Applying Quantum Technology to Problems in Particle Physics

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QUANTUM TECHNOLOGY INITIATIVE

17.11.2021

Talk Outline

1. **Brief introduction to Quantum Computing**
   - What are the general guiding principles?
   - How is it implemented in practice, with real-world hardware?

2. **Simulating the Standard Model and Beyond (far-future)**
   - Difficulties with constructing Hamiltonian formulation of gauge theories
   - Example of a new U(1) Formulation

3. **Quantum Machine Learning For Monte Carlo Event Generation (near-future)**

There are many interesting applications of quantum technology for sensing and metrology but I unfortunately will not have time to talk about this

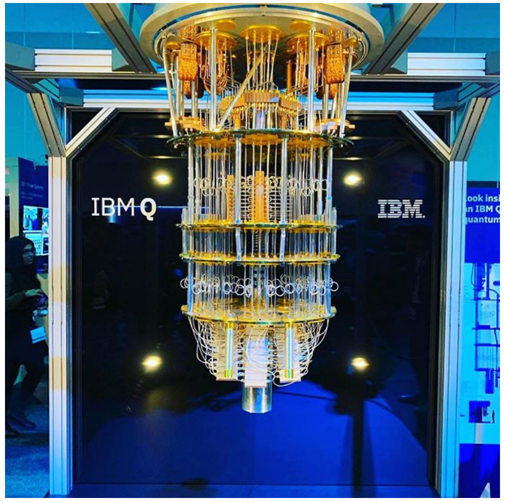
Take Home Messages

In this talk, I try to highlight four main point

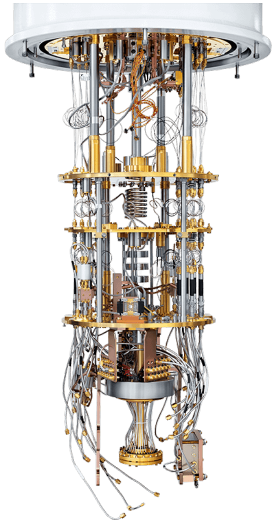
1. Quantum computing has the potential to probe theories currently inaccessible via classical methods
2. The time to start setting down the foundations for far-future work is right now
   - Currently, there is much active collaboration between academia and industry
3. There are particle physics problems that are currently amenable to quantum approaches, despite the Noisy Intermediate Scale Quantum (NISQ)-era hardware
4. Dream big….but also realistic!

Existing Quantum Hardware

Superconducting Qubits



IBM Q



Rigetti

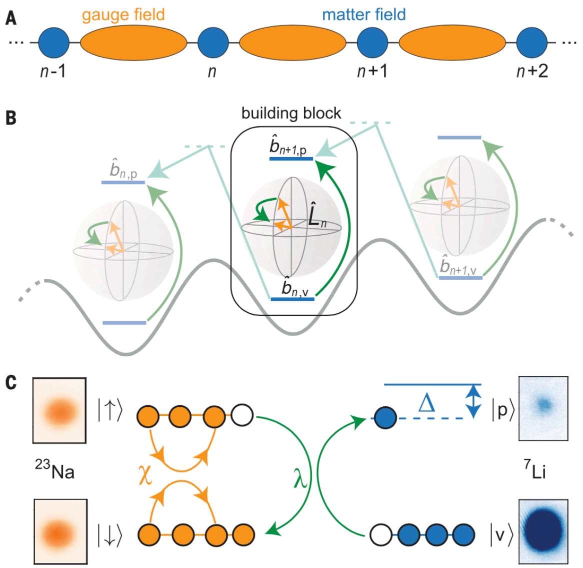
Honeywell

Trapped Ion

D-Wave

Annealer

IonQ



Academic Table Top

Cold Atom

Mil A. et al., Science 367:1128-1130 (2020)

What is Quantum Computing?

General Idea: Utilize the collective properties of quantum states (superposition, interference, entanglement) to perform calculations

Expectation/Hope: Dramatic improvement in run-time scaling for problems that are exponentially slow on classical machine

Shor's algorithm: Method for factoring large numbers (backbone of many encryption schemes)

Quantum Algorithm Run-Time Scaling: $\mathcal{O}\left((\log N)^2(\log\log N)(\log\log\log N)\right)$

N*: Size of Integer*

Classical Algorithm Run-Time Scaling: $\mathcal{O}\left(e^{1.9(\log N)^{1/3}(\log\log N)^{2/3}}\right)$

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First Paper that Provided the Theoretical Underpinnings of Quantum Computing

The Computer as a Physical System: A Microscopic Quantum Mechanical Hamiltonian Model of Computers as Represented by Turing Machines

Benioff, P, Journal of Statistical Physics Volume 22, 563-591 (1980)

Quantum mechanical model of Turing machines

Analog Quantum Computer

General Idea: Use one controllable quantum system to simulate the behavior of another

- Continuous time evolution of the system of interest
- Generally are built from cold atoms on an optical lattice
- Non-universal and need to be tuned to reflect the desired physics

Analog quantum computer are like an effective field theory for a more fundamental quantum field theory, but made physical

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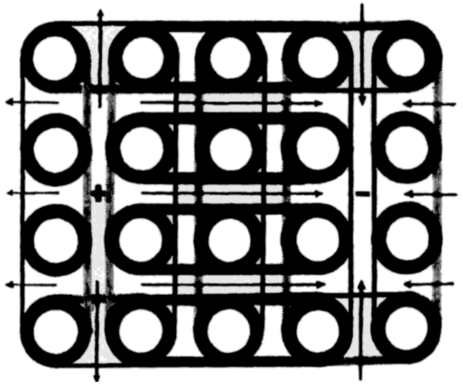
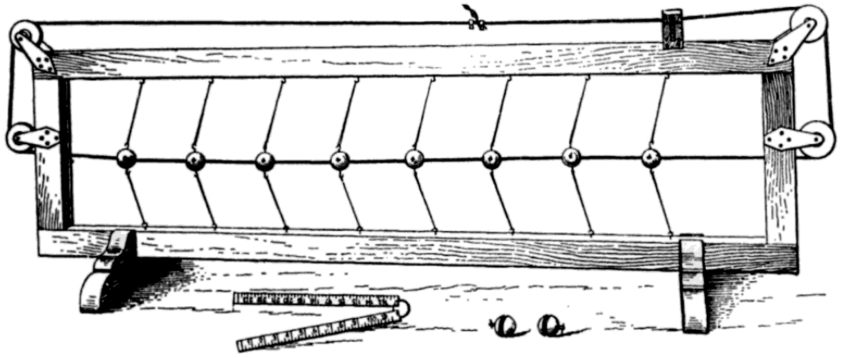
Method of using physical toy models to understand more complicated system has a long history in physics

Ex: Physical systems made of rollers, bands and string used to understand Maxwell's law and the Luminiferous Aether



