

# Free-streaming and Coupled Dark Radiation Isocurvature Perturbations

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# Overview

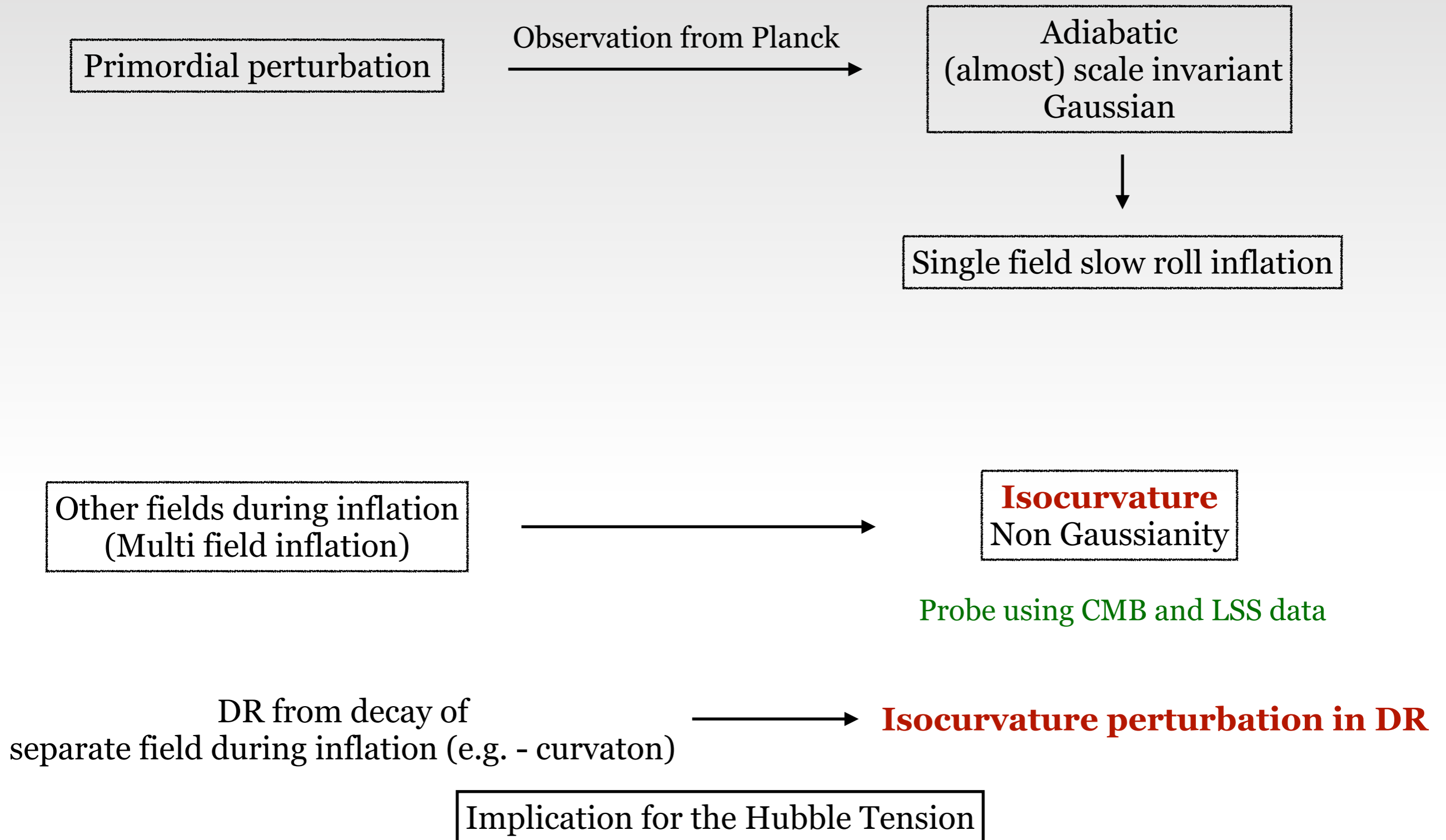
Dark Radiation (DR) Isocurvature

Features of **Free streaming DR Isocurvature (FDR)**  
&  
**coupled DR Isocurvature (CDR)**

Bounds from Cosmological Datasets

Implication for the Hubble tension

# Motivation: Isocurvature Perturbation in CMB



# Dark Radiation (DR)

Parametrized by  $\Delta N_{\text{eff}}$

Free-streaming DR (FDR)

Similar to (SM/free-streaming) neutrinos

**Non zero** anisotropic stress

Coupled/fluid DR (CDR)

Similar to (strongly) self-interacting neutrinos

**Zero** anisotropic stress

Additional variables to define the initial Isocurvature power spectrum

Isocurvature parameters

$A_{\text{iso}}(k_*)$  [or  $f_{\text{iso}} \equiv A_{\text{iso}}/A_{\text{adia}}$ ]

$n_{\text{iso}}$



Or

$P_{II}^{(1)}$  ( $\equiv A_{\text{iso}}(k_1)$ )

$P_{II}^{(2)}$  ( $\equiv A_{\text{iso}}(k_2)$ )

Additional parameters

$N_{\text{dr}}$



Amount of Dark Radiation

$N_{\text{ur}}$



Amount of Neutrinos

$k_1 = 0.002 \text{ Mpc}^{-1}$   
 $k_2 = 0.1 \text{ Mpc}^{-1}$

$$N_{\text{dr}} + N_{\text{ur}} \equiv N_{\text{tot}} = N_{\text{eff}}$$

# Isocurvature Perturbation studies with CMB

Planck Collaboration

Baryon Isocurvature  
CDM Isocurvature  
Neutrino Isocurvature

Akrami et. al., arXiv:1807.06211

FDR Isocurvature  
CDR Isocurvature

generalized

SG, Soubhik Kumar, Yuhsin Tsai: arXiv:2107.09076

DR  $\rightarrow$  Adiabatic + Isocurvature  
Neutrinos  $\rightarrow$  Adiabatic

