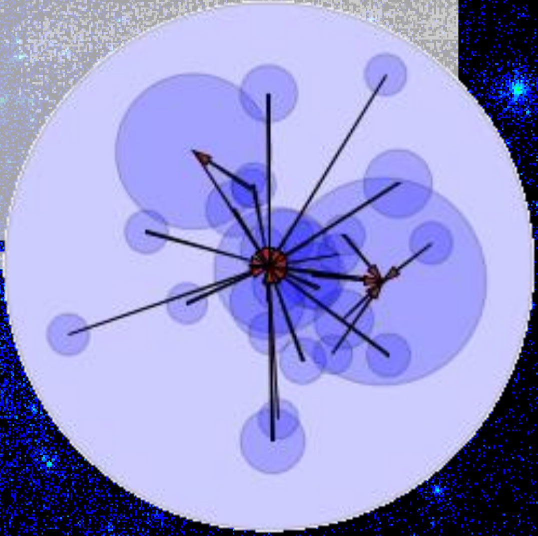
Related to: 22XX.XXXX, in ten days and 2205.03392

Pheno 2022
University of Pittsburgh

Graphs of dark matter halos from cosmological simulation and preferential attachment



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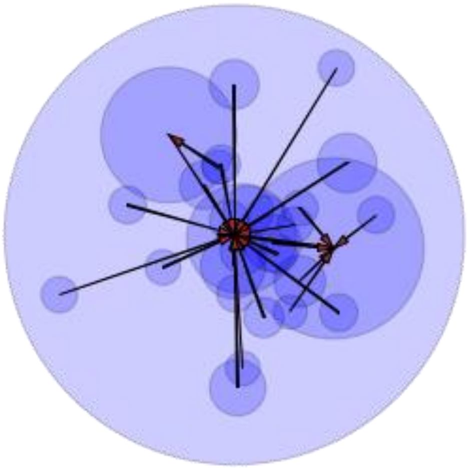
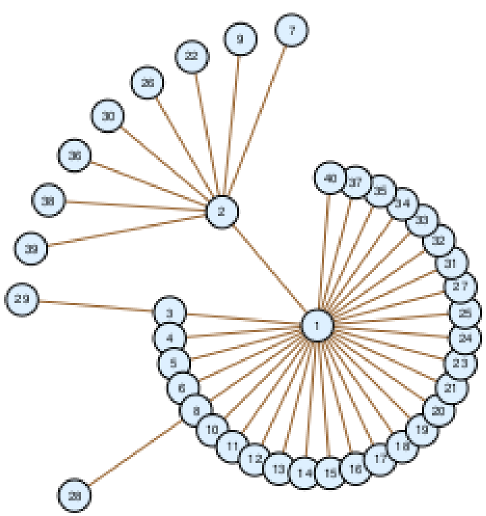
in collaboration with:

Hai-bo Yu (UCR)

May 10th, 2022

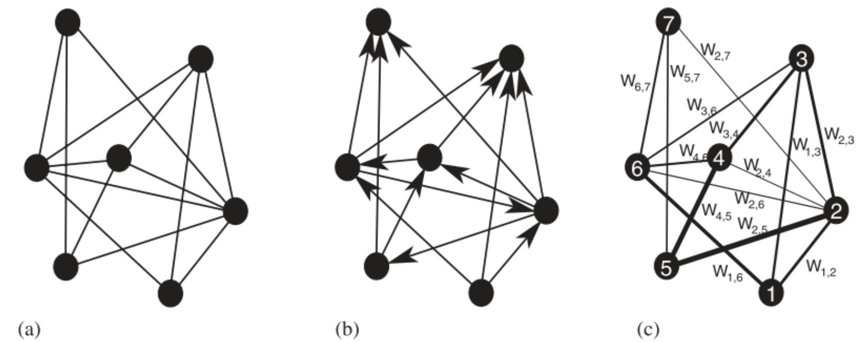




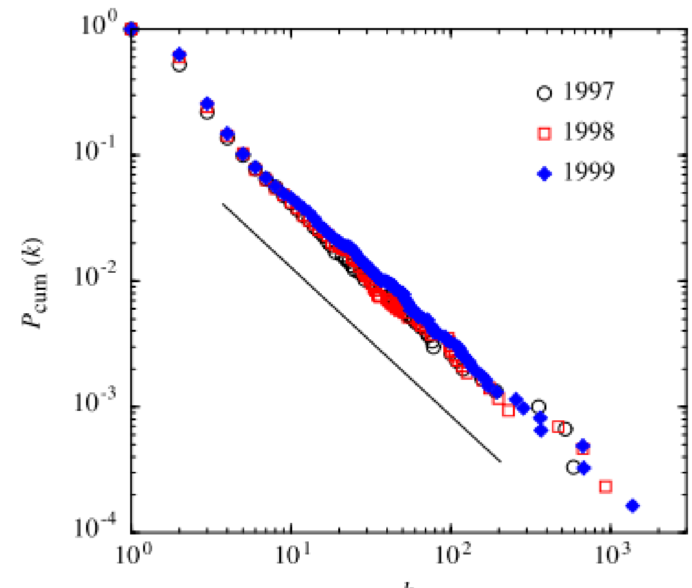
OUTLINE

- Power-law networks from complex systems
- Power-law network from dark matter halos
- A modified Barabasi-Albert model for the hierarchical halo distribution.
- Application and outreach

