Dirac Masses and CKM Mixings in the (geo)SM(EFT) & Beyond

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[2107.03951]

JHEP w/ M. Trott

+ future work!

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Outline

All-Orders Formulae*

all-orders prelude

Applicability

Outlook and WIP

The geoSMEFT, Intuited

[1605.03602]
[1803.08001]
[1909.08470] $+\dots$
[geoSMEFT,2001.01453]

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

$$\mathcal{L}_{\rm SMEFT} = \sum_i G_i\left(I, A, \phi, \dots\right)\, f_i$$

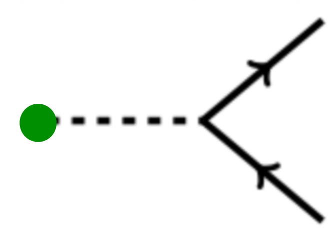
G: '*field space connections*' built from successive insertions of Higgs fields

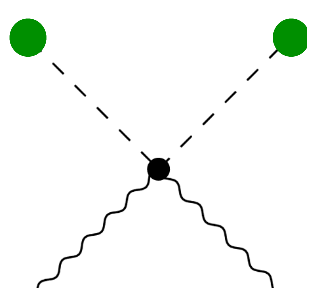
f: operator forms composed of Lorentz-index-carrying building blocks of the Lagrangian

$$H\left(\phi_I\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

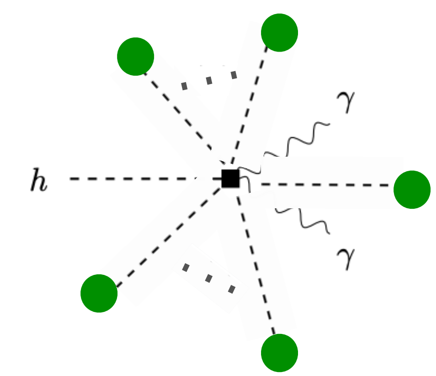
$$\bar{v}_T \equiv \sqrt{2\langle H^\dagger H\rangle}$$

$D \leq 4$





$D > 4$



[M. Trott KITP Talk]

vev -> fermion masses *-> boson masses* *-> geometries*

Gauge Field-Strength Terms at D=6 (e.g.)

$$\mathcal{L}_{\rm WB} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{C_{HW}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu},$$

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$\equiv -\frac{1}{4} g_{AB}(H) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu}$$

$$g_{ab} = \left(1 - 4\frac{C_{HW}}{\Lambda^2} H^\dagger H\right)\delta_{ab}$$

$$g_{a4} = g_{4a} = -2\frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma_a H.$$

$$g_{44} = 1 - 4\frac{C_{HB}}{\Lambda^2} H^\dagger H$$

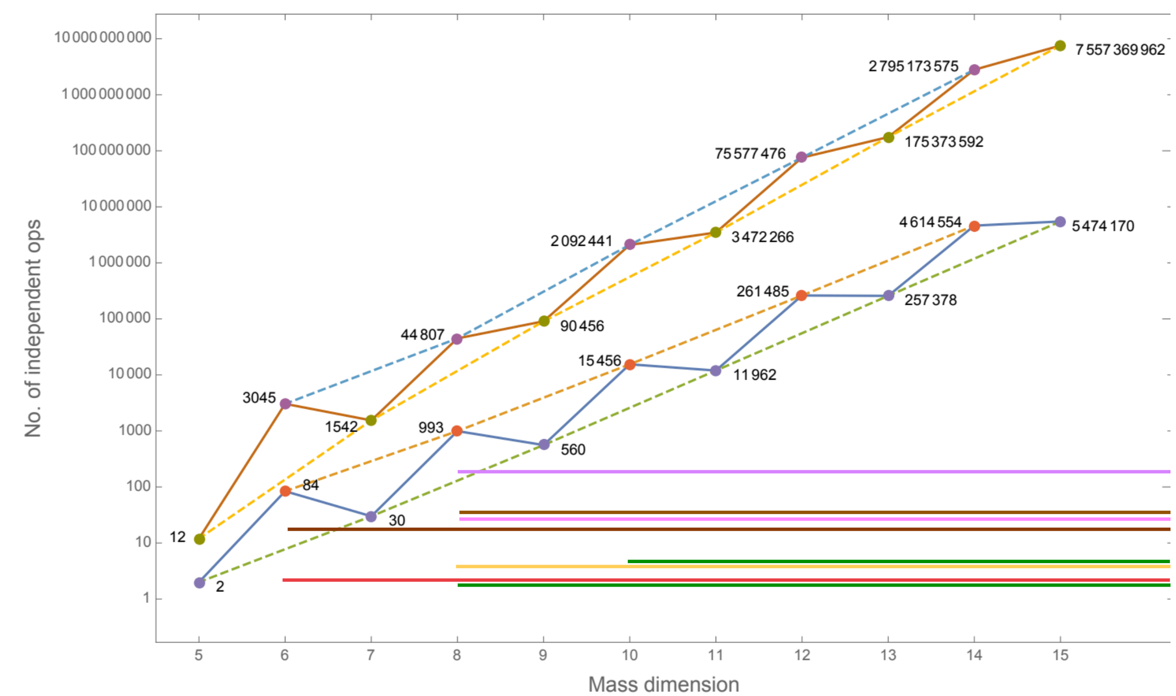
*Connection amounts to **metric in field space**, whose degree of curvature depends on size of v/Λ. The **SM** is therefore a **FLAT** direction!*

The geoSMEFT at 2 & 3 pts

[2001.01453]

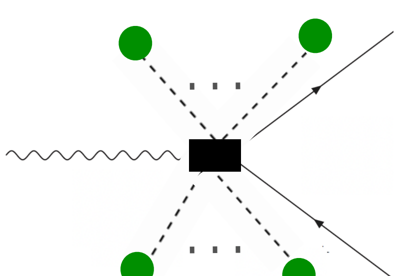
- EOM / Hilbert Series techniques allows for proof of ***all*** 2- and 3-pt field space connections!

Field space connection	Mass Dimension 6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u +$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d +$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e +$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A +$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A +$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A +$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$



[M. Trott KITP Talk] [1512.03433]

- All-orders connections *field-redefinition invariant* & yield large reduction in operators (EFT parameters)!
- Lagrangian parameters & Feynman rules obtained at all v/Λ orders ***before*** physical amplitude calculated!
- This is more than reorganization. It allows for all-orders amplitudes of fundamental processes:



$$\bar\Gamma_{Z\to\bar\psi\psi} = \sum_\psi \frac{N_c^\psi}{24\pi}\sqrt{\bar m_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar M_\psi^2}{\bar m_Z^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar g_Z}{2}\left[(2s^2_{\theta_Z}\,Q_\psi - \sigma_3)\delta_{pr} + \bar v_T\langle L^{\psi,pr}_{3,4}\rangle + \sigma_3\bar v_T\langle L^{\psi,pr}_{3,3}\rangle\right]$$

defined at all orders in v/Λ !!

