Model-dependent results on the energy dependence of the optical point of elastic pp scattering

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Day of Femtoscopy 2021

October 28, 2021, Gyöngyös, Hungary

Optical point (OP) and total cross section

- **optical theorem:**

$$\boxed{\sigma_{tot}(s) = 2ImT(s, t = 0)}$$

- **optical point (OP):**

$$\boxed{\frac{d\sigma}{dt}(s, t = 0) = \frac{1}{4\pi}\left[\left(ReT(s, t = 0)\right)^2 + \left(ImT(s, t = 0)\right)^2\right]}$$

- **connection between OP and total cross section:**

$$\boxed{\sigma_{tot}^2(s) = \frac{16\pi}{1 + \rho_0^2(s)}\frac{d\sigma}{dt}(s, t = 0)}$$

with

$$\boxed{\rho_0(s) = \frac{ReT(s, t = 0)}{ImT(s, t = 0)}}$$

Bialas-Bzdak p=(q,d) model

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2\vec{s}_q\, d^2\vec{s}^{\,\prime}_q\, d^2\vec{s}_d\, d^2\vec{s}^{\,\prime}_d\; D(\vec{s}_q,\vec{s}_d)\; D(\vec{s}^{\,\prime}_q,\vec{s}^{\,\prime}_d)\; \sigma(\vec{s}_q,\vec{s}_d;\vec{s}^{\,\prime}_q,\vec{s}^{\,\prime}_d;\vec{b})$$

- **quark-diquark distribution inside the proton:**

$$D(\vec{s}_q,\vec{s}_d) = \frac{1+\lambda^2}{R_{qd}^2\pi} e^{-\frac{s_q^2+s_d^2}{R_{qd}^2}} \delta^2(\vec{s}_q + \lambda\vec{s}_d)$$

$$\lambda = \frac{m_q}{m_d}$$

$$\vec{s}_d = -\lambda\vec{s}_q$$

$$\vec{s}^{\,\prime}_d = -\lambda\vec{s}^{\,\prime}_q$$

A. Bialas, A. Bzdak Acta Phys.Polon. B 38, 159-168 (2007)

- **interaction probability of the constituents:**

$$\sigma(\vec{s}_q,\vec{s}_d;\vec{s}^{\,\prime}_q,\vec{s}^{\,\prime}_d;\vec{b}) = 1 - \prod_a \prod_b [1 - \sigma_{ab}(\vec{b} + \vec{s}^{\,\prime}_a - \vec{s}_b)]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-|\vec{s}|^2/S_{ab}^2}$$

$$S_{ab}^2 = R_a^2 + R_b^2$$

$$a, b \in \{q,d\}$$

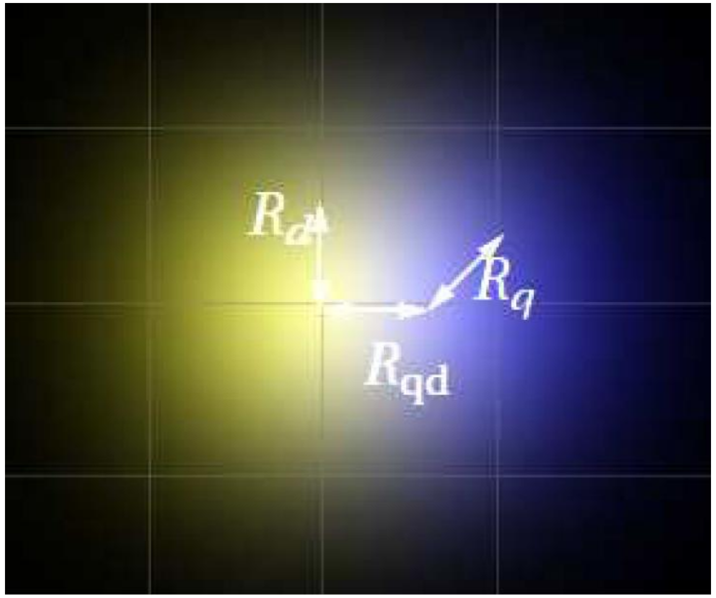
- **inelastic cross-sections of quark, diquark scatterings :**

$$\sigma_{ab,in} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sigma_{ab}(\vec{s})\, d^2\vec{s}$$

$$\sigma_{qq,in} : \sigma_{qd,in} : \sigma_{dd,in} = 1 : 2 : 4$$

- **free parameters:**

$$A_{qq}, \lambda, R_q, R_d, R_{qd},$$ $(A_{qq} = 1\ and\ \lambda = 0.5\ can\ be\ fixed)$



Proton-(anti)proton scattering in the quark-diquark model.

Unitarily Real Extended Bialas-Bzdak (ReBB) model

- **elastic scattering amplitude in the impact parameter space:**

$$\boxed{t_{el}(s,\vec{b}) = i\left[1 - e^{-\Omega(s,\vec{b})}\right]}$$

arXiv:1505.01415

F. Nemes, T. Csörgő, M. Csanád, Int. J. Mod. Phys. A Vol. 30 (2015) 1550076

- **the opacity function:**

$$\boxed{\Omega(s,\vec{b}) = Re\Omega(s,\vec{b}) + i\, Im\Omega(s,\vec{b})}$$

$Im\Omega \neq 0$ as the real part of the amplitude is not negligibly small

$$\boxed{Re\Omega(s,\vec{b}) = -\frac{1}{2} ln\left[1 - \tilde{\sigma}_{in}(s,\vec{b})\right]}$$

$$\boxed{Im\Omega(s,\vec{b}) = -\alpha\, \tilde{\sigma}_{in}(s,\vec{b})}$$

NEW FREE PARAMETER

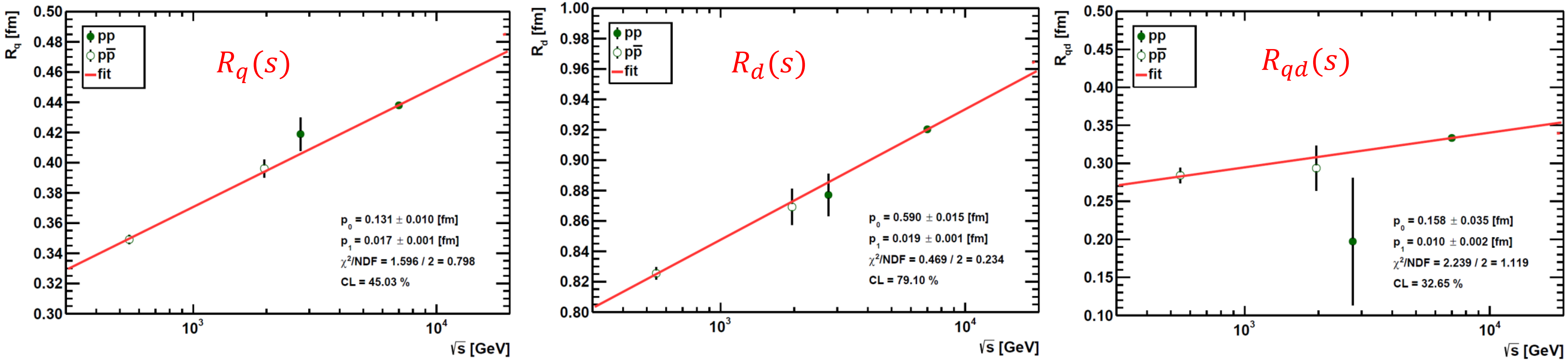
- **elastic scattering amplitude in momentum space:**

$$\boxed{T(s,t) = 2\pi \int_0^\infty t_{el}(s,|\vec{b}|)\, J_0(|\vec{\Delta}||\vec{b}|)|\vec{b}|d|\vec{b}|}$$

$$\boxed{|\vec{\Delta}| \equiv \sqrt{-t} \quad as\ \sqrt{s} \to \infty}$$

(t is the squared momentum transfer)

Energy dependences of the scale parameters



The energy dependences of the scale parameters, R_q, R_d and R_{qd}, are the same for pp and p$\bar{\text{p}}$ processes!

$$\boxed{P(s) = p_0 + p_1 ln(s/s_0)}$$

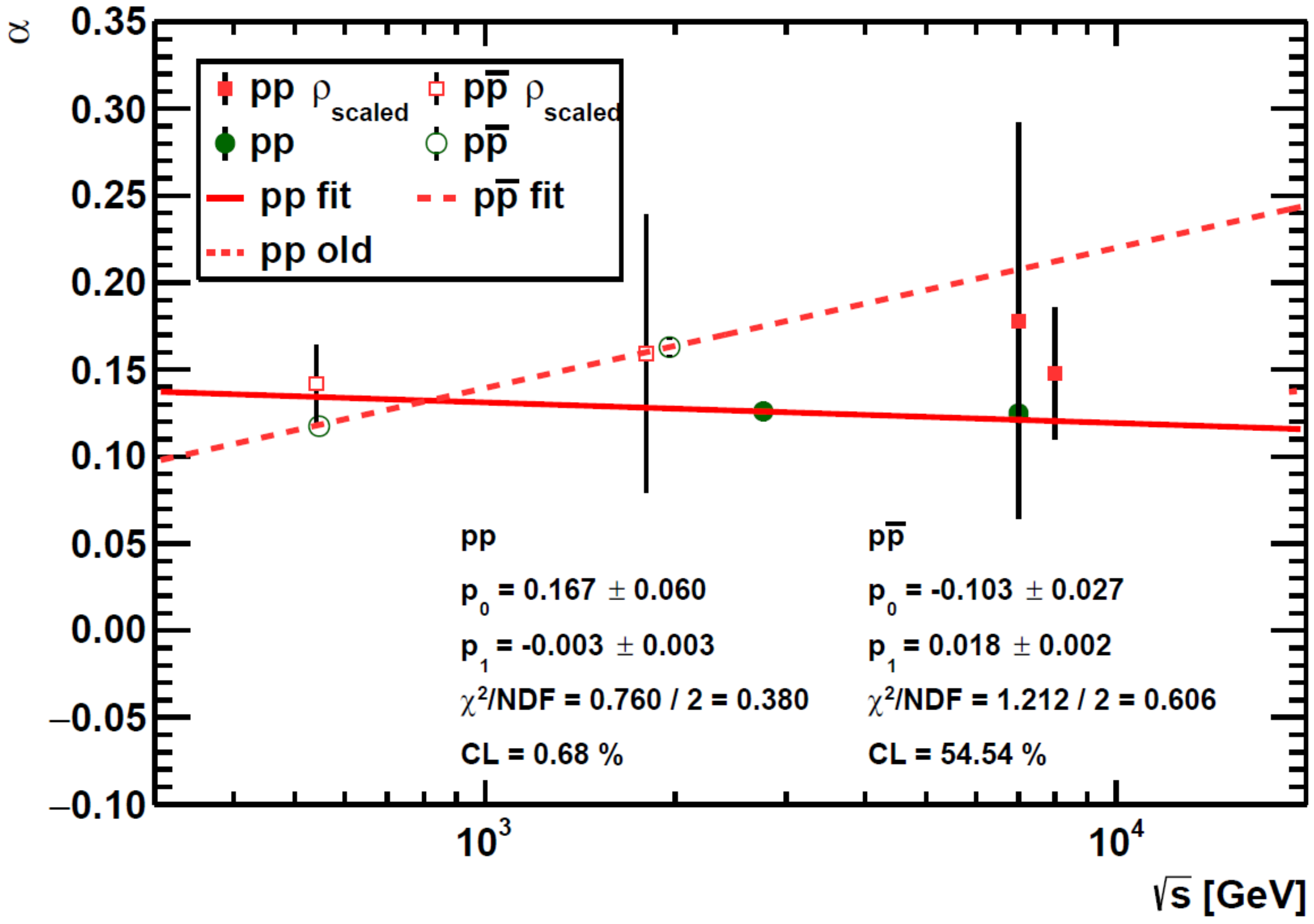
$$\boxed{P \in \{R_q, R_d, R_{qd}, \alpha\}}$$

$$\boxed{s_0 = 1\ GeV^2}$$

Parameter	$R_q\ [fm]$	$R_d\ [fm]$	$R_{qd}\ [fm]$
χ^2/NDF	$1.596/2$	$0.469/2$	$2.239/2$
CL [%]	45.03	79.10	32.65
p_0	0.131 ± 0.010	0.590 ± 0.015	0.158 ± 0.035
p_1	0.017 ± 0.001	0.019 ± 0.001	0.010 ± 0.002