4.3. FitzGerald's wheel-and-band model (strained and locked).

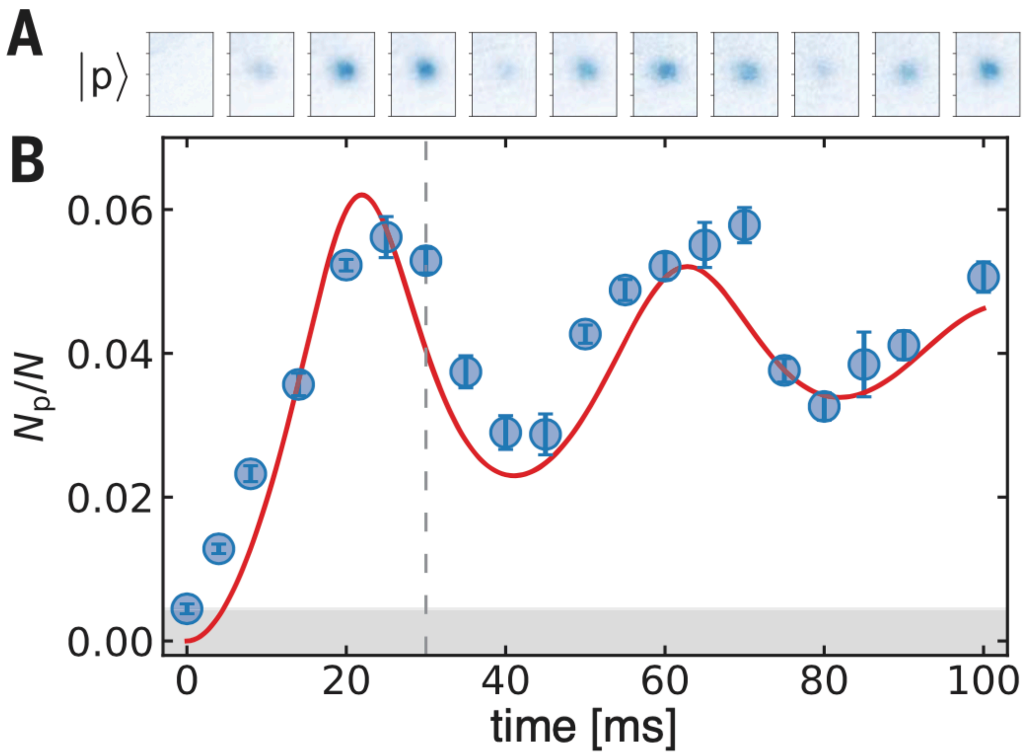


4.4. Lodge's string-and-button model of an electric circuit. The string runs through slots in the buttons, which are attached by rubber bands to the wooden frame. By tightening or loosening the screws holding the buttons to the string, the model can be made to represent either a dielectric or a conducting circuit.

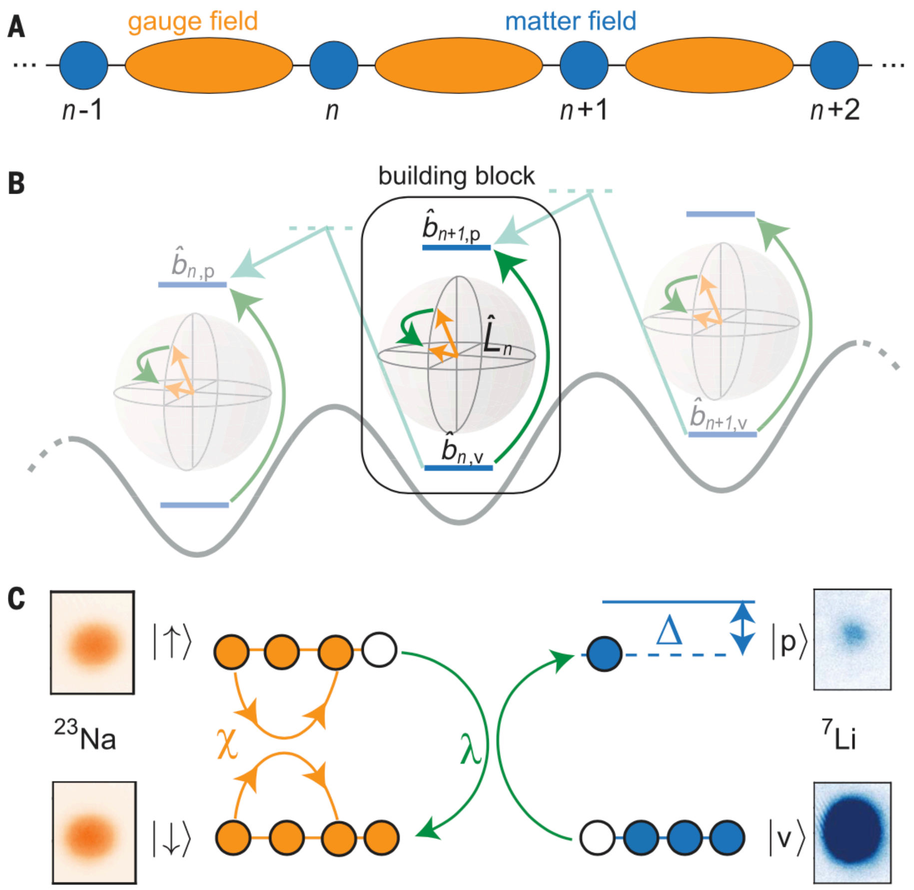
"The Maxwellians", Bruce J. Hunt, Cornell University Press (1991)

Analog Quantum Computer

Example of Analog Simulation: Lattice Schwinger model realized via cold atoms in a trapping potential



Key Observation*:* Time-dependent pair production



Experimental Set-Up: Two atomic Bose-Einstein Condensates

Mil A. et al., Science 367:1128-1130 (2020)

Digital Quantum Computer

General Idea: Construct a set of logic gates onto qubits and build a circuit from these components

- Only discrete time evolution of the system of interest
- Any two level quantum system can be made into a qubit
- Universal since any circuit built out of quantum logic gates will run on a digital quantum computer
  - The hard work is translating physical system into language of qubits and gates

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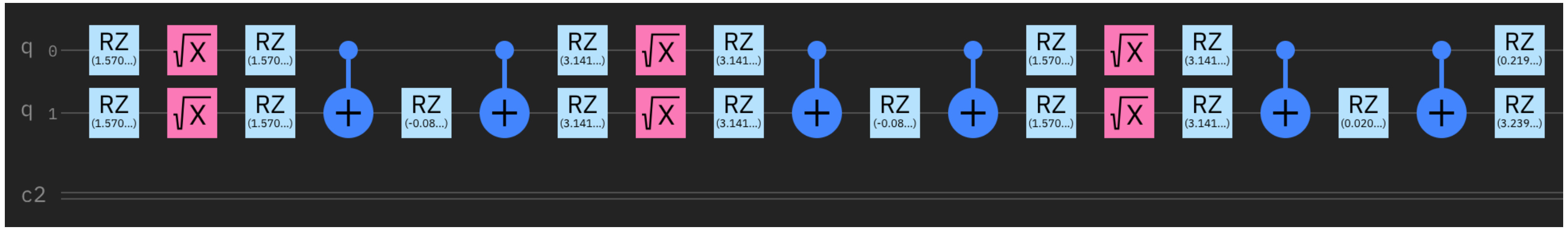
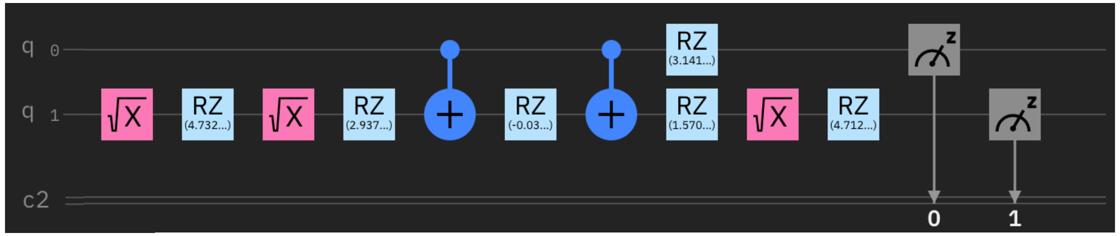
Ex: Circuit for single time step in 2-site Schwinger Model