Power-law networks from complex systems

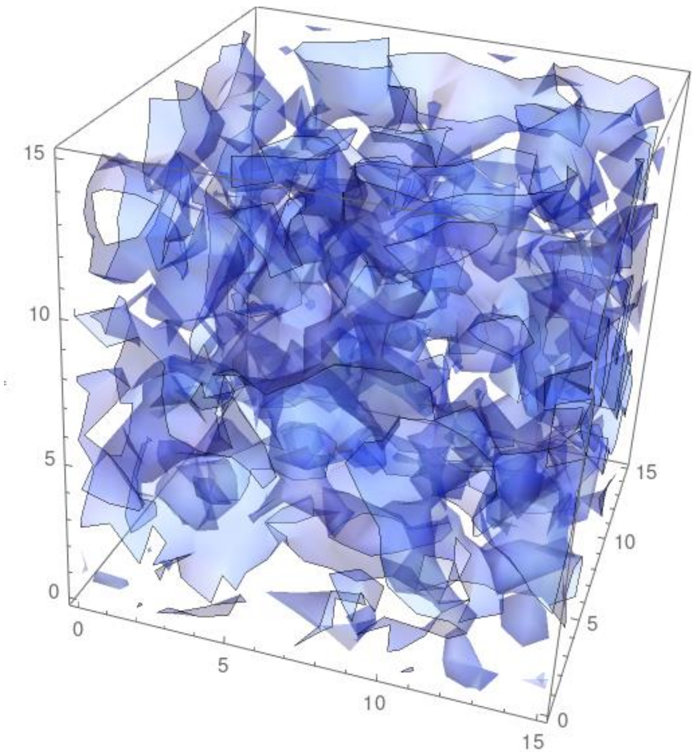


Network	N	γ
AS2001	11,174	2.38
Routers	228, 263	2.18
Gnutella	709	2.19
WWW	$\sim 2 \times 10^8$	2.1/2.7
Protein	2,115	2.4
Metabolic	778	2.2/2.1
Math1999	57, 516	2.47
Actors	225,226	2.3
e-mail	59,812	1.5/2.0



Cumulative degree distributions of the Internet graphs for three different years.

Sources: S. Boccaletti et al. / Physics Reports 424 (2006) 175–308



Cosmological simulations

Scale invariant initial power spectrum ($P_R \sim k^{-3}$)

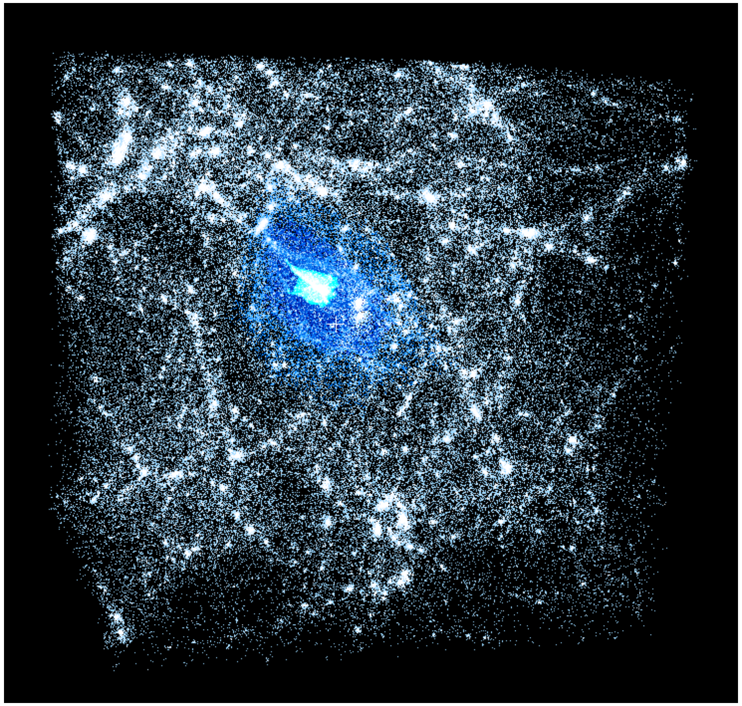
- $<\varphi(x)\varphi(x')> = <\varphi(\lambda x)\varphi(\lambda x')>$
- Different scales evolve independently

Structure formation

- Cosmic web of halos
- Density peaks where collapsed DM acquires kinetic energy and re-establishes pressure equilibrium
- At small scales: $P_\delta \sim k^4 T(k)^2 P_R \sim k^{-3}$

Hierarchical substructure

- A subhalo has also a number of subhalos
- *Zoom-in* one host to resolve more subhalos

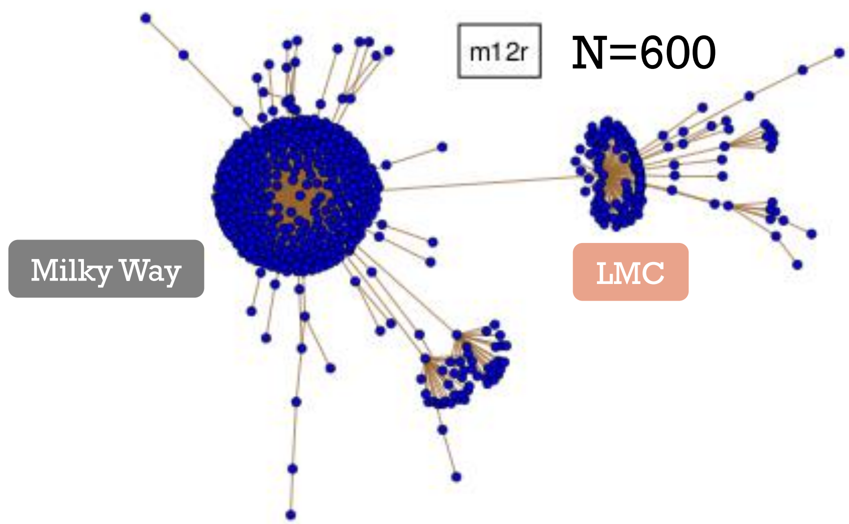


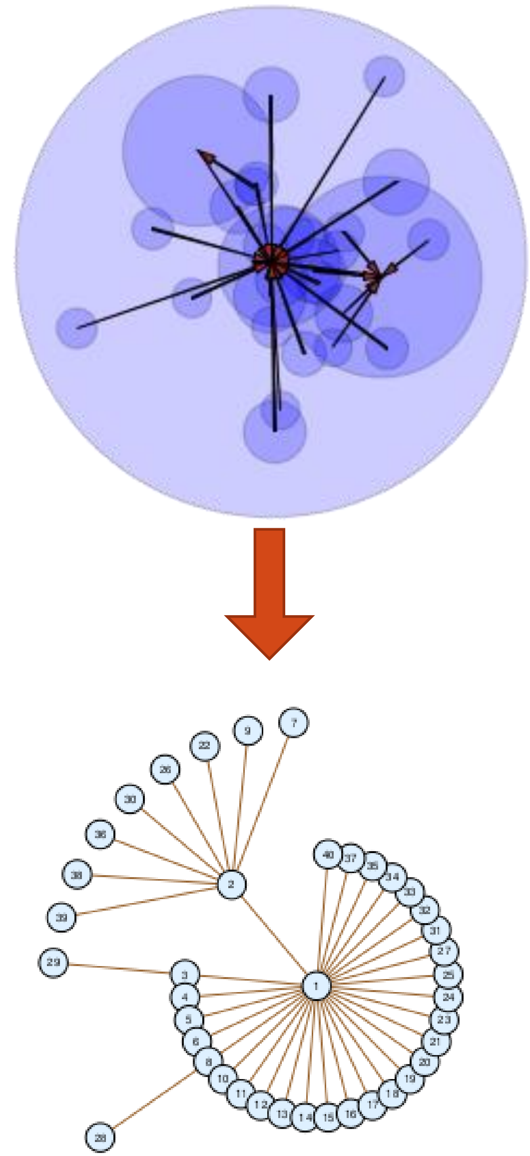
Power-law networks from dark matter halos