# Recipe

Derive DR Isocurvature **initial conditions**



Calculate the effects on the **CMB spectrum**

(Using CLASS)



Perform an **MCMC analysis** to find the constraints

(Using Montepython)

Results for **un-correlated** DR Isocurvature



No correlation between isocurvature and adiabatic spectrum

# Isocurvature Initial conditions

SG, Soubhik Kumar, Yuhsin Tsai: arXiv:2107.09076

(In synchronous gauge)

Adiabatic initial condition :  $\delta_\gamma = \delta_\nu = \delta_{\text{DR}}$

$$\delta_i = \frac{\delta\rho_i}{\bar{\rho}_i}$$

Isocurvature initial conditions:  $\sum_i R_i \delta_i = 0$

$$R_i = \bar{\rho}_i / (\bar{\rho}_\gamma + \bar{\rho}_\nu + \bar{\rho}_{\text{DR}})$$

variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(\omega k^2 \tau^3)$
$\delta_\gamma$	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
$\theta_\gamma/k$	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
$\delta_\nu$	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
$\theta_\nu/k$	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
$\sigma_\nu$	0	0	$-\frac{19R_{\text{DR}}}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
$\delta_{\text{DR}}$	1	0	$-\frac{1}{6}$	
$\theta_{\text{DR}}/k$	0	$\frac{1}{4}$	0	
$\sigma_{\text{DR}}$	0	0	$\frac{15-15R_{\text{DR}}+4R_\nu}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
$\eta$	0	0	$\frac{-R_{\text{DR}}+R_{\text{DR}}^2+R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
$h$	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$
$\delta_b$	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	
$\delta_c$	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$

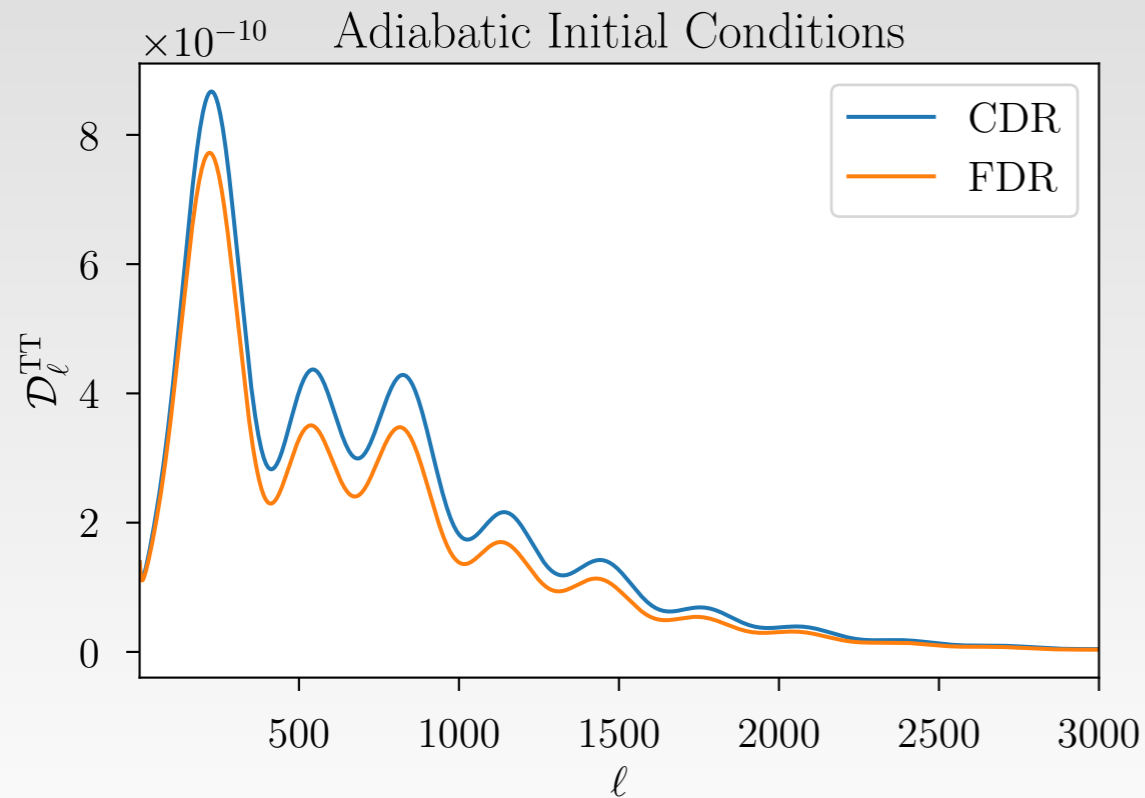
FDR - Isocurvature

variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(\omega k^2 \tau^3)$
$\delta_\gamma$	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
$\theta_\gamma/k$	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
$\delta_\nu$	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
$\theta_\nu/k$	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
$\sigma_\nu$	0	0	$-\frac{R_{\text{DR}}}{2(1-R_{\text{DR}})(15+4R_\nu)}$	
$\delta_{\text{DR}}$	1	0	$-\frac{1}{6}$	
$\theta_{\text{DR}}/k$	0	$\frac{1}{4}$	0	
$\eta$	0	0	$\frac{R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_\nu)}$	
$h$	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$
$\delta_b$	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	
$\delta_c$	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$

CDR - Isocurvature

$$\sigma_{\text{DR}} = 0$$

# FDR vs CDR Isocurvature spectrum



Adiabatic :  $\delta_\gamma = \delta_\nu = \delta_{\text{DR}}$

FDR free-streams out of potential well  
 $\rightarrow$  Smaller potential  $\rightarrow$  Smaller CMB anisotropy