Parameters which define the energy dependence of the ReBB model scale parameters

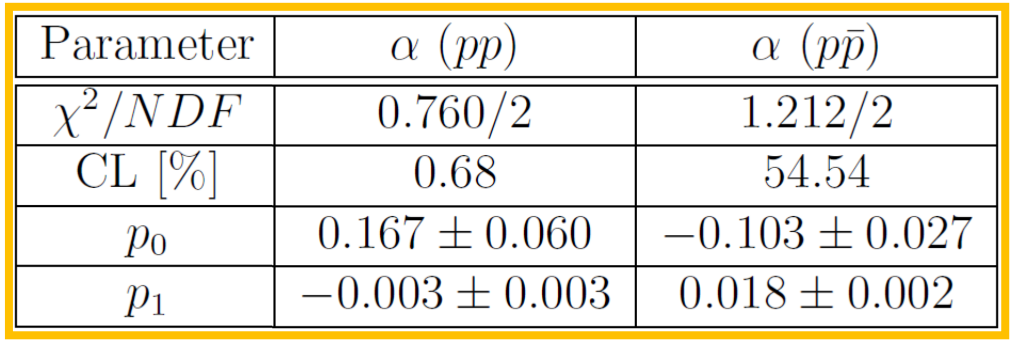
Energy dependence of the α parameter



$$\boxed{P(s) = p_0 + p_1 ln(s/s_0)}$$

$$\boxed{P \in \{R_q, R_d, R_{qd}, \alpha\}}$$

$$\boxed{s_0 = 1\ GeV^2}$$

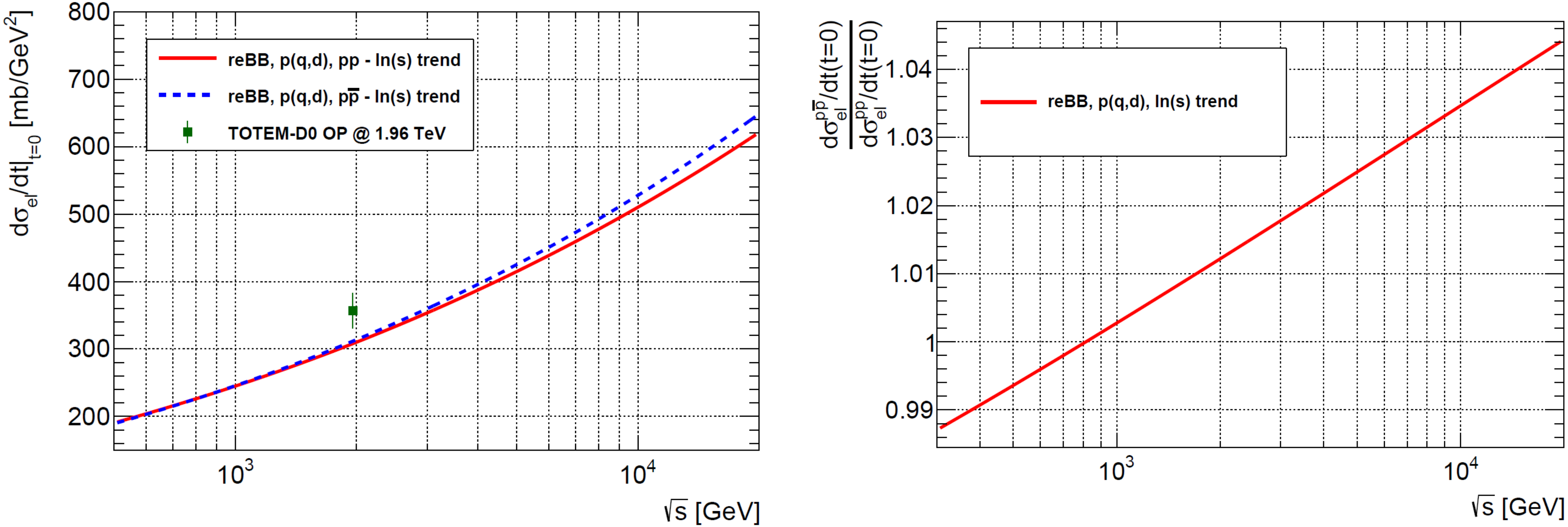


Parameter	$\alpha\ (pp)$	$\alpha\ (p\bar{p})$
χ^2/NDF	$0.760/2$	$1.212/2$
CL [%]	0.68	54.54
p_0	0.167 ± 0.060	-0.103 ± 0.027
p_1	-0.003 ± 0.003	0.018 ± 0.002

Parameters which define the energy dependence of the ReBB model α parameters for pp and pp̄

The energy dependence of the α parameter is not the same for pp and pp̄ processes
→ the Odderon is characterized by a single parameter, α!

Optical point in the ReBB model



$\sqrt{s}$, TeV	1.96	2.76	7.0	8.0
OP for pp, mb/GeV^2	307.962	344.291	459.2	477.774
OP for $p\bar{p}$, mb/GeV^2	311.629	350.018	472.807	492.827
$p\bar{p}$-pp ratio	1.01191	1.01663	1.02963	1.03151

H(x) scaling of the ReBB model

- **conditions:**
  - energy independence for the α parameter (or ρ_0)

$$\boxed{\alpha(s) = \alpha(s_0)} \qquad \left(\text{or } \boxed{\rho_0(s) = \rho_0(s_0)}\right)$$

  - the energy dependence of the scale parameters is determined by the same factorizable $b(s)$ scaling function

$$\boxed{R_q(s) = b(s)R_q(s_0)} \qquad \boxed{R_d(s) = b(s)R_d(s_0)} \qquad \boxed{R_{qd}(s) = b(s)R_{qd}(s_0)}$$

($\sqrt{s_0}$ is a reference energy to be chosen)

- **scaling of the measurables:**

$$\boxed{\frac{d\sigma}{dt}(s,t) = b^2(s)\frac{d\sigma}{dt}\left(s_0, t_0 = \frac{t}{b^2(s)}\right)} \qquad \boxed{\sigma_{el}(s) = b^2(s)\sigma_{el}(s_0)}$$

$$\boxed{B_0(s) = b^2(s)B_0(s_0)} \qquad \boxed{\sigma_{tot}(s) = b^2(s)\sigma_{tot}(s_0)}$$

b(s) scaling function

- **experimental determination of b(s):**

$$\boxed{b(s) = \sqrt{\frac{\sigma_{el}(s)}{\sigma_{el}(s_0)}}} \qquad \boxed{b(s) = \sqrt{\frac{\sigma_{tot}(s)}{\sigma_{tot}(s_0)}}}$$

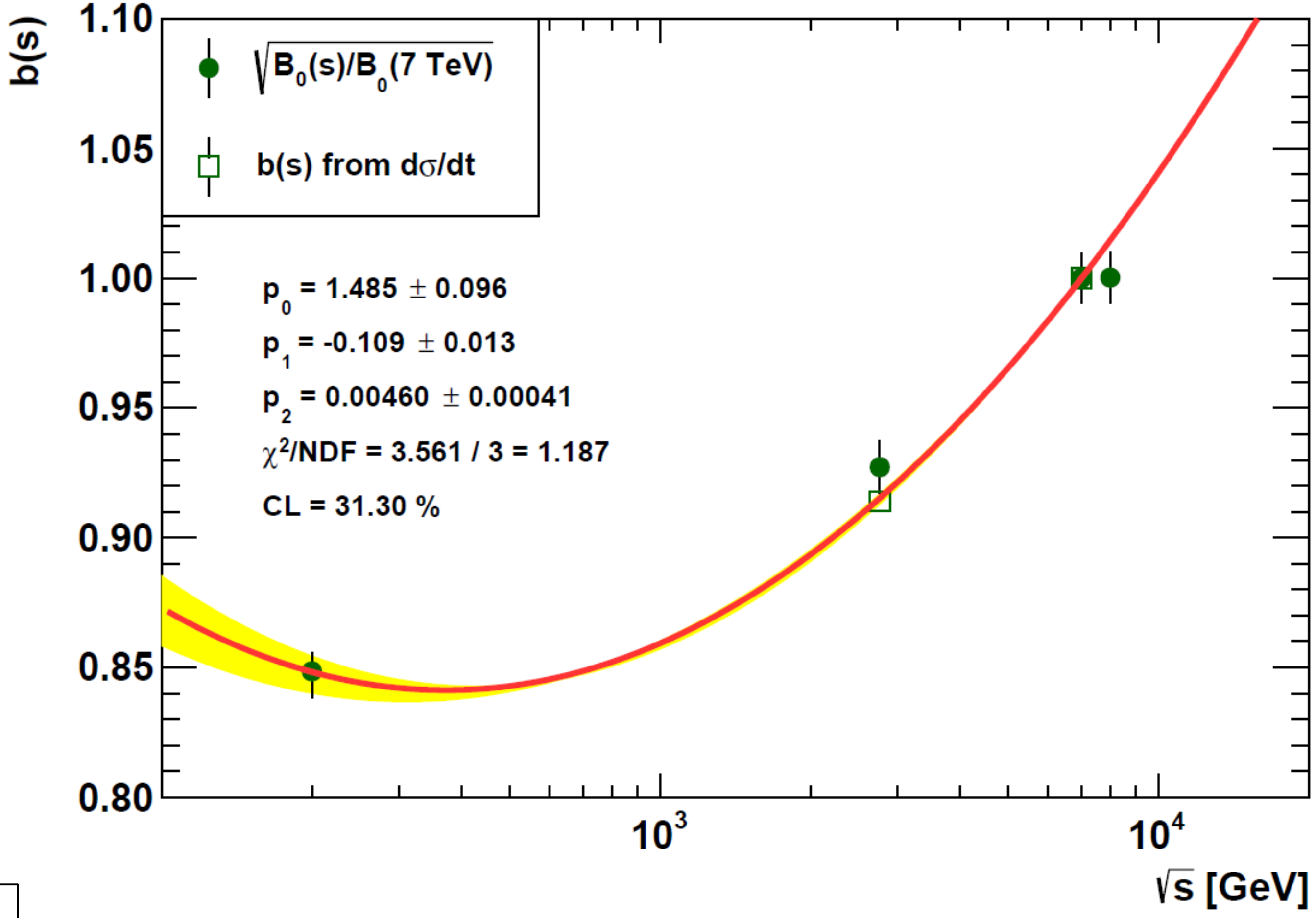
$$\boxed{b(s) = \sqrt{\frac{B_0(s)}{B_0(s_0)}}} \qquad \boxed{b(s) = \sqrt{\frac{d\sigma/dt(s,t)}{d\sigma/dt\left(s_0, t_0 = t/b^2(s)\right)}}}$$

  - the reference energy is chosen to be $\sqrt{s_0} = 7\ \mathrm{TeV}$

- **determination of the energy dependence:**

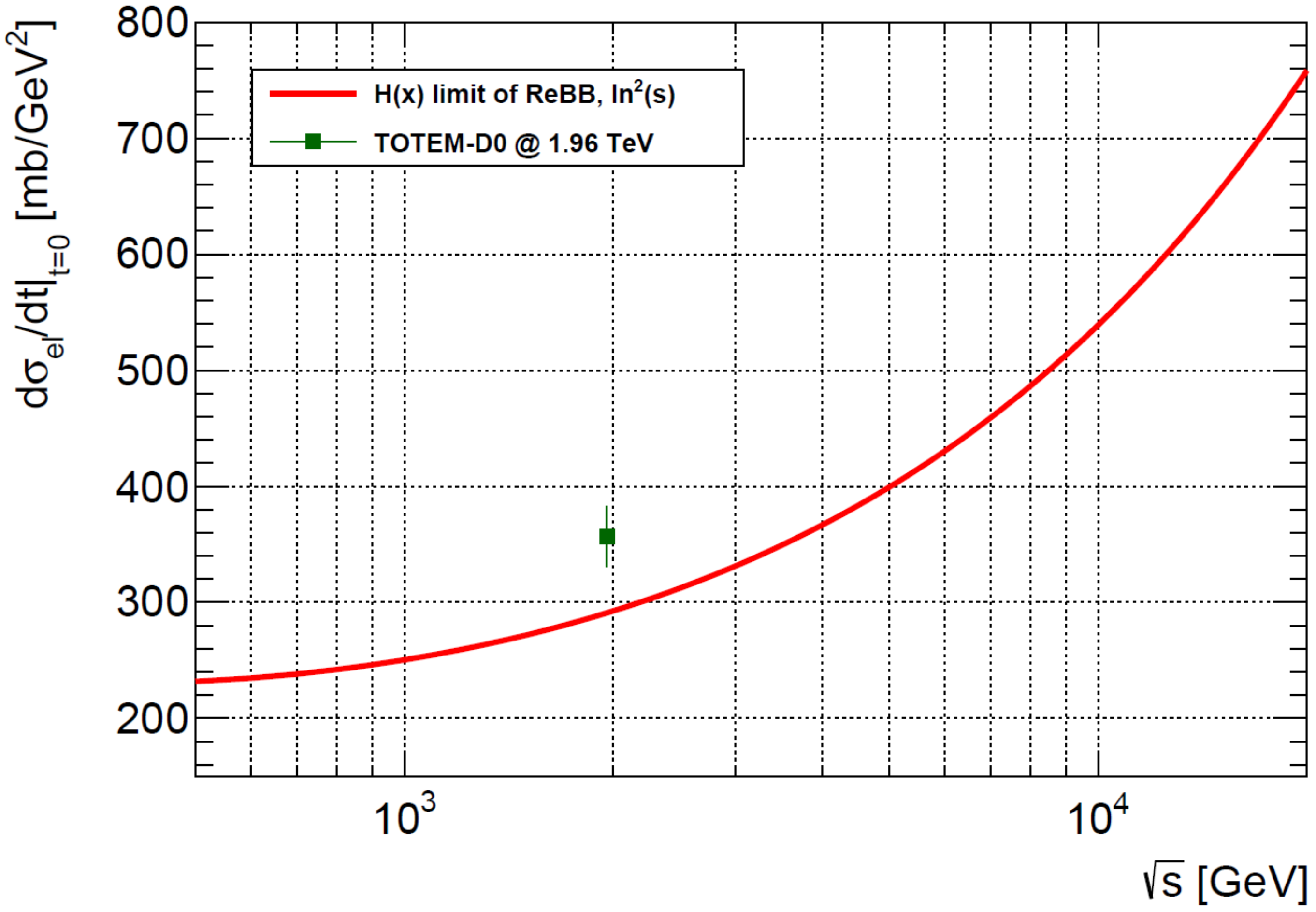
$$\boxed{b(s) = p_0 + p_1 \ln(s/s_0) + p_2 \ln^2(s/s_0)}$$

$$\boxed{s_0 = 1\ GeV^2}$$



The energy dependence of the $b(s)$ scaling function utilizing the $B_0(s)$ and $d\sigma/dt(s,t)$ data and fitting with a squared logarithmic function.