Consistent SMEFT Phenomenology @ dim-8: [2007.00565][2107.07470][2102.02819][2203.11976]

Towards loop calculations at all v/Λ orders : [2106.10284]

Flavoring the geoSMEFT

[2107.03951]
[2001.01453]

- Yukawa-like operators of the SMEFT are given by

$$Q^{6+2n}_{\substack{\psi H\\pr}} = \left(H^\dagger H\right)^{n+1} \left(\bar\psi_{L,p}\psi_{R,r} H\right) \quad \text{with} \quad n \geq 0$$

- In the geoSMEFT formalism this all-order tower in v/Λ is captured by Yukawa field space connections:

$$Y^\psi_{pr}\left(\phi_I\right) = -H\left(\phi_I\right)\left[Y_\psi\right]^\dagger_{pr} + H\left(\phi_I\right)\sum_{n=0}^{\infty} C^{(6+2n)}_{\substack{\psi H\\pr}} \left(\frac{\phi^2}{2}\right)^n$$

	Mass Dimension				
Field space connection	6	8	10	12	14
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$

- From this one can immediately derive the all-orders effective Yukawa couplings, in terms of SM and SMEFT contributions:

$$\left[\mathcal{Y}^\psi\right]_{rp} = \frac{\delta (Y^\psi_{pr})^\dagger}{\delta h}\bigg|_{\phi_i\to 0} = \frac{\sqrt{h}^{44}}{\sqrt{2}}\left(\left[Y_\psi\right]_{rp} - \sum_{n=3}^{\infty}\frac{2n-3}{2^{n-2}}\tilde{C}^{(2n),\star}_{\substack{\psi H\\pr}}\right)$$

$$\left[M_\psi\right]_{rp} = \langle (Y^\psi_{pr})^\dagger\rangle$$

What about actual mass eigenstates and mixing parameters?

$$[U^\dagger_{\psi L}]_{ir}\,[\mathcal{Y}^\psi\mathcal{Y}^{\psi,\dagger}]_{rp}\,[U_{\psi L}]_{pj} = \text{diag}\left(y^2_{\psi 1}, y^2_{\psi 2}, y^2_{\psi 3}\right)$$

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Basis of Invariants for Quarks

[0907.4763]
[1507.00328]

Do you know how to write $y^2(Y)$, $\theta(Y)$, $\delta(Y)$?

*Calculate **invariants** under U(3)!*

$$Y^\psi Y^{\psi\dagger} \to U^\dagger Y^\psi Y^{\psi\dagger} U \quad U(3)_{Q_L}$$

*Structure given by (known) **Hilbert Series**!*

$$H(q) = h(q,q) = \frac{1+q^{12}}{(1-q^2)^2(1-q^4)^3(1-q^6)^4(1-q^8)}$$

- A set of 11 invariants can be found to fully parameterize the theory, including six 'unmixed' *I*

$$YY^\dagger \equiv \mathbb{Y}$$

$$I_1 \equiv \mathrm{tr}\left(\mathbb{Y}_u\right), \quad \hat{I}_3 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\right), \quad \hat{I}_6 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_u\right) = 3\det\mathbb{Y}_u$$

$$I_2 \equiv \mathrm{tr}\left(\mathbb{Y}_d\right), \quad \hat{I}_4 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_d\right), \quad \hat{I}_8 \equiv \mathrm{tr}\left(\mathbb{Y}_d\,\mathrm{adj}\,\mathbb{Y}_d\right) = 3\det\mathbb{Y}_d$$

- as well as four 'mixed' *I*, relevant for extracting information about the CKM (overlap) matrix

$$\hat{I}_5 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathbb{Y}_d\right), \quad \hat{I}_7 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathbb{Y}_d\right), \quad \hat{I}_9 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right), \quad \hat{I}_{10} \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right)$$

- and finally one mixed, CP-odd invariant relevant to pinning down the overall sign of CP violation:

$$I_{11}^{-} = -\frac{3i}{8}\det\left[\mathbb{Y}_u, \mathbb{Y}_d\right]$$

proportional to to the Jarlskog Invariant J!

- The fundamental geoSMEFT object we can construct at all-orders is then given by

$$\mathbb{Y}_{rp} = \frac{\hbar}{2}\left(Y_{ri}Y^\star_{pi} - \sum_{n'}^{\infty} f(n')Y_{ri}\tilde{C}^{(2n')}_{ip} - \sum_{n}^{\infty} f(n)\tilde{C}^{(2n),\star}_{ir}Y^\star_{pi} + \sum_{n,n'}^{\infty} f(n)f(n')\tilde{C}^{(2n),\star}_{ir}\tilde{C}^{(2n')}_{ip}\right)$$

All-Orders Formulae: Masses

[2107.03951]

- Unmixed invariants can be solved to obtain exact formulae for Yukawa couplings / masses:

$$y_i^2 = \frac{(-2)^{1/3}}{3\,\psi_u}\left(I_1^2 - 3\,\hat{I}_3 + (-2)^{-1/3}\; I_1\,\psi_u + (-2)^{-2/3}\;\psi_u^2\right)\,,$$

$$y_{j,k}^2 = \frac{1}{12\psi_u}((-2)^{4/3}\; I_1^2 - 3\cdot(-2)^{4/3}\;\hat{I}_3 + 4\,I_1\,\psi_u$$

$$\mp\,\psi_u\sqrt{24\left(I_1^2 - 3\,\hat{I}_3\right) + \frac{6\cdot(-2)^{5/3}\left(I_1^2 - 3\,\hat{I}_3\right)^2}{\psi_u^2} - 3\cdot(-2)^{4/3}\,\psi_u^2 + (-2)^{2/3}\,\psi_u^2})$$

$$\psi_u = \left(-2\,I_1^3 + 9\,I_1\hat{I}_3 - 9\,\hat{I}_6 + 3\sqrt{-3\,I_1^2\hat{I}_3^2 + 12\,\hat{I}_3^3 + 4\,I_1^3\hat{I}_6 - 18\,I_1\hat{I}_3\hat{I}_6 + 9\,\hat{I}_6^2}\right)^{1/3}$$

Valid for up-quark masses. Send $I_{1,3,6}$ to $I_{2,4,8}$ for down quark masses.

- Of course, the only distinction between fermions of the same family are their (measured) mass eigenvalues….

$$y_u^2 \equiv \min\{y_i^2, y_j^2, y_k^2\}\,, \qquad y_c^2 \equiv \mathrm{mid}\{y_i^2, y_j^2, y_k^2\} \qquad y_t^2 \equiv \max\{y_i^2, y_j^2, y_k^2\}$$

All-Orders Formulae: Mixings & CP

[2107.03951]

