Stasis in an Expanding Universe: A Recipe for Stable Mixed-Component Cosmological Eras

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in collaboration with

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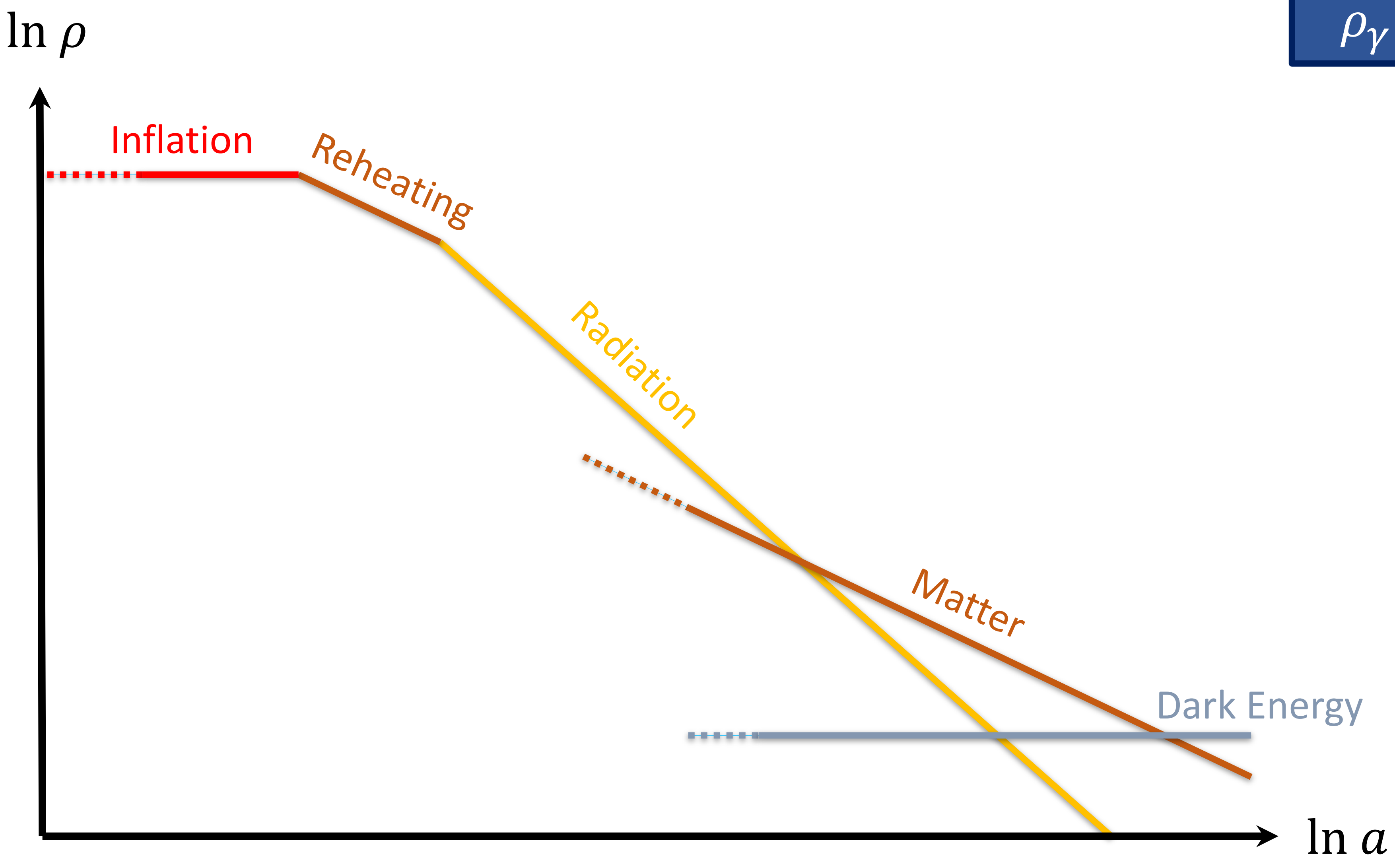
The Standard Lore: ΛCDM

$$\rho_\Lambda \sim a^0$$
$$\rho_M \sim a^{-3}$$
$$\rho_\gamma \sim a^{-4}$$



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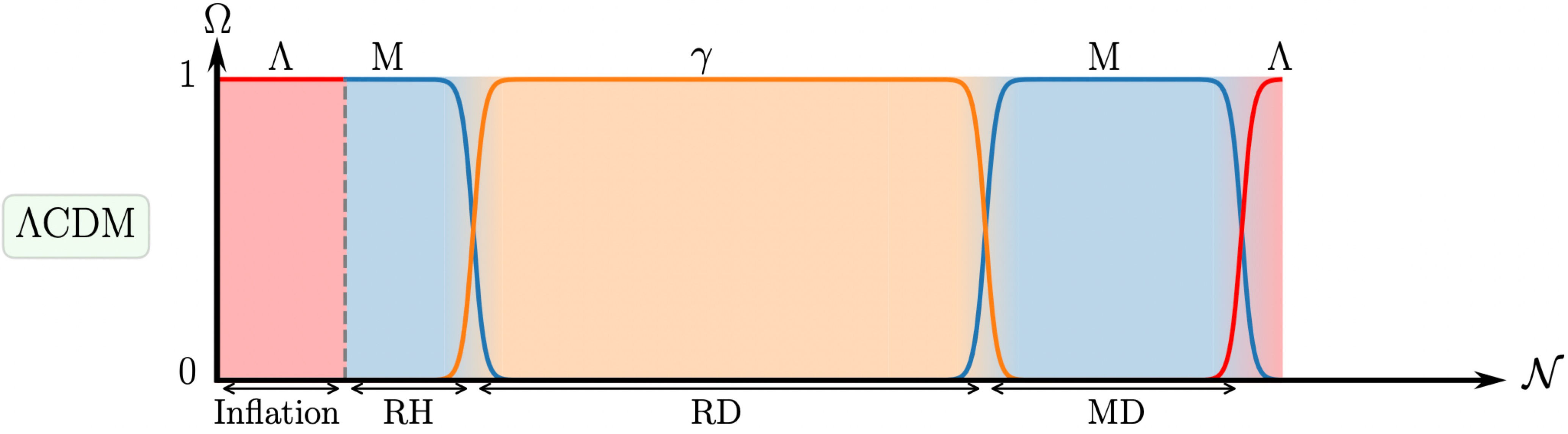