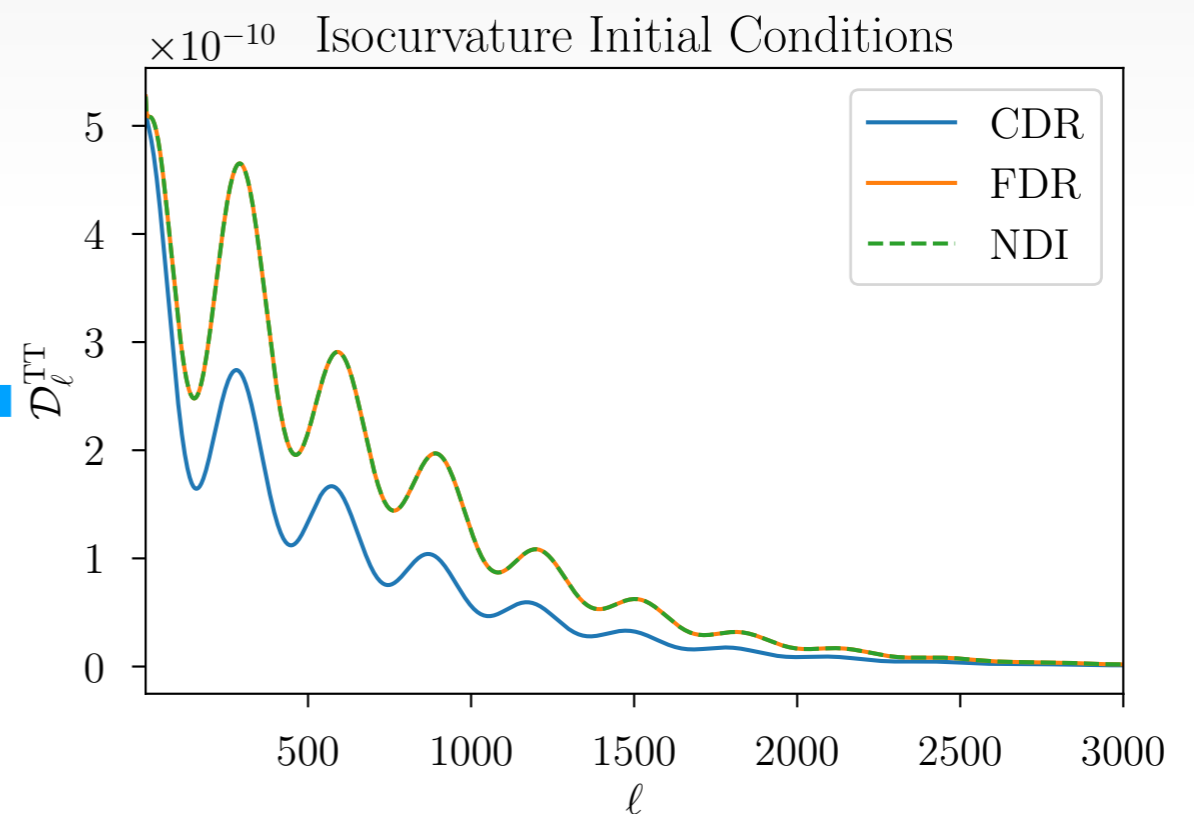
CDR does not free-stream  
 $\rightarrow$  larger CMB anisotropy

$$\sum_i R_i \delta_i = 0$$

For isocurvature metric fluctuation is sourced by anisotropic stress ( $\sigma$ ) at leading order

FDR:  $\sigma_{\text{tot}} > 0 \rightarrow$  More anisotropy

CDR:  $\sigma_{\text{tot}} < 0 \rightarrow$  Less anisotropy





# FDR vs CDR Isocurvature spectrum

$$\text{In Newtonian gauge: } \phi + \psi = -\frac{2\sigma}{(k\tau)^2}$$

$$\text{FDR: } \sigma_{\text{tot}} > 0 \rightarrow \sigma + \psi < 0$$

$$\text{CDR: } \sigma_{\text{tot}} < 0 \rightarrow \sigma + \psi > 0$$

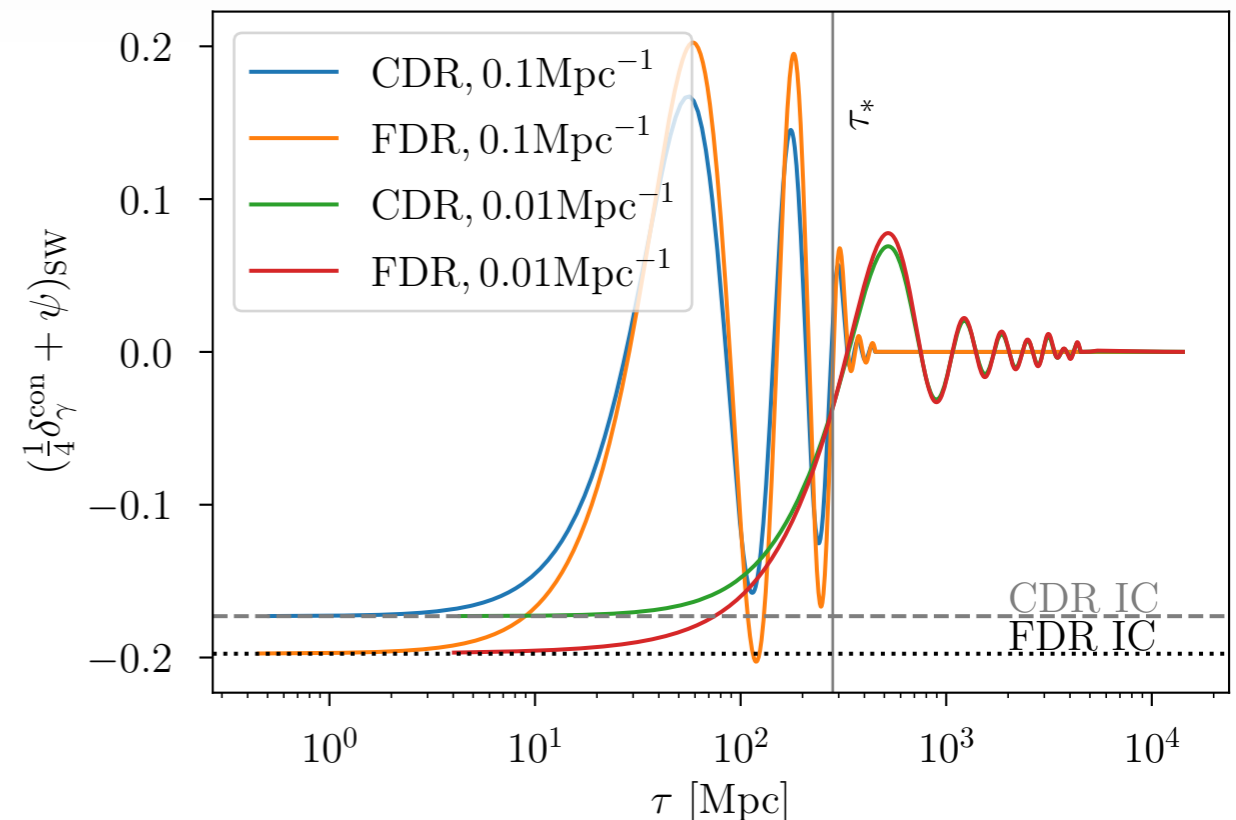
$$\text{Metric potential affects CMB via Sachs - Wolfe redshifting: } \frac{1}{4}\delta_{\gamma}^{\text{con}} + \psi = \xi_{\gamma} + \phi + \psi$$