Optical point in the H(x) limit of ReBB model



$\sqrt{s}$, TeV	1.96	2.76	7.0	8.0
OP for pp, mb/GeV^2	290.983	322.322	459.2	486.811

Dipole pomeron model

L. L. Jenkovszky and A. N. Wall,Czech. J. Phys. B26, 447 (1976)
L. L. Jenkovszky, Fortsch. Phys.34, 791 (1986)

- **assumptions:**
  - the asymptotic behaviour of the scattering amplitude $T(s,t)$ is determined by an isolated *j*-plane pole of the second order (dipole)
  - (motivated by duality) the residue at the pole is independent of t, i.e. the partial wave amplitude has the form

$$a_j(t) \equiv a(j,t) = \frac{d}{d\alpha(t)}\left[\frac{\beta(j)}{j-\alpha(t)}\right] = \frac{\beta(j)}{[j-\alpha(t)]^2}$$

- **Sommerfeld-Watson transform of partial wave expanson:**

$$T(s,t) = \sum_{j=0}^{\infty}(2j+1)\,a_j(t)P_j(1+2s/t),$$

$$\mathrm{T(s,t)} = \frac{1}{2i}\oint_C dj(2j+1)\frac{1+e^{-i\pi j}}{\sin\pi j}\frac{d}{d\alpha(t)}\frac{\beta(j)}{j-\alpha(t)}P_j(1+2s/t)\,.$$

Dipole pomeron model

- in the Regge limit i.e. $s \gg |t|$:

$$\mathrm{T}(s,t) = \frac{d}{d\alpha}\left[e^{-\frac{i\pi\alpha}{2}} G(\alpha)\left(\frac{s}{s_0}\right)^{\alpha}\right] = e^{-\frac{i\pi\alpha}{2}}\left(\frac{s}{s_0}\right)^{\alpha}\left[G'(\alpha) + \left(L - \frac{i\pi}{2}\right)G(\alpha)\right]$$

$$L \equiv ln\frac{s}{s_0}$$

$$\alpha = \alpha(t)$$

- since t-dependence enters through the trajectory, $G'(\alpha)$ is releated to $G(\alpha)$

by integration

- motivated by the shape of the diffraction cone (exponential decrease):

$$G'(\alpha) = Ae^{b[\alpha-1]} \quad ➡ \quad G(\alpha) = \int G'(\alpha)d\alpha + \gamma = \frac{A}{b}e^{b[\alpha-1]} + \gamma$$

- introducing that $\varepsilon = -\frac{b\gamma}{A}$, one has

$$\boldsymbol{T}(\mathrm{s},\mathrm{t}) = i\frac{A\ s}{bs_0}\left[r_1^2(s)e^{r_1^2(s)[\alpha-1]} - \varepsilon\ r_2^2(s)e^{r_2^2(s)[\alpha-1]}\right]$$

$$\mathrm{r}_1^2(\mathrm{s}) = b + L - \mathrm{i}\pi/2$$

$$\mathrm{r}_2^2(\mathrm{s}) = L - \mathrm{i}\pi/2$$

Dipole (DP) pomeron+odderon Regge model

$$T(s,t)^{\bar{p}p}_{pp} = \mathbf{T_P(s,t)} \pm \mathbf{T_O(s,t)}$$

I. Szanyi, N. Bence, L. Jenkovszky:
J. Phys. G 46, 055002 (2019)

- **Dipole pomeron and odderon:**

$$\mathbf{T_P}(s,t) = i\frac{a_P s}{b_P s_{0P}}\left[r_{1P}^2(s)e^{r_{1P}^2(s)[\alpha_P - 1]} - \varepsilon_P r_{2P}^2(s)e^{r_{2P}^2(s)[\alpha_P - 1]}\right]$$

$$\mathbf{T_O}(s,t) = -i\mathbf{A}_{\mathbf{P}\to\boldsymbol{O}}(s,t)$$

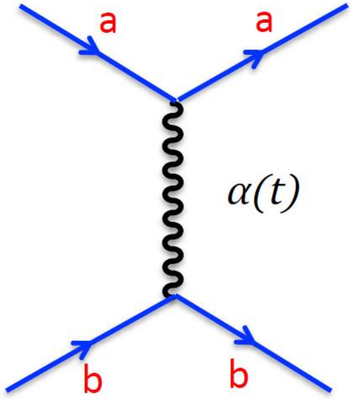
(with free parameters labeld by "**O**")

$$r_{1P}^2(s) = b_P + L_P - i\pi/2 \qquad r_{2P}^2(s) = L_P - i\pi/2 \qquad L_P \equiv \ln(s/s_{0P})$$

- **Pomeron and odderon trajectories:**

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P}t - \alpha_{2P}\left(\sqrt{4m_\pi^2 - t} - 2m_\pi\right)$$

$$\alpha_O \equiv \alpha_O(t) = 1 + \delta_O + \alpha_{1O}t - \alpha_{2O}\left(\sqrt{9m_\pi^2 - t} - 3m_\pi\right)$$



Exchange of a Regge trajectory in the t-channel.

Regge trajectories

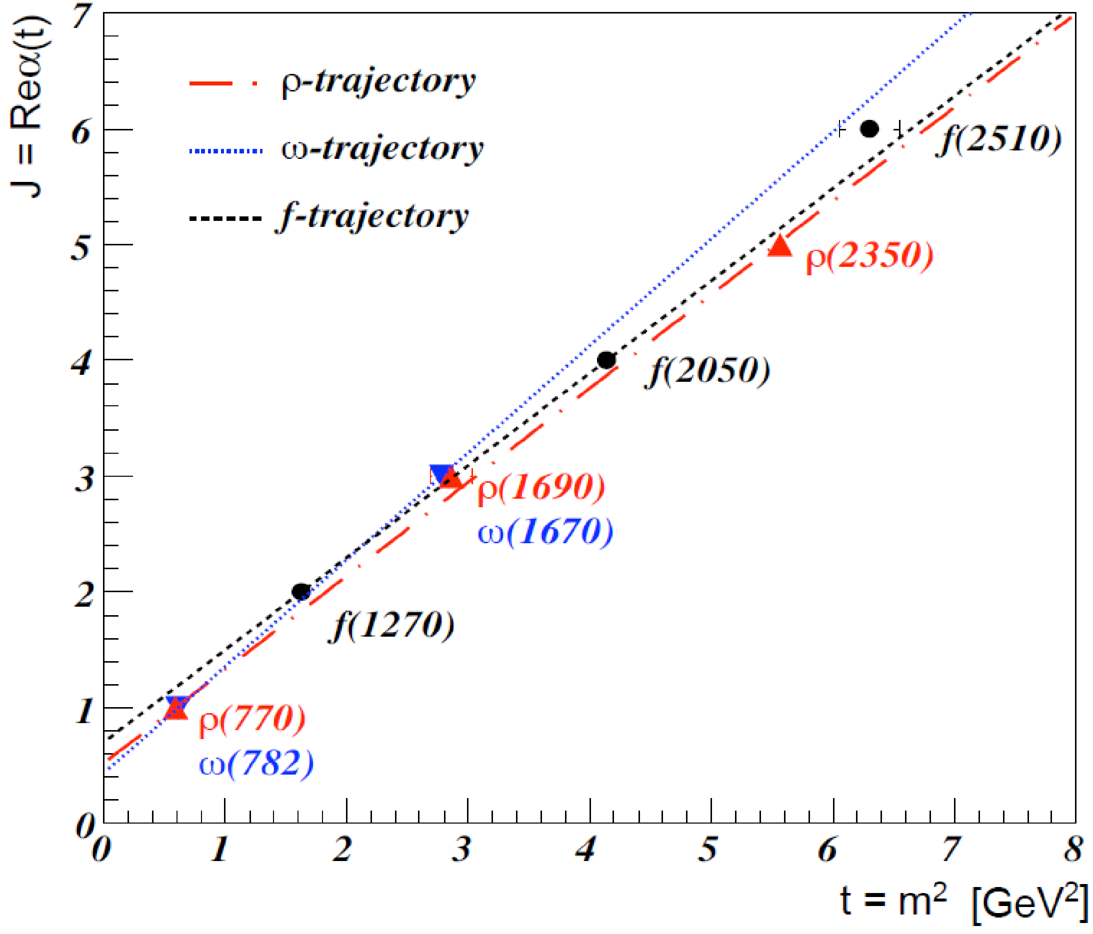
- Reggeon (in general):

  → virtual particle with continuously varying spin ($J = \mathrm{Re}\,\alpha(t)$) and virtuality ($t = m^2$) lying on the relevant trajectory (scattering at $-t$)

  → at certain values of virtuality there are real particles (spectroscopy at $+t$)

- Secondary reggeons (f, ω, ρ, …) → mesonic exchanges and meson spectra

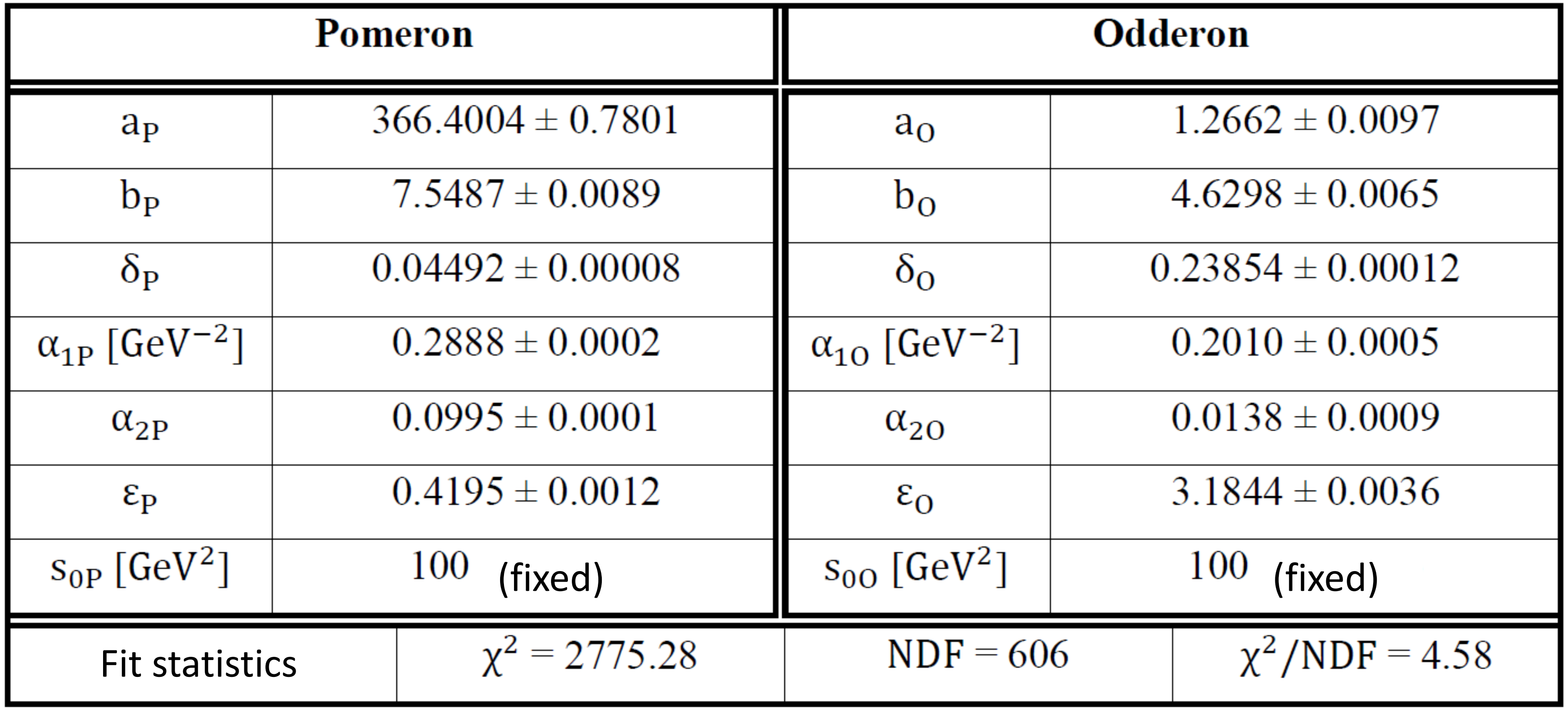
- Pomeron (P) and odderon (O) → gluonic exchanges and glueball spectra



Chew-Frautschi plot for the ρ, ω and f-trajectory with the corresponding experimentally measured mesonic spectra.

A mért adatok illesztése

- the free parameters of the model were fitted to pp and $p\bar{p}$ $d\sigma_{el}/dt$, σ_{tot} and ρ_0 data in the $1 \leq \sqrt{s} \leq 13$ TeV an $0.01 \leq -t \leq 2.5$ GeV^2 kinematical range



Pomeron		**Odderon**	
a_P	366.4004 ± 0.7801	a_O	1.2662 ± 0.0097
b_P	7.5487 ± 0.0089	b_O	4.6298 ± 0.0065
δ_P	0.04492 ± 0.00008	δ_O	0.23854 ± 0.00012
α_{1P} [GeV^{-2}]	0.2888 ± 0.0002	α_{1O} [GeV^{-2}]	0.2010 ± 0.0005
α_{2P}	0.0995 ± 0.0001	α_{2O}	0.0138 ± 0.0009
ε_P	0.4195 ± 0.0012	ε_O	3.1844 ± 0.0036
s_{0P} [GeV^2]	100 (fixed)	s_{0O} [GeV^2]	100 (fixed)
Fit statistics	$\chi^2 = 2775.28$	NDF = 606	$\chi^2/\text{NDF} = 4.58$

The fitted values of the free parameters of the DP P+O Regge model
and the fit statistics

