Gravitational Waves and Particle Physics

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Based on collaborations/discussions with
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1. Introduction

1-1. What is Gravitational Waves (GWs)?

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Einstein's General Theory of Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_P^2} T_{\mu\nu}$$

GWs are described as a tensor perturbation (h_{ij}) in Friedmann-Robertson-Walker metric:

$$ds^2 = a(\tau)^2 \left[-d\tau^2 + \left(\delta_{ij} + 2h_{ij} \right) dx^i dx^j \right]$$

with transverse & traceless conditions: $\partial^i h_{ij} = 0$ & $h^i_i = 0$

—> Two polarization states: + & x

GW’s EOM: $$h''_{ij} + 2\mathscr{H} h'_{ij} - \nabla^2 h_{ij} = \frac{1}{M_P^2}\Pi_{ij}^{TT}$$

Hubble parameter in τ: $\mathscr{H} = a'/a$

Transverse-traceless projection of T_{ij}: Π_{ij}^{TT}

In Fourier space:

$$h_{ij}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \left[\boxed{h_{\mathbf{k}}^{+}(\tau)} e_{ij}^{+}(\mathbf{x}) + \boxed{h_{\mathbf{k}}^{\times}(\tau)} e_{ij}^{\times}(\mathbf{x}) \right]$$

same as a massless scalar

Power spectrum: $$P_T(\tau, \mathbf{x}) = \frac{k^3}{2\pi^2}\left(|h_{\mathbf{k}}^{+}(\tau)|^2 + |h_{\mathbf{k}}^{\times}(\tau)|^2 \right)$$

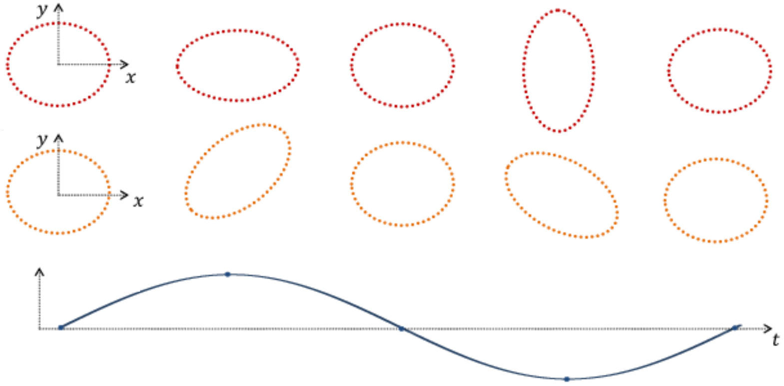
Energy spectrum: $$\Omega_{GW}(\tau, k) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k},$$

where $\rho_{GW}(\tau, k) = \frac{1}{a^2} M_P^2\, h'_{ij} h'^{ij}$ and $\rho_c = 3HM_P^2$

Displacement of test masses by GWs propagating in z-direction:

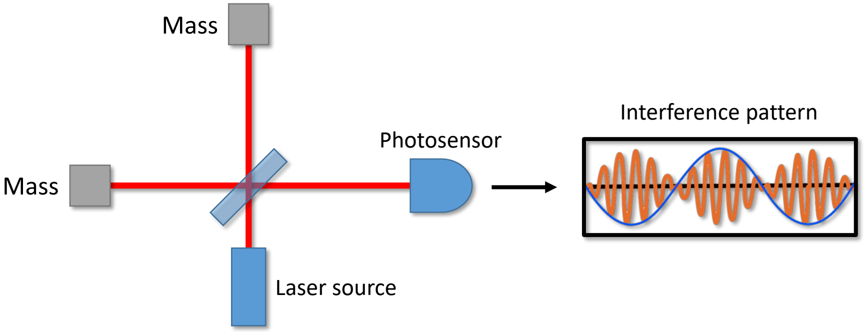
Polarizations +:

x:



Detection of GWs

- Direct: Interferometers



- Indirect: B-mode polarization of CMB (GWs from inflation)
  Pulsar timing arrays: GW effects on pulsar timing

We know GW signals are extremely weak.

Is it really possible to detect GWs?

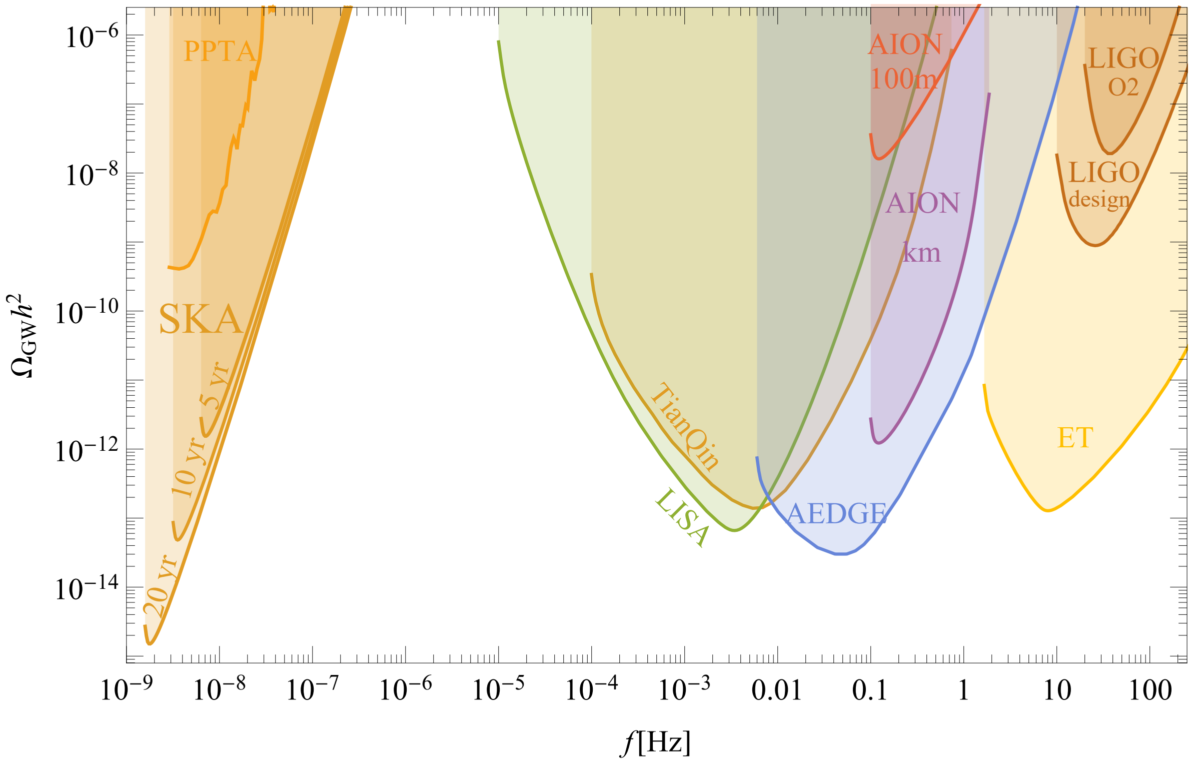
We know GW signals are extremely weak.

Is it really possible to detect GWs?

—> **YES!** GW150914 detection at LIGO!

It has opened up a possibility to detect GWs in a variety of frequencies.

On-going and planned GW detection experiments



1-2. Impact on Particle Physics?

$$h_{ij}'' + 2\mathscr{H} h_{ij}' - \nabla^2 h_{ij} = \frac{1}{M_P^2} \boxed{\Pi_{ij}^{TT}} \quad \text{<— source}$$

Sources of GWs?

- Astronomical sources: binaries, supernove

1-2. Impact on Particle Physics?

$$h''_{ij} + 2\mathscr{H} h'_{ij} - \nabla^2 h_{ij} = \frac{1}{M_P^2} \boxed{\Pi_{ij}^{TT}}$$ <— source

Sources of GWs?