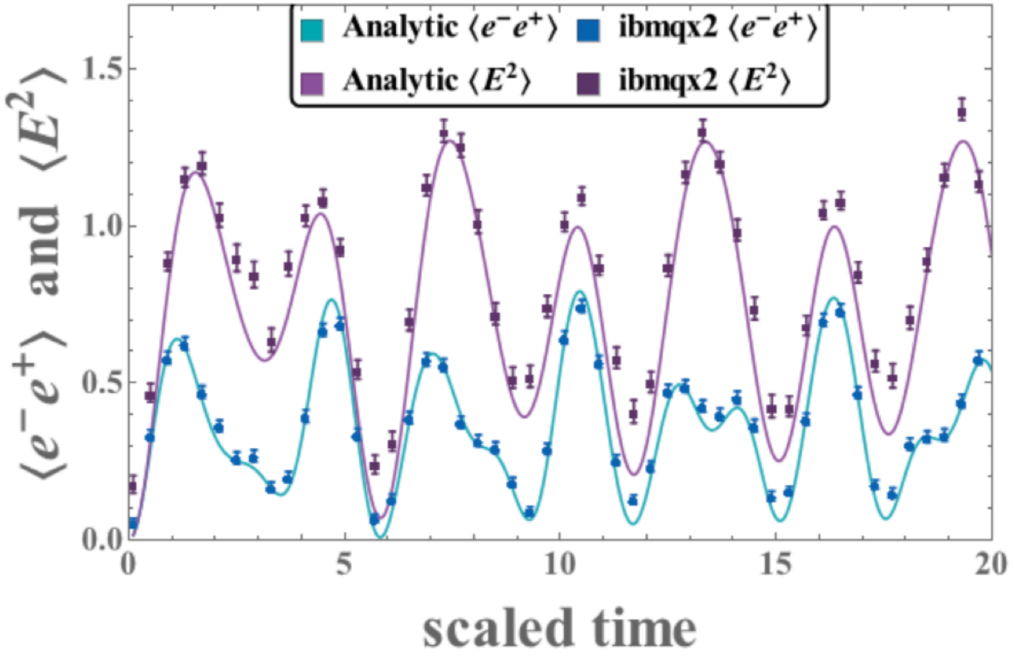




Circuit needs to be run multiple times to build up expectation value of the observable

Digital Quantum Computer

Example of Digital Simulation: Lattice Schwinger model realized via quantum circuit utilizing superconducting qubits



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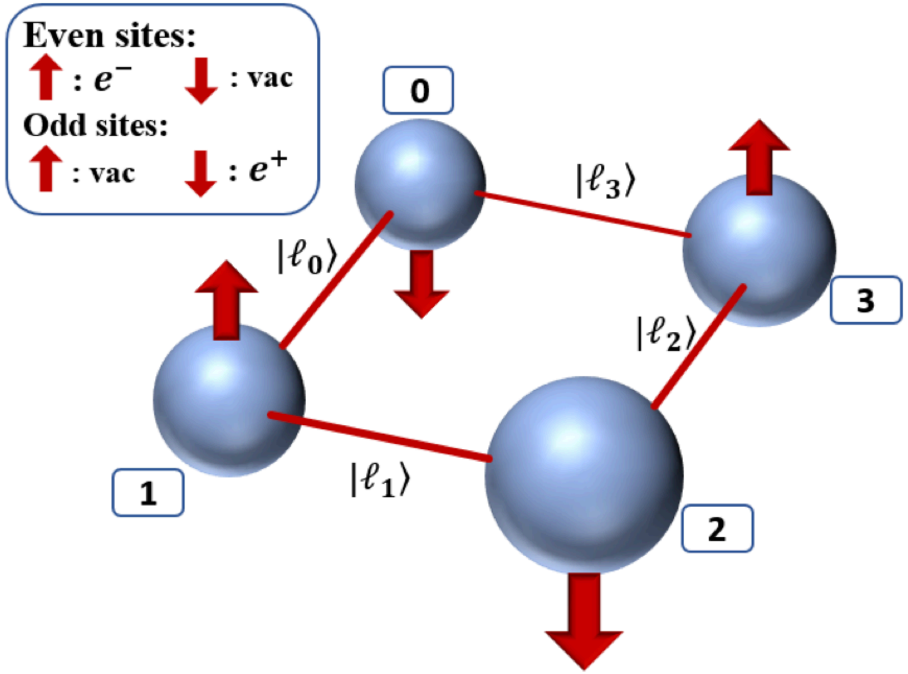


Circuit Mapping: Two-site system with mapping onto qubits

Klco, N. et al (Phys. Rev. A **98**, 032331 (2018)

Quantum Computation for Particle Physics, “Work Flow”

Theoretical Developments

Algorithmic Developments

All three are currently necessary

Optimization and Benchmarking

Quantum Computational Strategies for HEP

General Idea: Quantum Computing is still in its infancy and so we need to think carefully about what HEP problems would be most amenable to this novel computational strategy

Two General Criteria

Is this calculation exponentially hard with classical computing methods?

OR

Can NISQ-era hardware already provide an advantage?

Real-Time Dynamics

Finite-Density Nuclear Matter

Chiral Gauge Theories

Augmentation of Classically-Generated Data Sets

Would love to find some more!

Simulation of Lattice Gauge Theories

Classical Simulations of Gauge Theories

Lattice QCD: Highly advanced field, utilizing high-performance computing to carry out physical point pion mass calculations of light hadron physics

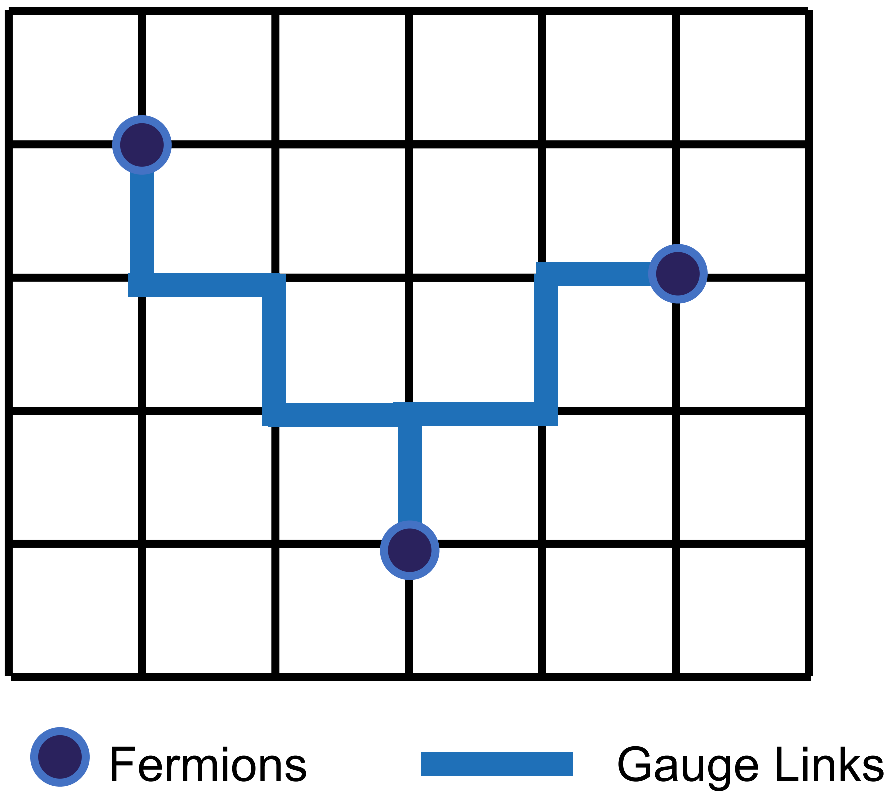
Methodology

- Work with Path-Integral Formulation
- Analytically continue to Euclidean Spacetime

$$\mathcal{Z} = \int [DU] \det D_F(U)\, e^{-S[U]}$$

- Use Monte Carlo methods to sample path integral

Method fails for real-time dynamics and theories where the fermion determinant is not real and positive