Example: N = 40 most massive halos FIRE2 m12r

$$\{N_{\mathrm{sub},1}, N_{\mathrm{sub},2}, N_{\mathrm{sub},3}, N_{\mathrm{sub},8}\} = \{29, 8, 1, 1\}$$

We use 11 MW-like zoomin FIRE2 simulations + BolshoiP cosmological simulation





Scaling behaviors of the N_{sub} (or k-1): from simulations & our model construction

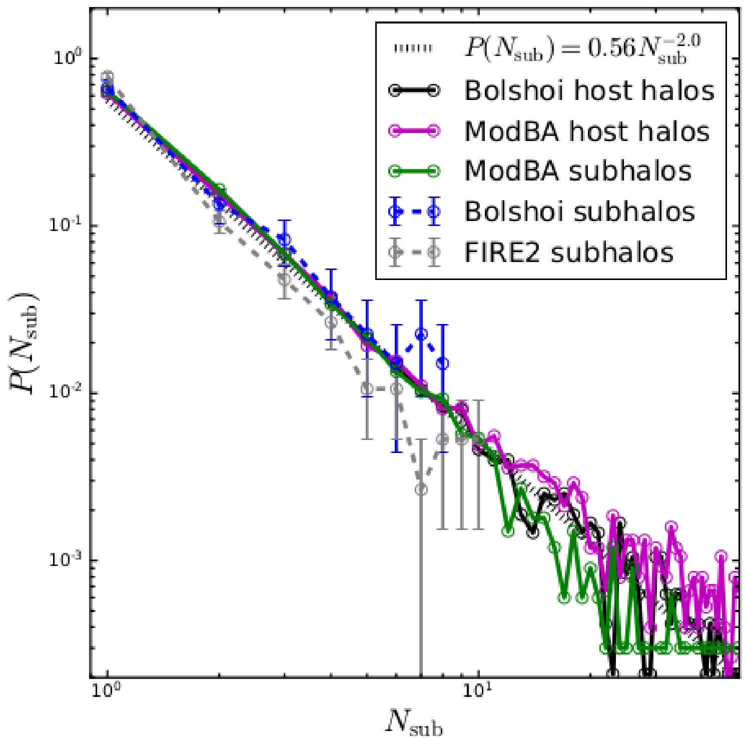
Press-Schechter 1974: fraction of mass locked in halo = P($\delta_M > \delta_c(t)$) → $\frac{dn}{dM} \propto \frac{1}{M^2}$ → $N_{sub} \propto \left(\frac{M_h}{m_0}\right)$

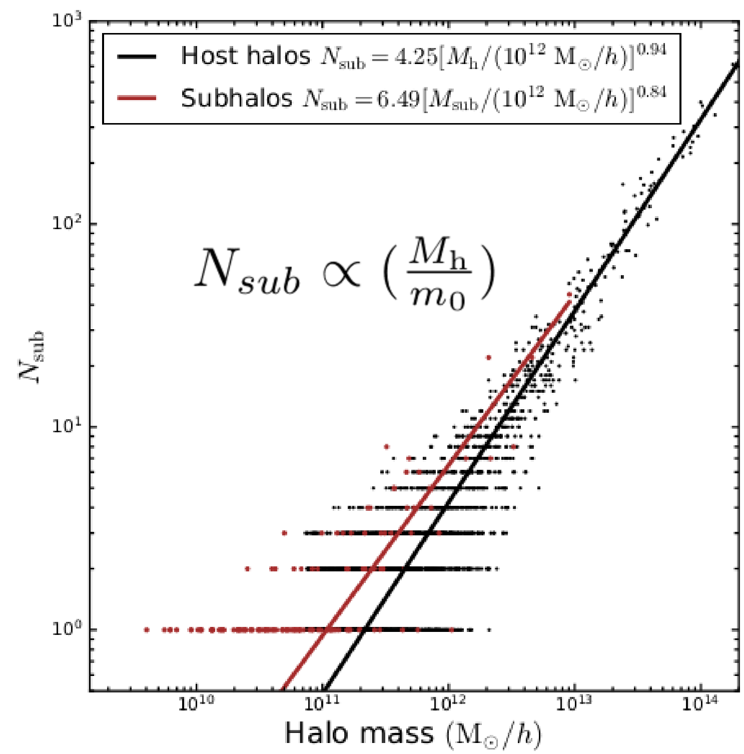
→ $\frac{dn}{dN_{sub}} \propto \frac{1}{N_{sub}^2}$