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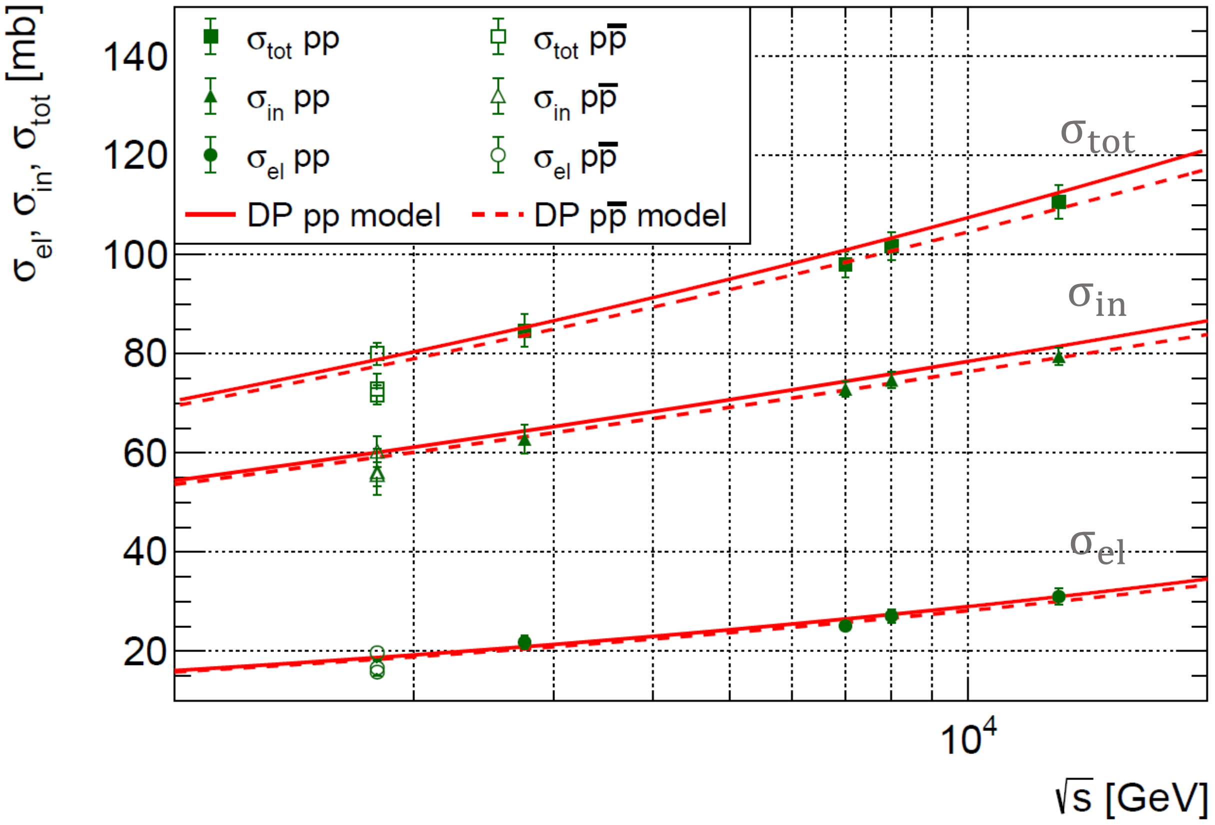
Fit statistics	$\chi^2 = 2775.28$	$NDF = 606$	$\chi^2/NDF = 4.58$

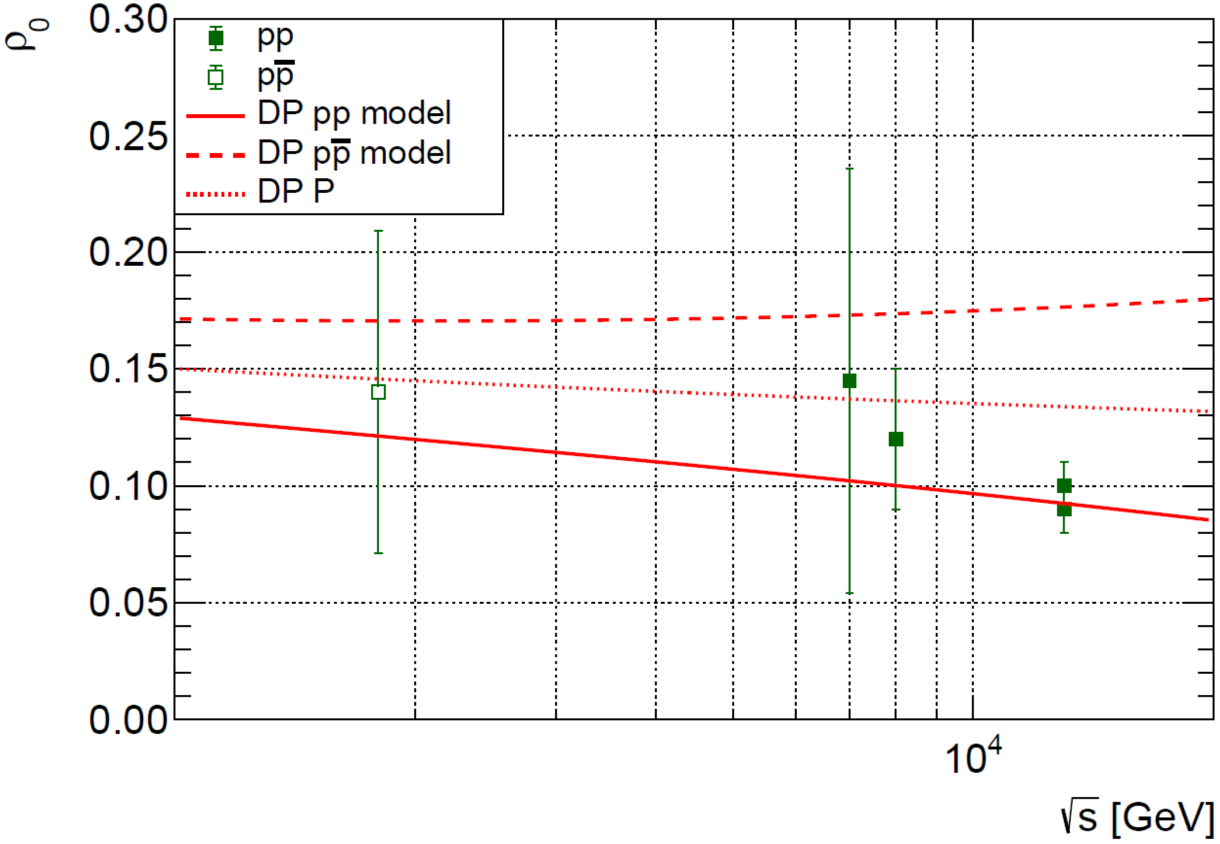
$\chi^2/NDF = 4.58$ → although the model must be improved further, it reproduces the main features of the measured pp and $p\bar{p}$ data in the TeV range with 12 free parameters

The fitted values of the free parameters of the DP P+O Regge model
and the fit statistics

The pp and p$\bar{p}$ $\sigma_{tot}(s)$, $\sigma_{el}(s)$, $\sigma_{in}(s)$ and $\rho_0(s)$



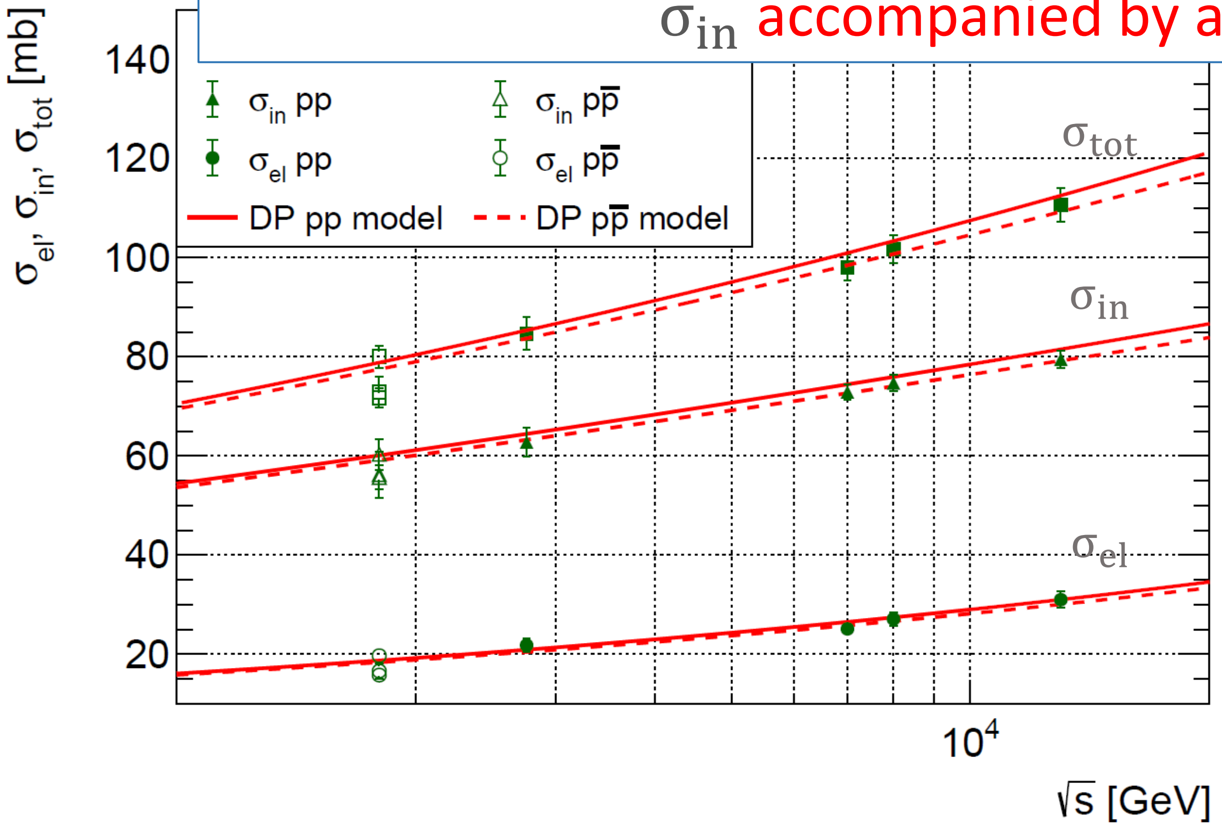
The DP P+O Regge model description of the proton-proton and proton-antiproton total, elastic, and inelastic cross-section data in the TeV energy range



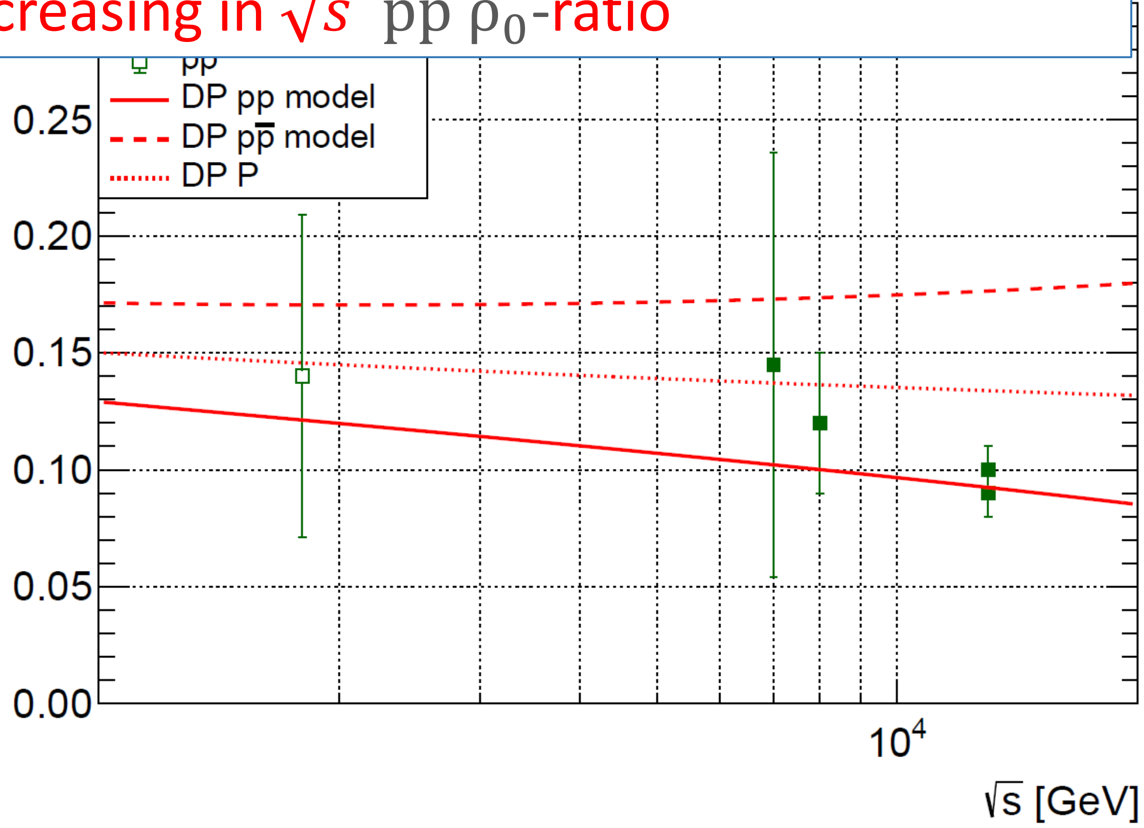
The DP P+O Regge model description of the proton-proton and proton-antiproton ρ_0-ratio data in the TeV energy range

The pp and p$\bar{p}$ $\sigma_{tot}(s)$, $\sigma_{el}(s)$, $\sigma_{in}(s)$ and $\rho_0(s)$

the model reproduces the experimental fact that the rising in $\sqrt{s}$ pp σ_{tot}, σ_{el} , and σ_{in} accompanied by a decreasing in $\sqrt{s}$ pp ρ_0-ratio



The DP P+O Regge model description of the proton-proton and proton-antiproton total, elastic, and inelastic cross-section data in the TeV energy range