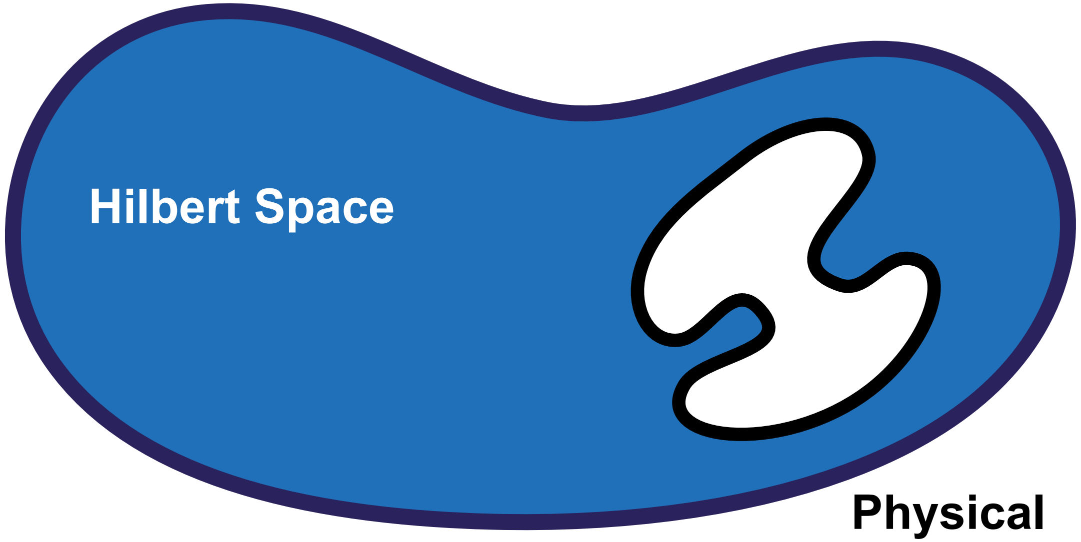


Lattice Gauge Theories on a Digital Quantum Computer

Need to Address Two Key Aspects

Gauge Invariance and Redundancies

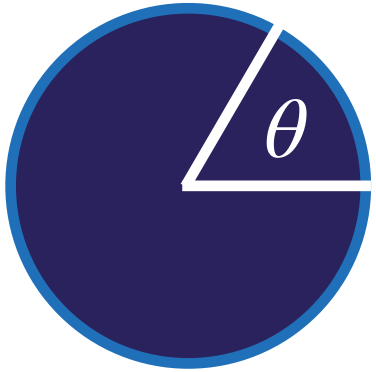
Physical Hilbert space is significantly smaller than full Hilbert space



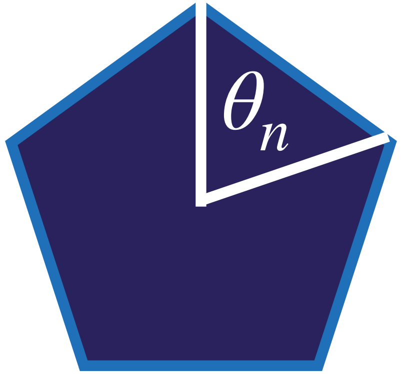
Physical Hilbert space

Truncation and Digitization of Fields

Hamiltonian operators are mapped onto a finite number of discrete basis states



$$\theta = [-\pi, \pi]$$



$$\theta_n = \frac{2\pi n}{5} \qquad n \in \mathbb{Z}$$

Digitization Example: Quantum Harmonic Oscillator

This simple toy model clearly demonstrates the pitfalls of unwise digitization choices

Goal: Using only $2L+1$ states, how well can we replicate the low-lying states of the QHO?

$$H = \frac{1}{2}X^2 + \frac{1}{2}P^2$$

1) Working in the X basis, it is trivial to digitize X

$$X_k = -X_{\text{max}} + k\delta X \qquad \delta X = \frac{X_{\text{max}}}{2L+1}$$

X_{max} is a free parameter

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2) Question: How to digitizing P, as its not diagonal in this basis?

Option One: Use finite difference version

$$P^2 = \frac{1}{\delta X^2}\begin{pmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}$$

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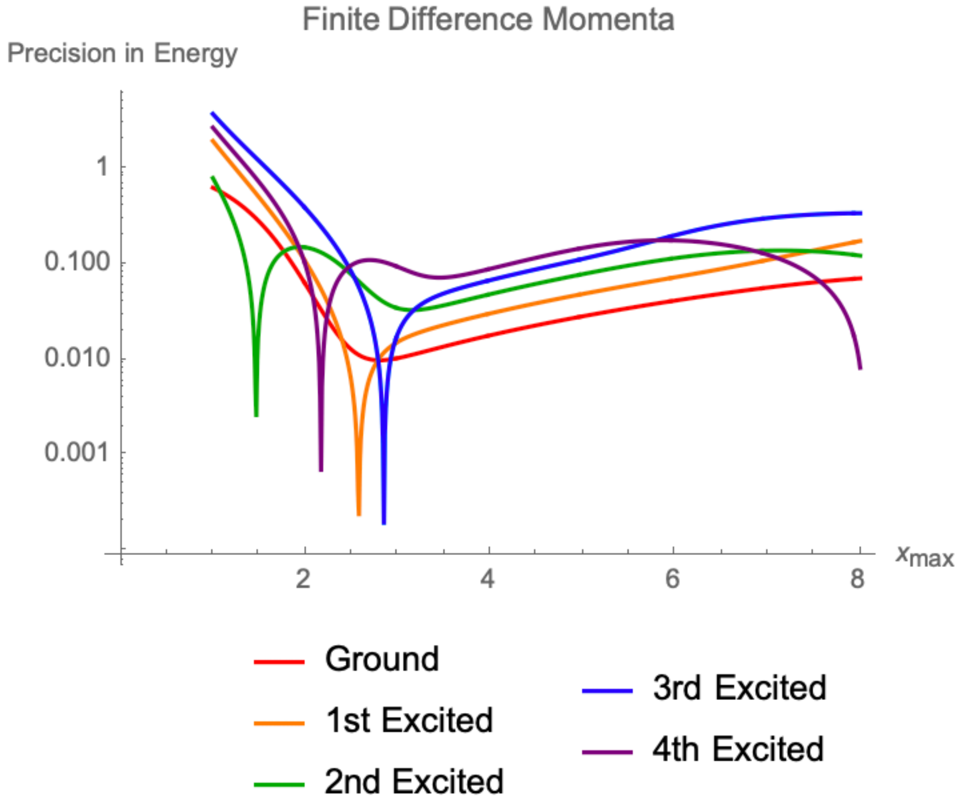

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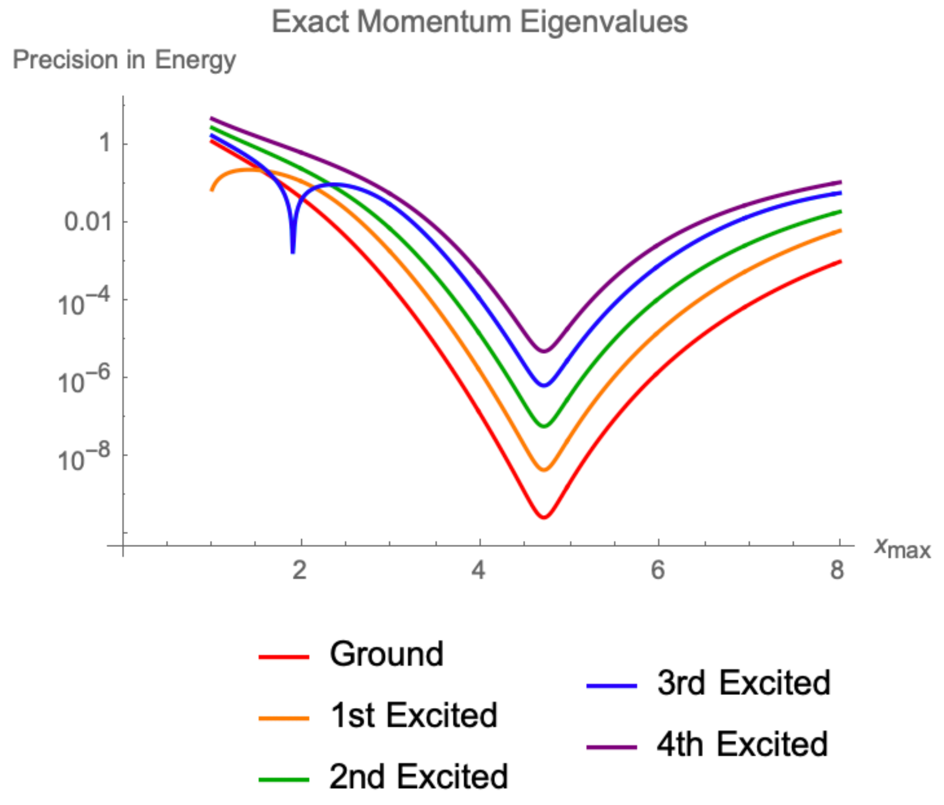
Option One: Use finite difference version
Option Two: Use exact form and Fourier transform to change basis

$$P_k = -P_{\text{max}} + k\delta P \qquad \delta_P = \frac{1}{\delta X}\frac{2\pi}{2L+1}$$