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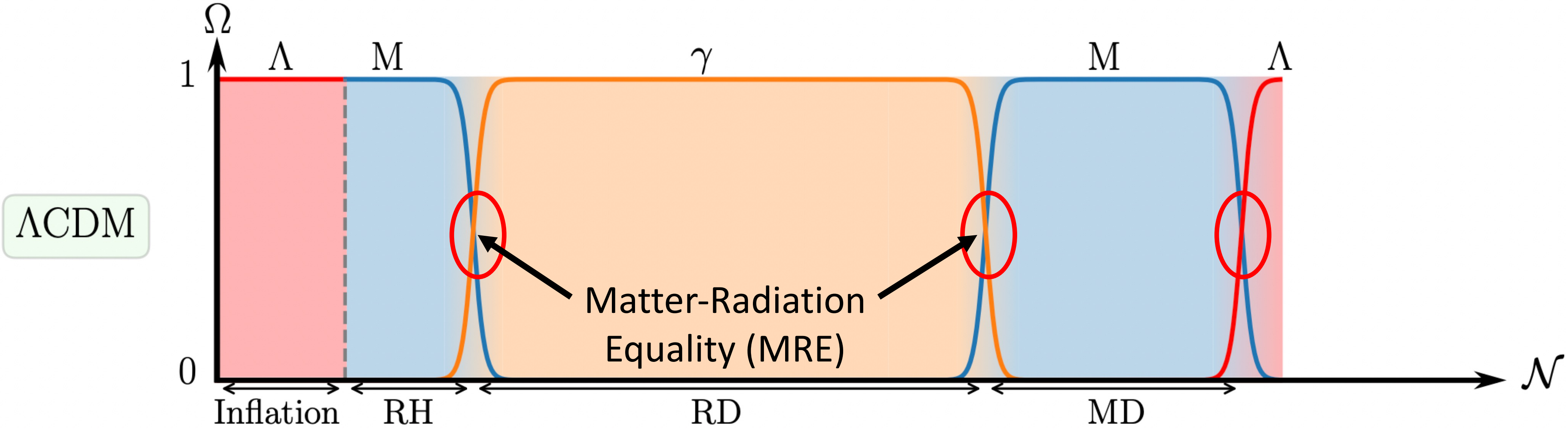

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Goals and Challenges

- In the standard ΛCDM cosmology, critical moments like MRE are fleeting.
- What if the MRE is not just a point, but a sustained era in the cosmological timeline?
- More generally, is it possible to maintain a constant Ω_M and Ω_γ over a sizable period of time?

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Particle decays!

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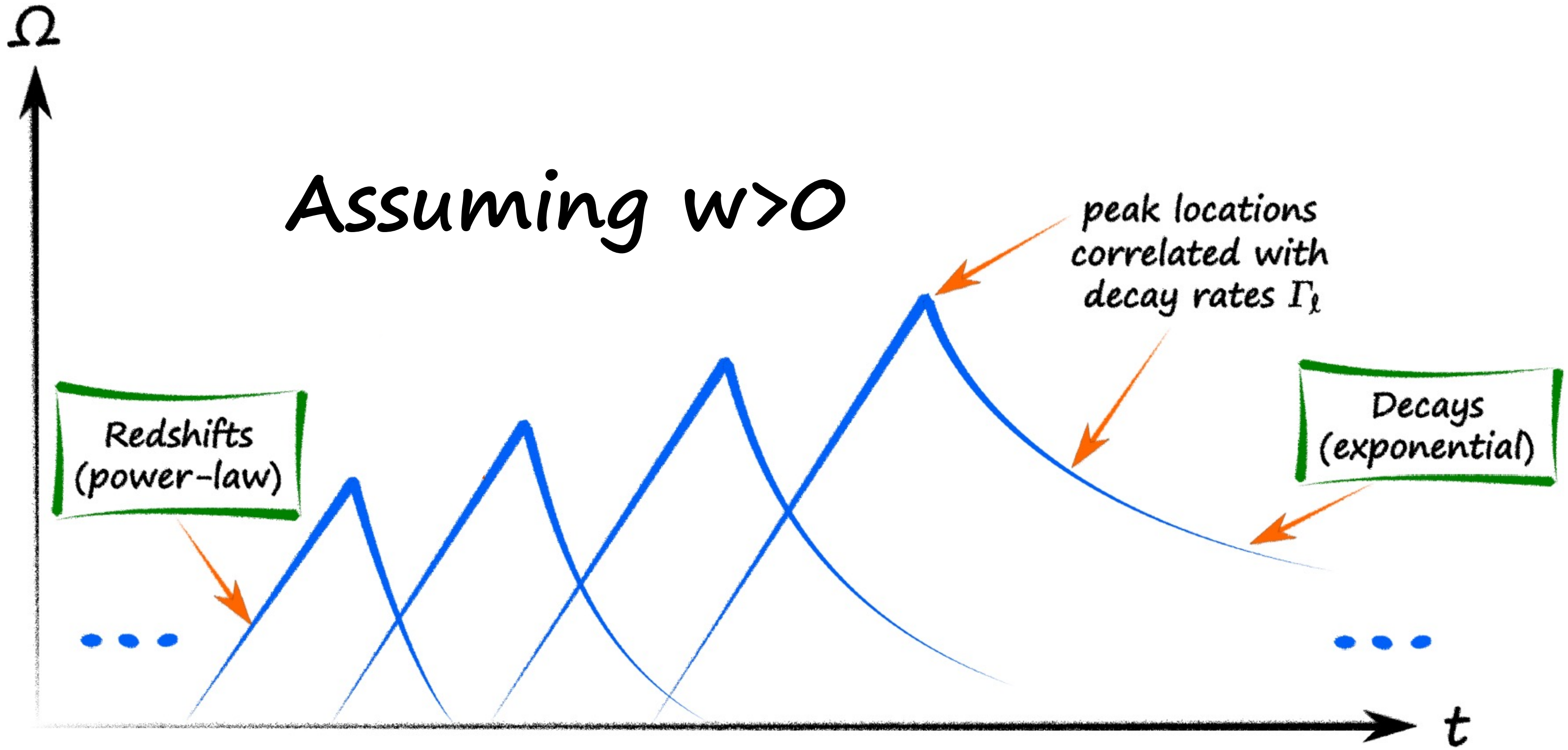