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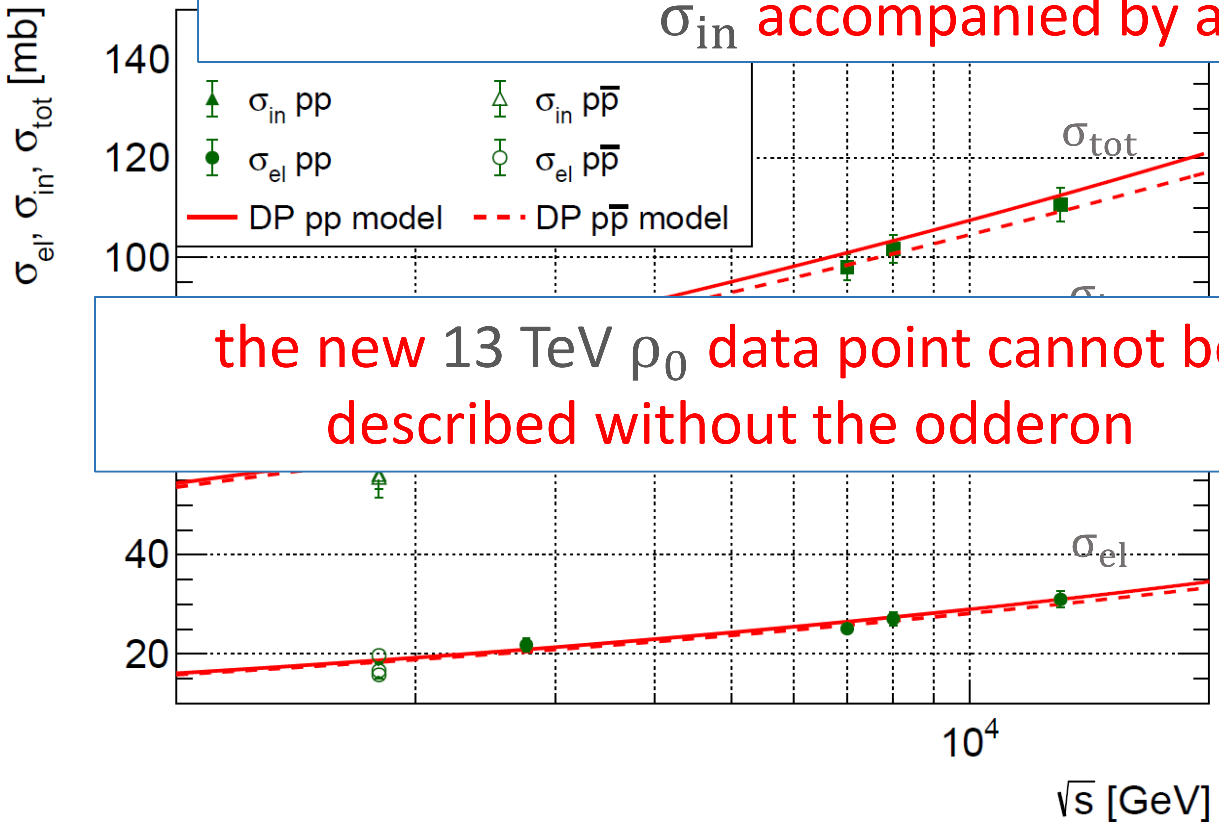
the model reproduces the experimental fact that the rising in $\sqrt{s}$ pp σ_{tot}, σ_{el} , and σ_{in} accompanied by a decreasing in $\sqrt{s}$ pp ρ_0-ratio

the new 13 TeV ρ_0 data point cannot be described without the odderon



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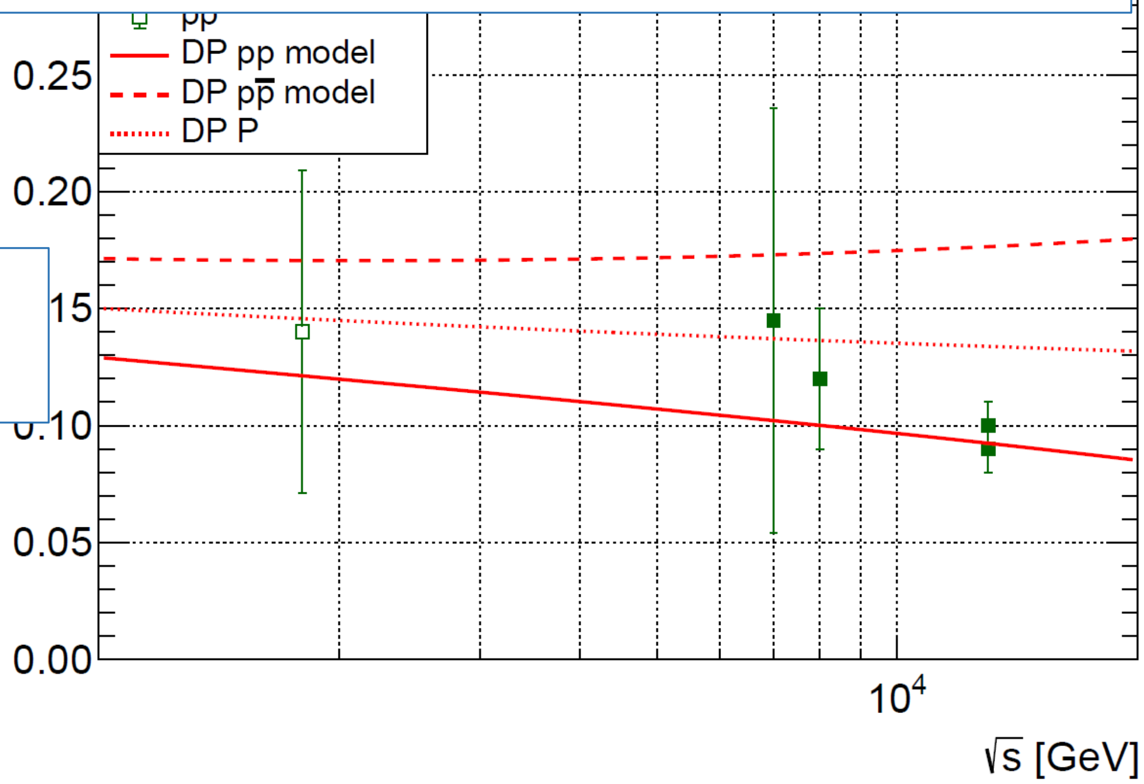
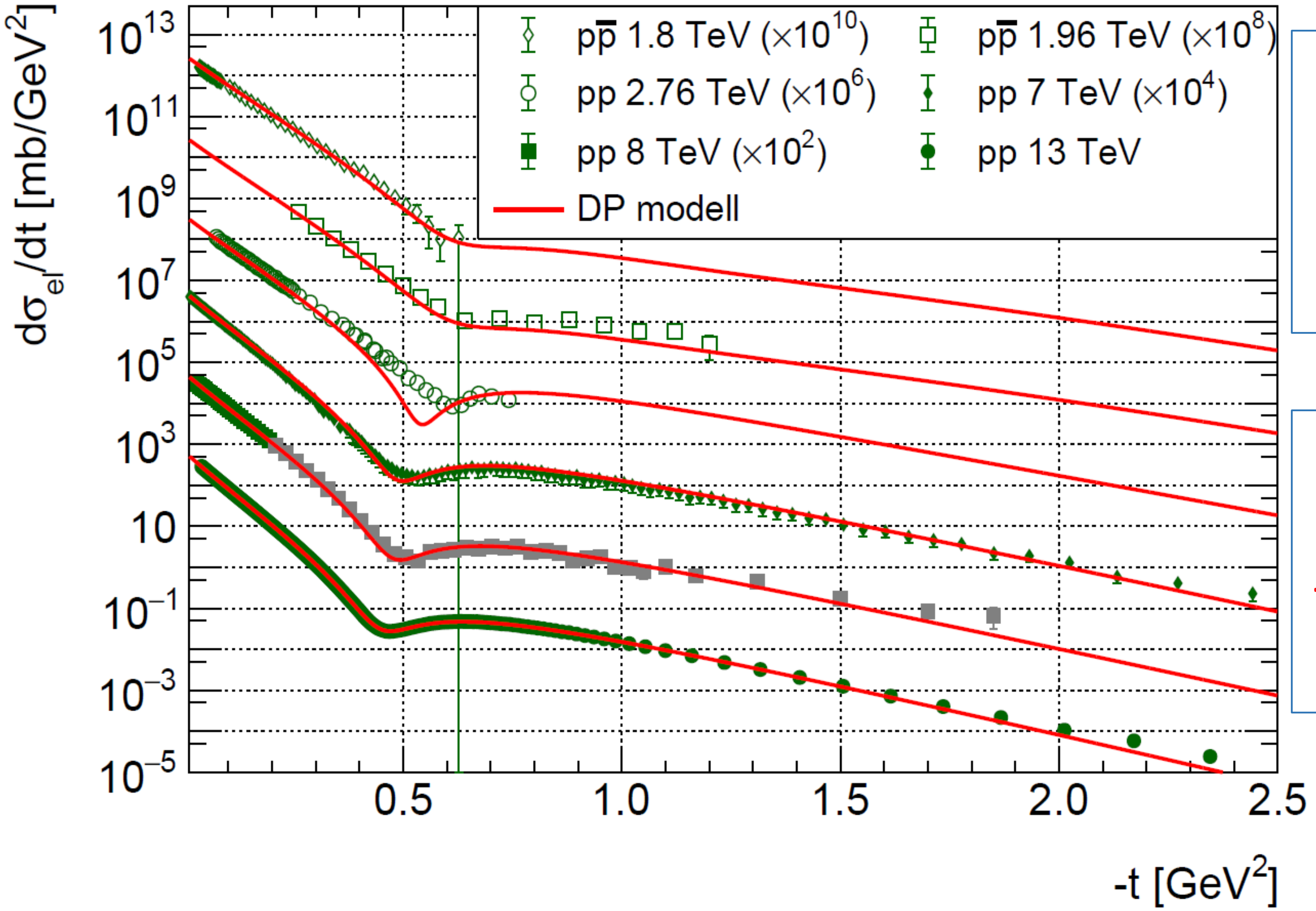


The DP P+O Regge model description of the proton-proton and proton-antiproton ρ_0 -ratio data in the TeV energy range

The pp and $p\bar{p}$ $d\sigma_{el}/dt(s,t)$

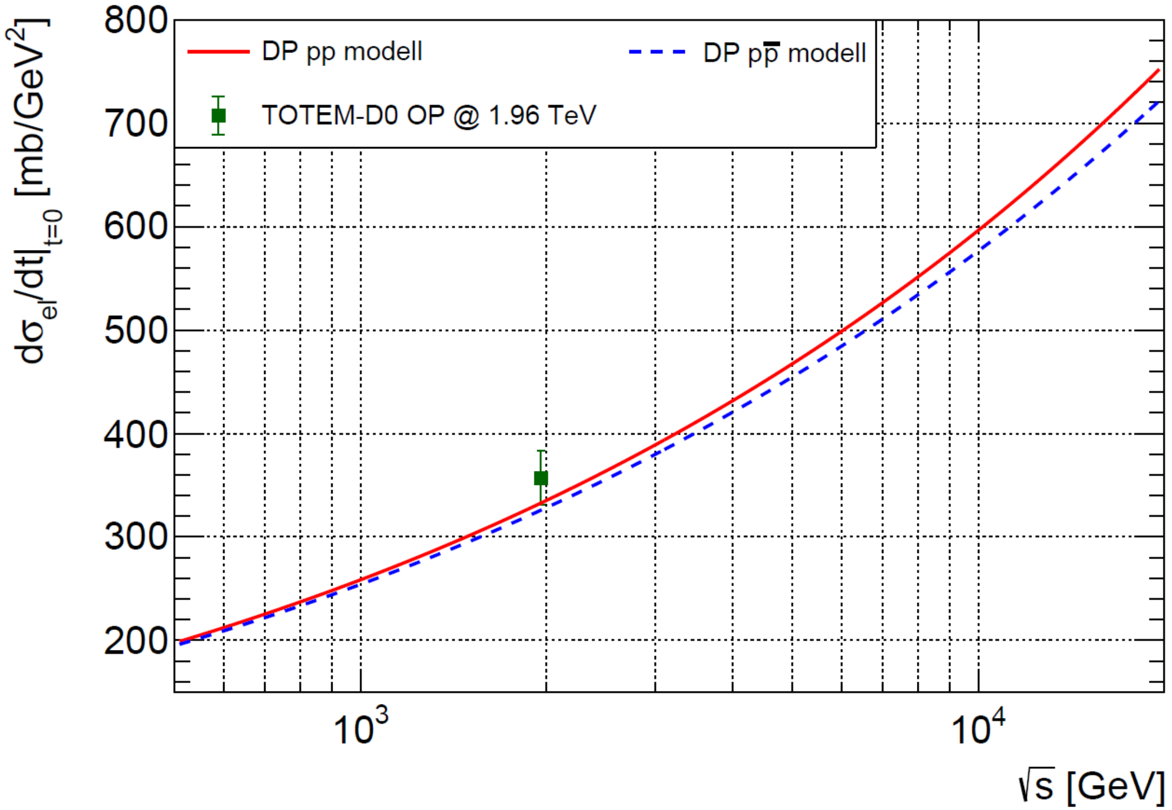


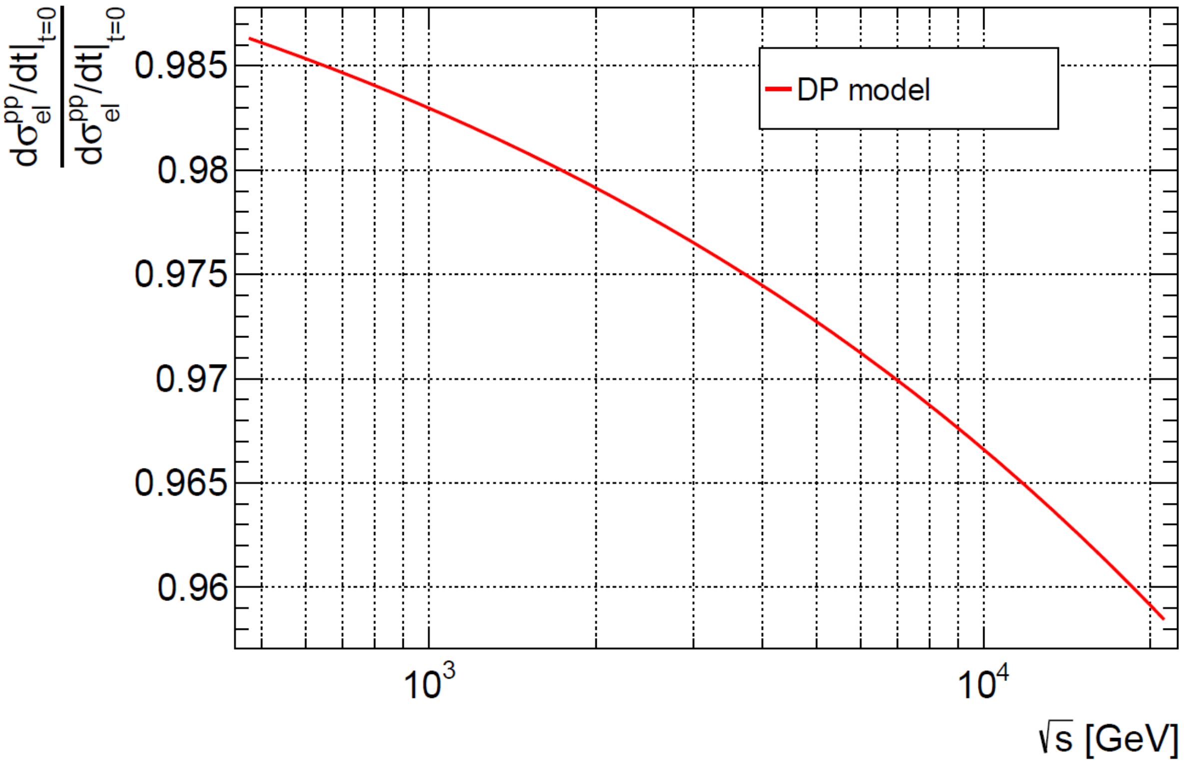
The DP P+O Regge model description of the proton-proton and proton-antiproton differential cross section data in the TeV energy range

the model reproduces the main features of the differential cross section: dip-bump structure in pp $d\sigma_{el}/dt(s,t)$ and shoulder structure in $p\bar{p}$ $d\sigma_{el}/dt(s,t)$

problems with the energy evolution of the positions of dip and bump in $-t$, further improvements are needed in this direction