Gauge invariant photon perturbation

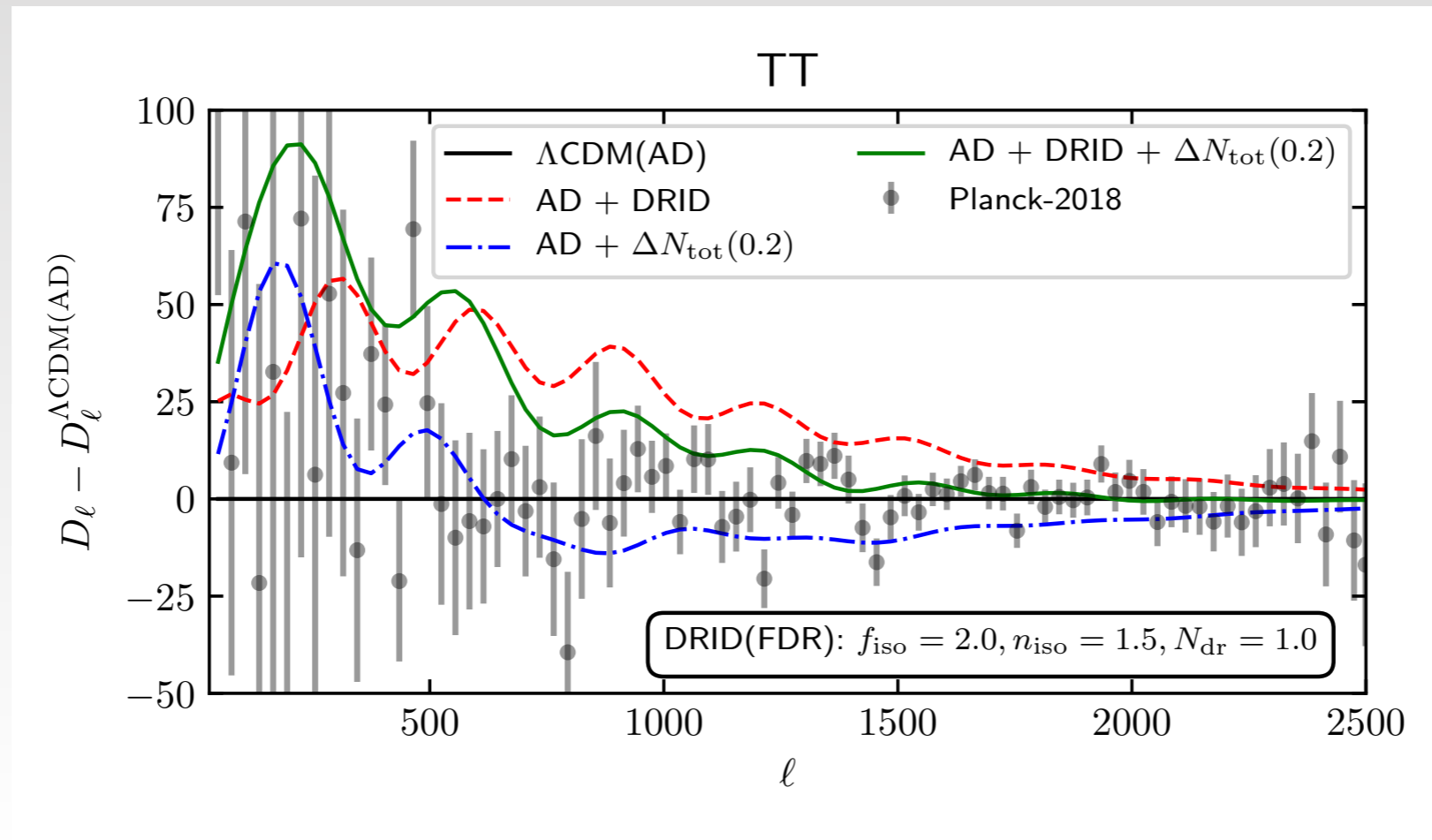
Isocurvature initial condition:  $\xi_{\gamma} \approx \delta_{\gamma} < 0$

$$\text{FDR: } \sigma_{\text{tot}} > 0 \rightarrow \sigma + \psi < 0 \rightarrow \text{larger } |\xi_{\gamma} + \phi + \psi|$$