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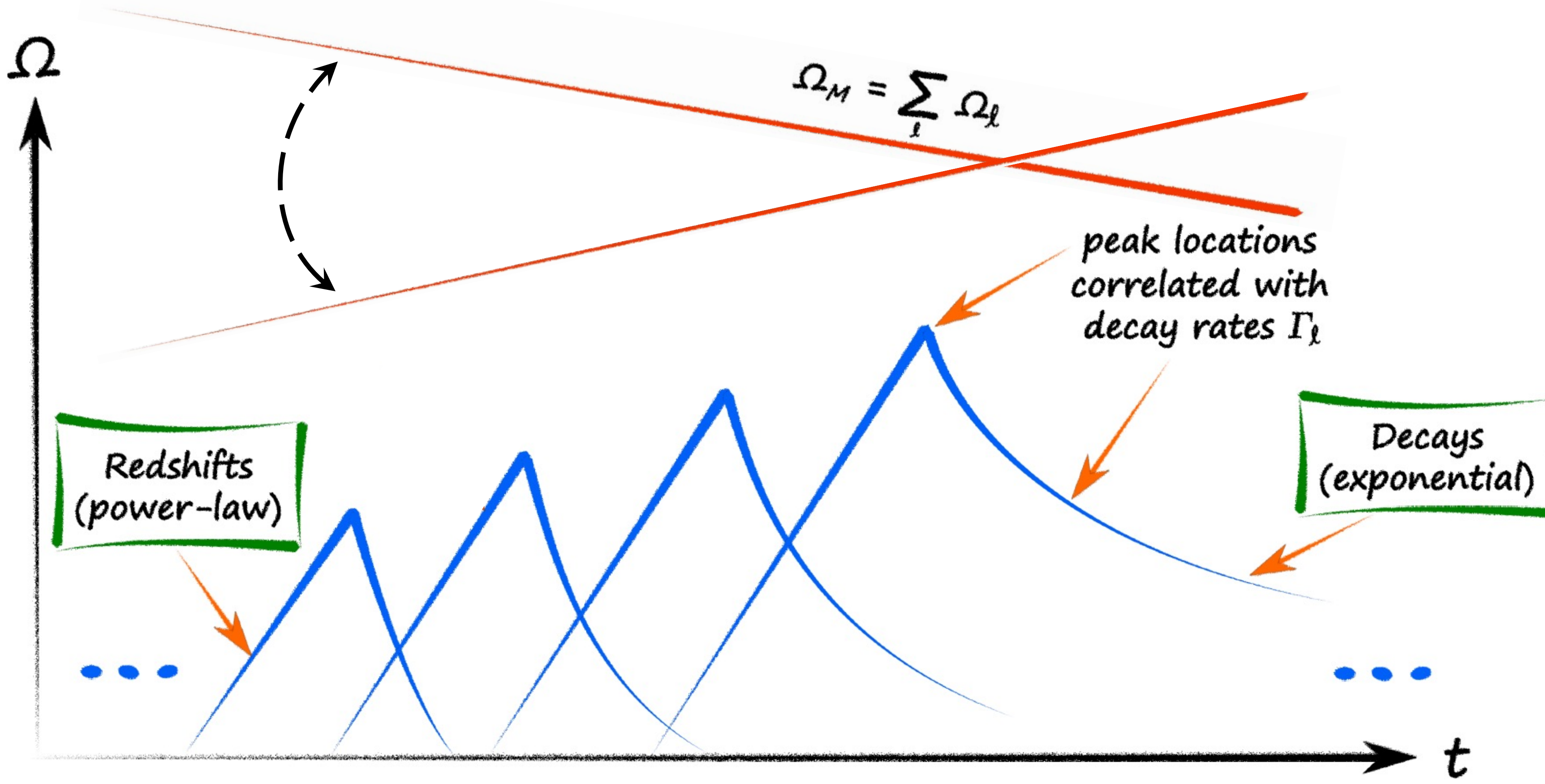

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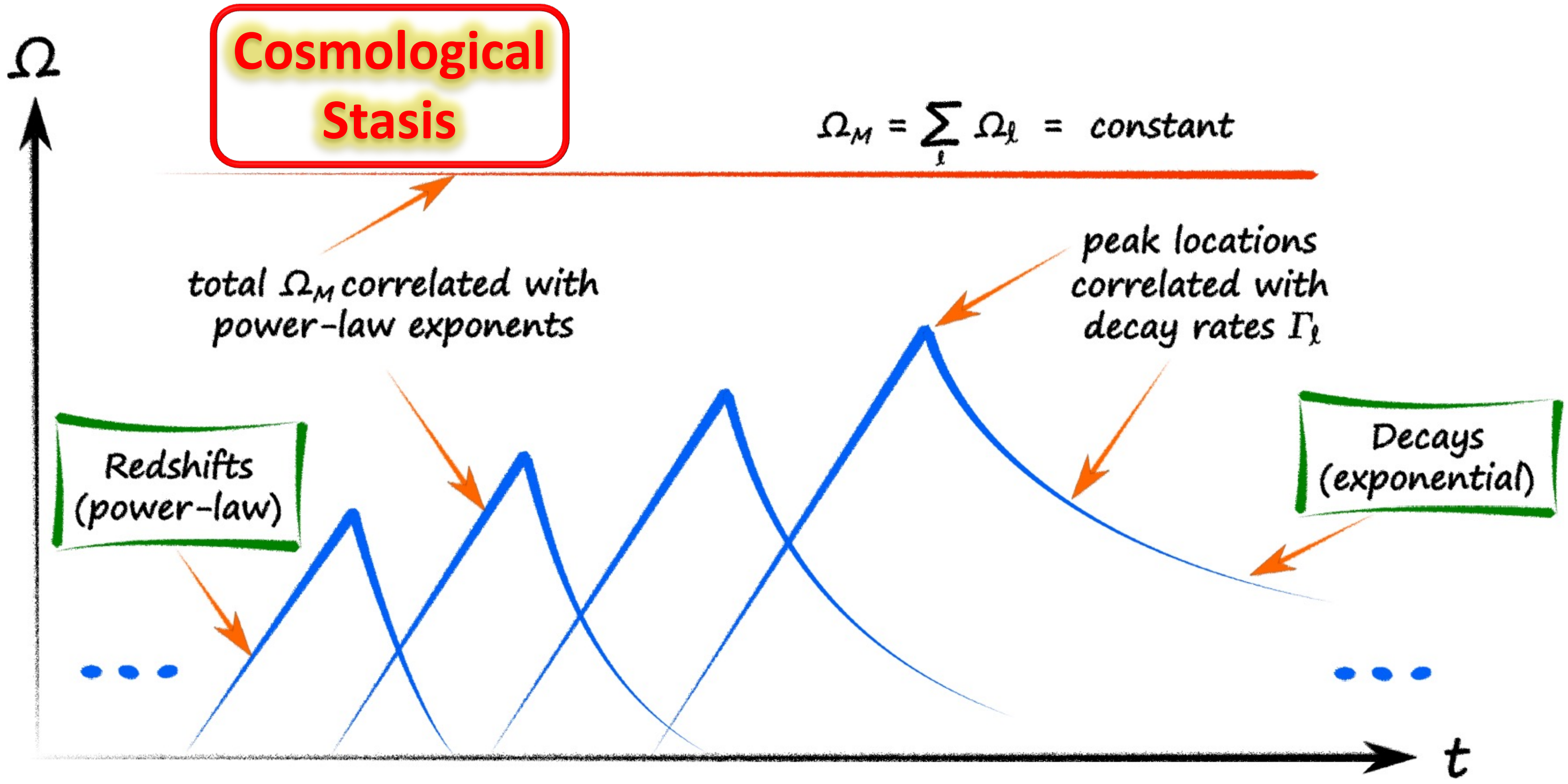
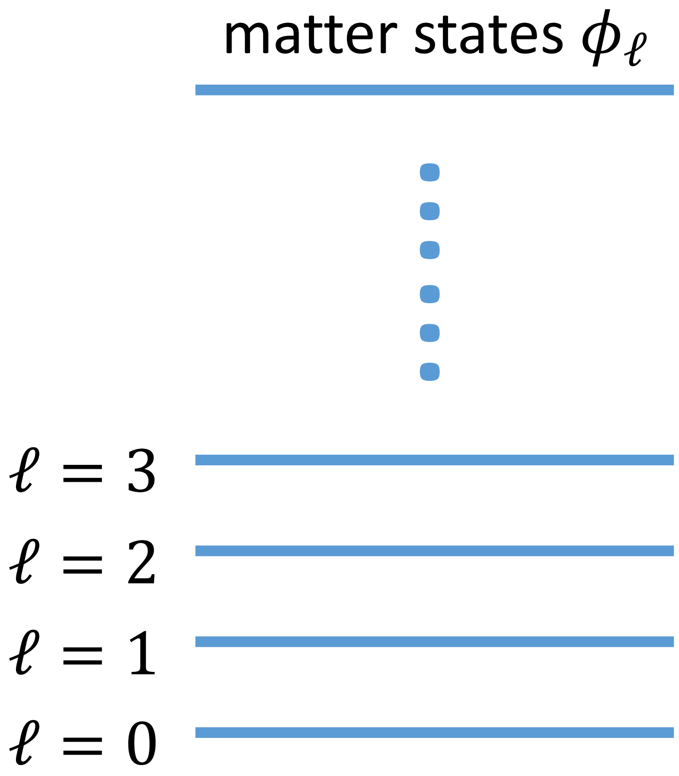
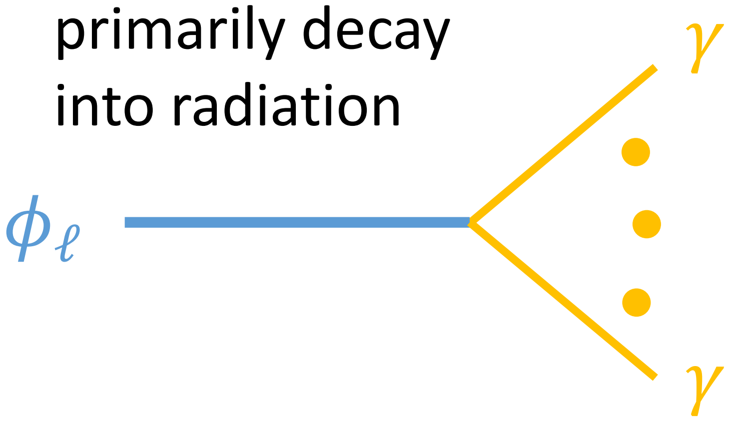