- Similarly, the mixed invariants give predictions for (CKM) mixing angles:

$$s_{13} = \left[\frac{-\hat{I}_{10} - y_b^2 \left(\hat{I}_7 - \Delta_{ds}^+ \Delta_{uc}^+ \Delta_{ut}^+ \right) - y_u^2 \left(\hat{I}_9 + y_b^2 \left(\hat{I}_5 - y_b^2 \Delta_{ct}^+ \right) - y_d^2 \, y_s^2 \, \Delta_{ct}^+ \right)}{\Delta_{bd}^- \Delta_{bs}^- \Delta_{cu}^- \Delta_{ut}^-} \right]^{1/2} \qquad \Delta_{ij}^{\pm} \equiv y_i^2 \pm y_j^2$$

$$s_{23} = \left[\frac{\Delta_{tu}^- \left(-\hat{I}_{10} + y_c^2 \left(-\hat{I}_9 + (y_b^4 + y_d^2 y_s^2)\, \Delta_{ut}^+ \right) + y_b^2 \left(-\hat{I}_7 + y_c^2 \left(-\hat{I}_5 + \Delta_{ct}^+ \Delta_{ds}^+ \right) + y_u^2 \Delta_{ct}^+ \Delta_{ds}^+ \right) \right)}{\Delta_{ct}^- \left(\hat{I}_{10} + y_u^2 \, \hat{I}_9 + y_b^2 \left(\hat{I}_7 + y_u^2 \left(\hat{I}_5 - 2\Delta_{ct}^+ \Delta_{ds}^+ \right) \right) - (y_u^4 + y_c^2 y_t^2)\,(y_b^4 + y_d^2 y_s^2) \right)} \right]^{1/2}$$

$$s_{12} = \left[\frac{\Delta_{db}^- \left(\hat{I}_{10} + y_s^2 \left(\hat{I}_7 - y_c^2 y_t^2 \Delta_{db}^+ \right) \right) + y_u^2 \, \Delta_{bd}^- \left(-\hat{I}_9 - y_s^2 \, \hat{I}_5 + \Delta_{sb}^+ \Delta_{ct}^+ \Delta_{ds}^+ \right) + y_u^4 y_s^2 \, (y_b^4 - y_d^4)}{\Delta_{ds}^- \left(\hat{I}_{10} + y_u^2 \, \hat{I}_9 + y_b^2 \left(\hat{I}_7 + y_u^2 \left(\hat{I}_5 - 2\Delta_{ct}^+ \Delta_{ds}^+ \right) \right) - (y_b^4 + y_d^2 y_s^2)\,(y_u^4 + y_c^2 y_t^2) \right)} \right]^{1/2}$$

- When combined with the CP-odd 11th invariant, one also can derive the Dirac CP-violating phase (and its sign!)

$$s_\delta = \frac{4}{3} \, I_{11}^- \left[\Delta_{tc}^- \Delta_{tu}^- \Delta_{cu}^- \Delta_{bs}^- \Delta_{bd}^- \Delta_{sd}^- \, s_{12} s_{13} s_{23} \left(1 - s_{23}^2\right)^{1/2} \left(1 - s_{12}^2\right)^{1/2} \left(1 - s_{13}^2\right) \right]^{-1}$$

Here one notices the proportionality to the Jarlskog as well!

These formulae…

- are **exact**, and **analytically** relate the fundamental Lagrangian parameters to the \`physical' masses, mixings, and phase (*for the first time, to my knowledge*).
- **complete** the list of all-orders Lagrangian parameters in the Dirac flavor sector of the geoSMEFT.
- are **basis independent** (as long as the information required is present in the basis in question).
- are applicable to explicit **(B)SM models and EFTs**, when global $U(3)_Q$ flavor rotations control flavor parameters.

Powerful tools in the description of (B)SM flavor physics!

Applicability

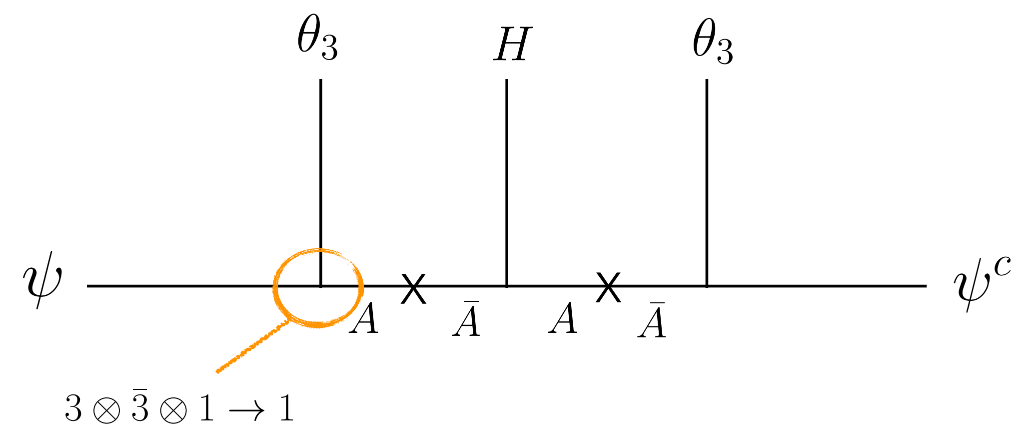
Applications: UV-completing flavor

[2107.03951]

The Universal Texture Zero Model

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

[de Medeiros Varzielas, Ross, Talbert: 1710.01741]



$$\mathcal{L}_{\rm UTZ} \supset \psi_p \left(\frac{1}{M^2_{3,f}} \theta^p_3 \theta^r_3 + \frac{1}{M^3_{23,f}} \theta^p_{23} \theta^r_{23} \Sigma + \frac{1}{M^3_{123,f}} (\theta^p_{123} \theta^r_{23} + \theta^p_{23} \theta^r_{123}) S \right) \psi^c_r H + \mathcal{O}(1/M^4) + \ldots$$

- After flavor- and EW-symmetry breaking, the EFT/model shapes Yukawa/mass matrices of the form

$$\mathcal{M}^D_f = \begin{pmatrix} 0 & a\,e^{i\gamma} & a\,e^{i\gamma} \\ a\,e^{i\gamma} & \left(b\,e^{-i\gamma} + 2a\,e^{-i\delta}\right) e^{i(\gamma+\delta)} & b\,e^{i\delta} \\ a\,e^{i\gamma} & b\,e^{i\delta} & 1 - 2a\,e^{i\gamma} + b\,e^{i\delta} \end{pmatrix}_f$$

- Proof-in-principle fits to global flavor data yield post-dictions for mass (ratios) and CKM mixing angles:

$$\frac{m_u}{m_t} = 7.16 \cdot 10^{-6}\,, \quad \frac{m_c}{m_t} = 0.0027\,, \quad \frac{m_d}{m_b} = 0.00090\,, \quad \frac{m_s}{m_b} = 0.020$$

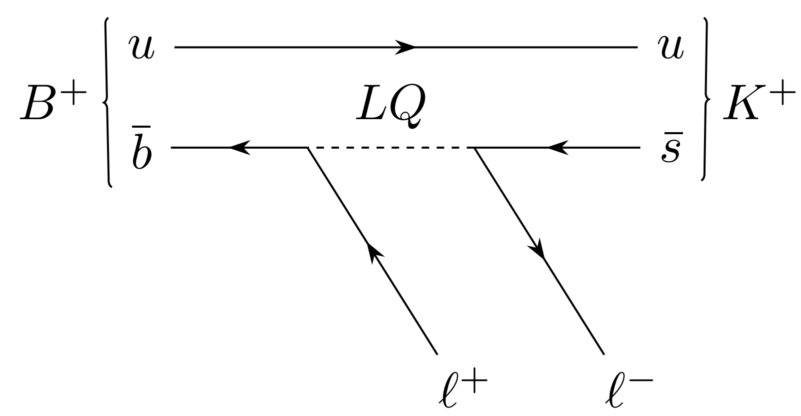
$$s_{12} = 0.226, \quad s_{23} = 0.0191, \quad s_{13} = 0.0042, \quad s_\delta = 0.5609$$

✅ currently working on an MCMC fit to the UTZ!