at all times

Linear accretion rate for *minor mergers*

$$N_{sub} + \Delta N_{sub} = \text{const.} \times N_{sub}$$





Linear attachment leads to power-law networks of known exponents

Attachment Probability = $\frac{A_i}{\sum_j A_j}$

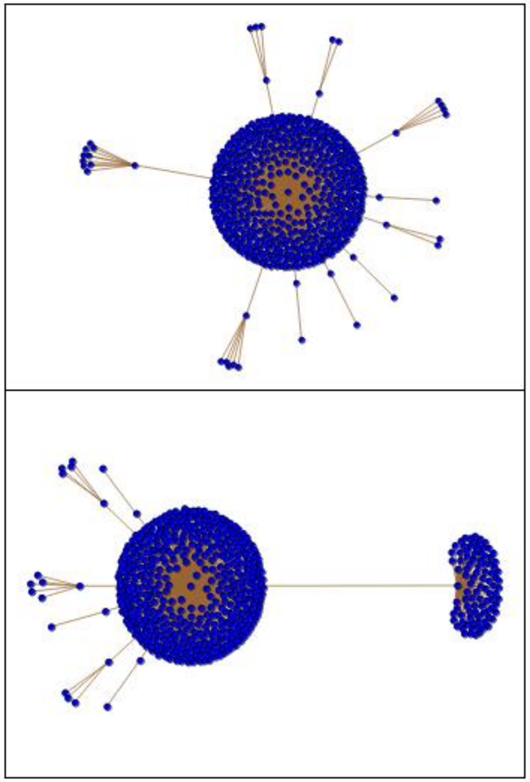
Barabasi-Albert model

- $A_k = k, k = 1,2,3, \ldots, N$
- *Power index k=-3 at large k*

P. L. Krapivsky, S. Redner, and F. Leyvraz, PRL 85 (2000) no. 21, 4629–4632

- $A_1 = \alpha' \ll 1$
- $A_k = k, k = 2,3, \ldots, N$
- power index = $-(3 + \sqrt{1 + 8\alpha'})/2$ that approaches -2 as α' approaches zero

Look alike, but not right… (α' =1/60)

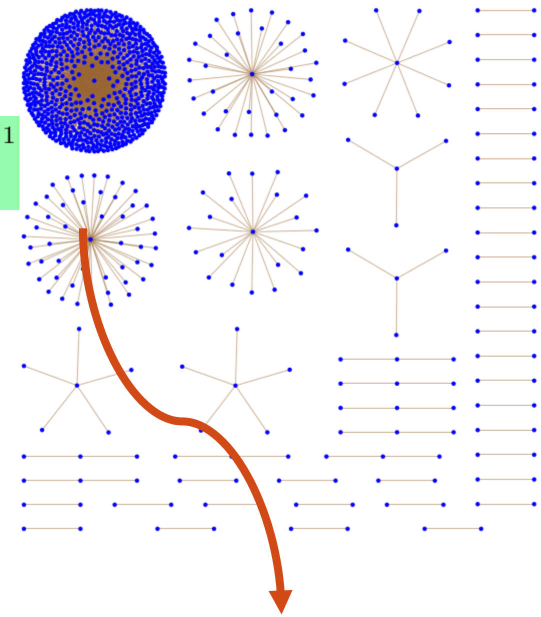


$$A_{j,h} = \left(k_j^{-1} + (\alpha k_j^{\beta})^{-1}\right)^{-1}$$

Can explore this to generate the subhalo occupation number

We exploit the linear attachment rate to construct a network model

$$A_{j,h} = \left(k_j^{-1} + (\alpha k_j^{\beta})^{-1}\right)^{-1}$$



Two step ModBA

α=0.05 && $\beta = 2.2, \gamma(N)$ in general >2

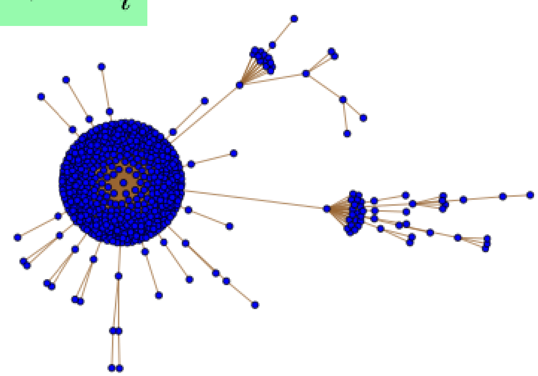
Major mergers do not contribute to N_{sub}

- Low N_{sub} halos have their attachment rate suppressed
- Linear attachment at large N_{sub}

Tidal stripping

- Sub-subhalos erased or released to the host
- Linear attachment at low N_{sub}

$$A_{i,s} = k_i + \alpha k_i^{\gamma}$$

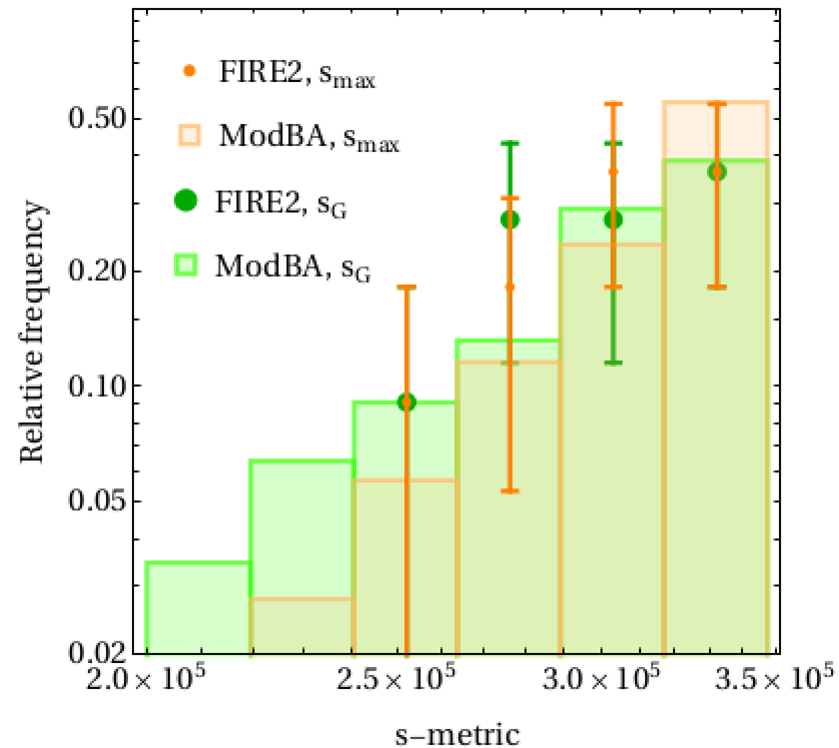


Scale-free networks

Properties (Li Lun, et al 2006)