Conditions for Stasis





$$\Omega_\ell = \frac{8\pi G}{3H^2}\rho_\ell \qquad \Omega_M = \sum_\ell \Omega_\ell \qquad \Omega_M + \Omega_\gamma = 1$$

Conditions for Stasis

Boltzmann equations

$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell \rho_\ell \qquad \frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell$$

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Friedmann equation

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Necessary *condition for Stasis, but* ***NOT sufficient***

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

To have Ω_M remain constant in time, one would require

$$\frac{d^n\Omega_M}{dt^n} = 0$$

⟹ an infinite set of constraints

Conditions for Stasis

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Necessary condition for Stasis, but not sufficient

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Instead, we shall do this in a quicker way 😎

Assuming $\frac{d\Omega_M}{dt} = 0$ is satisfied at some ***fiducial time*** t_*, demand that ***both sides*** evolve with time in the same manner.

Conditions for Stasis

$$\sum_{\ell} \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

$$\sim \frac{1}{t}$$

For a universe with only matter and radiation

$$\frac{dH}{dt} = -\frac{1}{2} H^2 (4 - \Omega_M)$$

During Stasis $\Omega_M = \overline{\Omega}_M$

$$H(t) = \left(\frac{2}{4 - \overline{\Omega}_M}\right)\frac{1}{t}$$

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$\Omega_\ell = \rho_\ell / \rho_{tot}$

$\rho_\ell \sim a^{-3} e^{-\Gamma_\ell (t-t_*)}$

$\rho_{tot} \sim H^2 \sim t^{-2}$

$a \sim t^{2/(4-\bar{\Omega}_M)}$

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$$\Omega_\ell(t) = \Omega_\ell^* \left(\frac{t}{t_*}\right)^{2-6/(4-\bar{\Omega}_M)} e^{-\Gamma_\ell (t-t_*)}$$

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Stasis (Eternal)

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{2\bar{\Omega}_M (1 - \bar{\Omega}_M)}{4 - \bar{\Omega}_M} \frac{1}{t}$$

$$\sum_\ell \Omega_\ell(t) = \bar{\Omega}_M$$

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{2(1 - \bar{\Omega}_M)}{4 - \bar{\Omega}_M} \frac{1}{t}$$

A Model of Stasis

Mass Spectrum $$m_\ell = m_0 + (\Delta m)\ell^\delta$$

Decay Widths $$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m}\right)^\gamma$$

Initial Abundances $$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m}\right)^\alpha$$

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Depends on particle physics model

- KK excitations of a 5-d scalar field compactified on a circle of radius R
  - $\delta \sim 1$ for $mR \ll 1$
  - $\delta \sim 2$ for $mR \gg 1$
- Bound states of strongly-coupled gauge theory
  - $\delta \sim 1/2$

Depends on decay mode

- if ϕ_ℓ decays to photons through contact operator $\mathcal{O}_\ell \sim c_\ell \phi_\ell \mathcal{F}/\Lambda^{d-4}$, $\gamma = 2d - 7$, e.g., $\gamma \sim \{3,\ 5,\ 7\}$

Depends on the production mechanism

- $\alpha < 0$ for misalignment production
- both $\alpha > 0$ or $\alpha < 0$ for thermal freeze-out
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Free parameters

$$\{\alpha, \gamma, \delta, m_0, \Delta m, \Gamma_0, \Omega_0^{(0)}, t^{(0)}\}$$

initial conditions

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$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}$$

With our particular model, we can work out the LHS

If we consider the production of states ϕ_ℓ before stasis

$$\Omega_\ell(t) = \Omega_\ell^{(0)} h(t^{(0)}, t) e^{-\Gamma_\ell (t - t^{(0)})}$$

with non-trivial redshift effect encoded in h(t⁽⁰⁾, t)

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \Gamma_0 \Omega_0^{(0)} h\left(t^{(0)}, t\right) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma (t - t^{(0)})}$$

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Continuous limit

$$= \frac{\Gamma_0 \Omega_0^{(0)}}{\delta (\Delta m)^{1/\delta}} h(t^{(0)}, t) \int_0^\infty dm\, m^{1/\delta - 1} \left(\frac{m}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0 \left(\frac{m}{m_0}\right)^\gamma (t - t^{(0)})}$$

$$= \frac{\Gamma_0 \Omega_0^{(0)}}{\gamma \delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} \Gamma\left(\frac{\alpha + 1/\delta + \gamma}{\gamma}\right) h(t^{(0)}, t) \left[\Gamma_0 (t - t^{(0)})\right]^{-(\alpha + 1/\delta + \gamma)/\gamma}$$

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Stasis condition can be satisfied as long as