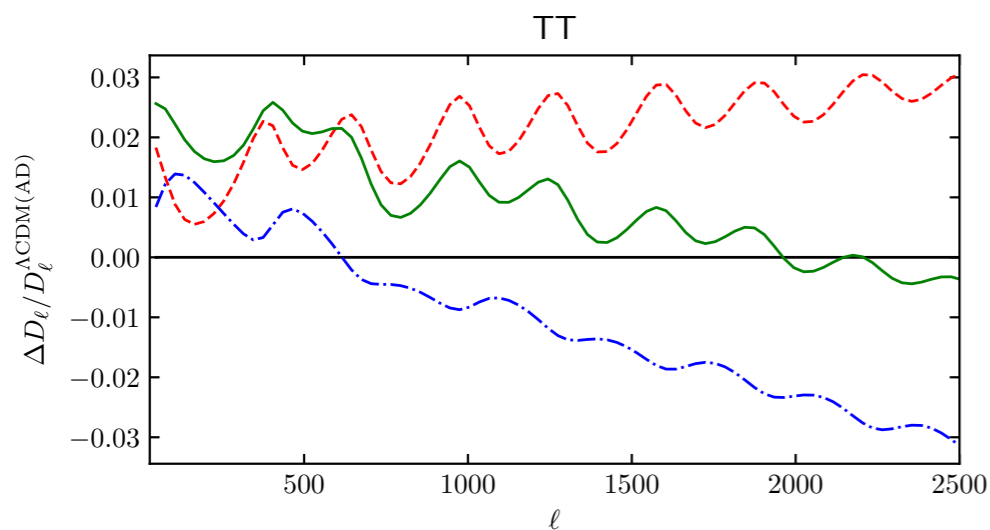
$$\text{CDR: } \sigma_{\text{tot}} < 0 \rightarrow \sigma + \psi > 0 \rightarrow \text{smaller } |\xi_{\gamma} + \phi + \psi|$$



# Isocurvature accommodate larger $N_{\text{eff}}$ ( $\equiv N_{\text{tot}}$ )



Blue tilted ( $n_{\text{iso}} > 1$ ) isocurvature compensates for the larger silk damping due to higher  $N_{\text{eff}}$



DRID  $\equiv$  Dark Radiation density Isocurvature

# MCMC variables

New parameter w.r.t.  $\Lambda$ CDM

Fixed  $N_{\text{ur}} (= 3.046)$  [FN]

→  $P_{II}^{(1)}, P_{II}^{(2)}, N_{\text{dr}}$  → 3 new parameters

Varying  $N_{\text{ur}}$  [VN]

→  $P_{II}^{(1)}, P_{II}^{(2)}, N_{\text{dr}}, N_{\text{ur}}$  → 4 new parameters

Isocurvature initial conditions

$$\delta_\gamma, \theta_\gamma, h, \eta \propto \frac{R_{\text{dr}}}{1 - R_{\text{dr}}} \approx R_{\text{dr}} \propto N_{\text{dr}}$$



$$C_\ell \text{ (DRID)} \propto A_{\text{iso}} N_{\text{dr}}^2$$

Physical isocurvature parameters:  $N_{\text{dr}}^2 P_{II}^{(1)}$  and  $N_{\text{dr}}^2 P_{II}^{(2)}$

Primary parameters used for MCMC runs

Fixed  $N_{\text{ur}}$



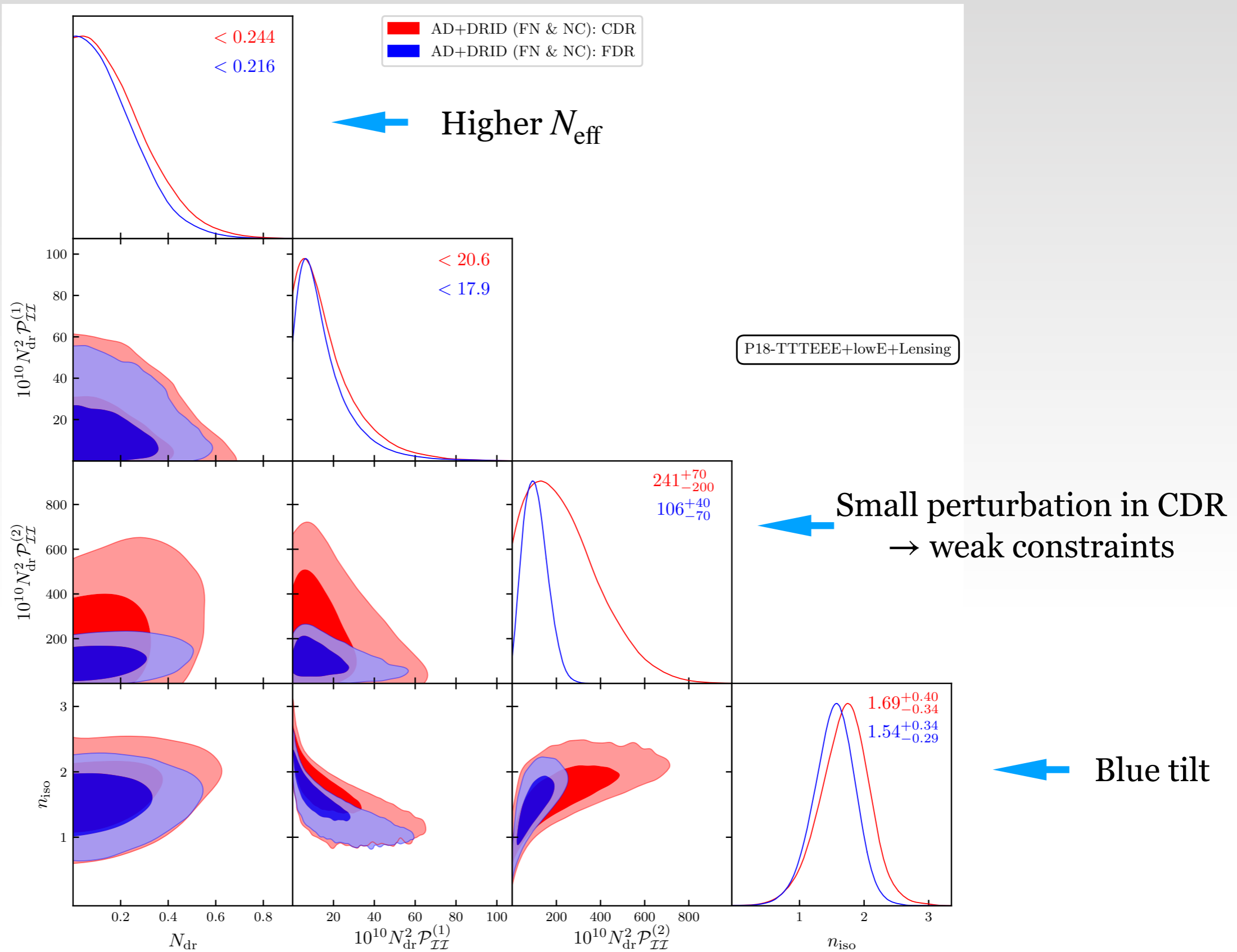
$$N_{\text{dr}}^2 P_{II}^{(1)}, N_{\text{dr}}^2 P_{II}^{(2)}, N_{\text{dr}}$$