- Astronomical sources: binaries, supernove
- Primordial/Cosmological sources

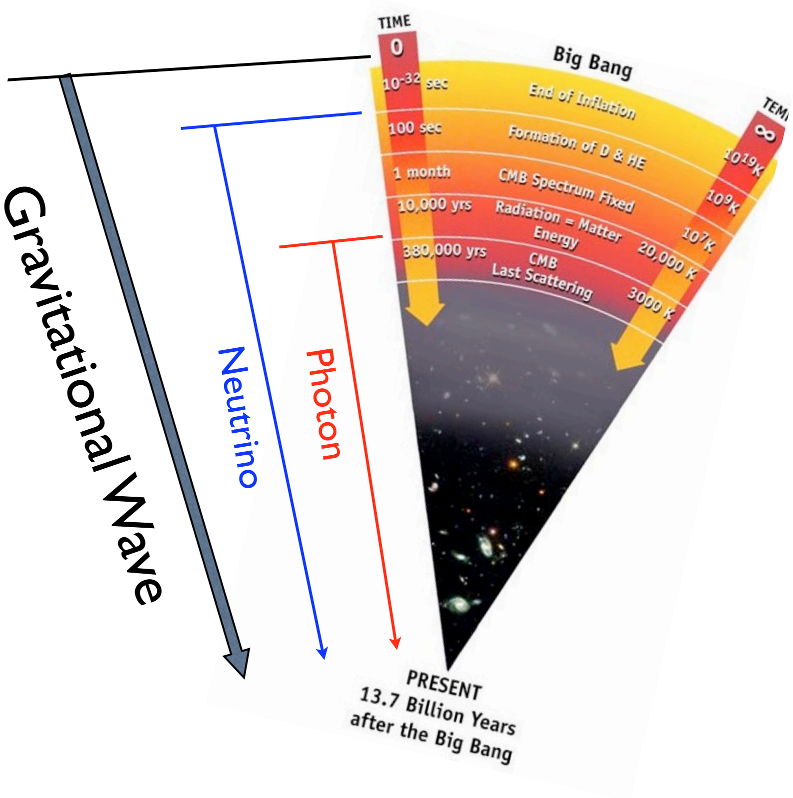
Examples: Cosmic Infaltion
1st order Phase transition
Topological defect
(Cosmic strings, Domain Walls, monopoles, etc)
Primordial Black Hole (formation)
Preheating/Fragments after inflation….

Sources = Physics Beyond the Standard Model

1-3. Exploring Early Universe (BSM in cosmology)

GWs carry the information from the "earliest Universe"!

GW detections as a probe of BSM!



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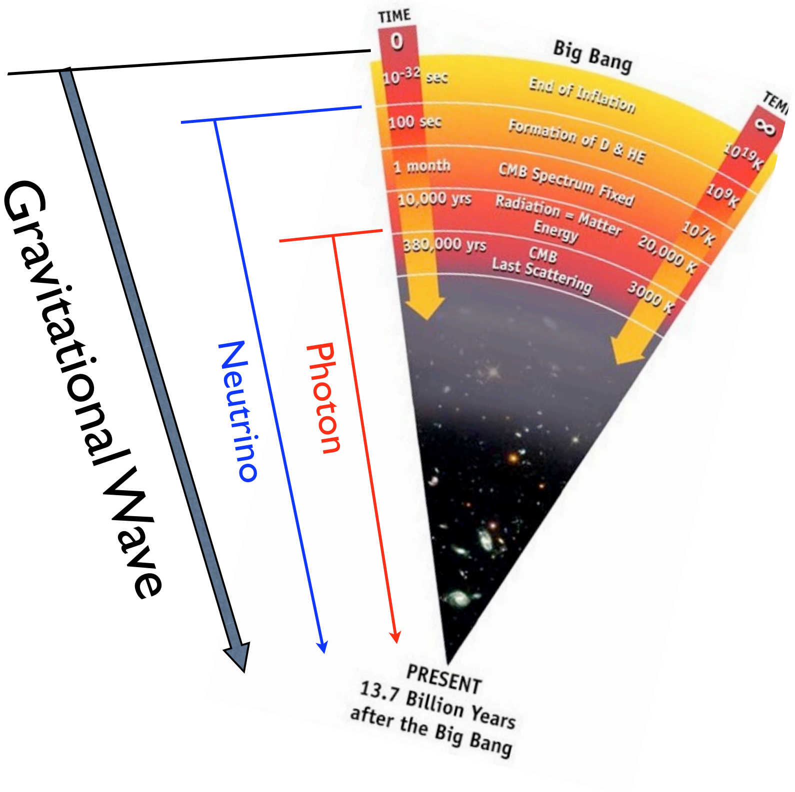


GWs carry the information
from the "earliest Universe"!

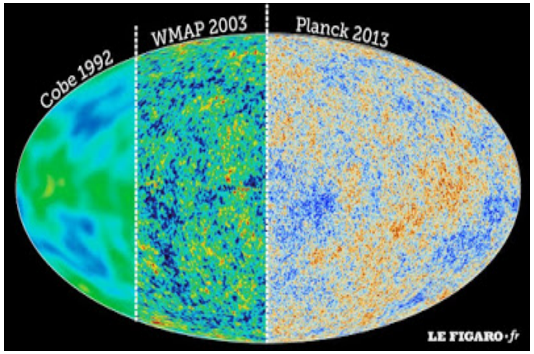
GW detections as a probe of BSM!

In the following, we will focus on
1. Cosmic Inflation
2. 1st order PT
3. Topological defect (cosmic strings)

2. GWs from Cosmic Inflation

Cosmic Inflation is the standard paradigm of the modern cosmology which can solve problems in Big Bang Cosmology

- Flatness problem
- Horizon problem
- Origin of the primordial density fluctuations



$$\boxed{\frac{\delta T}{T} \simeq 10^{-5}}$$

Seeds of the large scale structure

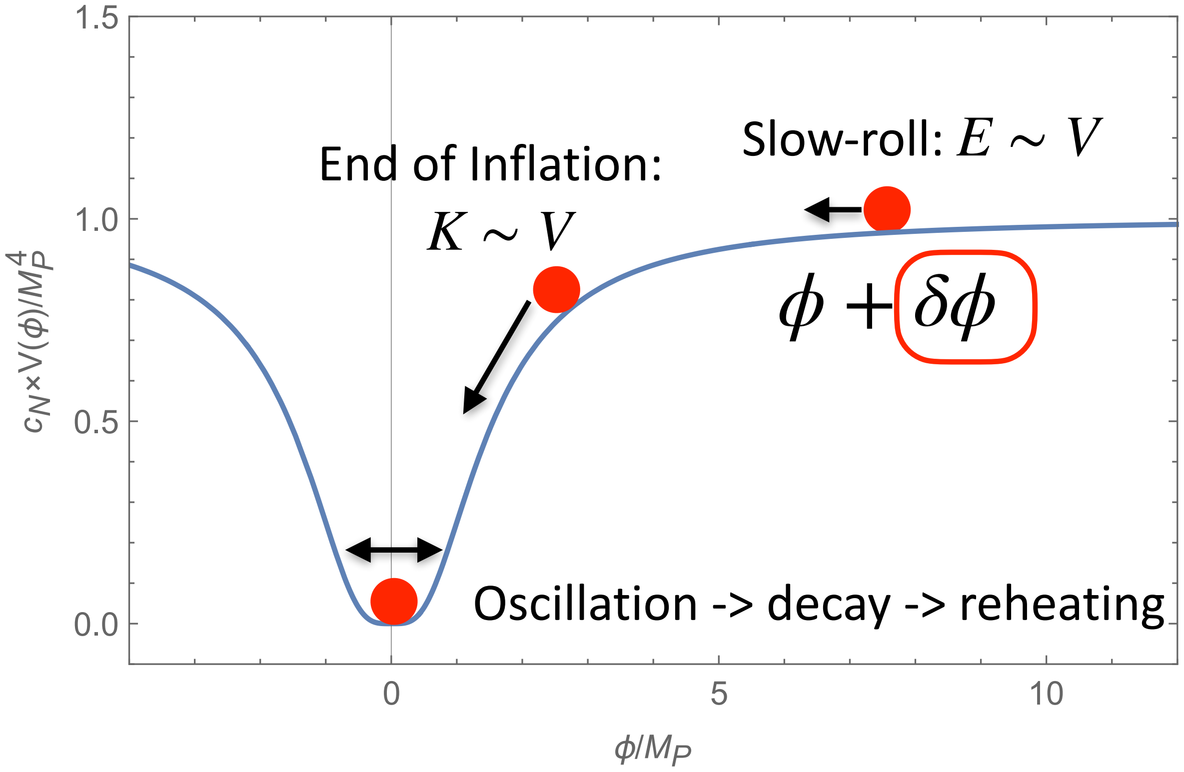
- Primordial GWs produced during inflation