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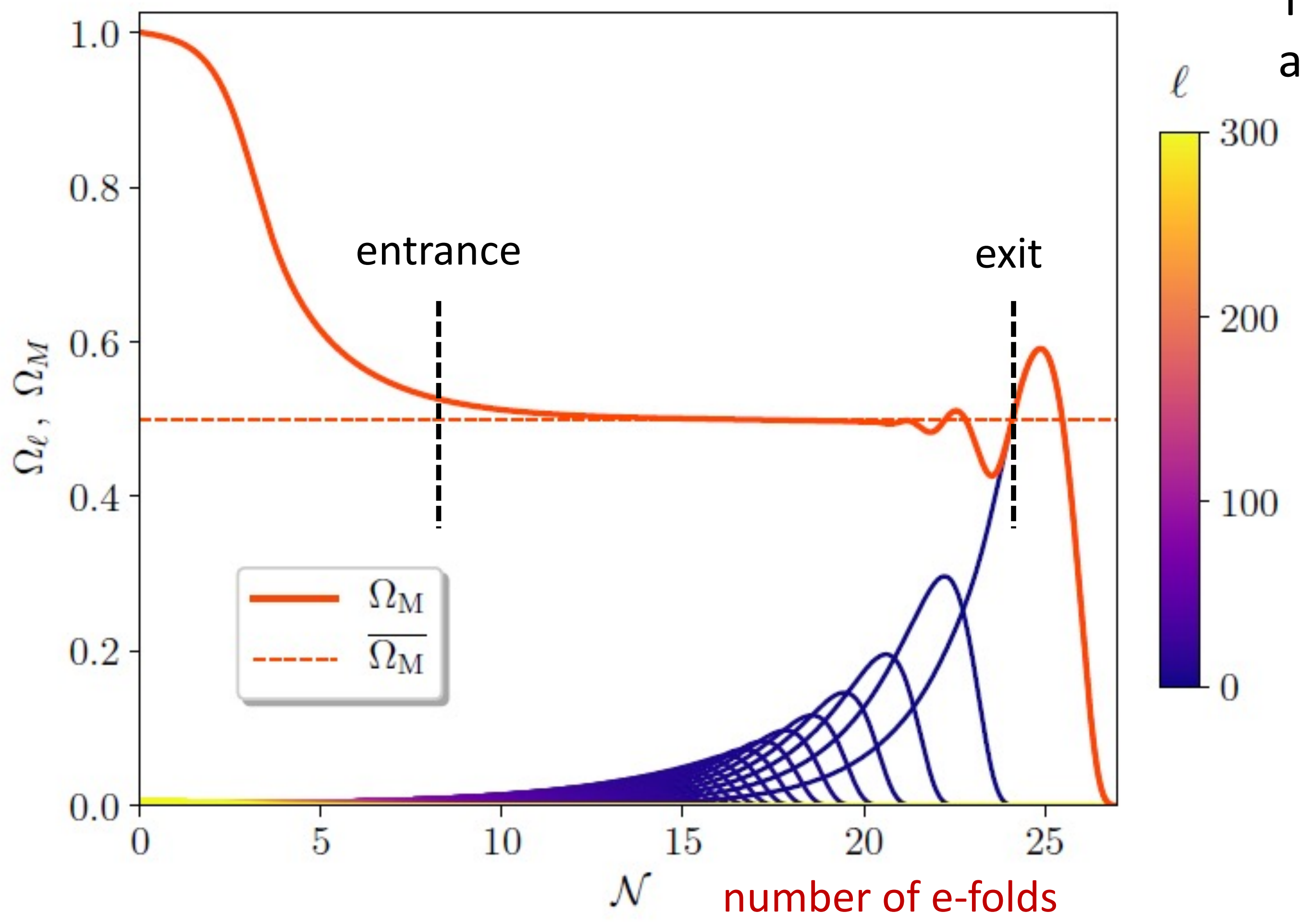
Matter abundance during stasis is determined by model parameters

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}$$

It's time to test it numerically…

Examples

An "Extended MRE"

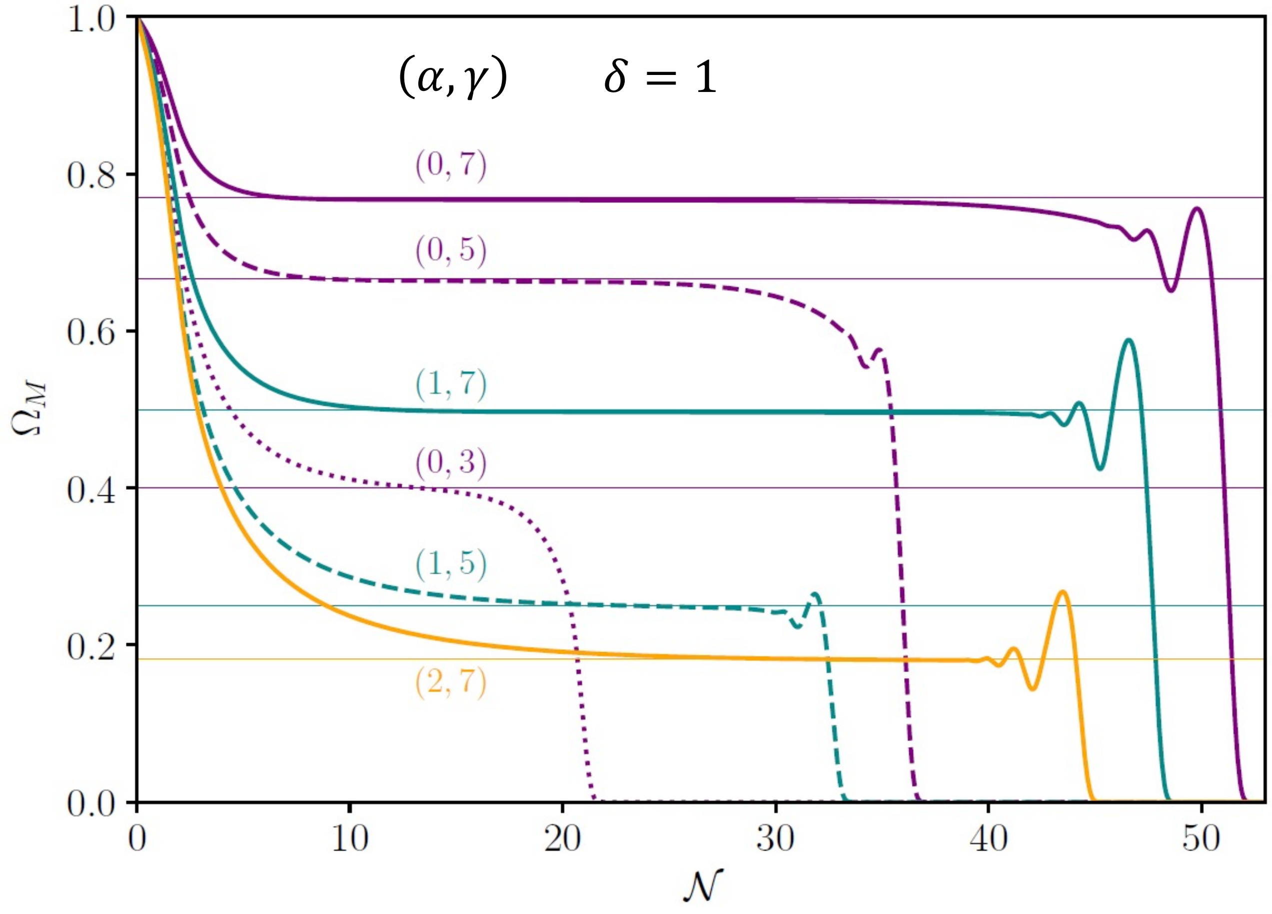


The actual model has an entrance and an exit

$$(\alpha, \gamma, \delta) = (1{,}7{,}1) \to \overline{\Omega}_M = 1/2, \Delta m = m_0, \Gamma_{N-1}/H^{(0)} = 0.01, N = 300$$