Applications: BSM States in the IR

[2107.03951]

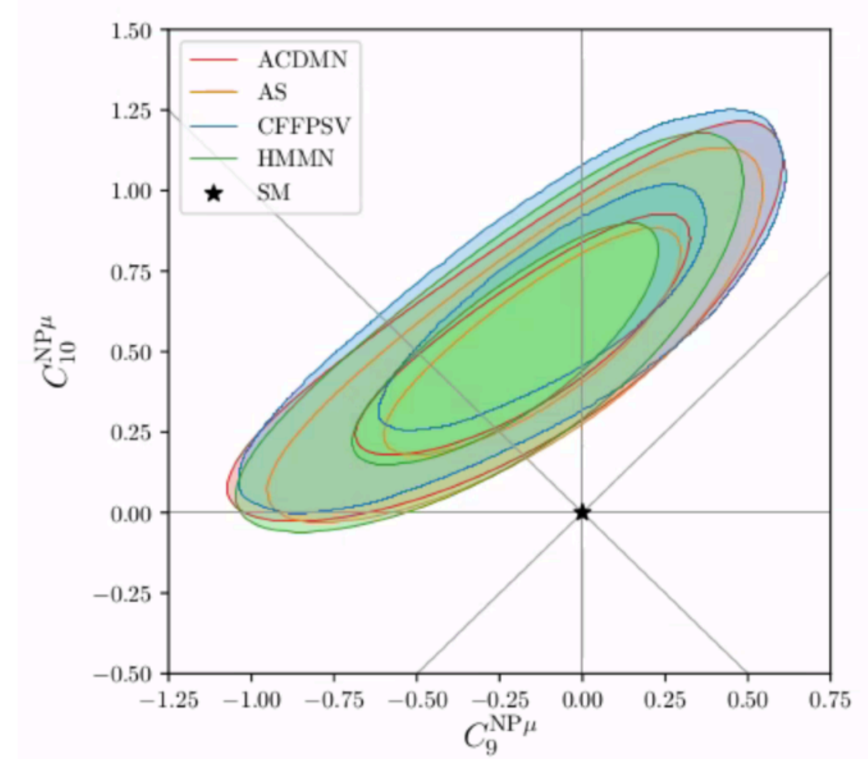
$$R_H \equiv \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\mathcal{B}(B\to H\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\mathcal{B}(B\to He^+e^-)}{dq^2} dq^2}$$



$$\Delta_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$$

$$\mathcal{G}_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{L} \supset y^{LL}_{3,ij} \bar{Q}^{C\,i,a}_L \epsilon^{ab} (\tau^k \Delta^k_3)^{bc} L^{j,c}_L + z^{LL}_{3,ij} \bar{Q}^{C\,i,a}_L \epsilon^{ab} ((\tau^k \Delta^k_3)^\dagger)^{bc} Q^{j,c}_L + \text{h.c.}$$



fit to LFU observables + $B_s \to \mu\mu$

[Capdevila et al, 2021 Flavor Anomaly Workshop]

- Regardless of the introduction of new IR flavor violation, Dirac mass and mixing still predictable!

EFT for CKM + PMNS + Leptoquarks

	Q''^1	Q''^{23}	u''^1_R	u''^2_R	u''^3_R	d''^1_R	d''^2_R	d''^3_R	ϕ_u	ϕ_d
D_{15}	$\mathbf{1}_-$	$\mathbf{2}_\mathbf{1}$	$\mathbf{1}_-$	$\mathbf{1}$	$\mathbf{1}_-$	$\mathbf{1}_-$	$\mathbf{1}$	$\mathbf{1}_-$	$\mathbf{2}_\mathbf{1}$	$\mathbf{2}_\mathbf{1}$

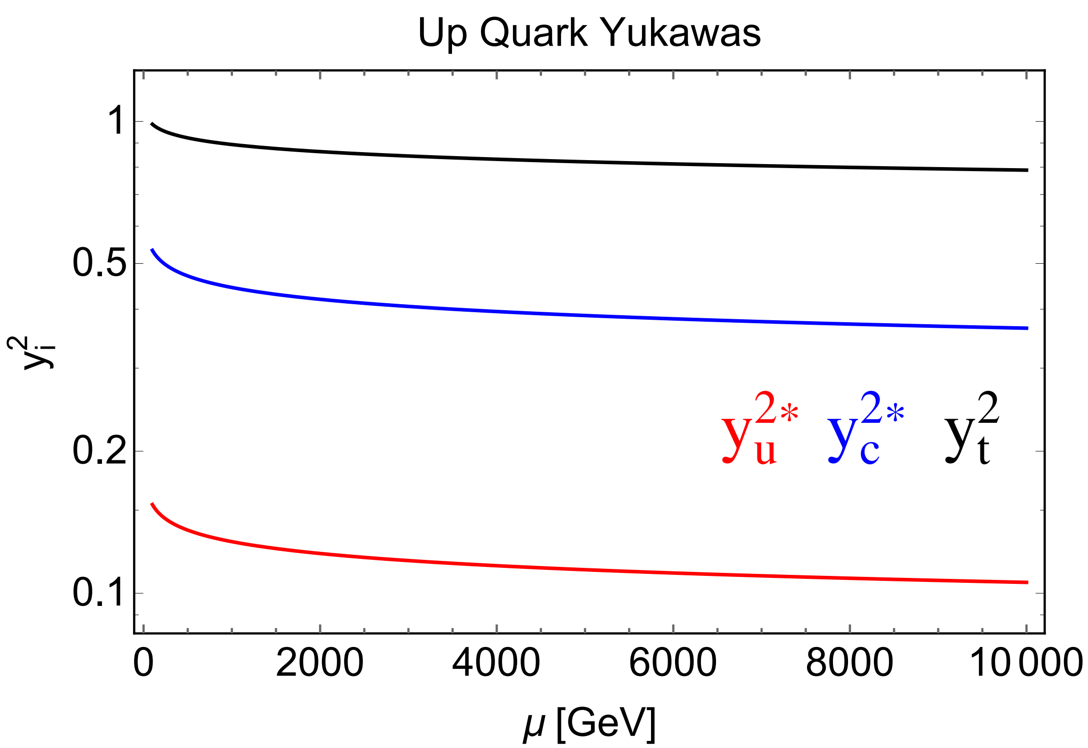
[Bernigaud, de Medeiros Varzielas, Talbert: 2005.12293]