Optical point in the DP P+O Reggemodel





$\sqrt{s}$, TeV	**1.96**	**2.76**	**7.0**	**8.0**
OP for pp, mb/GeV²	332.78	377.32	526.416	551.67
OP for p$\bar{p}$, mb/GeV²	325.883	368.675	510.582	534.416
p$\bar{p}$-pp ratio	0.979275	0.97709	0.96992	0.968723

Summary

- **OP by ReBB model:**

→slightly different energy dependence for the pp and p$\bar{\text{p}}$ optical point: crossing point at about 800 GeV and then slightly increasing difference as the energy grows

- **OP by H(x) limit of ReBB model:**

→ energy dependence only for pp scattering as H(x) scaling is valid only for pp scattering

→ compared to the original ReBB model, lower values at energies lower than $\sqrt{s_0}$ and higher values at energies higher than $\sqrt{s_0}$

- **OP by DP P+O Regge model**:

→ slight difference between the energy dependence for the pp and p$\bar{\text{p}}$ optical point

which increases as the energy grows

→ contrary to the original ReBB model, the DP P+O Regge model at TeV energies predicts lower values for the p$\bar{\text{p}}$ optical point than for the pp optical point

Summary

….but further improvements of the models are needed!

- **ReBB model**:

→ statistically acceptable description in the limited energy range of $0.546 \leq \sqrt{s} \leq 7(8)$ TeV and limited squared momentum transfer range of $0.37 \leq -t \leq 1.2$ GeV^2

→ need for improvement of the model both for lower and higher $\sqrt{s}$ and $-t$

→ a possibility: generalization from Gauss distribution to Levy distribution

- **DP P+O Regge model**:

→ statistically unacceptable i.e. qualitative description in the $\sqrt{s} \geq 1$ TeV and $0.01 \leq -\mathrm{t} \leq 2.5\ \mathrm{GeV}^2$ kinematic domain

→ the main features of the measured data are reproduced

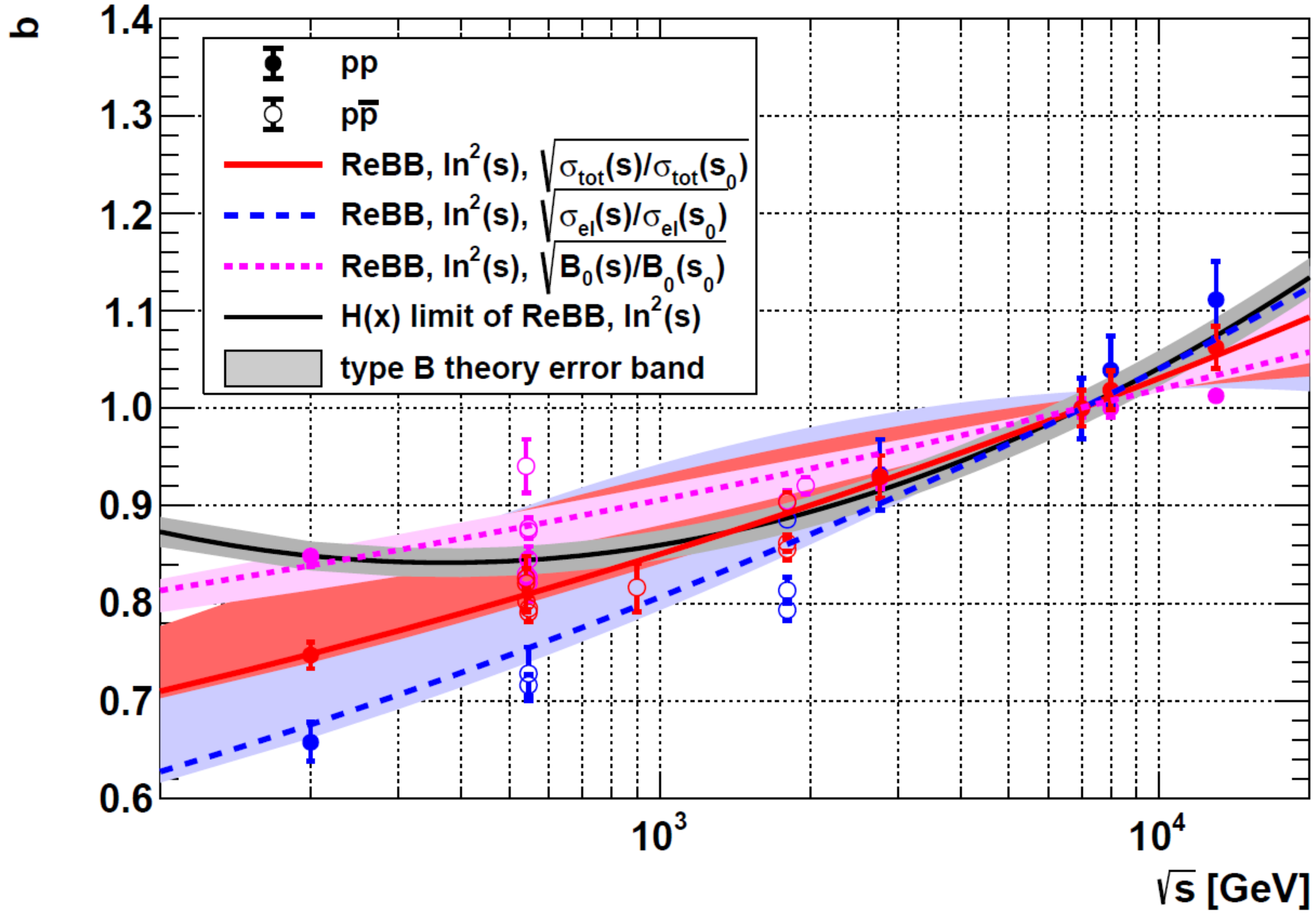
→ the main problem is the improper energy evolution of the positions of dip and bump in $-t$

→ some possibilities: unitarization and/or introduction of the spin degrees of freedom

Thank you for your attention!

Backup slides

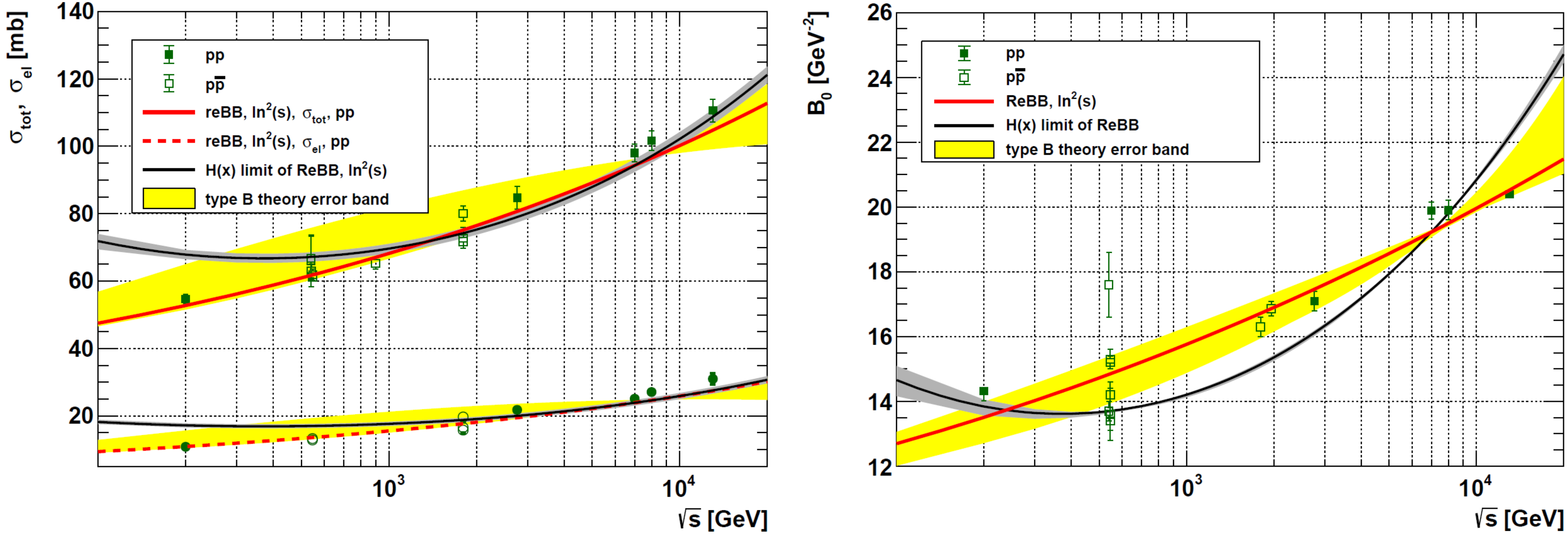
b(s) scaling function



The energy dependence of the $b(s)$ scaling function determined from the experimental data and compared to the original ReBB model as well as its H(x) scaling version

ReBB & H(x) limit of ReBB for pp $\sigma_{tot}(s)$, $\sigma_{el}(s)$, $B_0(s)$

- the ReBB & H(x) limit of ReBB curves, within the type B theory errors agree for $\sigma_{tot}(s)$ and $\sigma_{el}(s)$ down to about 300 GeV
- because of the problems with ReBB model at low $-t$ the results for $B_0(s)$ are not reliable (→ further improvement of the model is needed)