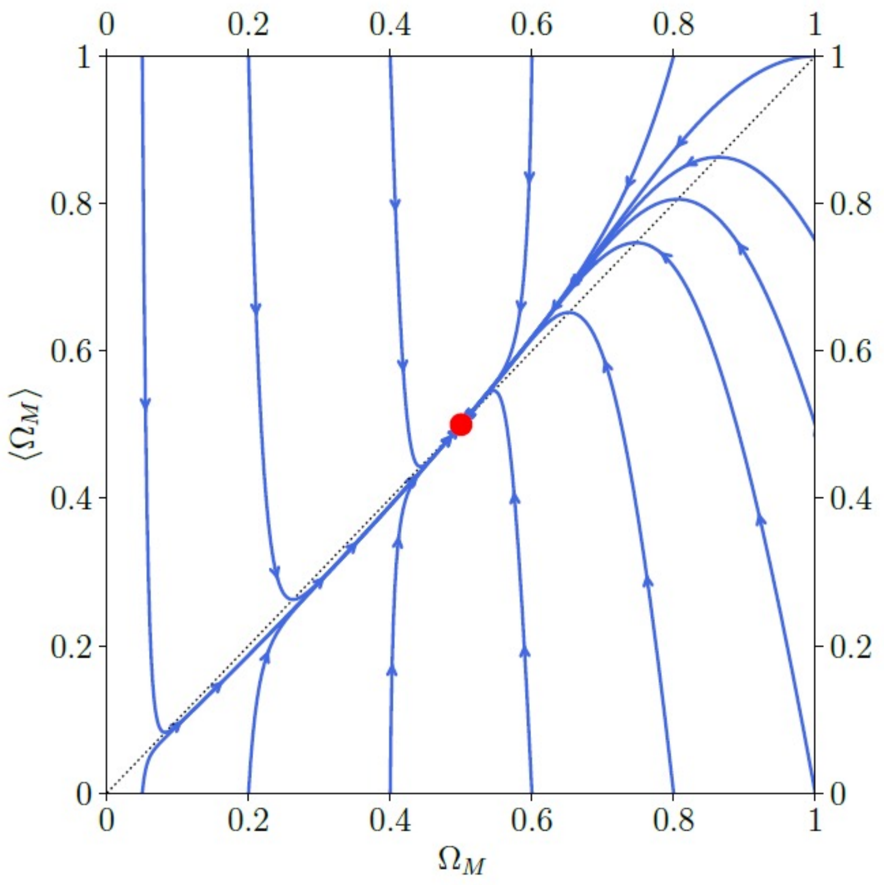
Examples

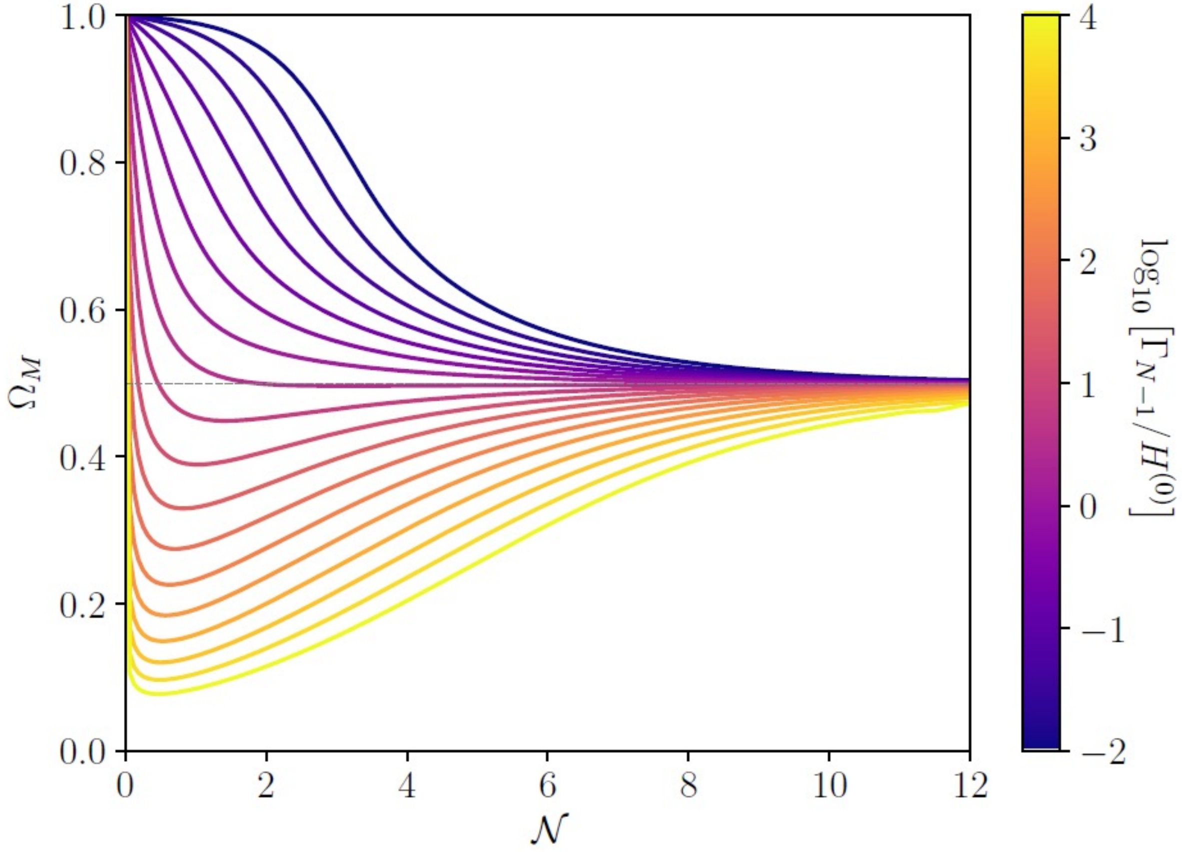
Fine tuned?



Prolonged epochs in which $\Omega_M \sim \bar{\Omega}_M$ can be achieved in multiple cases!