Power spectrum
of tensor perturbation:

$$\boxed{P_T = \frac{8}{M_P^2}\left(\frac{H_{inf}}{2\pi}\right)^2}$$

Simple scenario: ``Slow-roll Inflation''

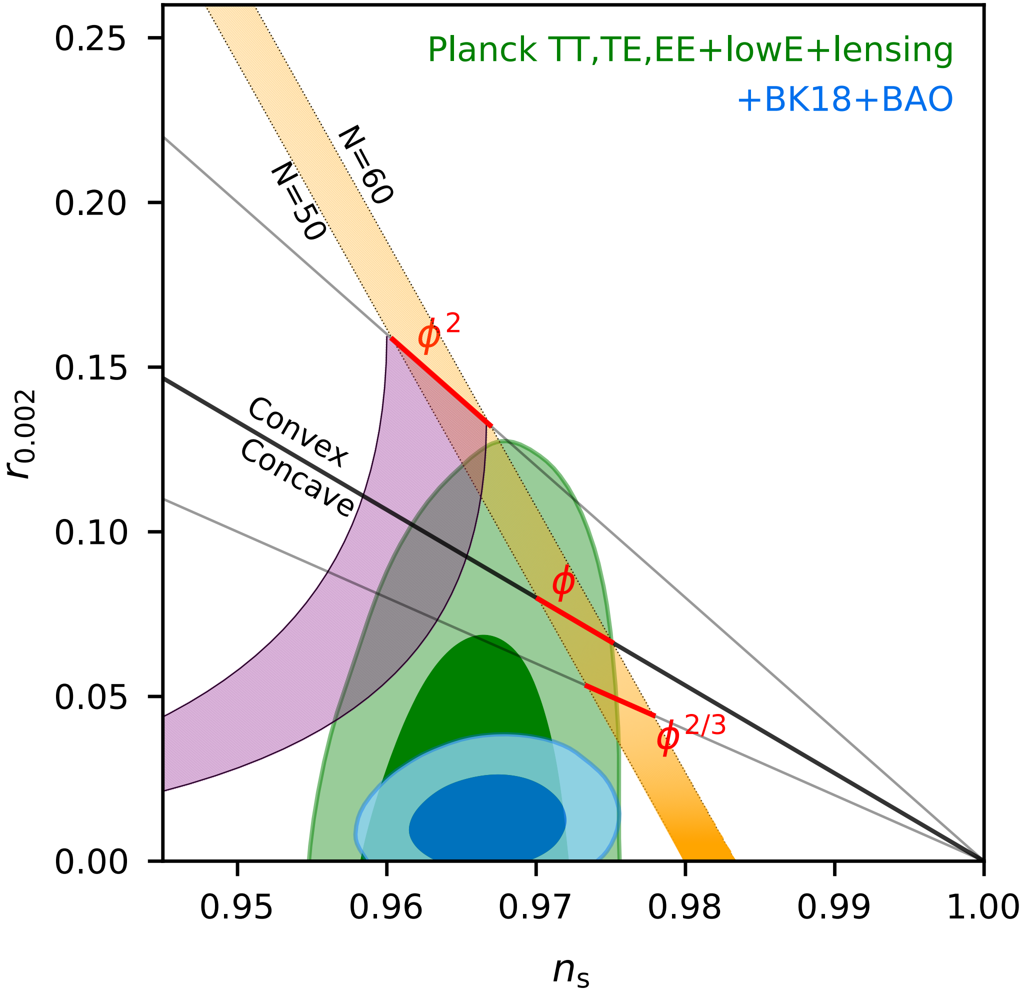
Inflation is driven by a slow-rolling scalar (Inflaton)



- Inflation takes place during slow-roll: $a(t) \propto e^{H_{inf} t}$
- Quantum fluctuation $\delta\phi$ is magnified to a macroscopic scale —> origin of the density fluctuation

Constraints on inflation scenario from CMB observations

BICEP/Keck 2018
PRL 127 (2021) 151301



Power spectrum of scalar perturbation:

$$\boxed{P_S(k_0) = 2.099 \times 10^{-9}}$$

$$k_0 = 0.05\ \mathrm{Mpc}^{-1}$$

Spectral index:

$$\boxed{n_s = 1 + \frac{d \ln P_S}{d \ln k} \simeq 0.965}$$

Tensor-to-scalar ratio:

$$\boxed{\frac{P_T}{P_S} = r \leq 0.036\ (95\%)}$$

Inflationary predictions of a slow-roll inflation

$$\mathscr{L}_{inf} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \boxed{V(\phi)}$$

Defining the slow-roll parameters (in Planck units $M_P = 1$)

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta = \frac{V''}{V}$$

the spectral index & tensor-to-scalar ratio:

$$n_s = 1 - 6\epsilon + 2\eta \, , \quad r = 16\epsilon$$

The power spectrum of scalar perturbation: $P_S = \frac{1}{12\pi^2}\frac{V^3}{(V')^2}$

The number of e-folds: $N_e = \int_{\phi_e}^{\phi_0} d\phi \frac{V}{V'}$

Here, $\phi = \phi_0$ at the horizon exit & the end of inflation $\epsilon(\phi_e) = 1$

Inflationary predictions of a slow-roll inflation

The power spectrum of scalar perturbation:

$$P_S = \frac{1}{12\pi^2}\frac{V^3}{(V')^2} \to 2.099 \times 10^{-9}$$

The number of e-folds: $N_e = \int_{\phi_e}^{\phi_0} d\phi \frac{V}{V'} \to$ Fix (say, 50-60)

$$\xrightarrow[\text{predictions}]{} \quad n_s \,\&\, r$$

Ex) A successful inflation scenario: non-minimal $\lambda\phi^4$ inflation

Action in the Jordan frame:

See, for example,
NO, Rehman & Shafi, PRD 82 (2010) 04352

$$\boxed{\mathcal{S}_J = \int d^4x\sqrt{-g}\left[-\frac{1}{2}f(\phi)\mathcal{R} + \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - V_J(\phi)\right]},$$