- Power-law degree distribution
- Hub-like core
- Preserved by random degree-preserving rewiring
- Self-similar

$$s_G = \sum_{(i,j)\in E} k_i k_j.$$



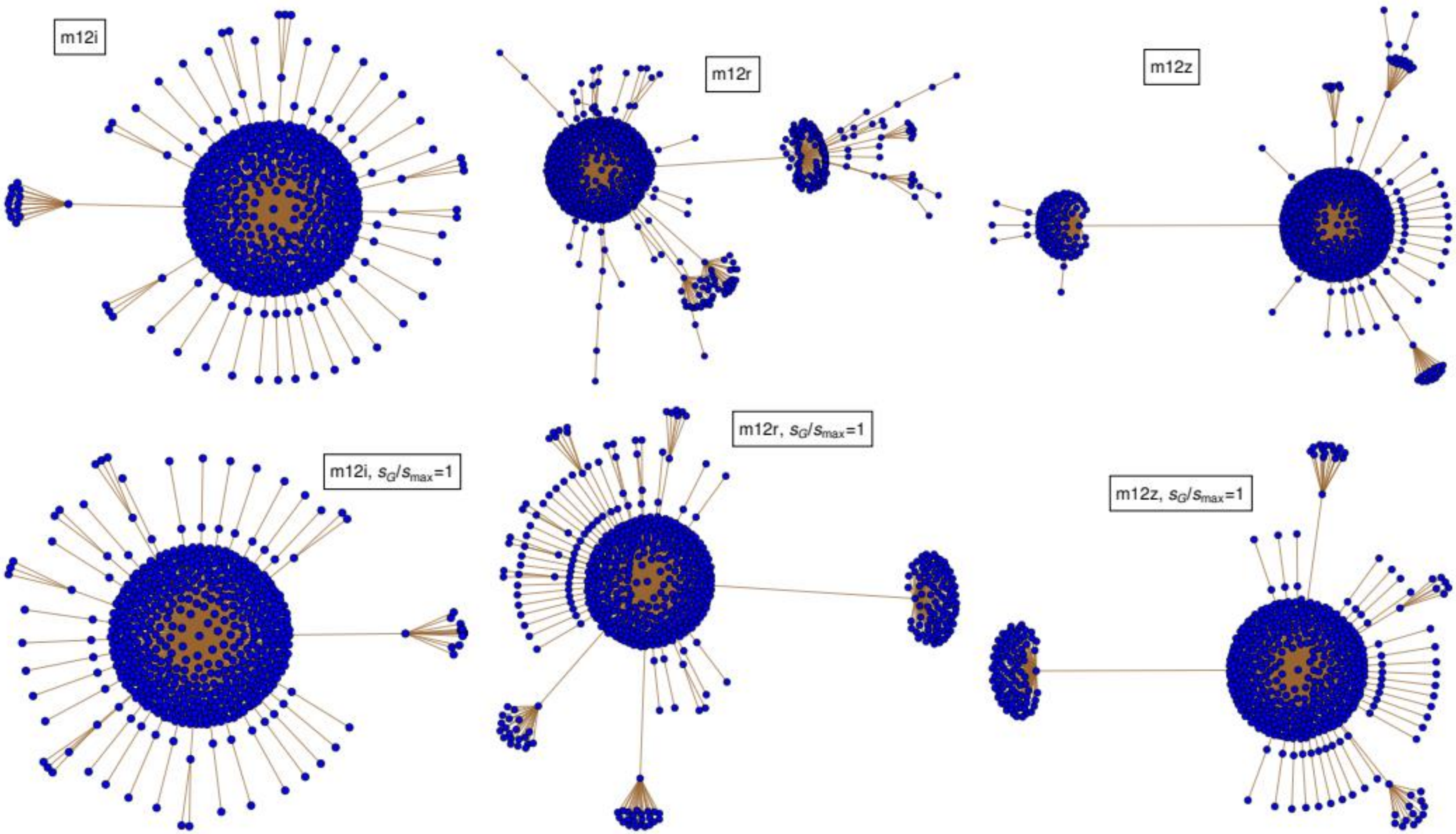
Given a degree sequence D={$k_1, k_2, \ldots, k_N$}, one can construct a graph maximizing s_G (Li Lun, et al 2006)

- s_G / s_{max}=0.98 (**FIRE2 simulations**)
- s_G / s_{max}=0.93 (**Model constructions**)

Preferential attachment naturally leads to scale-free networks: Early attachment advantage

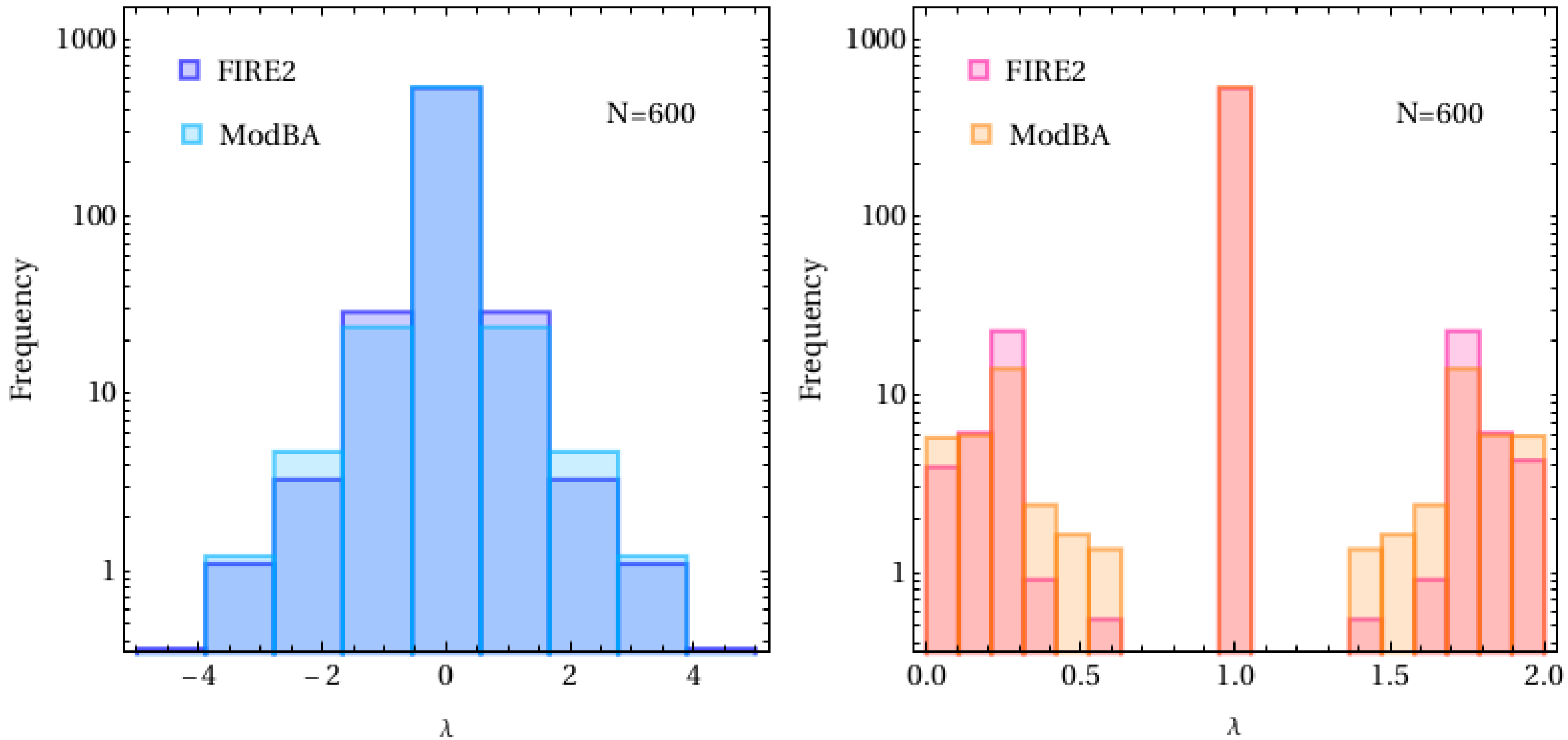
Top: graphs reconstructed from FIRE2 simulations