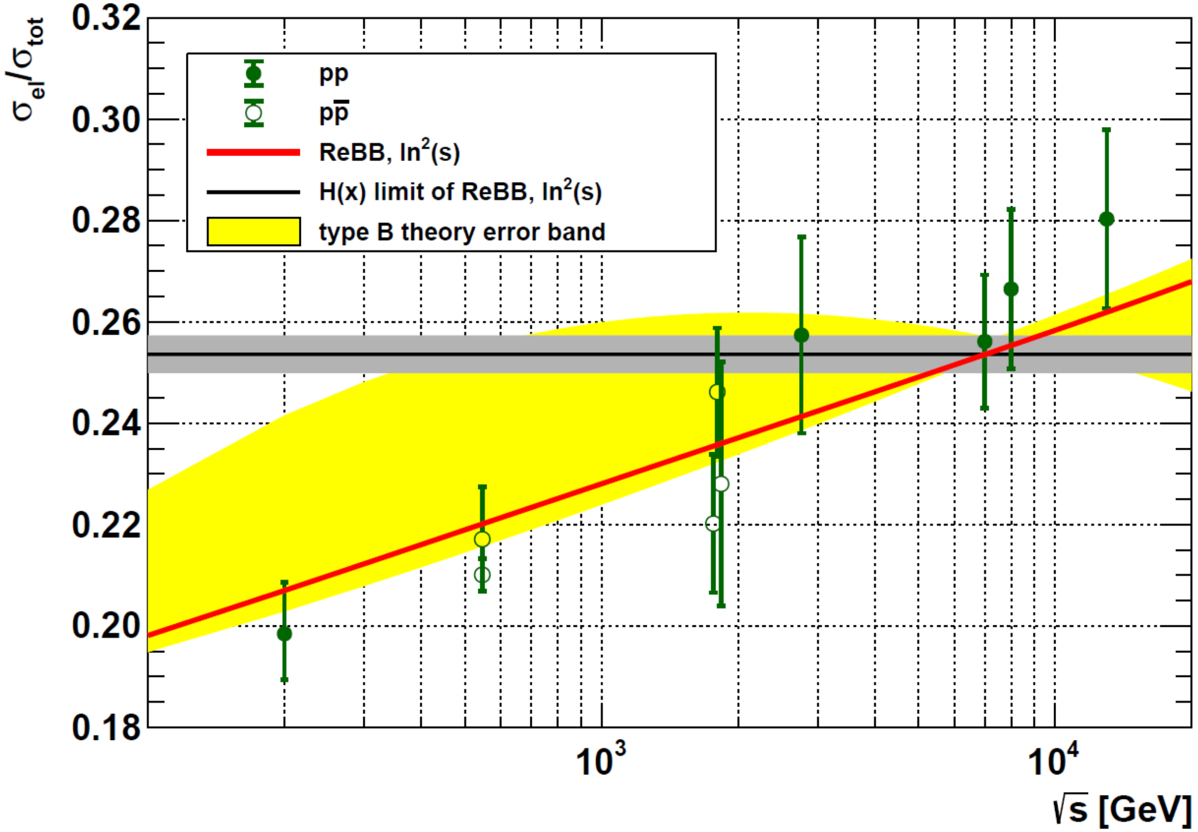
H(x) scaling in the ReBB model

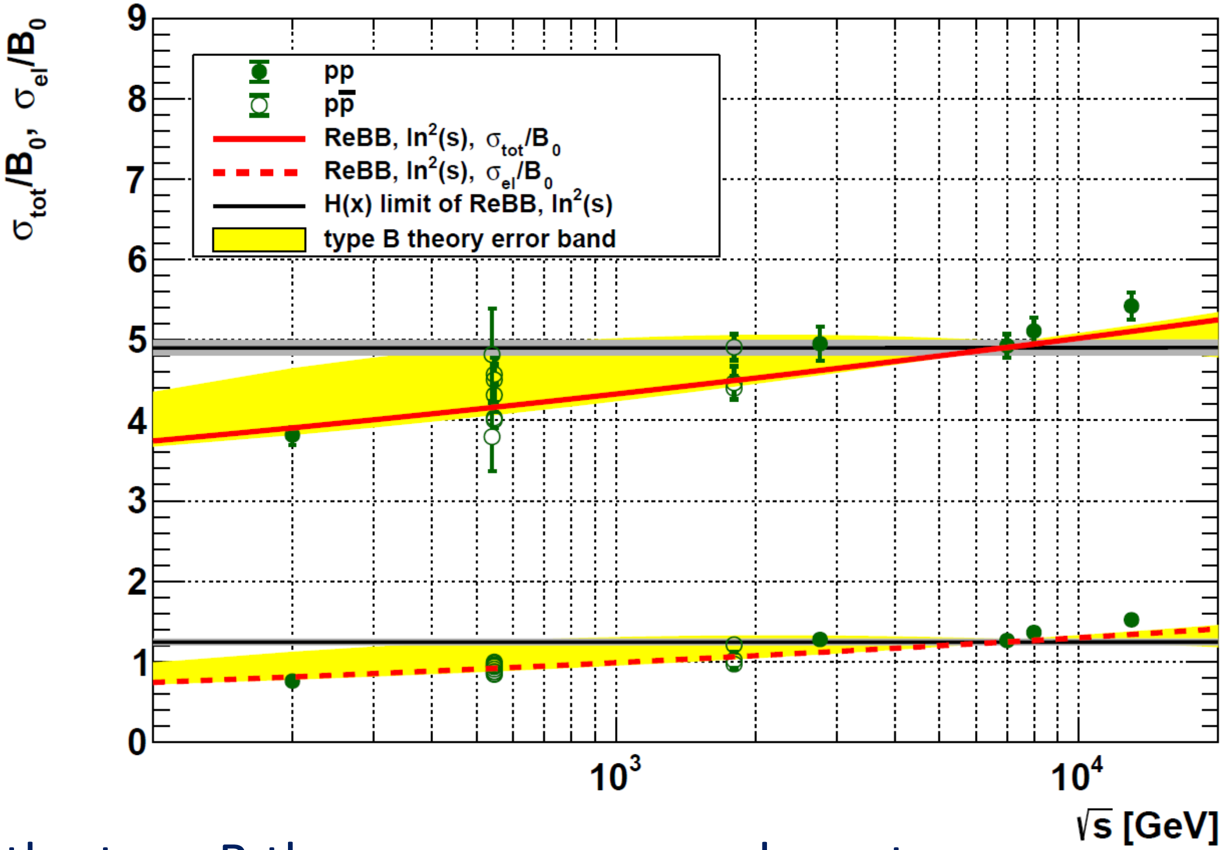
- **consequences of the H(x) scaling:**

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} = const$$

$$\frac{\sigma_{tot}(s)}{B_0(s)} = const$$

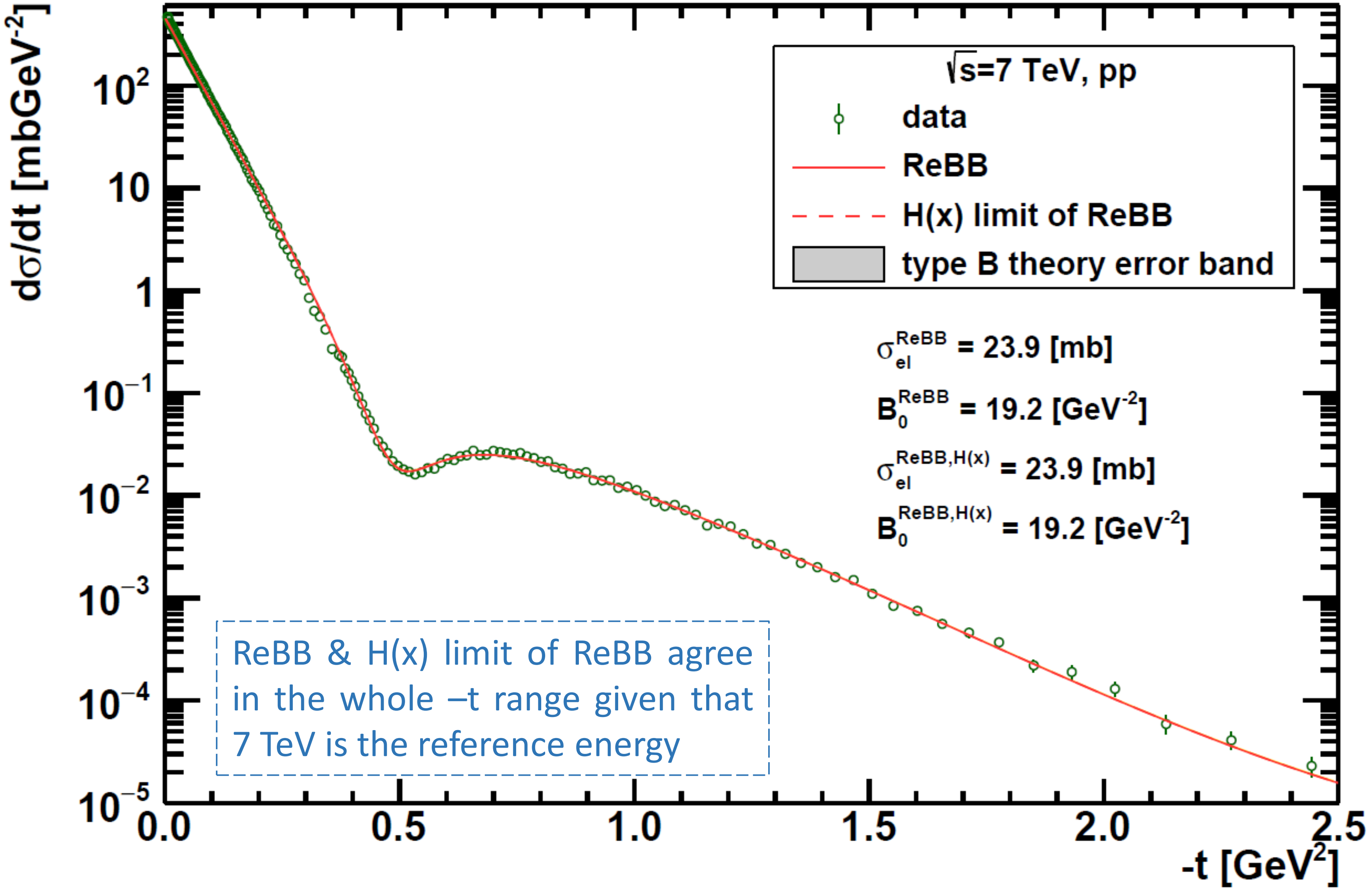
$$\frac{\sigma_{el}(s)}{B_0(s)} = const$$





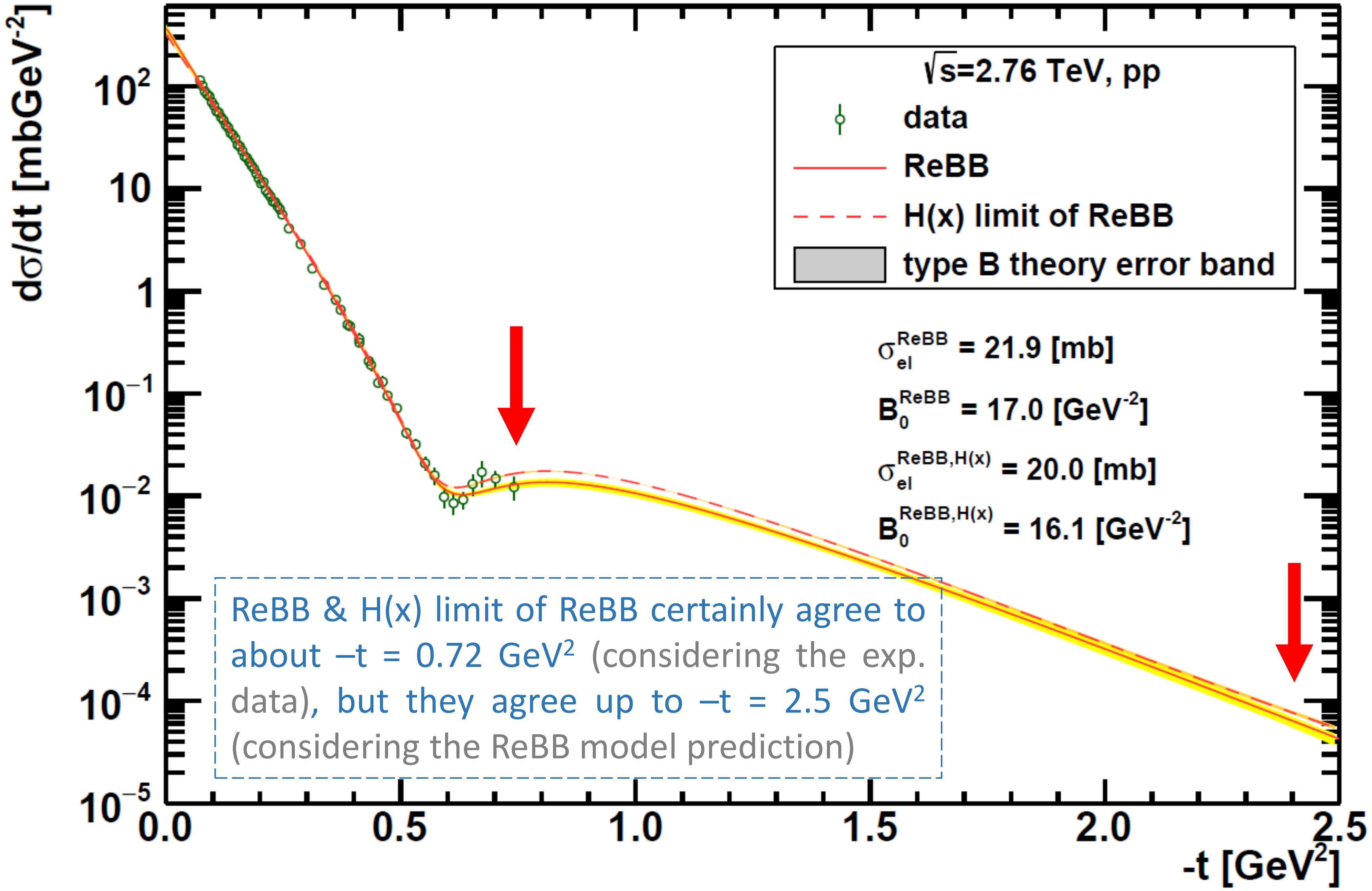
→ the ReBB & H(x) limit of ReBB curves, within the type B theory errors agree down to about 400 GeV

ReBB & H(x) limit of ReBB for pp dσ/dt @ 7 TeV



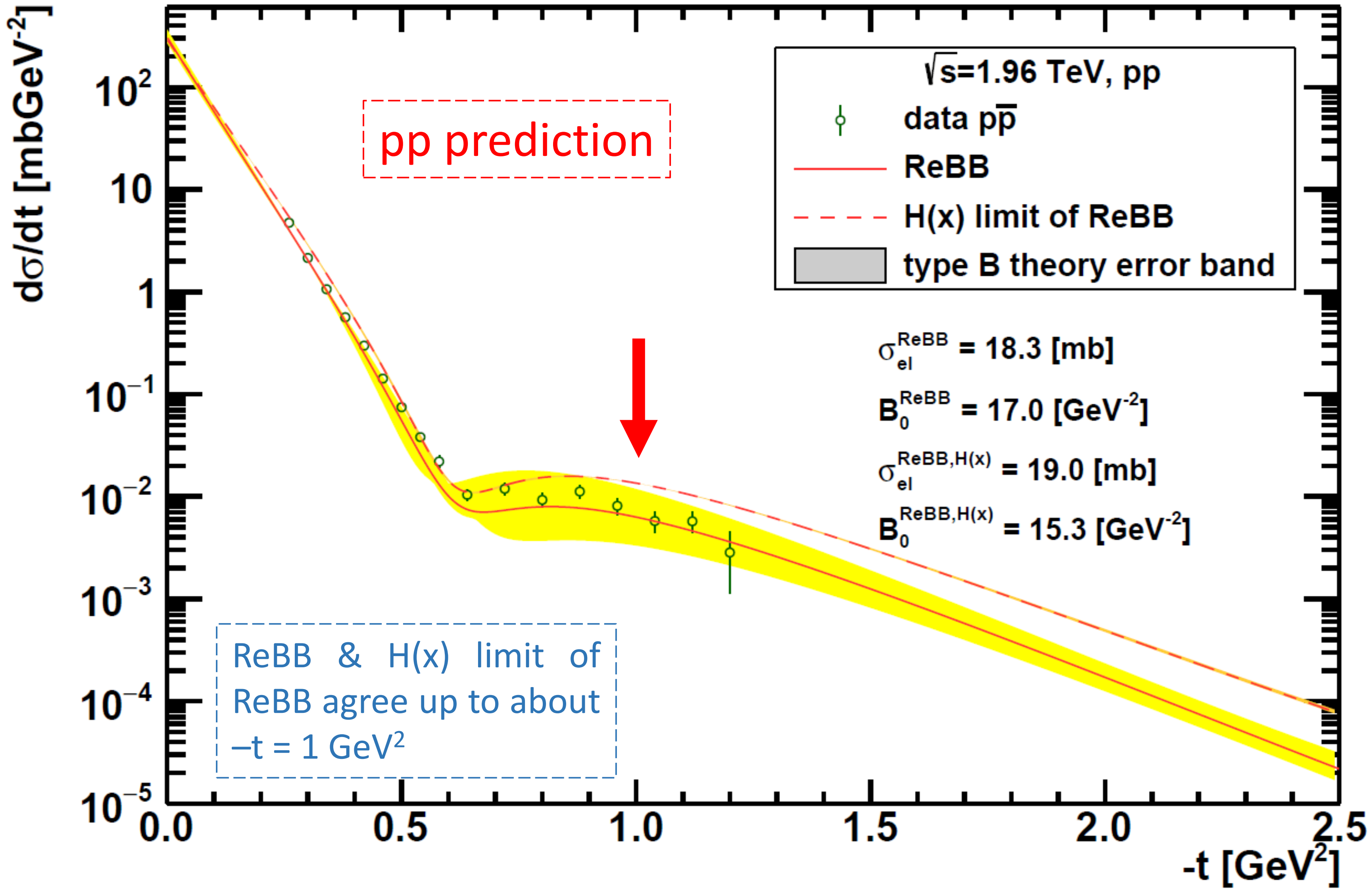
ReBB & H(x) limit of ReBB agree in the whole –t range given that 7 TeV is the reference energy