Stasis as a Global Attractor



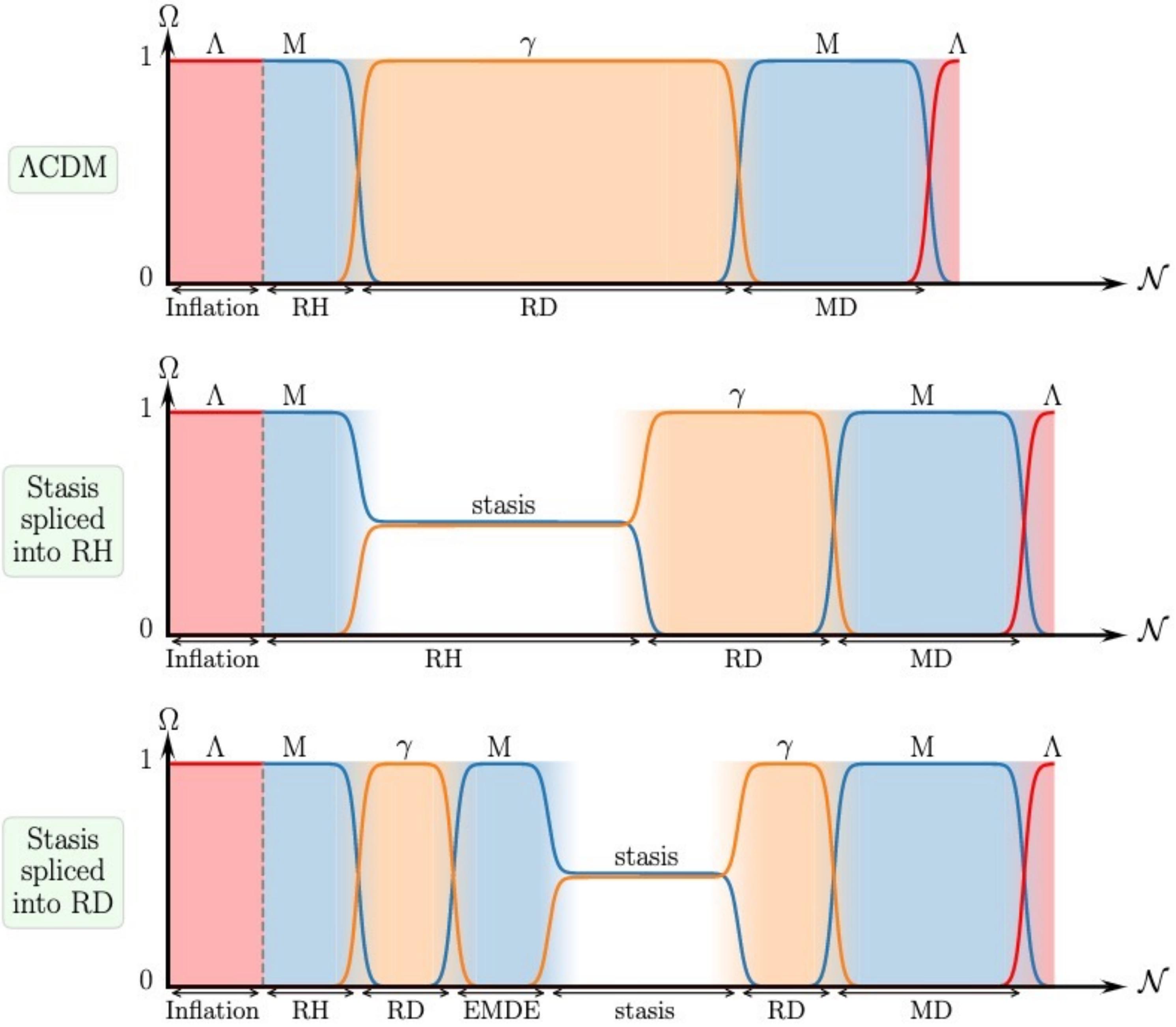


The attractor is GLOBAL!

Splicing Stasis into the Cosmological Timeline

Reheating occurs during the stasis epoch and results from the decays of ϕ_ℓ

The presence of multiple matter fields first leads to an early matter-dominated era (EMDE), then stasis occurs when decays start