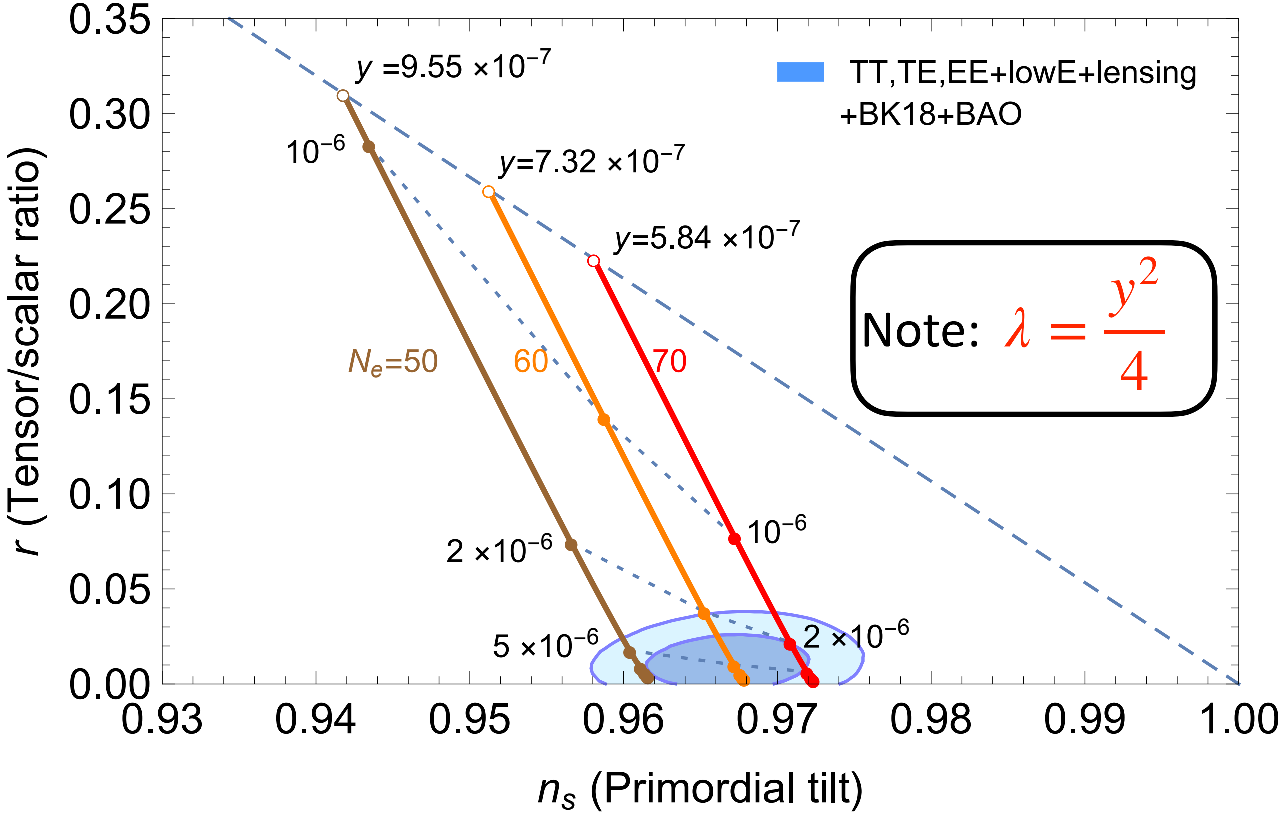
- Non-minimal gravitational coupling

$\boxed{f(\phi) = \left(1 + \xi\phi^2\right)}$ with a real parameter $\xi > 0$,

- Quartic coupling dominates during inflation

$$\boxed{V_J(\phi) = \frac{1}{4}\lambda\phi^4}$$

Inflationary Predictions VS Planck+BK18+BAO results



Note: $\lambda = \frac{y^2}{4}$

- Once N_e is fixed, only 1 free parameter (ξ) determines the predictions
- Predicted GWs are $r \gtrsim 0.003$

Future experiments (CMB-S4, LiteBIRD) will cover the region!

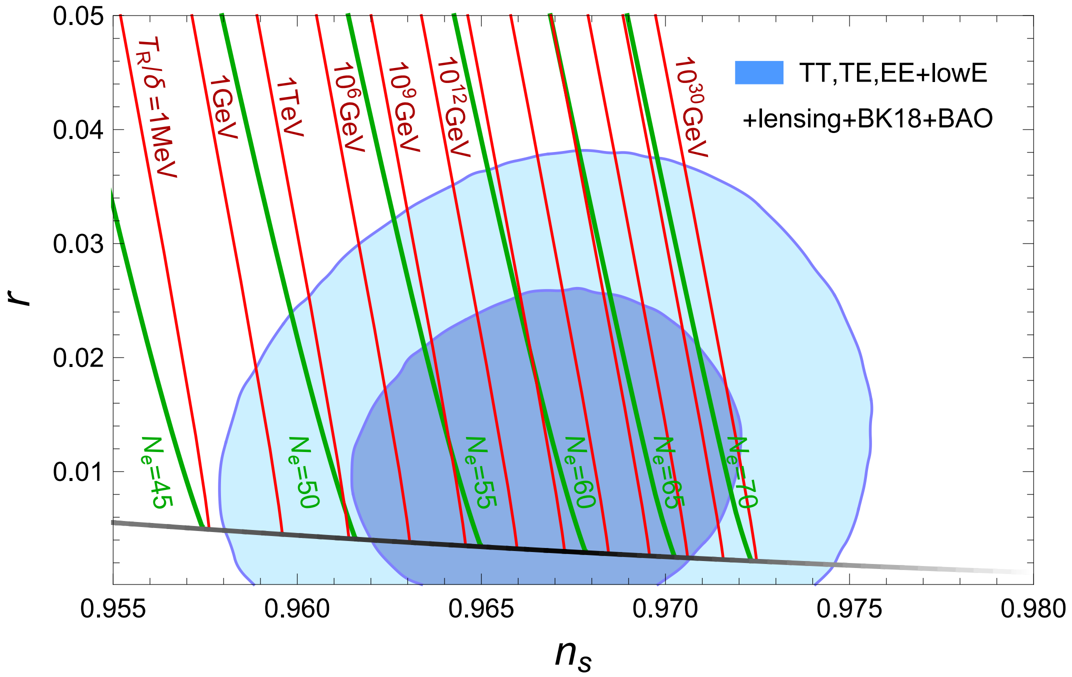
This scenario also predicts a relation between T_R & (n_s, r)

$$\boxed{N_k} \equiv \ln\frac{a_{\rm end}}{a_k} = 66.5 - \ln h - \ln\frac{k}{a_0 H_0} + \frac{1-3w}{12(1+w)}\ln\frac{\boxed{\rho_{\rm th}}}{\rho_{\rm end}} + \frac{1}{4}\ln\frac{V_k}{\rho_{\rm end}} + \frac{1}{4}\ln\frac{V_k}{M_{\rm P}^4} + \frac{1}{12}\left(\ln g_*^{\rm eq} - \ln g_*^{\rm th}\right),$$

e-folding number

reheating temperature: $\rho_{\rm th} = \frac{\pi^2}{30} g_*^{\rm th} T_R^4$

Shinsuke Kawai & NO, 2111.03645, to appear PRD



$$\delta = \frac{\rho_{\rm quad}}{\rho_{\rm end}}$$

For similar studies, see
Cheong, Lee & Park, 2111.00825;
Ellis, Garcia, Nanopoulos, Olive & Verner, 2112.04466

Non-minimal $\lambda\phi^4$ inflation

- Simple 1-field inflation with the introduction of $\xi|\phi|^2 R$
- Consistent with Planck + others with a suitable choice of quartic coupling $\lambda|\phi|^4$

Non-minimal $\lambda\phi^4$ inflation

- Simple 1-field inflation with the introduction of $\xi|\phi|^2 R$
- Consistent with Planck + others with a suitable choice of quartic coupling $\lambda|\phi|^4$
- Potentially, any scalar can play the role of inflaton

So, it may be more interesting if the inflaton in this scenario can play another important role in particle physics

An interesting possibility is the identification:

Inflaton = a Higgs field in a gauge extension of the SM

* SM Higgs is not likely the inflaton since its running quartic coupling runs into negative at high energies

Example: Minimal B-L Model

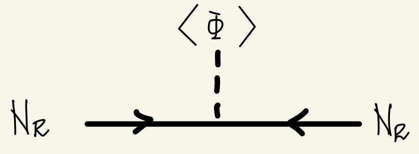
A simple & well-motivated U(1) gauge extension of the SM