Bottom: $s_{\max}$ graphs based on the corresponding degree sequence



Similarity => Scale free

Model vs. Simulation



Eigenvalue spectra of the adjacency matrix and the Laplacian matrix

- Adjacency matrix: $X_{ij} = X_{ji}$=1 if i,j nodes are connected
- Normalized Laplacian matrix: L=I-$D^{-1/2}XD^{-1/2}$

Possible applications

Topological data analysis

- Metrics based on the adjacency matrix
- Persistent homology
- Could have sensitivity to scale-violating physics

+

arXiv: 2205.03392
characteristic velocity and cross sections of SIDM

Graph neural network

- Observables can be encoded into *weights* of nodes/links
- Graph Fourier Transformation + Low pass filter

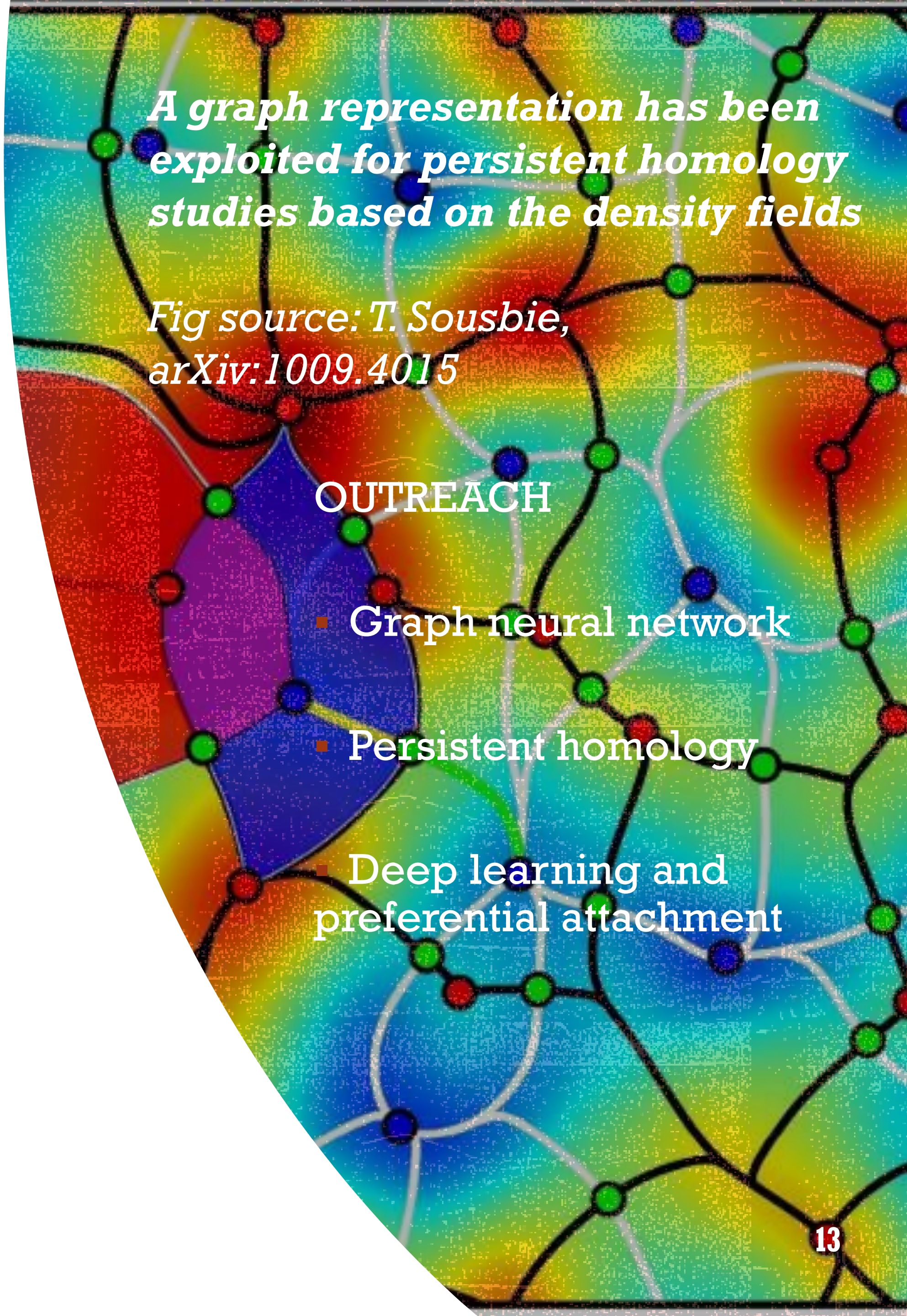
$$\mathbf{h}_u^{(k+1)} = \text{UPDATE}^{(k)}\left(\mathbf{h}_u^{(k)}, \text{AGGREGATE}^{(k)}(\{\mathbf{h}_v^{(k)}, \forall v \in \mathcal{N}(u)\})\right)$$

- Has been applied to study halo shape and orientation

(Jagvaral Y. et al, arXiv: 2204.07077)

SUMMARY

- Our graph model offers a simple way to understand the hierarchical structure of the dark matter distribution.
- Our knowledge of structure formation improves our understanding of other networks
- It is promising to apply graph-based techniques to explore small-scale physics.