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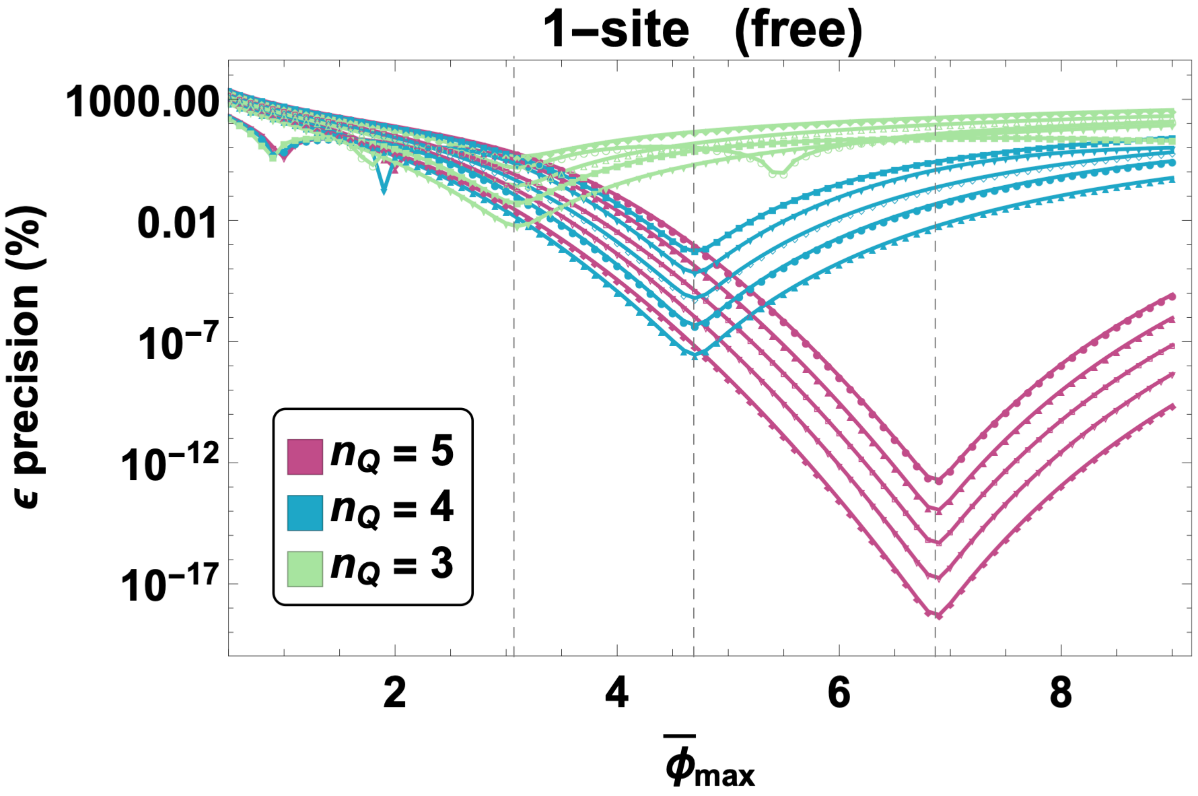
Optimal value can be calculated exactly

$$x_{\max} = L\sqrt{\frac{2\pi}{2L+1}}$$

Intuitive Understanding: *Eigenstate has the same width in both position and momentum space and so* $\delta x = \delta p$



Klco, N. and Savage, M.J.: Phys. Rev. A 99, 052335 (2019) [arXiv: 1808.10378]

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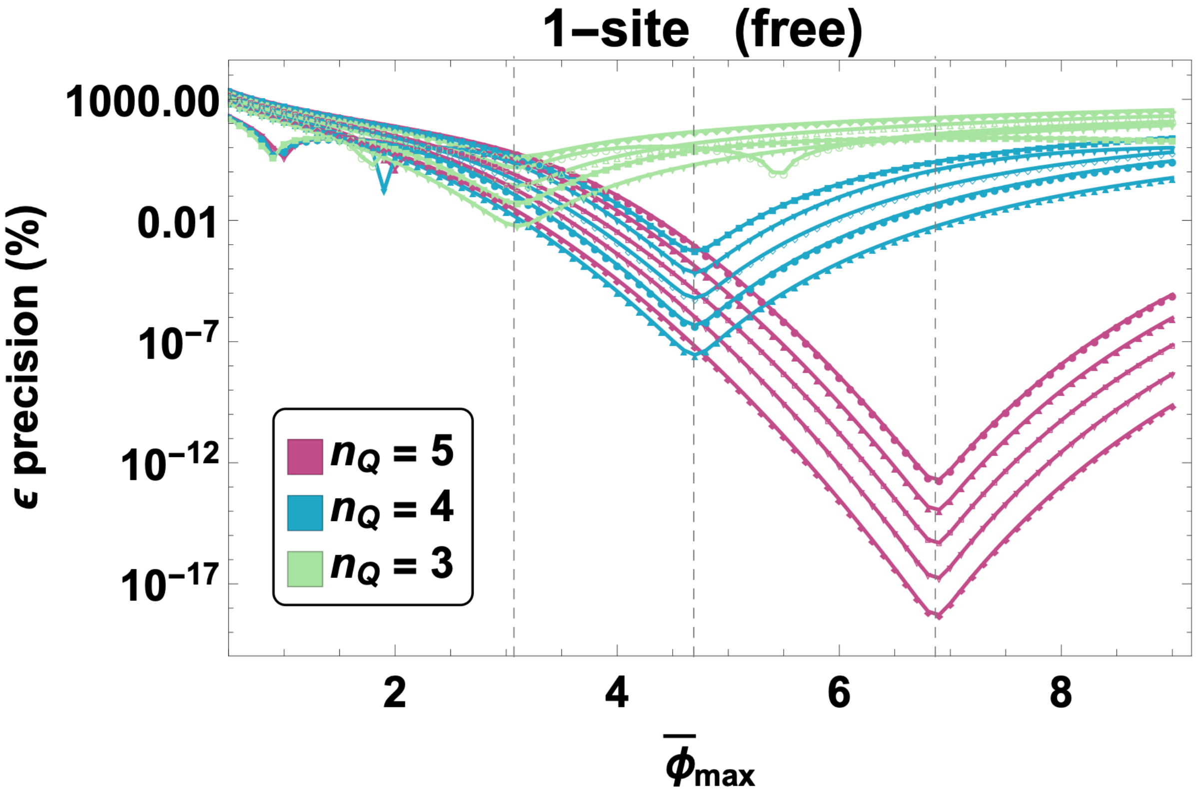
Value for optimal X_{max} can also be related to Nyquist–Shannon sampling theorem

Macridin, A.,Spentzouris, P., Amundson, J., and Harnik, R: Phys. Rev. Lett. 121, 110504 (2018) and Phys. Rev. A 98, 042312 (2018)



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Enforcing Gauge Invariance

Strategy One: Do not restrict to physics states and use other methods do deal with gauge violation