	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{B-L}$
q_L^i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$+1/3$
u_R^i	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$+1/3$
d_R^i	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$+1/3$
ℓ_L^i	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1
N_R^i	$\mathbf{1}$	$\mathbf{1}$	0	-1
e_R^i	$\mathbf{1}$	$\mathbf{1}$	-1	-1
H	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
Φ	$\mathbf{1}$	$\mathbf{1}$	0	$+2$

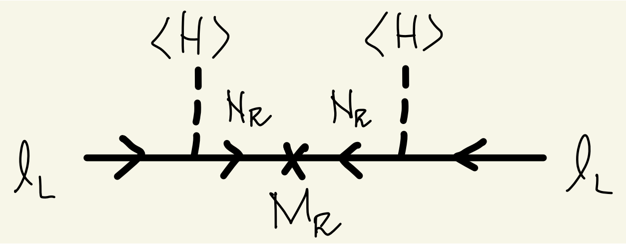
B-L Higgs field & Inflaton

Example: Minimal B-L Model

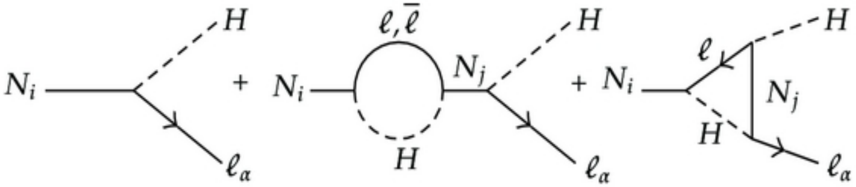
- Anomaly global B-L symmetry in the SM is gauged
- Right-handed neutrinos to cancel gauge/gravitational anomaly
- Spontaneous B-L gauge symmetry breaking to generate Majorana mass for RHNs



- Type-I seesaw mechanism after electroweak symmetry breaking



- Leptogenesis via CP-asymmetric out-of-equilibrium NR decay



Embedding into Grand Unified Theory is also possible

See for example
NO, Raut & Shafi, 1906.06869
NO, Seto & Uchida, 2006.01406

$$SO(10) \supset SU(5) \times U(1)_X$$

$$\mathbf{10} = \mathbf{5}(-2/5) + \mathbf{5}^*(2/5),$$

$$\boxed{\mathbf{16} = \mathbf{1}(1) + \mathbf{5}^*(-3/5) + \mathbf{10}(1/5),}$$

SM fermions + RHN

$$\mathbf{45} = \mathbf{1}(0) + \mathbf{10}(-4/5) + \mathbf{10}^*(4/5) + \mathbf{24}(0),$$

$$\mathbf{126} = \boxed{\mathbf{1}(2)} + \mathbf{5}^*(2/5) + \mathbf{10}(6/5) + \mathbf{15}^*(-6/5) + \mathbf{45}(-2/5) + \mathbf{50}^*(2/5).$$

U(1)x Higgs field & Inflaton

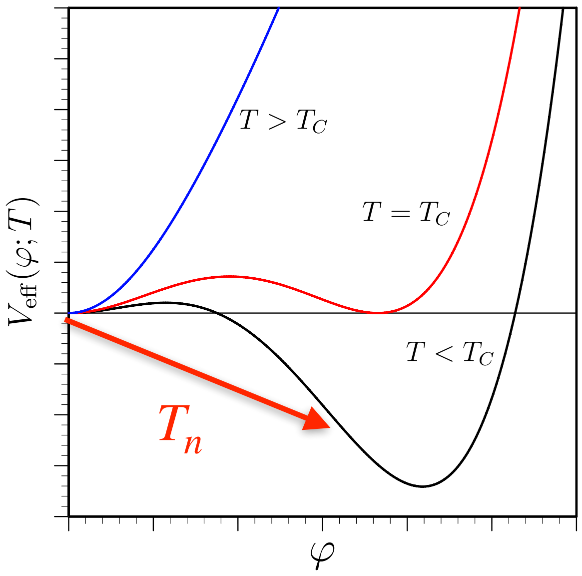
3. GWs from 1st order phase transition

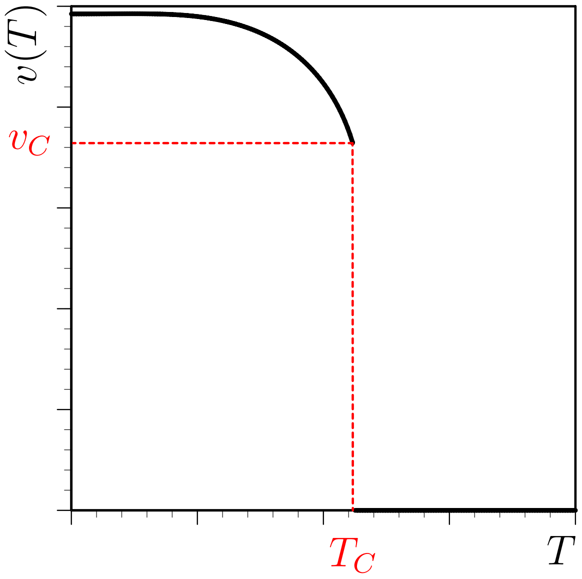
There are many well-motivated models beyond the SM, in which the SM gauge symmetry is extended.

We naturally expect that the universe experienced some phase transitions associated to the extended gauge symmetry breaking, in addition to the electroweak & QCD phase transitions in the SM.

If a gauge symmetry breaking exhibits 1st order phase transition, we may expect a large amplitude of GWs created by bubble dynamics.

1st order phase transition

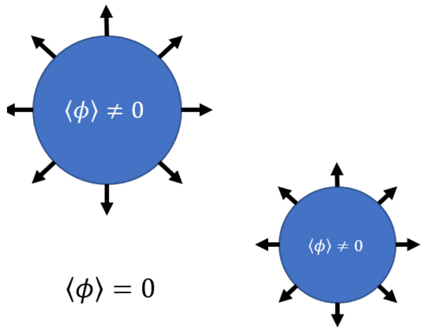




Bubble nucleation occurs at T_n (nucleation temp)
if the condition is satisfied:

Thermal bubble nucleation rate/vol

$$\boxed{\Gamma(T_n) \sim T_n^4 e^{-S_3/T_n} \sim H(T_n)^4}$$



Theory background: finite-temperature field theory

$$V_{\text{eff}}(\varphi, T) = V_0(\varphi) + \Delta V_{1-\text{loop}}(\varphi) + \Delta V_T(\varphi, T)$$

- Tree-level potential: $V_0(\varphi)$

- 1-loop effective potential:

$$\Delta V_{1-\text{loop}}(\varphi) = \sum_s g_s \frac{m_s^4}{64\pi^2}\left(\ln\frac{m_s^2}{Q^2} - c_s\right) - \sum_f g_f \frac{m_f^4}{64\pi^2}\left(\ln\frac{m_f^2}{Q^2} - c_f\right) + \sum_v g_v \frac{m_v^4}{64\pi^2}\left(\ln\frac{m_v^2}{Q^2} - c_v\right).$$

- Finite temperature corrections to the effective potential:

$$\Delta V_T(\varphi) = \sum_s g_s \frac{T^4}{2\pi^2} J_B(m_s^2/T^2) - \sum_f g_f \frac{T^4}{2\pi^2} J_F(m_f^2/T^2) + \sum_v g_v \frac{T^4}{2\pi^2} J_B(m_v^2/T^2)$$

$$J_{B,F}(y^2) = \int_0^\infty dx\, x^2 \log\left[1 \mp e^{-\sqrt{x^2+y^2}}\right]$$

Phase transition analysis

- Thermal bubble nucleation rate/vol

$$\Gamma(T) \sim T^4 e^{-S_3/T}$$

- 3-D Euclidean action

$$S_3 = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2} \left(\frac{d\varphi(r)}{dr} \right) + V(\varphi, T) \right]$$

with a <u>bounce solution</u> of $\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'$

$$\lim_{r\to\infty} \varphi(r) = 0 \;\&\; \lim_{r\to 0} \frac{d\varphi(r)}{dr} = 0$$