A graph representation has been exploited for persistent homology studies based on the density fields

Fig source: T. Sousbie, arXiv:1009.4015

OUTREACH

- Graph neural network
- Persistent homology
- Deep learning and preferential attachment

14 BACKUP

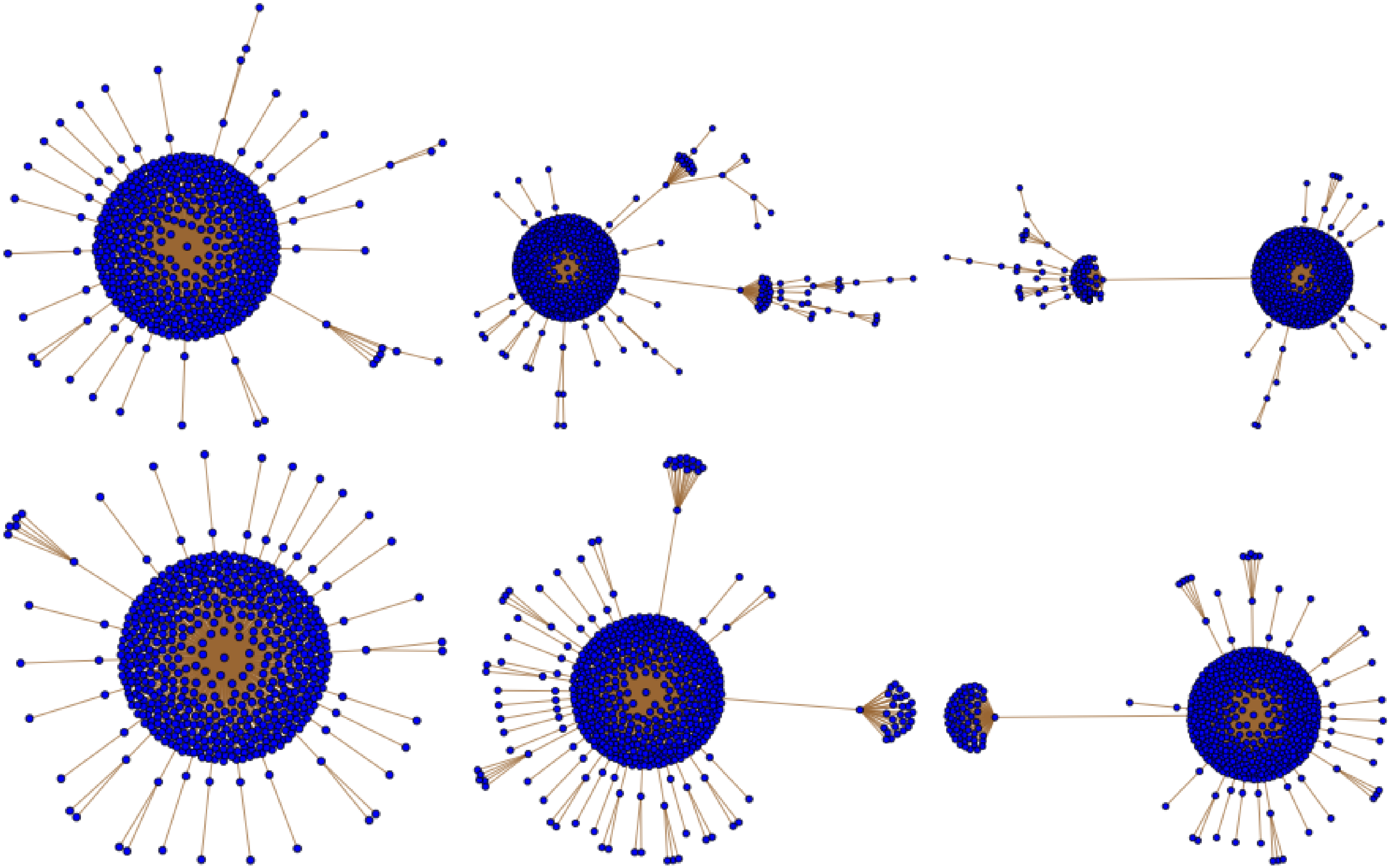
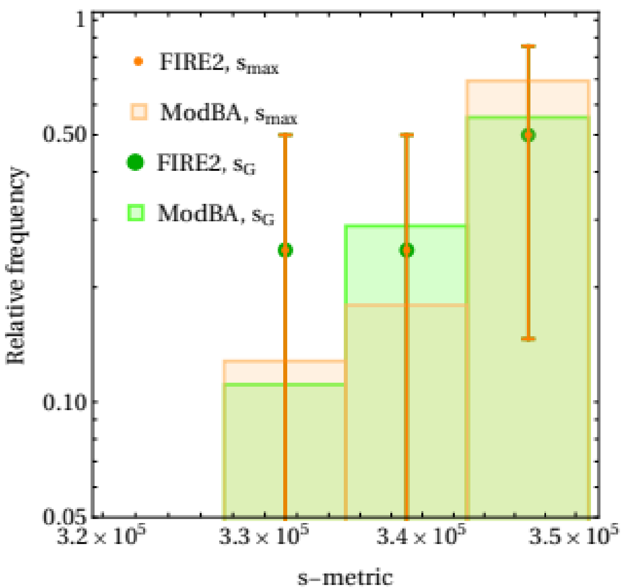


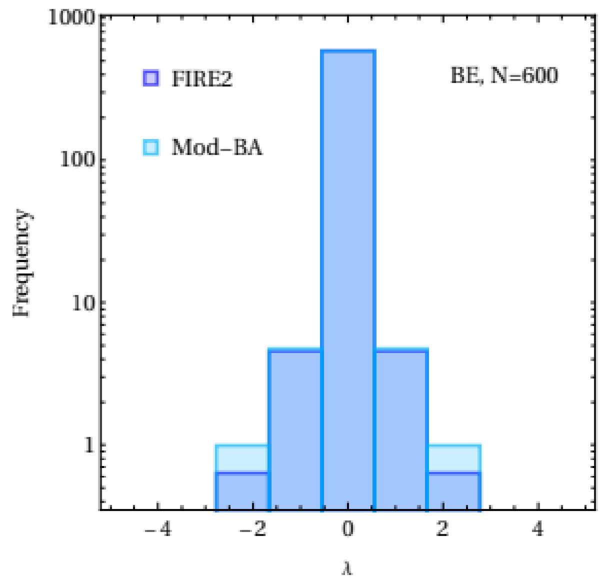
FIG. 5. Top: Example graphs of $N = 600$ generated using our Mod-BA model. They are selected by eye to mimic the ones in Fig.4 from a sample of a hundred graphs. Bottom: The $s_{\max}$ graphs constructed for the corresponding ones on the upper row.