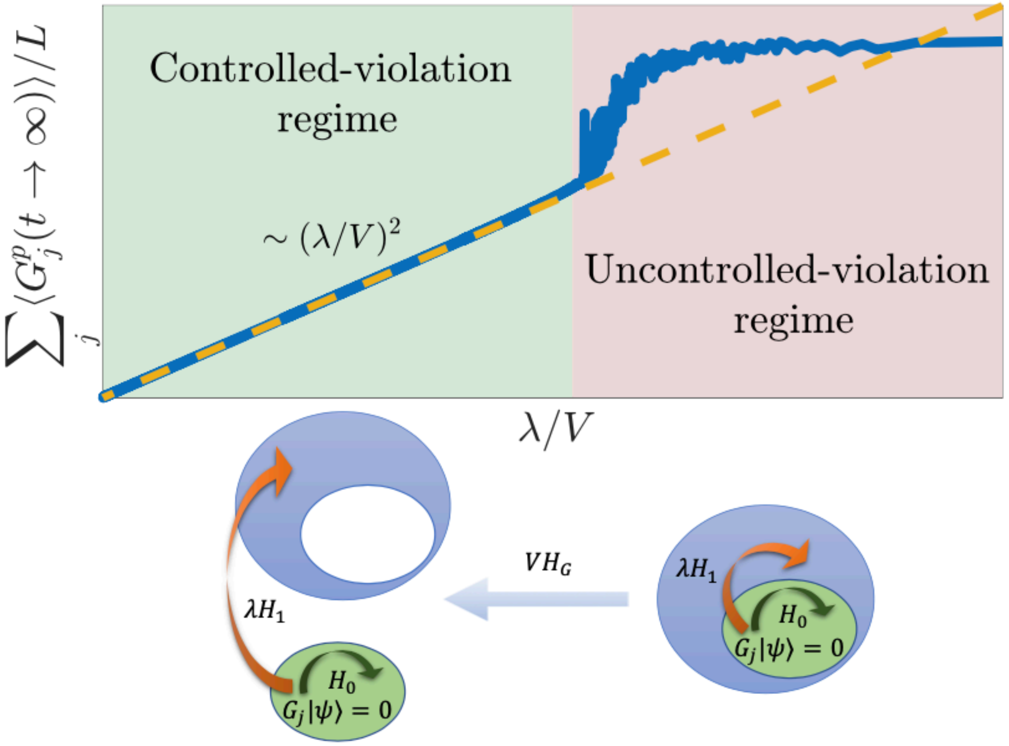
One Possible Method: Introduce energy penalty for gauge-violating transitions to Hamiltonian

Gauge Invariance can be written in terms of Gauss' Law

$$\hat{G}\,|\,\text{phys}\rangle = Q_{\text{st}}\,|\,\text{phys}\rangle$$

$$\Updownarrow$$

$$\left[\hat{G},\hat{H}\right] = 0$$



Halimeh, J.C. and Hauke, P. Phys. Rev. Lett. 125, 030503 (2020)

Enforcing Gauge Invariance

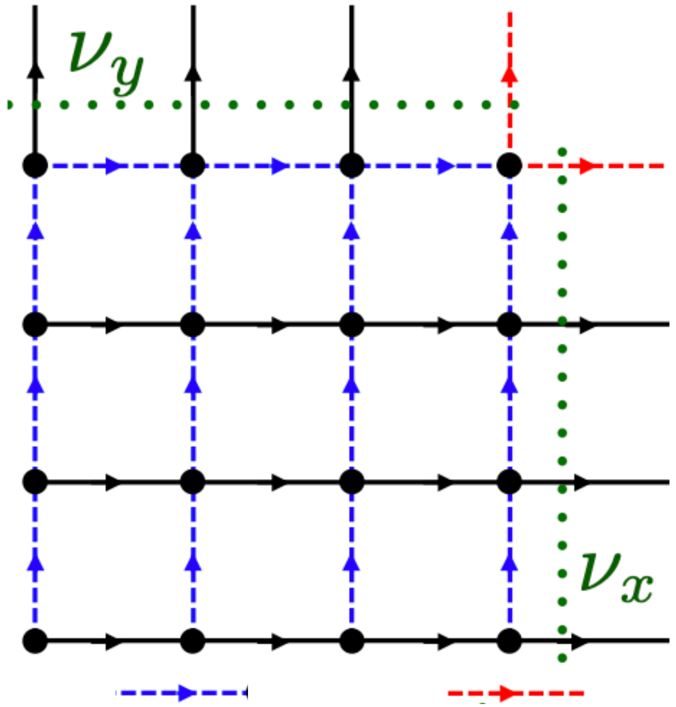
Strategy Two: Define Hamiltonian purely in terms of physics states

Method One: Enforce Gauss' Law by hand at each lattice site and eliminate redundant links

Can use a maximal tree to develop a systematic way of eliminating redundant links ( )

Kaplan, D.B. and Stryker, J:
Phys. Rev. D 102, 094515 (2020)

2D Example



Eliminate via Gauss Law

Constrain via Topology

Enforcing Gauge Invariance

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Kaplan, D.B. and Stryker, J:
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Method Two: Work with Dual Basis Formalism where Gauss' law is automatically satisfied

$$H = \frac{1}{2a}\sum_p \left[g^2(\vec{\nabla}\times R_p)^2 + \frac{1}{g^2}B_p^2 \right]$$

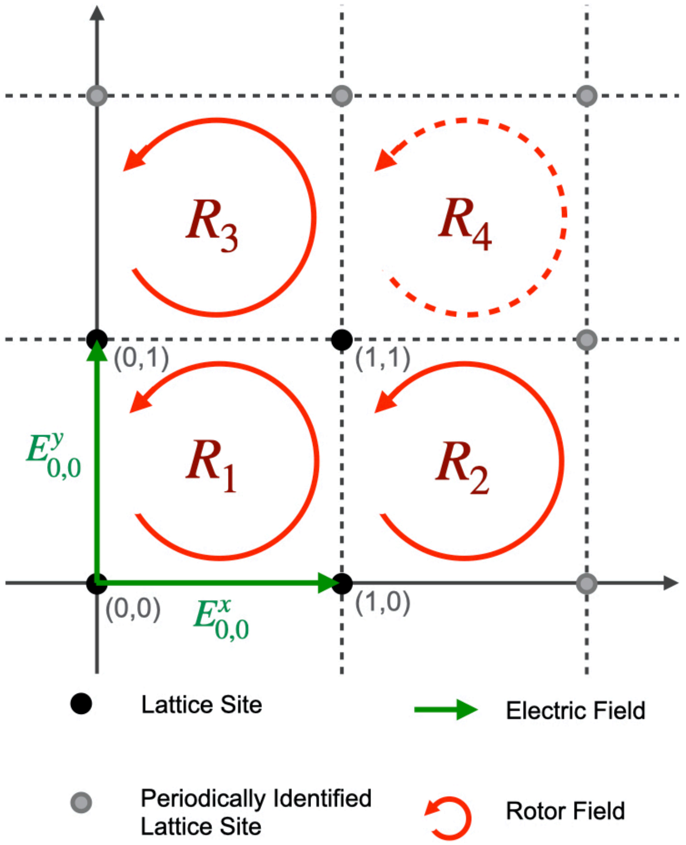
$$\vec{E}^T = \vec{\nabla}\times R$$

Drell, S.D, Quinn, H.R., Svetitsky, B. and Weinstein, M
*Phys. Rev. D **19**, 619 (1979)*

Can do same thing for compact theory

$$B_p \rightarrow \cos B_p$$

2D Example



Digitizing the Dual Formulation in the Magnetic Basis

General Idea: Combine the “gauge-redundancy free” dual representations with the QHO digitization methods

Step One: Digitize rotor and magnetic fields

$$b_p^{(k)} = -\, b_{\max} + k\,\delta b\,, \qquad \delta b = \frac{b_{\max}}{\ell} \qquad\qquad r_p^{(k)} = -\, r_{\max} + \left(k + \frac{1}{2}\right)\delta r \qquad\qquad \delta r = \frac{2\pi}{\delta b(2\ell+1)}\,, \qquad r_{\max} = \frac{\pi}{\delta b}$$

Step Two: Define digitized rotor and magnetic operators

$$\langle b_p^{(k)} | B_p | b_{p'}^{(k')} \rangle = b_p^{(k)} \delta_{kk'} \delta_{pp'} \qquad\qquad \langle b_p^{(k)} | R_p | b_{p'}^{(k')} \rangle = \sum_{n=0}^{2\ell} r_p^{(n)} \left(\mathsf{FT}\right)^{-1}_{kn} \left(\mathsf{FT}\right)_{nk'} \delta_{pp'}$$

Bauer, C.W. and Grabowska, D.M.
arXiv: 2111.08015

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Step Three: Determine optical value for b_{max}

$$b_{\max}^{\mathrm{NC}}(g,\ell) = g\ell \sqrt{\frac{\sqrt{8\pi}}{2\ell+1}} \qquad\qquad b_{\max}^{\mathrm{C}}(g,\ell) = \min\left[b_{\max}^{\mathrm{NC}}, \frac{2\pi\ell}{2\ell+1} \right]$$