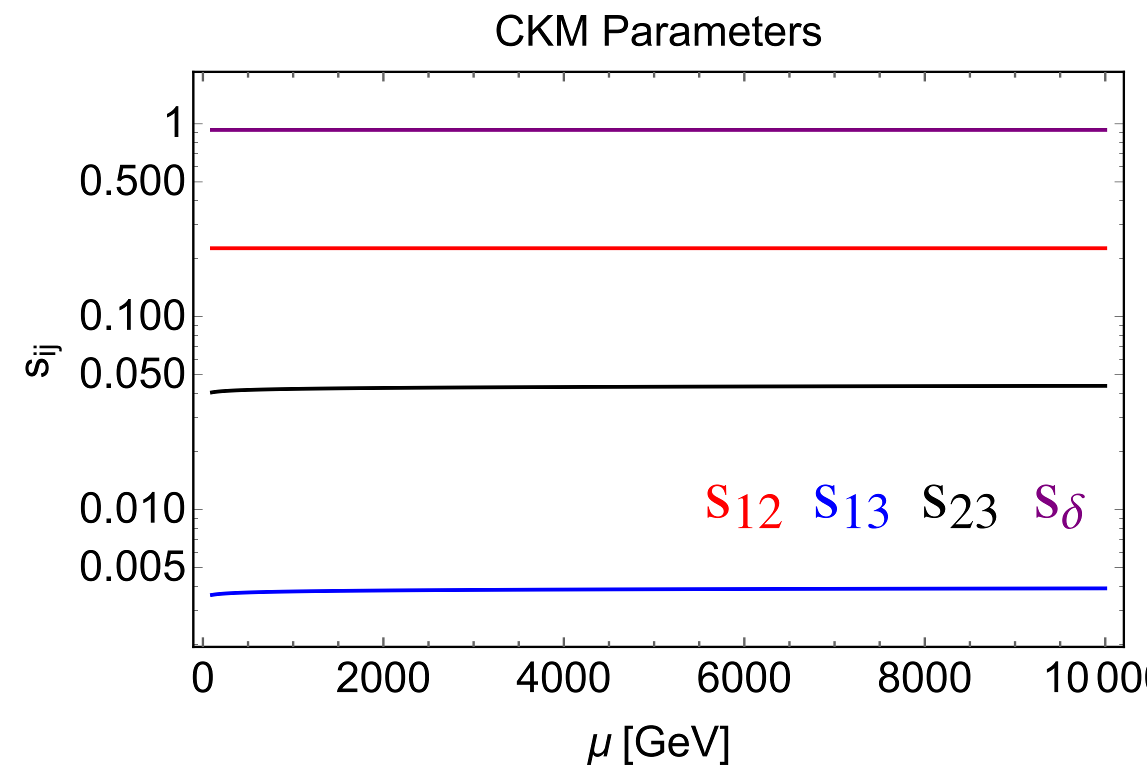
$$\mathcal{L}_Y \supset a_u \bar{Q}''^1_L u''^1_R + b_u \left[\bar{Q}''^{23}_L \phi_u\right]_\mathbf{1} u''^2_R + c_u \left[\bar{Q}''^{23}_L \phi_u\right]_{\mathbf{1}_-} u''^3_R$$
$$+ a_d \bar{Q}''^1_L d''^1_R + b_d \left[\bar{Q}''^{23}_L \phi_d\right]_\mathbf{1} d''^2_R + c_d \left[\bar{Q}''^{23}_L \phi_d\right]_{\mathbf{1}_-} d''^3_R,$$

$$Y''_u = P^\dagger \Lambda_d V^\dagger_{CKM} \cdot \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \cdot \Lambda^\dagger_U P, \quad Y''_d = P^\dagger \Lambda_d \cdot \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \cdot \Lambda^\dagger_D P$$

(even in an absurd model basis!)

Applications: Renormalization Group Flow





[2107.03951]

$$\dot{s}_\delta = s_\delta\left[\frac{\dot{I}^-_{11}}{I^-_{11}} - \sum_{(ij)\in \mathfrak{s}_2}\frac{\dot{\Delta}^-_{ij}}{\Delta^-_{ij}} - \dot{s}_{12}\frac{(1-2\,s_{12}^2)}{s_{12}c_{12}^2} - \dot{s}_{23}\frac{(1-2\,s_{23}^2)}{s_{23}c_{23}^2} - \dot{s}_{13}\frac{(1-3\,s_{13}^2)}{s_{13}c_{13}^2}\right]$$

(e.g)

$$\mu\frac{dI^-_{11}}{d\mu} \simeq (6a_0 + 6b_0 + 2a_1 I_1 + 2b_1 I_2)\, I^-_{11}$$

$$a_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_1 - I_2}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad a_1 = \frac{3}{16\pi^2}$$

$$b_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_2 - I_1}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad b_1 = \frac{3}{16\pi^2}$$

[1507.00328]

- Note however that latter formulae only hold for MFV theories (numerics done for SM limit)!
- Would be interesting to pursue more generic RGE studies in SMEFT (e.g. 2005.12283).

Speculative Applications & Ongoing WIP

- An obvious application of the flavored geoSMEFT would be to **higher-dimension fits of CKM** parameters in SMEFT (see e.g. 1812.08163 & backup slide):

$$O_\alpha = O_{\alpha,\mathrm{SM}}(W_j) + \delta O^{\mathrm{direct}}_{\alpha,\mathrm{NP}} = O_{\alpha,\mathrm{SM}}(\widetilde{W}_j) + \delta O^{\mathrm{indirect}}_{\alpha,\mathrm{NP}} + \delta O^{\mathrm{direct}}_{\alpha,\mathrm{NP}}$$

$$W_j \equiv \{\lambda, A, \bar{\rho}, \bar{\eta}\} \qquad \widetilde{W}_j = W_j \left(1 + \frac{\delta W_j}{W_j}\right)$$

$$\tilde{\lambda} = 0.22537 \pm 0.00046$$
$$\tilde{A} = 0.828 \pm 0.021$$
$$\tilde{\rho} = 0.194 \pm 0.024$$
$$\tilde{\eta} = 0.391 \pm 0.048$$

Extracted using dim-6 SMEFT in [1812.08163]

- In addition, I have recently shown that `**Weinberg connection**' ~LL saturates in mass dimension:

$$\mathcal{L} \supset \eta(\phi)_{\alpha\beta}\, \ell^\alpha \ell^\beta$$

Field space connection	Mass Dimension 5	7	9	11	13
$\eta(\phi)_{\alpha\beta}\, \ell^\alpha \ell^\beta + \mathrm{h.c.}$	$2 \cdot 2N_f$	?	?	?	?

✅

- …as do Yukawa and LNV Majorana mass terms in **(geo)νSMEFT**, which I am presently building!

$$\mathcal{L} \supset Y^\nu_{pr}(\phi)\, \overline{L}\, N + M_{pr}(\phi)\, \overline{N}\, N$$

Field space connection	Mass Dimension 6	8	10	12	14
$Y^\nu_{pr}(\phi)\, \overline{L}\, N + \mathrm{h.c.}$	$2\, N_f^2$	?	?	?	?
$M_{pr}(\phi)\, \overline{N}\, N + \mathrm{h.c.}$	$2 \cdot 2N_f$	?	?	?	?

✅

- With knowledge of Hilbert Series/invariants, all-orders flavor can be derived for neutrino mass+mixing!

