}(q_1\cdot p_1)^{n_2'}(q_1\cdot p_2)^{n_3'}(q_1\cdot r_\perp)^{n_4'}}{(q_1^2-m_t^2)^{l_1}[(q_1+p_2)^2-m_t^2][(q_1-p_1)^2-m_t^2]}$$

4) IBP reduction