$\longrightarrow$ We fix T_n by $\Gamma(T_n) \sim T_n^4 e^{-S_3/T_n} \sim H(T_n)^4$

Characterizing the GW spectrum

- Nucleation temperature: T_n
- Phase transition strength: $\alpha = \dfrac{\Delta\rho(T_n)}{\rho_{\text{rad}}(T_n)}$
- Hubble normalized transition time scale: $\dfrac{\beta}{H(T_n)} = T\dfrac{d(S_3/T)}{dT}\Big|_{T=T_n}$
- Bubble wall velocity: v_b

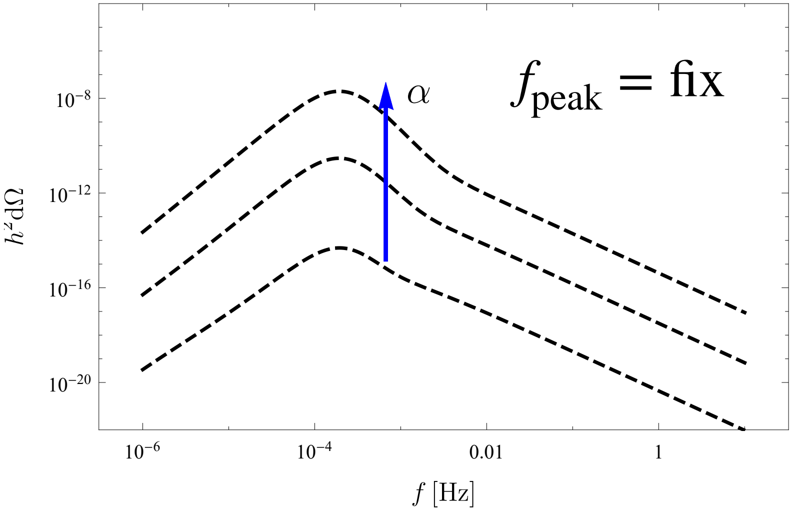
GW spectrum

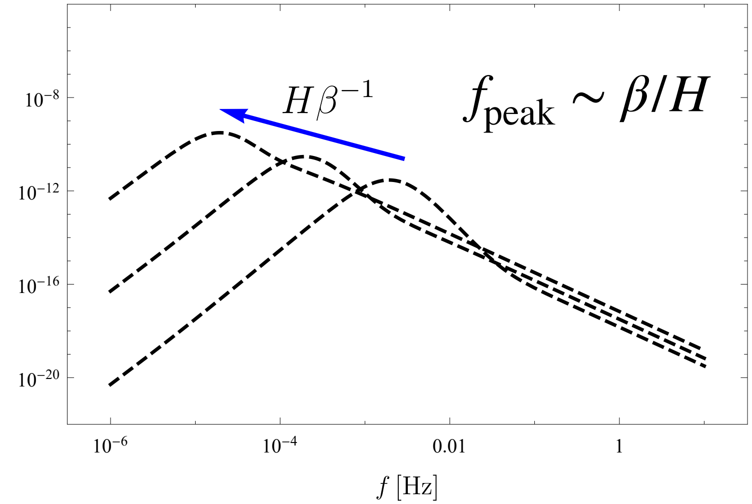
$$\Omega_{GW}(f) = \Omega_{GW}^{\text{coll}}(f) + \Omega_{GW}^{\text{sw}}(f) + \Omega_{GW}^{\text{turb}}(f)$$

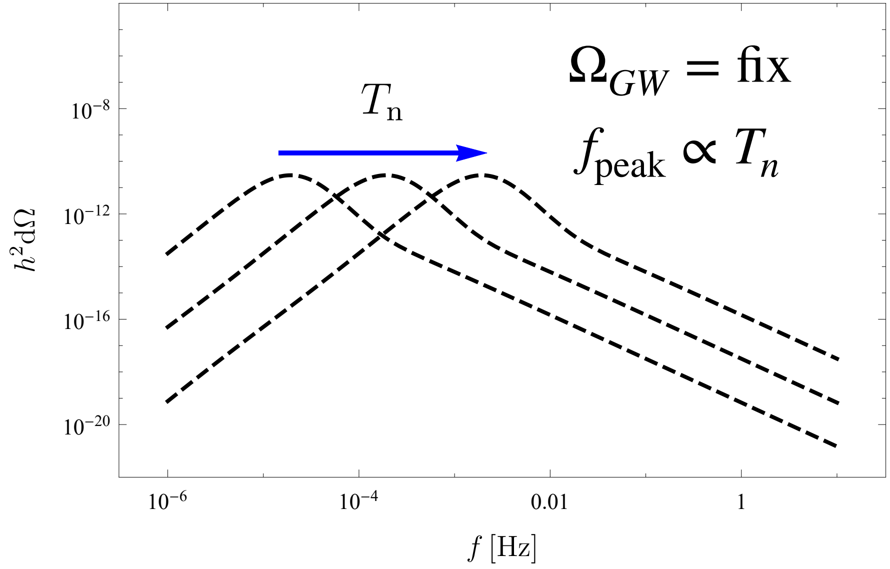
from 3 main sources: bubble collisions (coll), sound waves (sw) after bubble collisions, and turbulence (turn)

Fitting formulas for the spectrum are obtained by simulations

Huber et al., 0806.1828; Hindmarsh et al., 1504.03291; Caprini et al., 0909.0622, ..

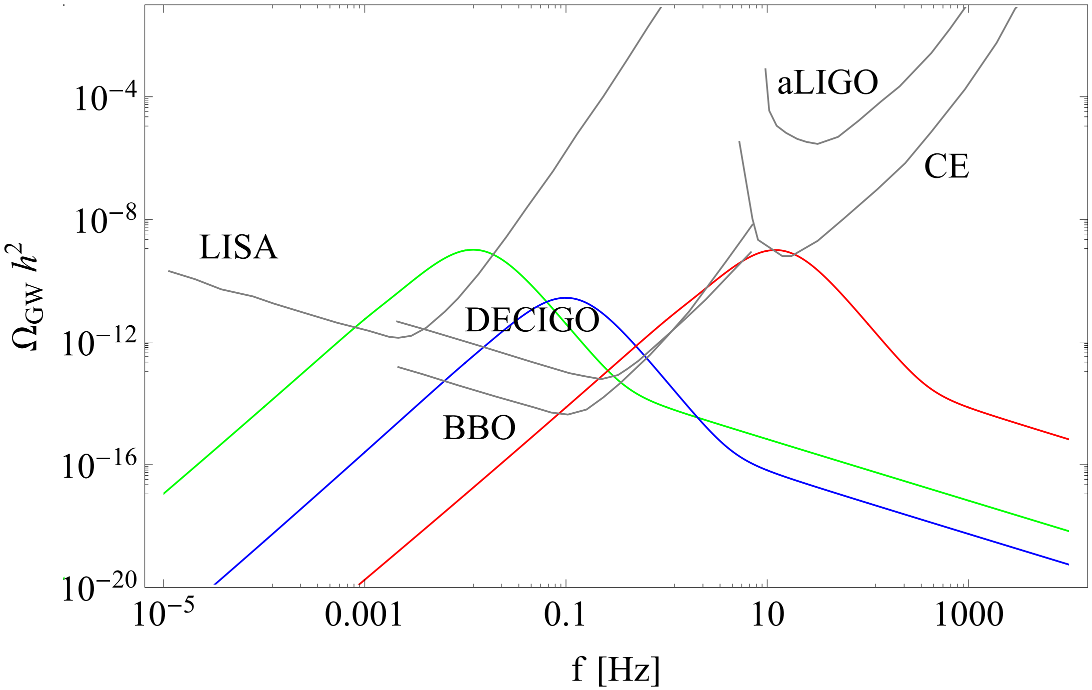

$f_{\text{peak}} = \text{fix}$
α
10^{-8}
10^{-12}
10^{-16}
10^{-20}
$h^2 d\Omega$
10^{-6}
10^{-4}
0.01
1
f [Hz]