[0907.4763] [2107.06274] [1010.3161]

Summary and outlook

- One can construct basis-independent flavor formalisms using invariant theory.
- These formalisms depend exclusively on flavor symmetry and free parameters.
- As a result, they hold at all-orders in effective field theories, e.g. the (geo)SMEFT.
- We have presented analytic formulae for the Dirac masses and CKM mixings present in the (geo)SM(EFT). They are useful in any number of (B)SM contexts.
- Phenomenological applications are obvious, including fits to mass and mixing.
- The extension of the formalism to neutrino physics is ongoing, and rich in application.

THANK YOU!

Backup Slides

Numerical Checks

[2107.03951]

- To test the validity of our formulae, we wrote a script to compare values of Dirac parameters predicted from our formulae vs. those extracted with numerical techniques. It did so by…

(a) computing the eigenvectors of $\left[\mathcal{Y}^{(u,d)}\mathcal{Y}^{(u,d),\dagger}\right]$. These are normalized to unit vectors $\mathbb{v}_i$ and then the numerical matrices are defined by $U_{(u,d)} \equiv \left(\mathbb{v}_1^{(u,d),T}, \mathbb{v}_2^{(u,d),T}, \mathbb{v}_3^{(u,d),T}\right)$.

(b) computing the CKM matrix as $V_{CKM} = U_u^\dagger \cdot U_d$.

(c) uniquely extracting the s_{13} mixing angle from V_{13}, $s_{13} = |V_{13}|$.

(d) uniquely extracting the s_{23} mixing angle from V_{23}, $s_{23} = |V_{23}|/\sqrt{1 - s_{13}^2}$.

(e) uniquely extracting the s_{12} mixing angle from V_{12}, $s_{12} = |V_{12}|/\sqrt{1 - s_{13}^2}$.

(f) uniquely extracting the s_δ phase in a phase-convention-independent manner from the Jarlskog invariant J.

- Computations done in arbitrary flavor/weak bases (i.e. with full but arbitrary 3D structure in matrices)

mass dimension up to n = 10

new physics between 2-10 TeV

- **Conclusion**: complete agreement up to numerical tolerance of 10^{10} !! ✅

- Note that numerical checks in environments with $\{\mathbb{Y}_u', \mathbb{Y}_d'\} = \{U_\chi^{u\,\dagger}\, \mathbb{Y}_u\, U_\chi^u, U_\chi^{d\,\dagger}\, \mathbb{Y}_d\, U_\chi^d\}$ also confirm LACK of ability to predict CKM angles with our formulae in this instance, as expected!

Building Up the $g_{AB}(\phi)$ Metric

[2001.01453]
[2203.06771]

- Consider the higher-order operators that can connect two gauge field strengths:

Dim 6+

$$Q_{HB}^{(6+2n)} = (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW}^{(6+2n)} = (H^\dagger H)^{n+1} W_a^{\mu\nu} W_{\mu\nu}^a,$$
$$Q_{HWB}^{(6+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu}$$

Dim 8+

$$Q_{HW,2}^{(8+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu}$$

That the operator forms saturate at all orders can be seen with **Hilbert Series** techniques:

	Mass Dimension				
Field space connection	6	8	10	12	14
$g_{AB}(\phi)\mathcal{W}_{\mu\nu}^A \mathcal{W}^{B,\mu\nu}$	3	4	4	4	4

- Expanding in terms of real scalar fields, and combining into a single gauge field (A,B = 1,2,3,4), one can write

$$H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$g_{AB}(\phi)\mathcal{W}_{\mu\nu}^A \mathcal{W}^{B,\mu\nu}$$

$$\begin{aligned} g_{AB}(\phi_I) = & \left[1 - 4\sum_{n=0}^{\infty}\left(C_{HW}^{(6+2n)}(1-\delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ & + \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)}\left(\frac{\phi^2}{2}\right)^{n}\left(\phi_I \Gamma_{A,J}^I \phi^J\right)\left(\phi_L \Gamma_{B,K}^L \phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^{n}\right]\left(\phi_I \Gamma_{A,J}^I \phi^J\right)(1-\delta_{A4})\delta_{B4}, \end{aligned}$$

- This *field-space connection* is therefore valid at **all-orders in v/Λ**! In the Higgsed phase the connection reduces to a number + emissions of *h*.

Partial Sq. vs. Full Dim-8: Fermionic Z Decay

- Consider all-order geoSMEFT width for Z-boson decay to fermions:

$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi} \sqrt{\bar{m}_Z^2} |g_{\rm eff}^{Z,\psi}|^2 \left(1 - \frac{4\bar{M}_{\psi}^2}{\bar{m}_Z^2}\right)^{3/2} \qquad g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}^2 Q_{\psi} - \sigma_3)\delta_{pr} + \bar{v}_T \langle L_{3,4}^{\psi,pr}\rangle + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi,pr}\rangle\right]$$

defined at all orders in v/Λ !!

- Expand complete dependence at dim-6, dim-8:

$$\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM} = \bar{g}_Z^{\rm SM}\left[(s_\theta^{\rm SM})^2 Q_\psi - \frac{\sigma_3}{2}\right]\delta_{pr}$$

SM

$$\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^2/\Lambda^2)} = \frac{\langle \bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}}{\bar{g}_Z^{\rm SM}} \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}\,\delta_{pr} + \bar{g}_Z^{\rm SM} Q_\psi \langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^2/\Lambda^2)}\,\delta_{pr} + \frac{\bar{g}_Z^{\rm SM}}{2}\left[\tilde{C}^{1,(6)}_{\substack{H\psi\\pr}} - \sigma_3\,\tilde{C}^{3,(6)}_{\substack{H\psi\\pr}}\right]$$

dim-6

$$\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^4/\Lambda^4)} = \frac{\langle \bar{g}_Z\rangle_{\mathcal{O}(v^4/\Lambda^4)}}{\bar{g}_Z^{\rm SM}} \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}\,\delta_{pr} + \bar{g}_Z^{\rm SM} Q_\psi \langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^4/\Lambda^4)}\,\delta_{pr} + \langle \bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}\langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^2/\Lambda^2)} Q_\psi \delta_{pr}$$
$$+ \frac{\langle \bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}}{2}\left[\tilde{C}^{1,(6)}_{\substack{H\psi\\pr}} - \sigma_3\,\tilde{C}^{3,(6)}_{\substack{H\psi\\pr}}\right] + \frac{g_Z^{\rm SM}}{4}\left[\tilde{C}^{1,(8)}_{\substack{H\psi\\pr}} - \sigma_3\,\tilde{C}^{2,(8)}_{\substack{H\psi\\pr}} - \sigma_3\,\tilde{C}^{3,(8)}_{\substack{H\psi\\pr}}\right]$$

dim-8

- Compare (e.g.) dependence on $(C^{(6)}_{HWB})^2$ using partial square vs. full dim-8 analysis:

Partial Square

$$|g_{\rm eff,pr}^{\mathcal{Z},\psi}|^2_{\rm partial\,square} \supset \frac{g_1^2\,g_2^2\,(\tilde{C}^{(6)}_{HWB})^2}{(g_Z^{\rm SM})^6}\delta_{pr}\left[g_Z^{\rm SM}\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM} + (g_2^2 - g_1^2)\,Q_\psi)\right]^2$$