Varying  $N_{\text{ur}}$

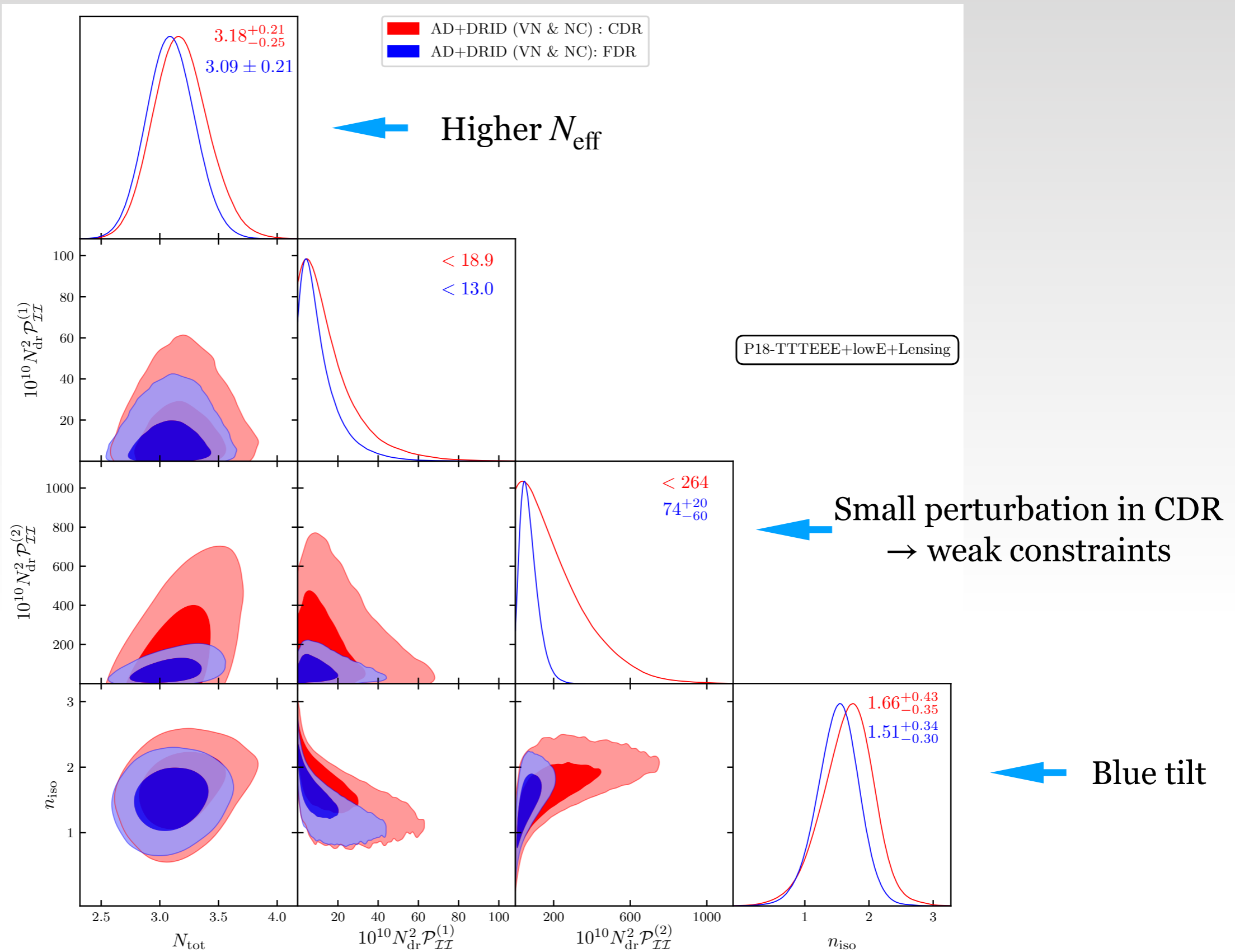


$$N_{\text{dr}}^2 P_{II}^{(1)}, N_{\text{dr}}^2 P_{II}^{(2)}, N_{\text{dr}}, N_{\text{ur}}$$