$H\beta^{-1}$
$f_{\text{peak}} \sim \beta/H$
10^{-8}
10^{-12}
10^{-16}
10^{-20}
10^{-6}
10^{-4}
0.01
1
f [Hz]


$\Omega_{GW} = \text{fix}$
$f_{\text{peak}} \propto T_n$
T_{n}
10^{-8}
10^{-12}
10^{-16}
10^{-20}
$h^2 d\Omega$
10^{-6}
10^{-4}
0.01
1
f [Hz]

Example: Minimal B-L Model

Hasegawa, NO & Seto,
1904.03020



Benchmarks: $(g_{BL}, v_{BL}, \lambda_\Phi)$

$=(0.44,\ 4\,\mathrm{TeV},\ 1.5\times 10^{-4}),$

$(0.4,\ 12\,\mathrm{TeV},\ 2.0\times 10^{-4}),$

$(0.46,\ 3.8\,\mathrm{PeV},\ 4.0\times 10^{-4})$

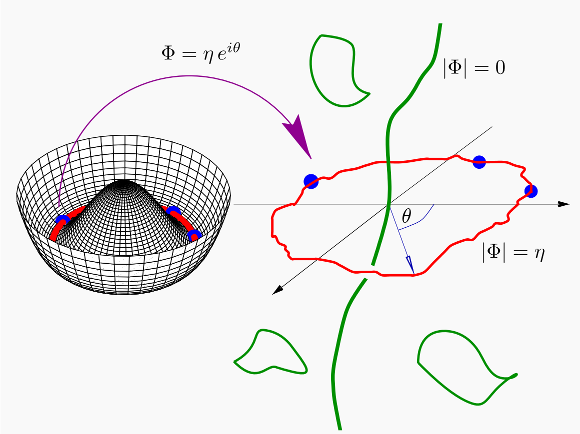
Probing the seesaw scale with GWs from 1st order PT!

4. GWs from Cosmic Strings

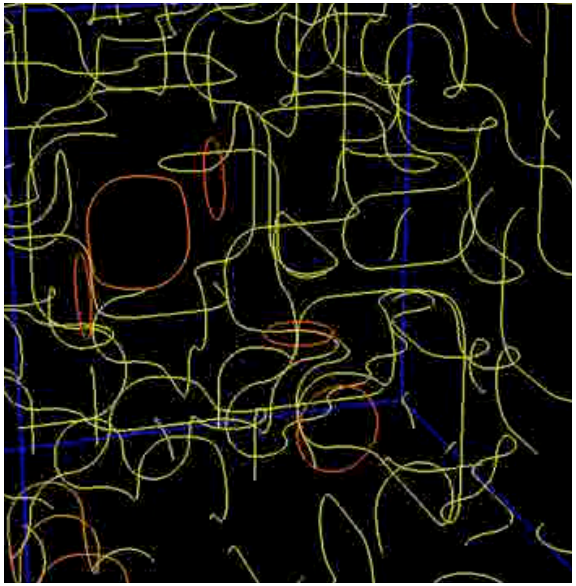
There are many well-motivated models beyond the SM, in which the SM gauge symmetry is extended.

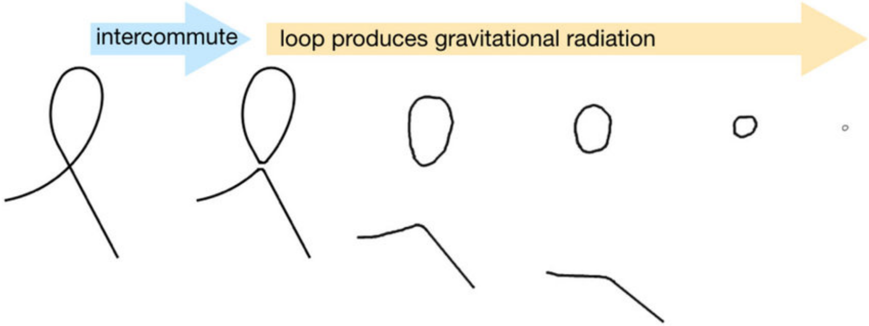
We naturally expect that the universe experienced some phase transitions associated to the extended gauge symmetry breaking, in addition to the electroweak & QCD phase transitions in the SM.