Complete Analysis

$$|g_{\rm eff,pr}^{\mathcal{Z},\psi}|^2_{\mathcal{O}(v^4/\Lambda^4)} \supset \frac{g_1^2\,g_2^2\,(\tilde{C}^{(6)}_{HWB})^2\,(g_2^2 - g_1^2)^2\,Q_\psi^2}{(g_Z^{\rm SM})^6}\delta_{pr} + (\tilde{C}^{(6)}_{HWB})^2 \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}^2\delta_{pr}$$

*There are even *cancellations* such that term ~Q doesn't exist in full expansion…*

Approach with Invariants

[…]
[0706.4313]
[0907.4763]

- A group ring ***C[x]ᴳ*** of polynomials ***x*** invariant under symmetry **G** is contained in the free ring ***C[x]***:

$$\mathbb{C}[x_1, \ldots, x_n]^G \subseteq \mathbb{C}[x_1, \ldots, x_n]$$

- For certain G, *C[x]ᴳ* is *finitely generated* by polynomial invariants ***I(x)***, such that any G-invariant polynomial **f(x)** can be written as a polynomial **g(I)**, a member **P** of the (not necessarily free) ring ***C[I]***:

$$f(x_1, \ldots, x_n) = g(I_1, \ldots, I_n) \qquad P \in \mathbb{C}[I_1, \ldots, I_r]$$

- Minimal basis of invariants can be enumerated with ***Hilbert series*** (well-known in SMEFT!):

$$H(q) = \sum_{r=0}^{\infty} c_r q^r$$

q = invariant 'spurion'

r = polynomial degree of invariant

c_r = # of invariants of degree r

- For semi-simple Lie groups further results can be derived:

$$H(q) = \frac{N(q)}{D(q)}$$

$$N(q) = 1 + c_1 q + \ldots c_{d_N - 1} q^{d_N - 1} + q^{d_N}$$

$$D(q) = \prod_{r=1}^{p} (1 - q^{d_r})$$

N & Q = polynomials

d_N= polynomial degree of N

d_D= polynomial degree of D = sum of d_r

p = # free parameters

Applications: Flavor Violation Pheno

[2005.12283]

LL RGE evolution for Yukawa and Wilson Coefficients known:

$$Y_d(\mu_{\rm EW}) = Y_d(\Lambda) - \delta Y_d \frac{3y_t^2}{32\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) + \ldots$$

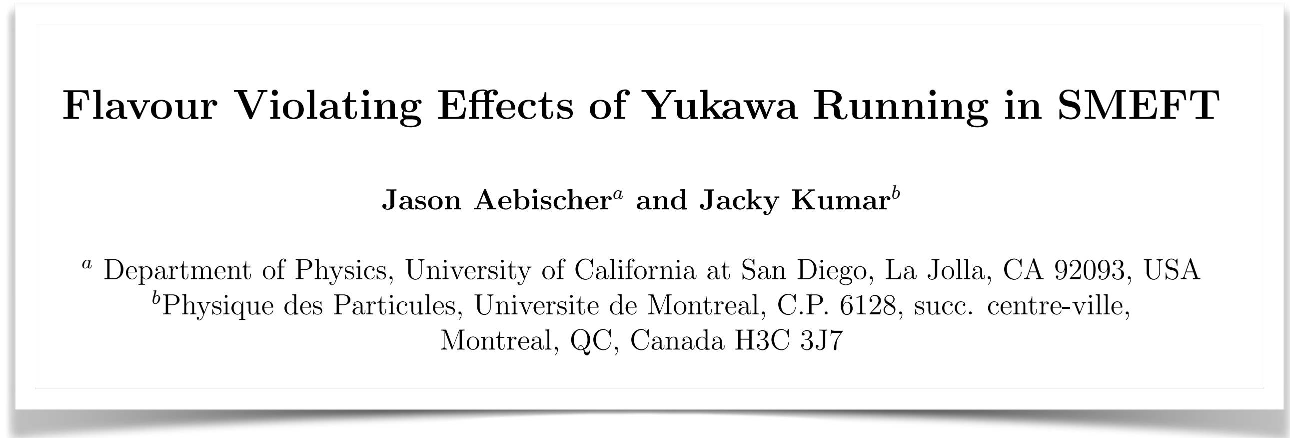
$$\left[\widetilde{\mathcal{C}}_a(\mu_{\rm EW})\right]_{ij} = \left[\mathcal{C}_a(\Lambda)\right]_{ij} + \frac{(\beta_{ab})^{ijkl}}{16\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) \left[\mathcal{C}_b(\Lambda)\right]_{kl}$$

At EW scale, Yukawa (and Wilson Coefficients) must be re-rotated to (physical) fermion mass-eigenstates!

$$\left[\mathcal{C}_a(\mu_{\rm EW})\right]_{ij} = U^\dagger_{ik} \left[\widetilde{\mathcal{C}}_a(\mu_{\rm EW})\right]_{kl} U_{lj}$$

$$U_{d_L} = \begin{pmatrix} -0.93 + 0.37i & 1.6\cdot 10^{-5} + 2.5\cdot 10^{-7}\, i & -3.8\cdot 10^{-4} \\ -1.2\cdot 10^{-5} + 1.1\cdot 10^{-5}\, i & -0.93 + 0.37i & 1.6\cdot 10^{-3} - 6.7\cdot 10^{-4}\, i \\ 2.7\cdot 10^{-4} - 2.6\cdot 10^{-4}\, i & -1.6\cdot 10^{-3} + 6.1\cdot 10^{-4}\, i & -0.93 + 0.37\, i \end{pmatrix}$$

compare to $\kappa^{ij}_{RGE} = \frac{\lambda_t^{ij}}{16\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) \approx 9\cdot 10^{-4} - 2\cdot 10^{-5} i.$