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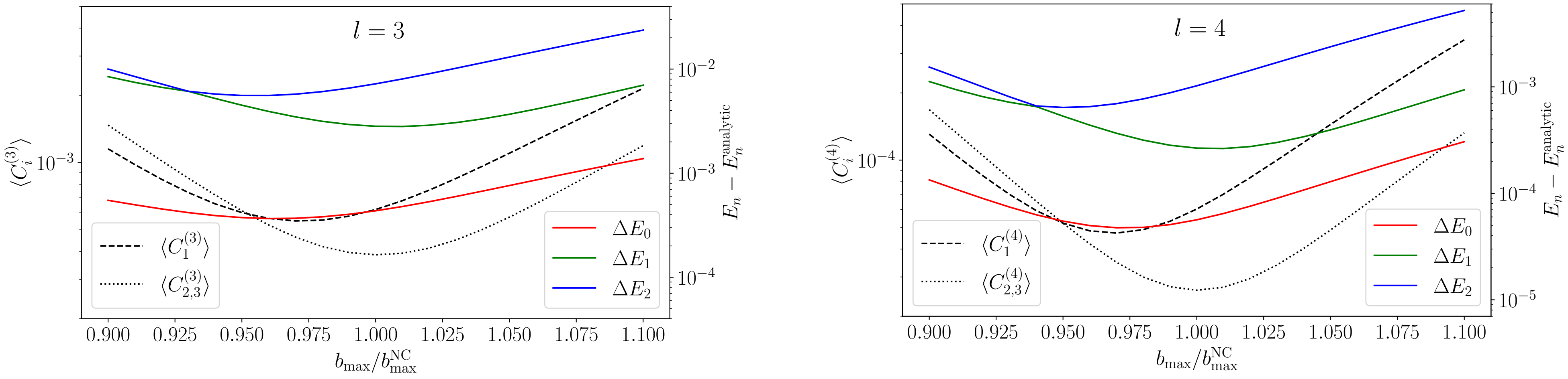

Take Away Message One: Achieve per-mille level accuracy with just seven states per site

Take Away Message Two: Canonical Commutation Relations are minimally violated for correctly chosen $b_{\max}$

Preserving the relations is key for creating a faithful representation

Bauer, C.W. and Grabowska, D.M. arXiv: 2111.08015

Feasibility of Near-Term QCD Simulations

Question: What are the quantum computing resource needs to simulate QCD?

Desired Simulation

Physical pion mass, 192×96^3 lattice points with lattice spacing of 0.064 fm

Quantum Hardware Naive Number of Qubits	***Classical Hardware*** (100 Configs) 2020
$\sim 20 \times 192 \times 96^3 \approx 3 \times 10^9$	20 million core hours

So maybe not something for the Noisy Intermediate-Scale Quantum (NISQ) era…

Key Point

We are far from doing physical point simulations, but the toy models are interesting in their own right!

(Chiral Gauge Theories)

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Key Point

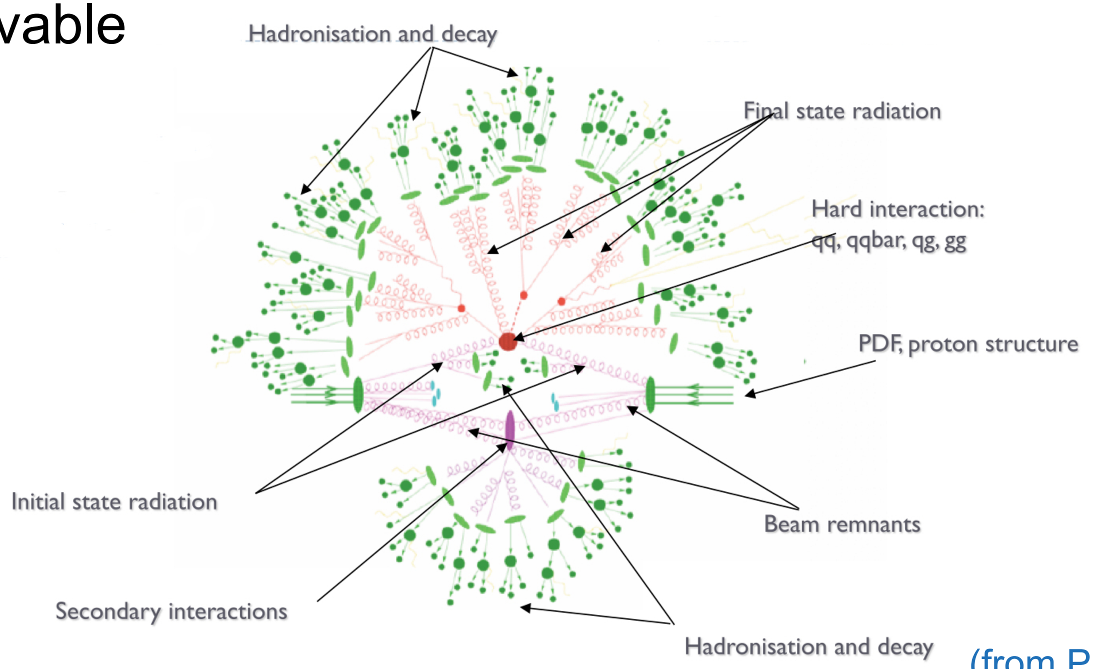
Algorithmic advancements can also dramatically increase the feasibility of a simulation

Quantum Machine Learning For Monte Carlo Event Generation

Simulations of LHC Events

Event Generation: Requires a multi-step process, with different energy scales, in order to extract useful physical observable

- Hard Matrix Element
- Parton Showering for soft radiation
- Hadronization
- Pile-up
- Detector Simulation



(from P. Skands and F. Krauss)

LHC produces $\mathcal{O}(10^9)$ ***collisions per second:*** this is a very complex environment and the simulation is quite computationally intensive

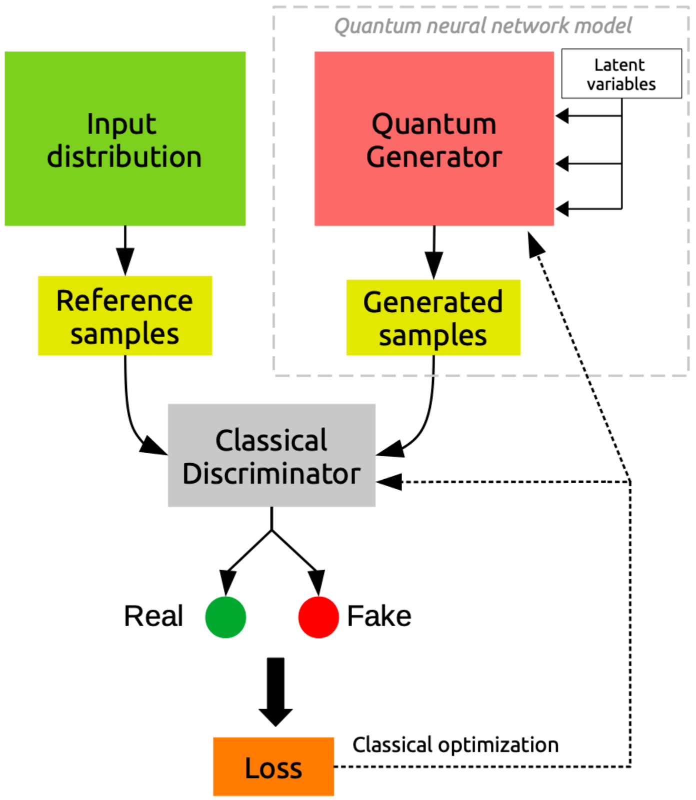
Can Quantum Machine Learning on NISQ-era hardware help?