Cosmic strings are created after U(1) symmetry breaking



Cosmic string network of long strings and closed loops



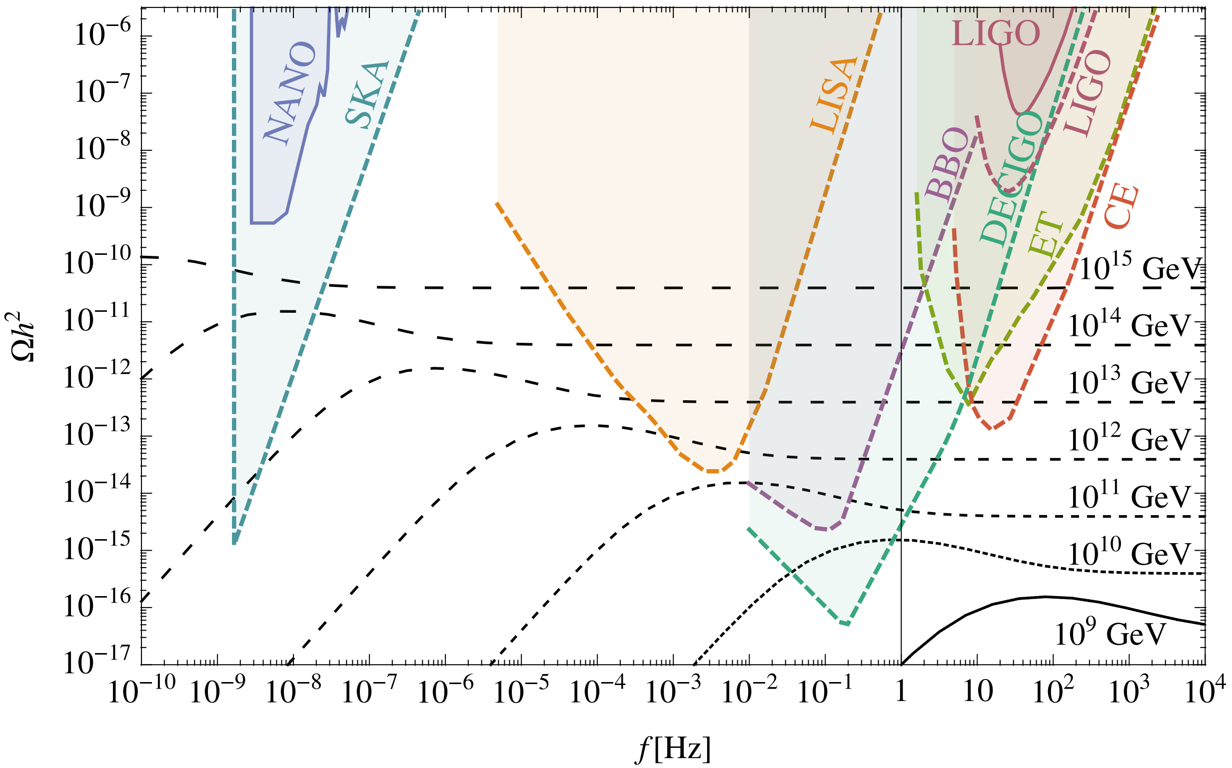


Oscillation of closed loops create GWs

GW spectrum is characterized by $G\mu \sim \frac{v_\phi^2}{M_{Pl}^2}$

GW spectrum is obtained by Lattice Simulations

Dror, Hiramatsu, Kohri, Murayama & White, 1908.03227

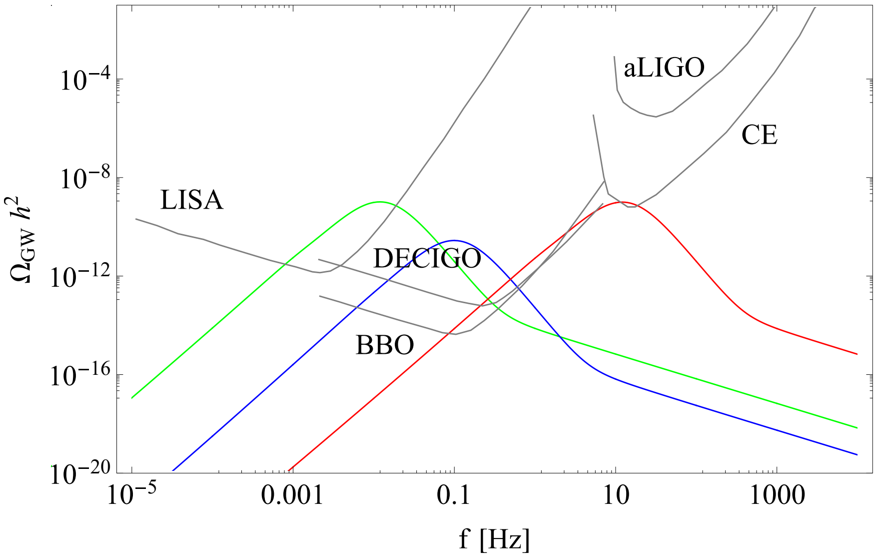


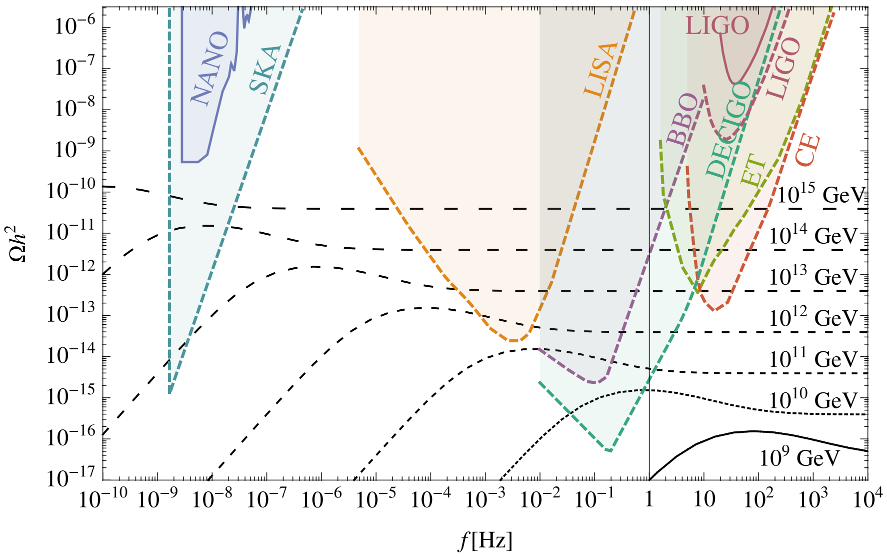
v_ϕ

- Spectrum of GWs from cosmic strings is flat
- Strength is going down as the VEV is decreases: $v_\phi \gtrsim 10^{10}$ GeV is necessary for detections
- If this is U(1) B-L string
  —> Probing the seesaw scale with GWs from Cosmic Strings!

Ex) Minimal B-L Model

GWs from 1st order PT VS. Cosmic strings





- Mountain-like shape
- Frequency must be matched
- $v_\Phi \lesssim 10^7\,\mathrm{GeV}$

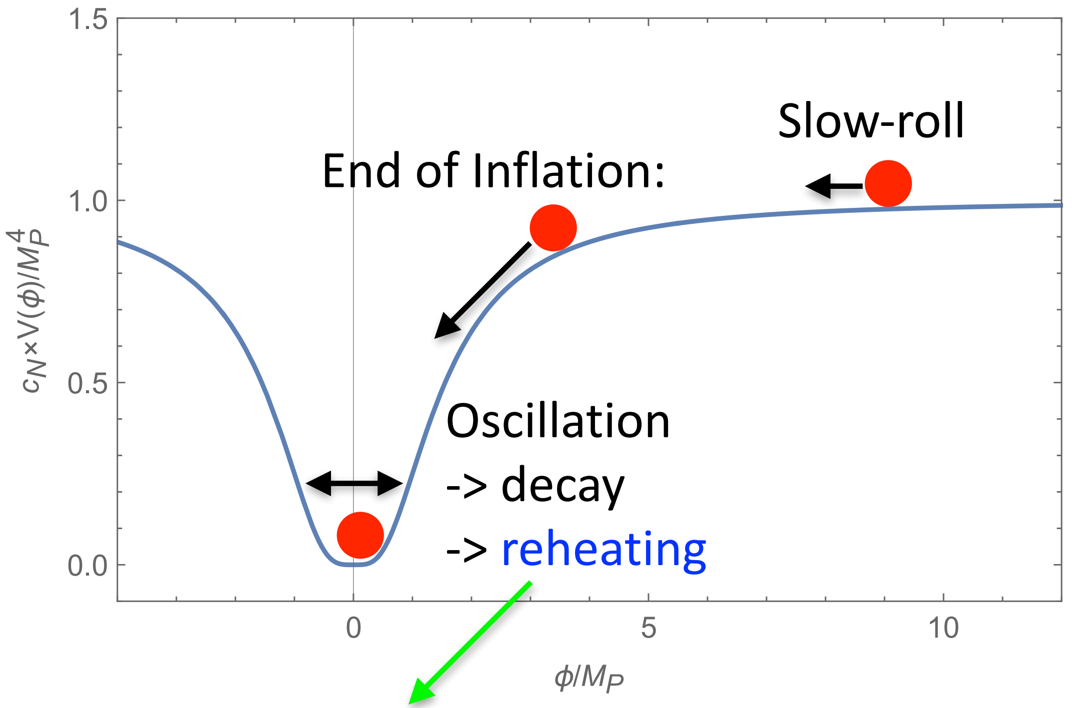
- Flat shape
- Easy to overlap
- $v_\Phi \gtrsim 10^{10}\,\mathrm{GeV}$

4. Summary

- GWs are messengers from very early universe
- GWs as a probe of BSM physics
- We have discussed 3 major sources:

  Inflation, 1st order phase transition & Cosmic string
- As a simple & well-motivated BSM, we consider the minimal B-L extended SM

Simple scenario based on the minimal B-L model



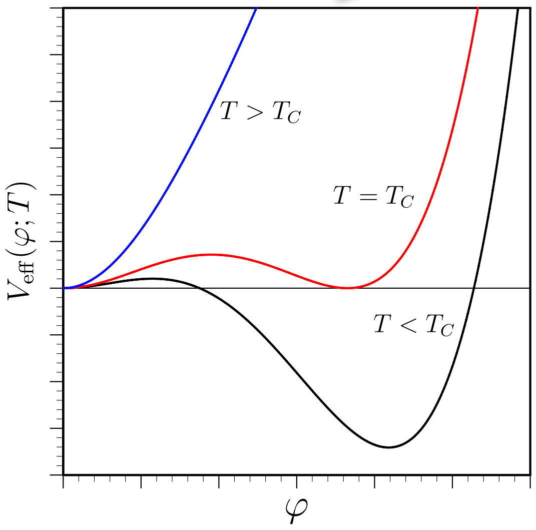
Slow-roll

End of Inflation:

Oscillation
-> decay
-> reheating

Non-minimal
B-L Higgs inflation

Primordial GWs
$r \gtrsim 0.003$



B-L symmetry is restored for $T_R > T_c$
1st order phase transition at T_n

GWs creations from
Bubble dynamics & Cosmic strings

Thank you

for your attention!