Flavour Violating Effects of Yukawa Running in SMEFT

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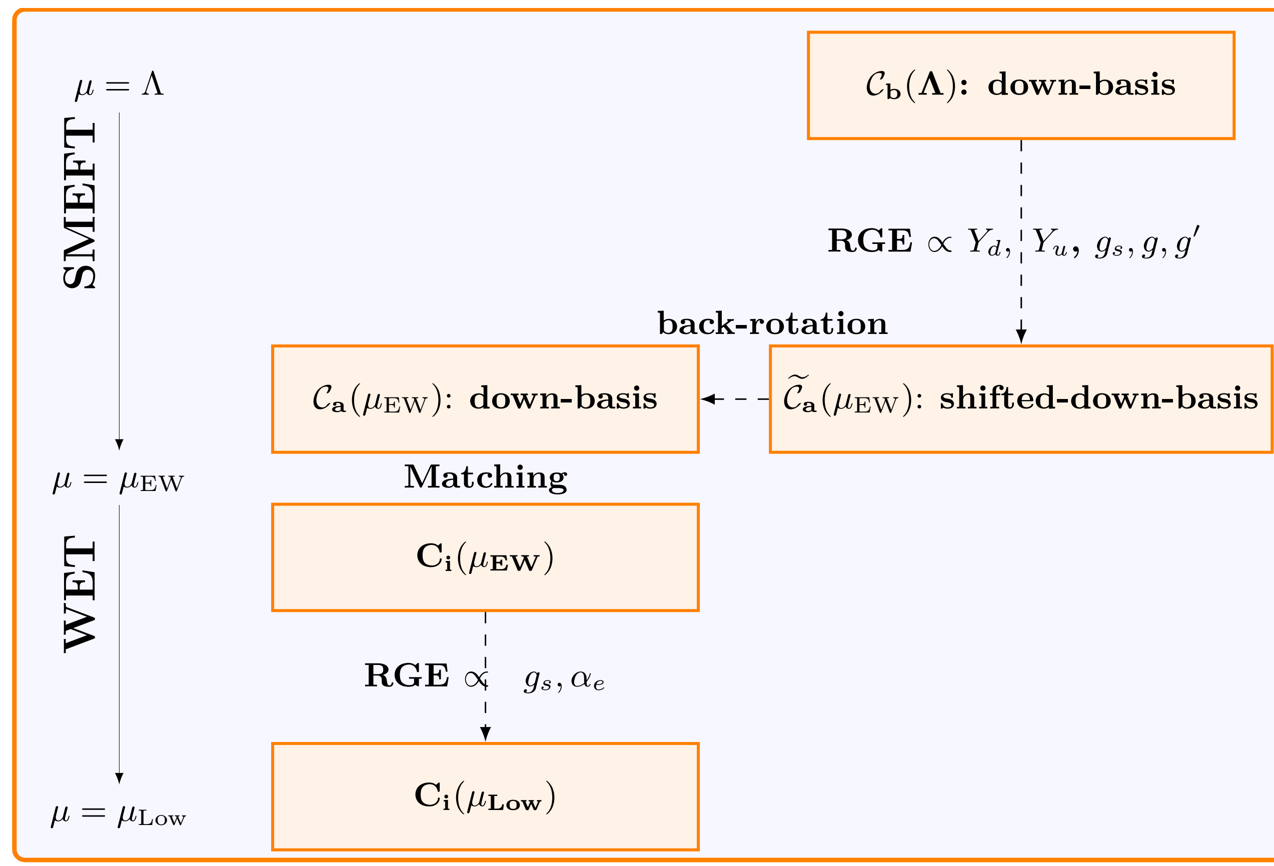

- The resulting NP bounds derived from (e.g.) ΔF=2 or b->sll processes are very important!
- *Q1: what is the correspondence between RGE of flavor invariants and (known) non-MFV relations?*
- *Q2: what is the phenomenological impact of higher-order RGE of physical parameters?*

Applications: CKM Fits?

Wolfenstein Parameterization

$$W_j \equiv \{\lambda, A, \bar{\rho}, \bar{\eta}\}$$

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(1+\frac{1}{2}\lambda^2)(\bar{\rho}-i\bar{\eta}) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\bar{\rho}-i\bar{\eta}) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\bar{\rho}-i\bar{\eta}) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

2020 PDG Global Fit

12. CKM Quark-Mixing Matrix

Revised March 2020 by A. Ceccucci (CERN), Z. Ligeti (LBNL) and Y. Sakai (KEK).

$$\begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

- CKM parameter fits big business in flavor physics — critical tests of the SM.
- However, as we have seen, BSM physics encoded in Wilson coefficients impacts the definition of the CKM matrix. A consistent treatment of such effects critical for interpretation of NP bounds.

$$O_i^{\text{input}} = O_{i,\text{SM}}^{\text{input}}(W_j)\big[(1+f(L_k)\big] = O_{i,\text{SM}}^{\text{input}}(W_j)\big[1+g(C_k)\big]$$

LEFT ***SMEFT***

➡

$$O_i^{\text{input}} = O_{i,\text{SM}}^{\text{input}}(\widetilde{W}_j)$$

$$\widetilde{W}_j = W_j\left(1+\frac{\delta W_j}{W_j}\right)$$

[1812.08163]

$$O_\alpha = O_{\alpha,\text{SM}}(W_j) + \delta O_{\alpha,\text{NP}}^{\text{direct}} = O_{\alpha,\text{SM}}(\widetilde{W}_j) + \delta O_{\alpha,\text{NP}}^{\text{indirect}} + \delta O_{\alpha,\text{NP}}^{\text{direct}}$$

$$\delta O_{\alpha,\text{NP}}^{\text{indirect}} = -\frac{\partial O_{\alpha,\text{SM}}}{\partial W_i}\delta W_i + \mathcal{O}(\Lambda^{-4})$$

'indirect' and 'direct' NP effects contribute at the same order in v/Λ!

Applications: CKM Fits?

[1812.08163]

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The CKM parameters in the SMEFT

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Martín González-Alonso[c] and Javier Virto[d,e]

$\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \quad \Gamma(B \to \tau\nu_\tau), \quad \Delta M_d, \quad \Delta M_s$

CKMfitter (SM) [14]	UTfit (SM) [15]	This work (SMEFT)
$\lambda = 0.224747^{+0.000254}_{-0.000059}$	$\lambda = 0.2250 \pm 0.0005$	$\tilde{\lambda} = 0.22537 \pm 0.00046$
$A = 0.8403^{+0.0056}_{-0.0201}$	$A = 0.826 \pm 0.012$	$\tilde{A} = 0.828 \pm 0.021$
$\bar{\rho} = 0.1577^{+0.0096}_{-0.0074}$	$\bar{\rho} = 0.148 \pm 0.013$	$\tilde{\rho} = 0.194 \pm 0.024$
$\bar{\eta} = 0.3493^{+0.0095}_{-0.0071}$	$\bar{\eta} = 0.348 \pm 0.010$	$\tilde{\eta} = 0.391 \pm 0.048$

- As expected, reabsorption of BSM effects into 'SM' parameters leads to non-trivial bounds on NP when calculating other flavored processes:

$$\Gamma(\pi \to \mu\nu) = \left|1 - \frac{\tilde{\lambda}^2}{2} - \frac{\tilde{\lambda}^4}{8}\right|^2 \frac{f_{\pi^\pm}^2 m_{\pi^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_\mu^2}{m_{\pi^\pm}^2}\right)^2 (1 + \delta_{\pi\mu})\left[1 + \widetilde{\Delta}_{\pi\mu 2}\right] \quad \textit{(e.g)}$$

$$\mathcal{B}(\pi \to \mu\nu) = 0.9998770(4) \quad + \quad \tau_\pi = 2.6033(5) \cdot 10^{-8} s \quad \Rightarrow \quad \widetilde{\Delta}_{\pi\mu 2} = 0.004 \pm 0.013$$

$$\widetilde{\Delta}_{\pi\mu 2} = 2\,\mathrm{Re}(\epsilon_A^{\mu ud}) - \frac{2m_{\pi^\pm}^2}{(m_u + m_d)m_\mu}\mathrm{Re}(\epsilon_P^{\mu ud}) + 4\frac{\delta v}{v} + 2\tilde{\lambda}(1 + \tilde{\lambda}^2)\delta\lambda + \mathcal{O}(\Lambda^{-4}, \tilde{\lambda}^6)$$

- Formalism with flavored geoSMEFT can potentially push fits to higher order in v/Λ.
- Of relevance to potential *Cabibbo Angle Anomaly* — see e.g. 2109.06065.