ReBB & H(x) limit of ReBB for pp $d\sigma/dt$ @ 2.76 TeV



ReBB & H(x) limit of ReBB certainly agree to about –t = 0.72 GeV^2 (considering the exp. data), but they agree up to –t = 2.5 GeV^2 (considering the ReBB model prediction)

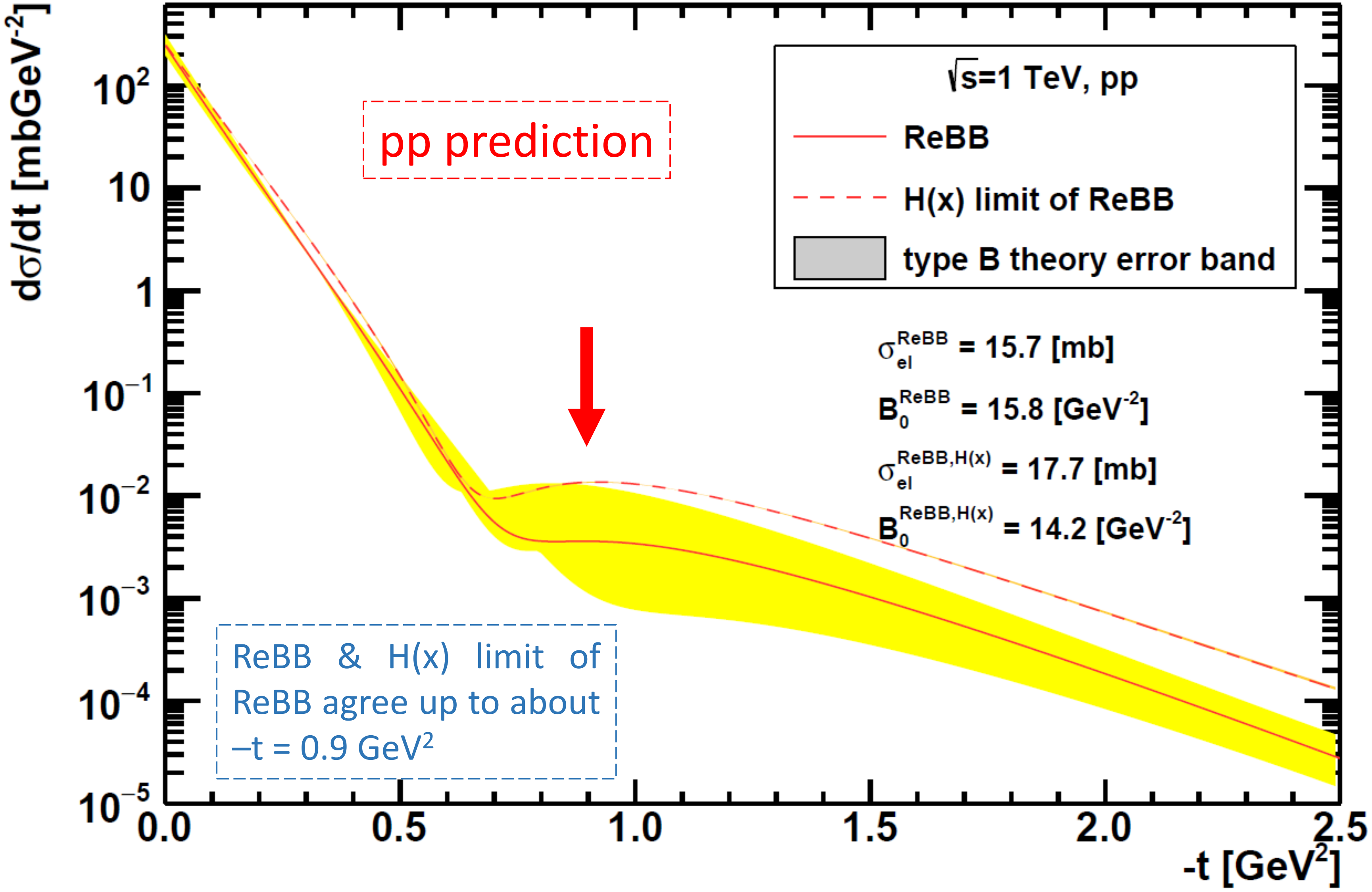
ReBB & H(x) limit of ReBB for pp $\mathrm{d}\sigma/\mathrm{d}t$ @ 1.96 TeV



pp prediction

ReBB & H(x) limit of ReBB agree up to about $-t$ = 1 GeV^2

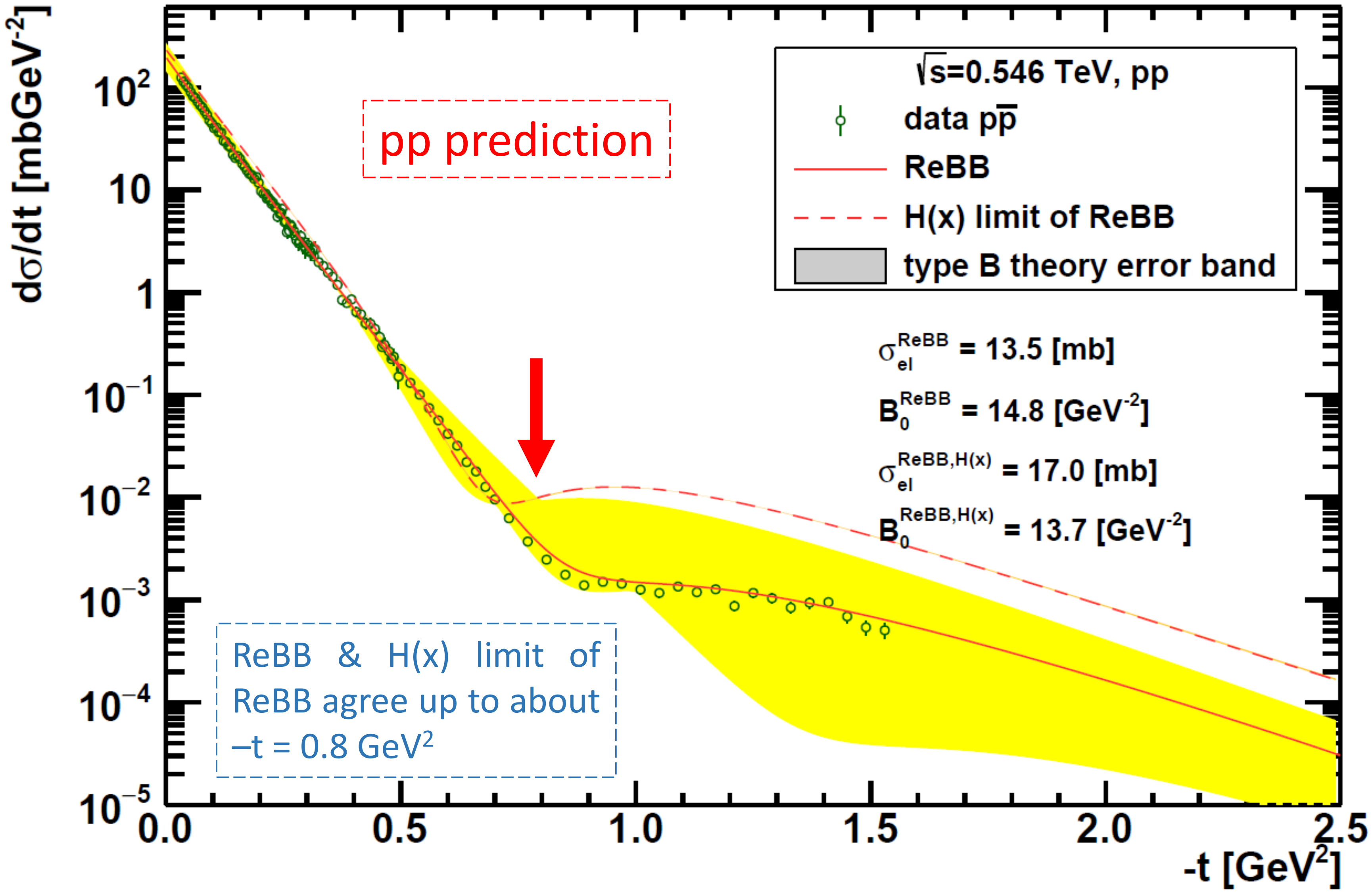
ReBB & H(x) limit of ReBB for pp $d\sigma/dt$ @ 1 TeV



pp prediction

ReBB & H(x) limit of ReBB agree up to about $-t = 0.9$ GeV^2

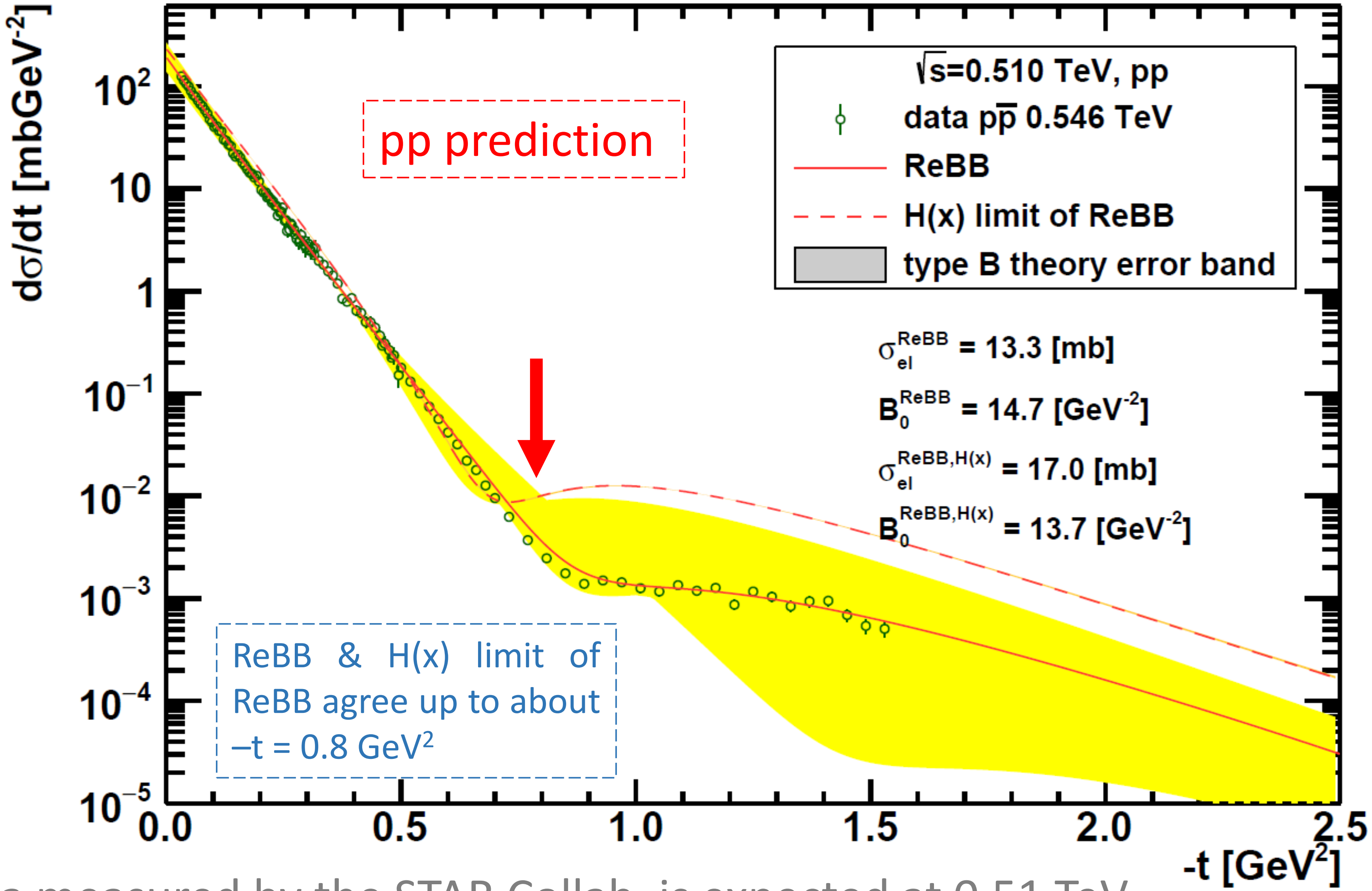
ReBB & H(x) limit of ReBB for pp $d\sigma/dt$ @ 0.546 TeV



pp prediction

ReBB & H(x) limit of ReBB agree up to about $-t = 0.8$ GeV^2

ReBB & H(x) limit of ReBB for pp $d\sigma/dt$ @ 0.51 TeV



pp prediction

ReBB & H(x) limit of ReBB agree up to about $-t = 0.8$ GeV^2

- pp data measured by the STAR Collab. is expected at 0.51 TeV