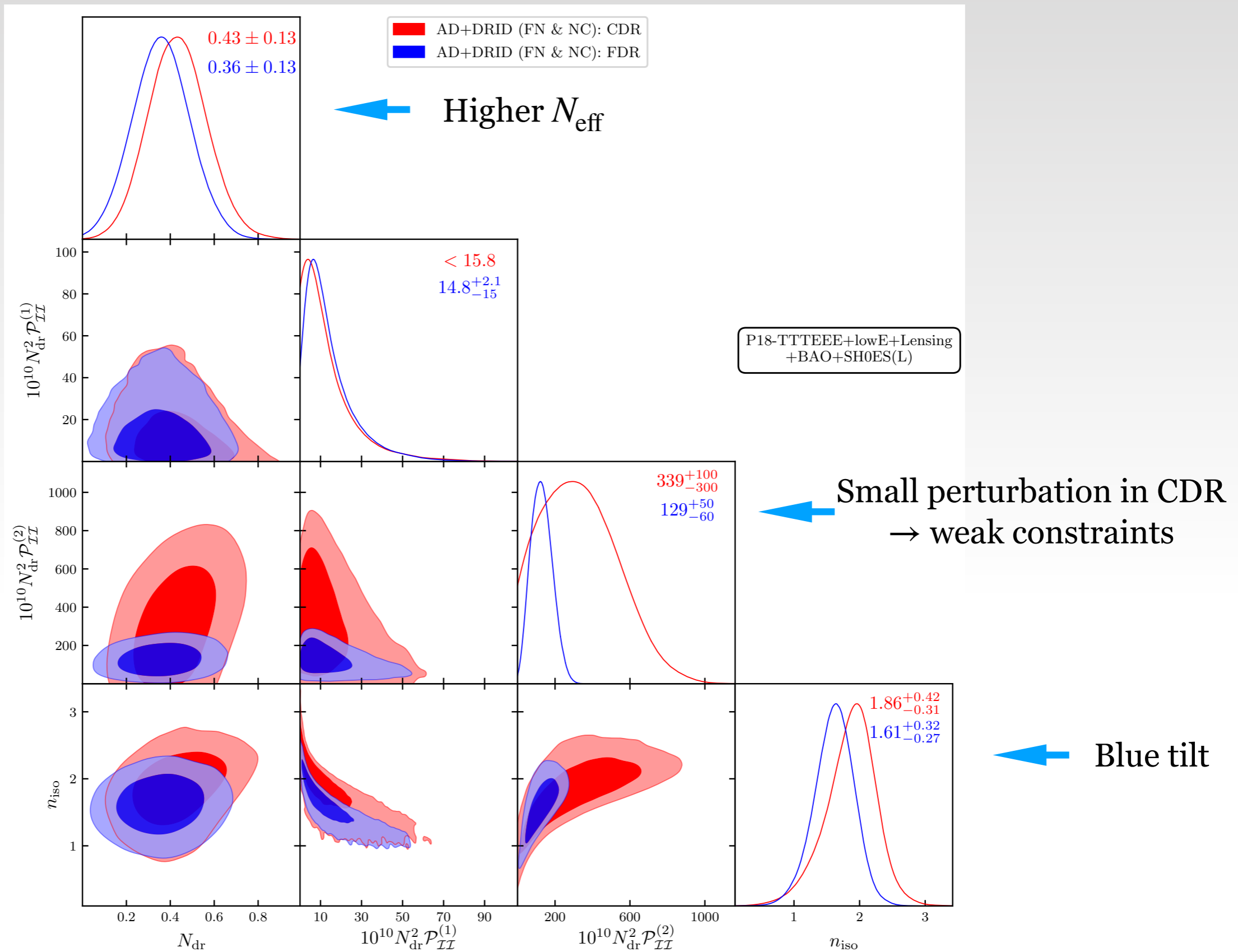
# MCMC results : FN - Planck



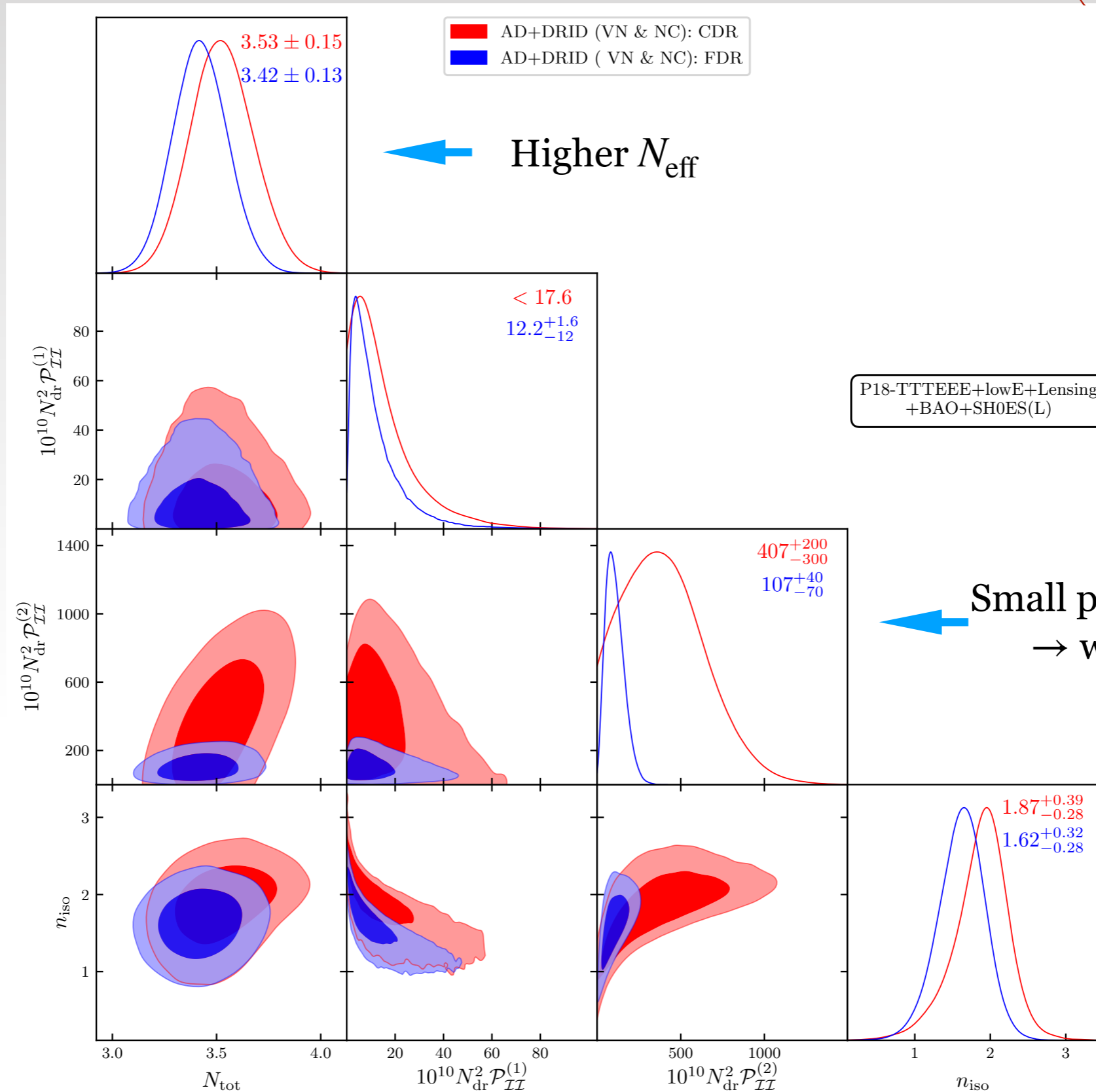
# MCMC results : VN - Planck



# MCMC results : FN - Planck + BAO+ SH0ES(Latest)