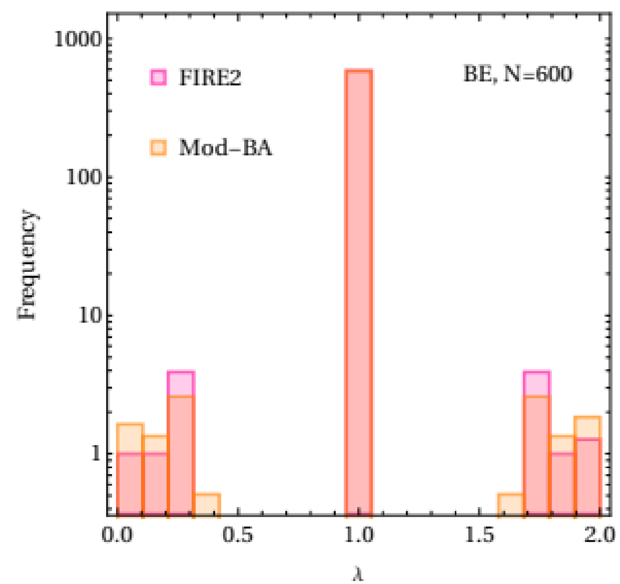
A binding energy based construction

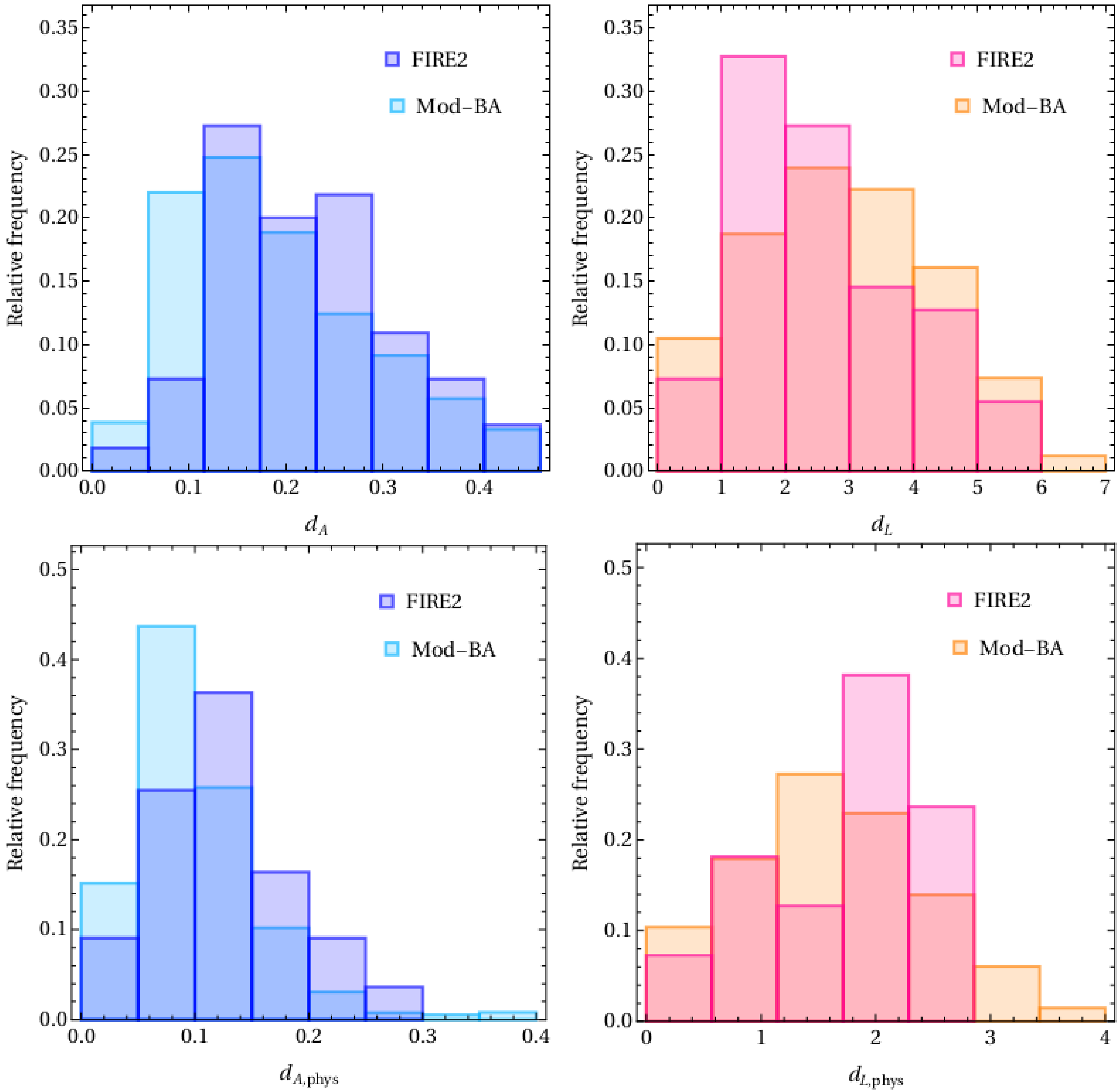
- Similar in-degree distributions
- Similar topological metrics

$$G\frac{m_H m_L}{r} > \mu \frac{\left|\mathbf{v}_L - \mathbf{v}_H\right|^2}{2}$$









Spectra distance	d_A	d_L
ModBA μ_M	0.21	2.9
ModBA σ_M	0.13	1.5
FIRE2 μ_F	0.22	2.6
FIRE2 σ_F	0.094	1.3