Quantum Machine Learning Strategy

General Idea: Use a trained neural network to augment data produced by classical Monte Carlo event generation

Generative Adversarial Network (GAN): Two networks compete against one another and through this competition, one network learns the underlying distribution

Part of this work included designing a novel generator, called style-qGAN



Bravo-Prieto, C., Baglio, J., Cè, M., Francis,A., Grabowska, D.M. and Carrazza, S.; arXiv: 2110.06933

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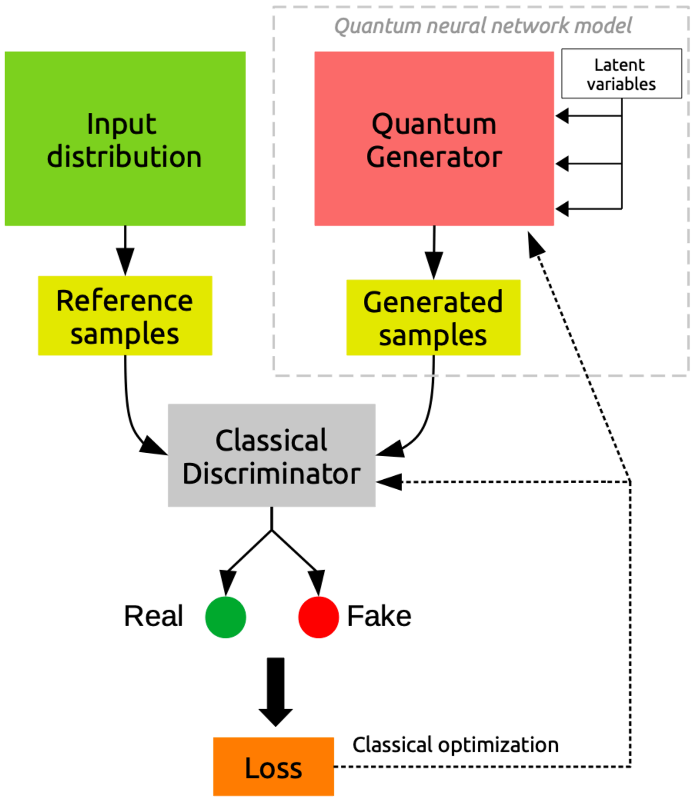
General Idea: Use a trained neural network to augment data produced by classical Monte Carlo event generation

Generative Adversarial Network (GAN): Two networks compete against one another and through this competition, one network learns the underlying distribution

Part of this work included designing a novel generator, called style-qGAN



Bravo-Prieto, C., Baglio, J., Cè, M., Francis,A., Grabowska, D.M. and Carrazza, S.; arXiv: 2110.06933

Analogy

Generator: Art Forger trying to pass fake art as authentic

Discriminator: Art Historian trying to detect fake art

Training: High-stakes film-noir "cat and mouse" game

Successful Training: Art forger learns enough to be able to not only replicate painting but create new ones in the same style

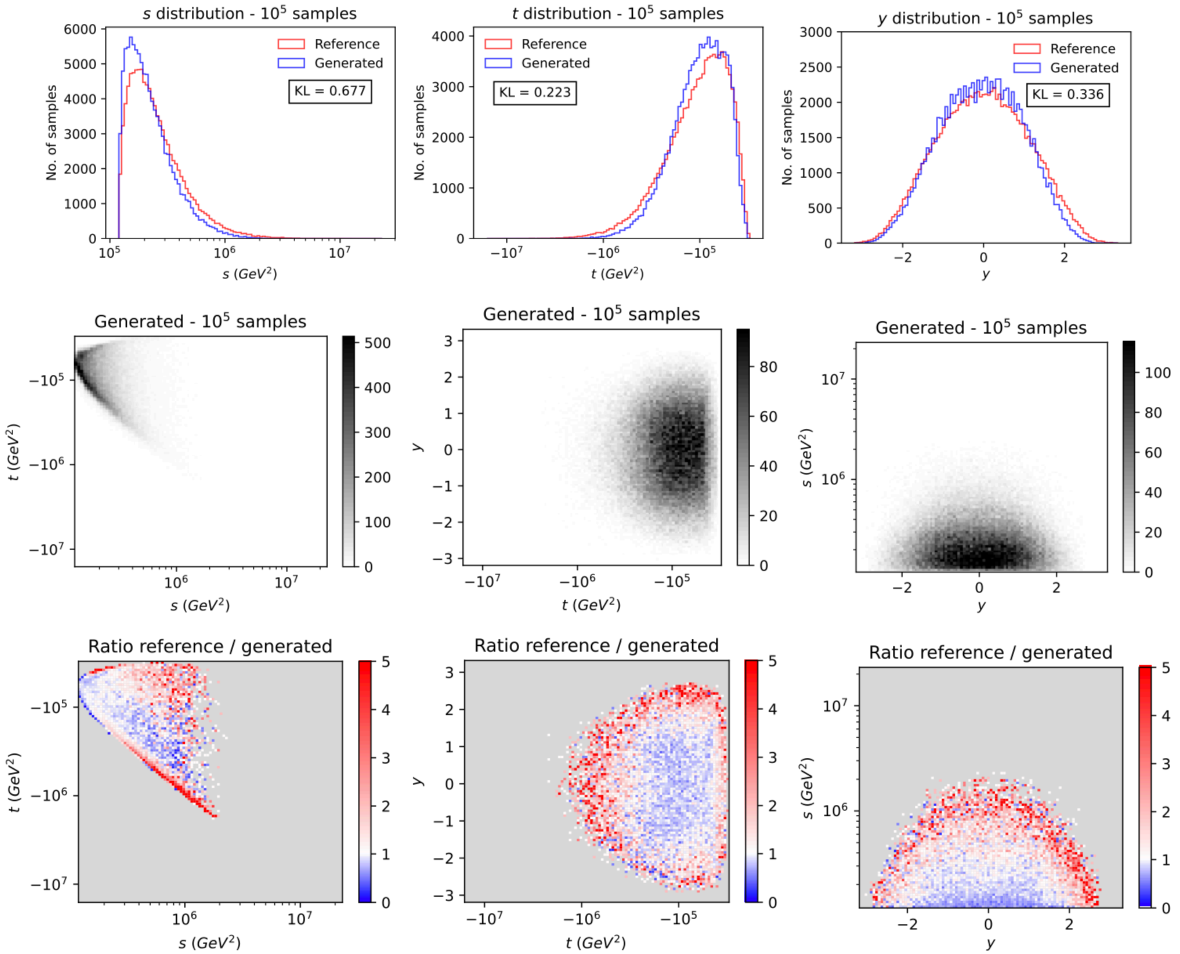
Results on running style-qGAN on $pp \to t\bar{t}$ Data

ibmq_santiago

Real Data, Real Machine: New qGAN architecture implemented onto 5-qubit quantum machine using Monte Carlo generated data for $pp \to t\bar{t}$ process

- Data is correlated and non-Gaussian
- Circuit only requires three qubits and does not have a high gate count
- Despite noise in machine, see successful data augmentation

If this piqued your interest, please see Julien's talk on Friday at 2pm!



Bravo-Prieto, C., Baglio, J., Cè, Marco, Francis,A., Grabowska, D.M. and Carrazza, S.; arXiv: 2110.06933

Conclusions

In this talk, I try to highlight four main point

1. **Quantum computing has the potential to probe theories currently inaccessible via classical methods**
   - Real-time simulations of gauge theories on quantum hardware using the Hamiltonian formulation
2. **The time to start setting down the foundations for far-future work is right now**
   - Exploring lower-dimensional gauge theories is scientifically valuable
   - Currently, there is much active collaboration between academia and industry
3. **There are particle physics problems that are currently amenable to quantum approaches, despite the Noisy Intermediate Scale Quantum (NISQ)-era hardware**
4. **Dream big….but also realistic!**

Theory + Simulation Branch of CERN's QTI

Four Top-Level Objects

Identify possible applications of quantum simulations and support worldwide experimental efforts to probe and measure both Standard Model and beyond the Standard Model physics.

Applications of Quantum Technology To Particle Physics

For details, please see CERN QTI Roadmap

Assist the computing and sensing activities in identifying theoretically promising regions of parameter space in which quantum technology could provide an advantage over classical methods.

Benchmark the current and potential performance of quantum simulations against state-of-the-art classical computations.

Host workshops, summer institutes and visitors, establishing global collaborations with other institutes, national labs and companies.