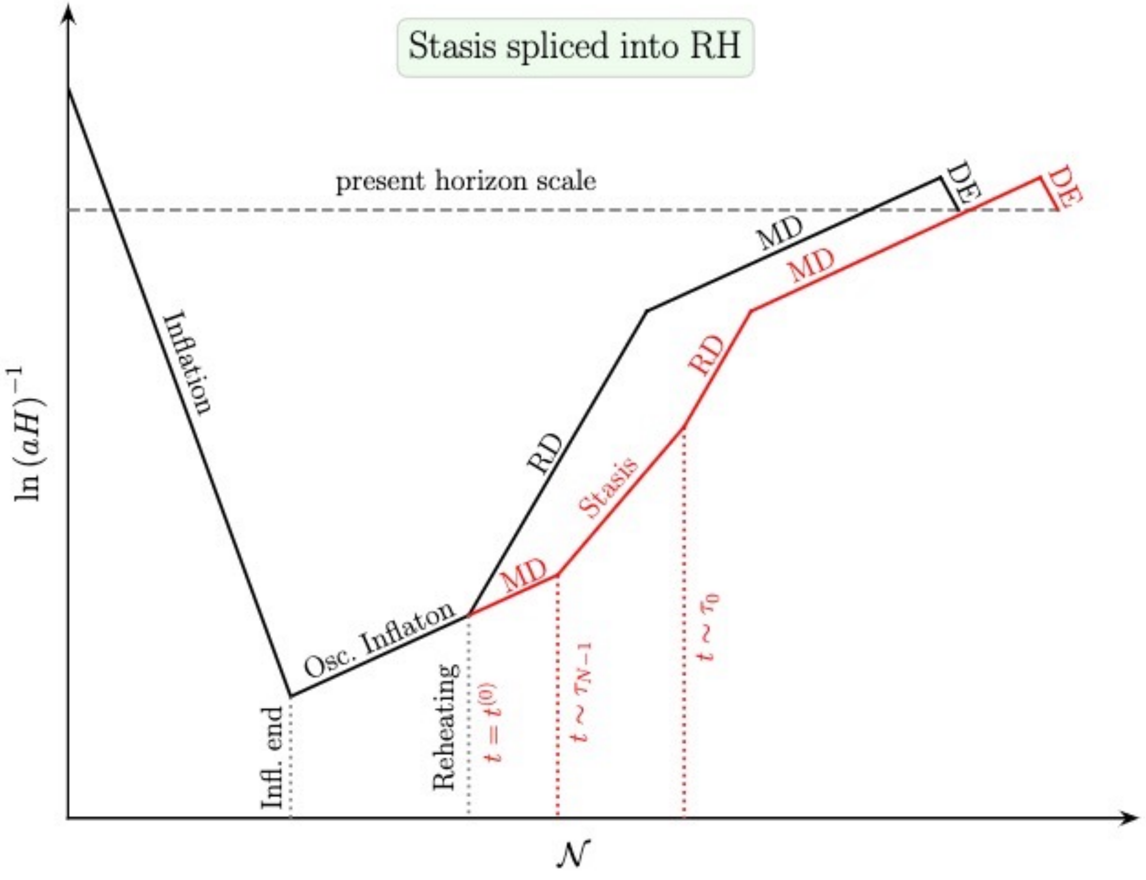
Summary and Future Directions

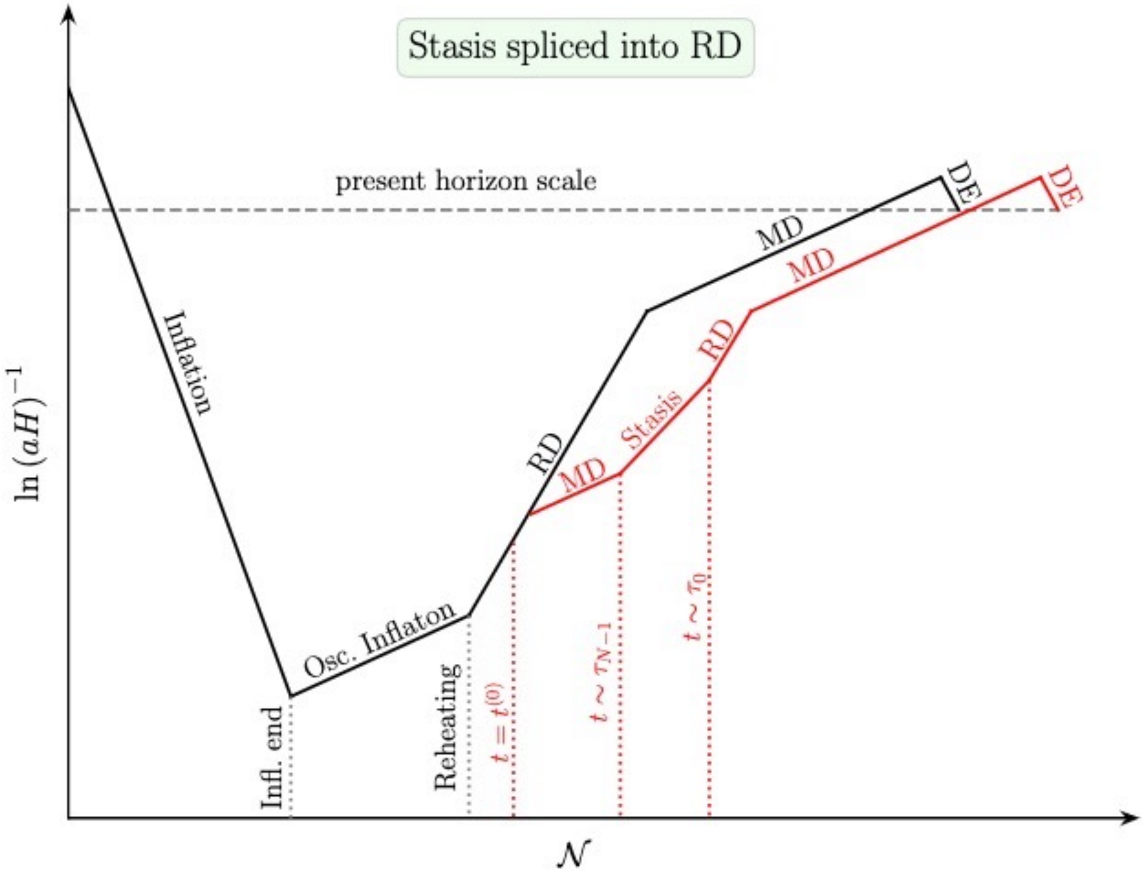
- Decays of a tower of states could result in an epoch in which the matter and radiation abundances are constant and can take any value.
- Such a scenario could occur naturally in models motivated by particle physics, like KK theory and strongly-coupled gauge theories.
- Stability analysis shows the stasis solution is a global attractor, No fine-tuning needed.

- Since the comoving Hubble radius grows more slowly, perturbation modes re-enter the horizon at a later time, can affect predictions for ***inflationary observables***
- ***Density perturbations*** grow faster during stasis than in ΛCDM, can result in formation of compact objects such as PBH or compact minihalos
- ***DM relic abundance*** would be affected if produced prior to or during Stasis due to different expansion history and injection of entropy. DM can also be the decay products of ϕ_ℓ or the lightest ϕ_ℓ with smaller decay width and parametrically smaller initial abundance.
- ***Generation of lepton or baryon asymmetry*** would also be affected if it occurs prior to or during Stasis, or if the asymmetry is produced through ϕ_ℓ decays
- Numerous directions to be explored…

Cosmological Implications

The insertion of a stasis period delays the subsequent timeline relative to ΛCDM expectations





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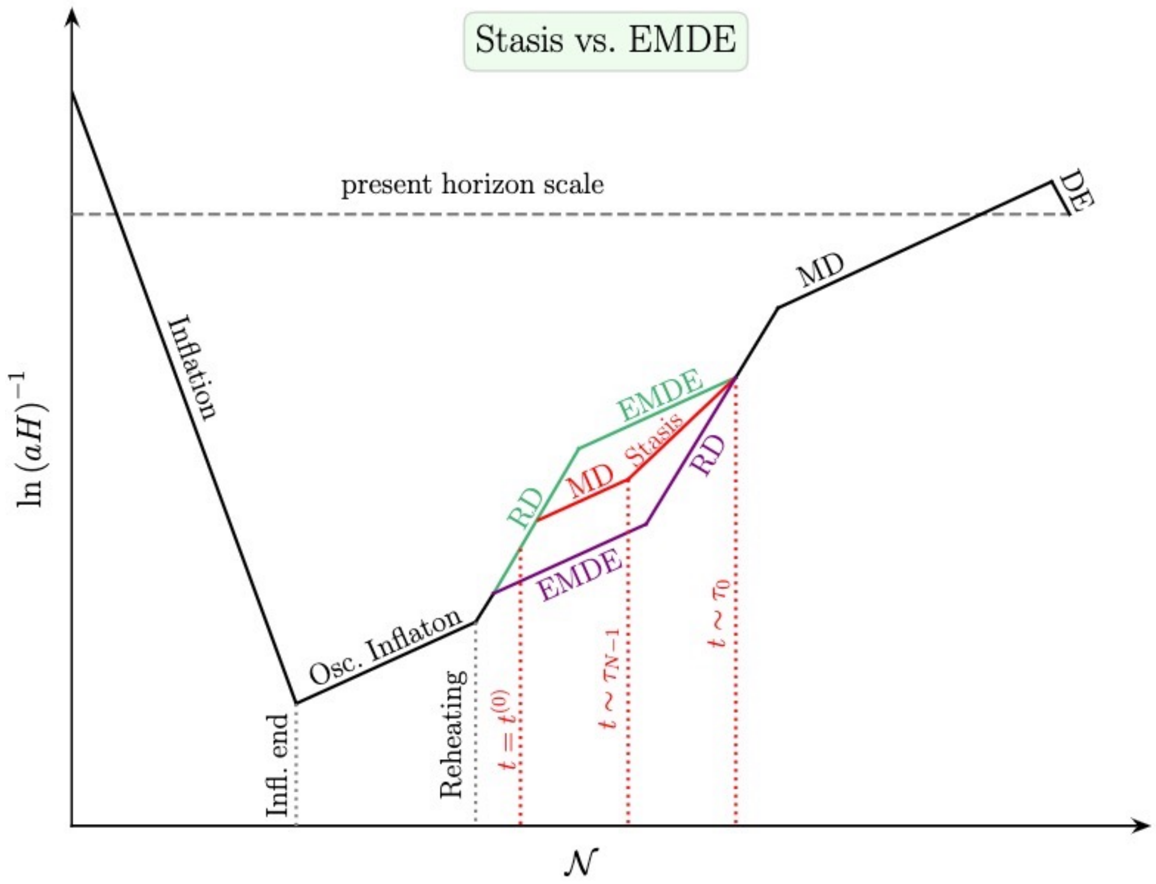
Cosmological Implications

Stasis and EMDE can place the universe on the same cosmological timeline



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