# MCMC results : VN - Planck + BAO+ SH0ES(Latest)



# Isocurvature accommodates larger $N_{\text{eff}} \rightarrow$ larger $H_0$ : Planck Data

FDR

CDR

FN

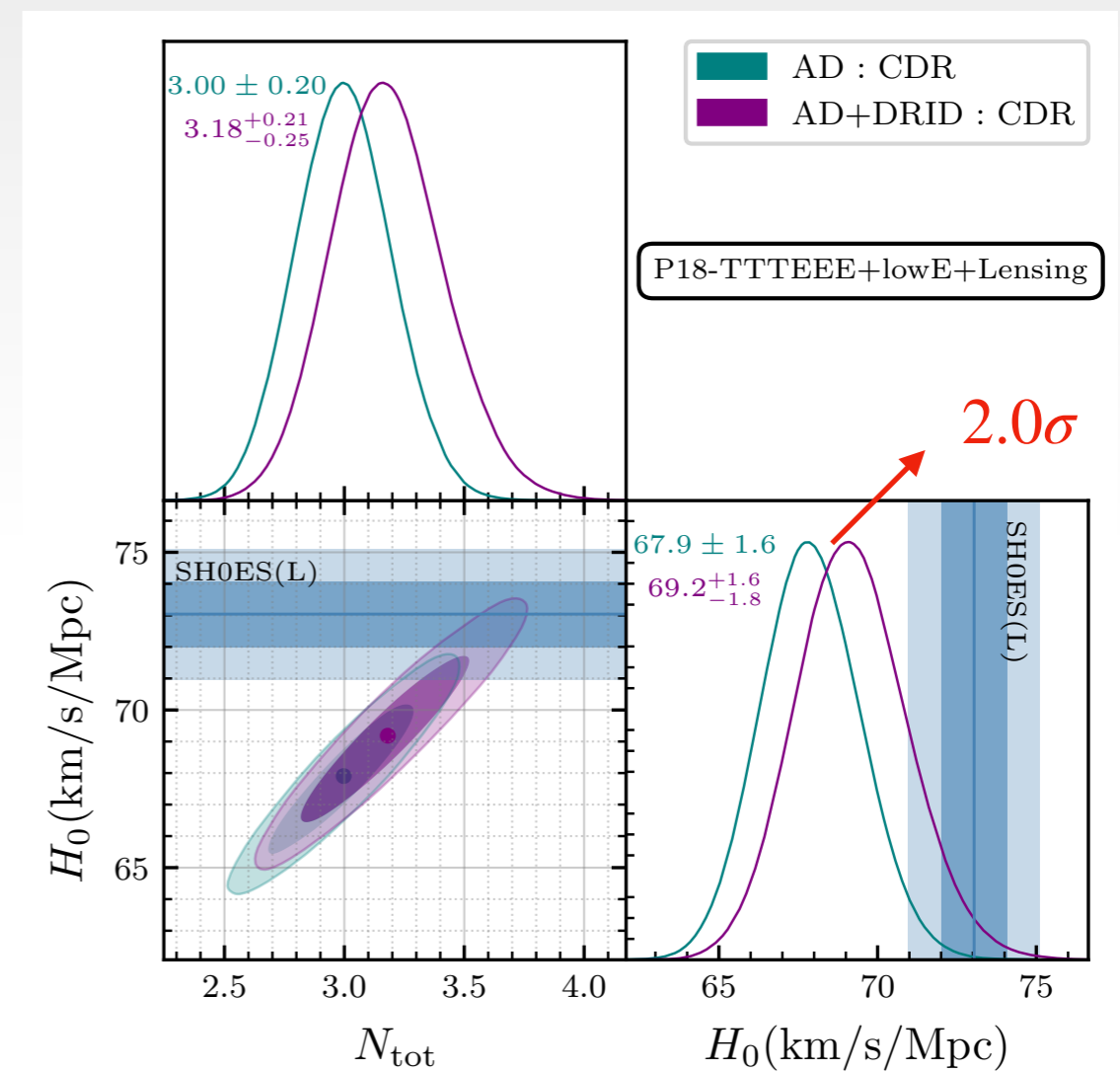
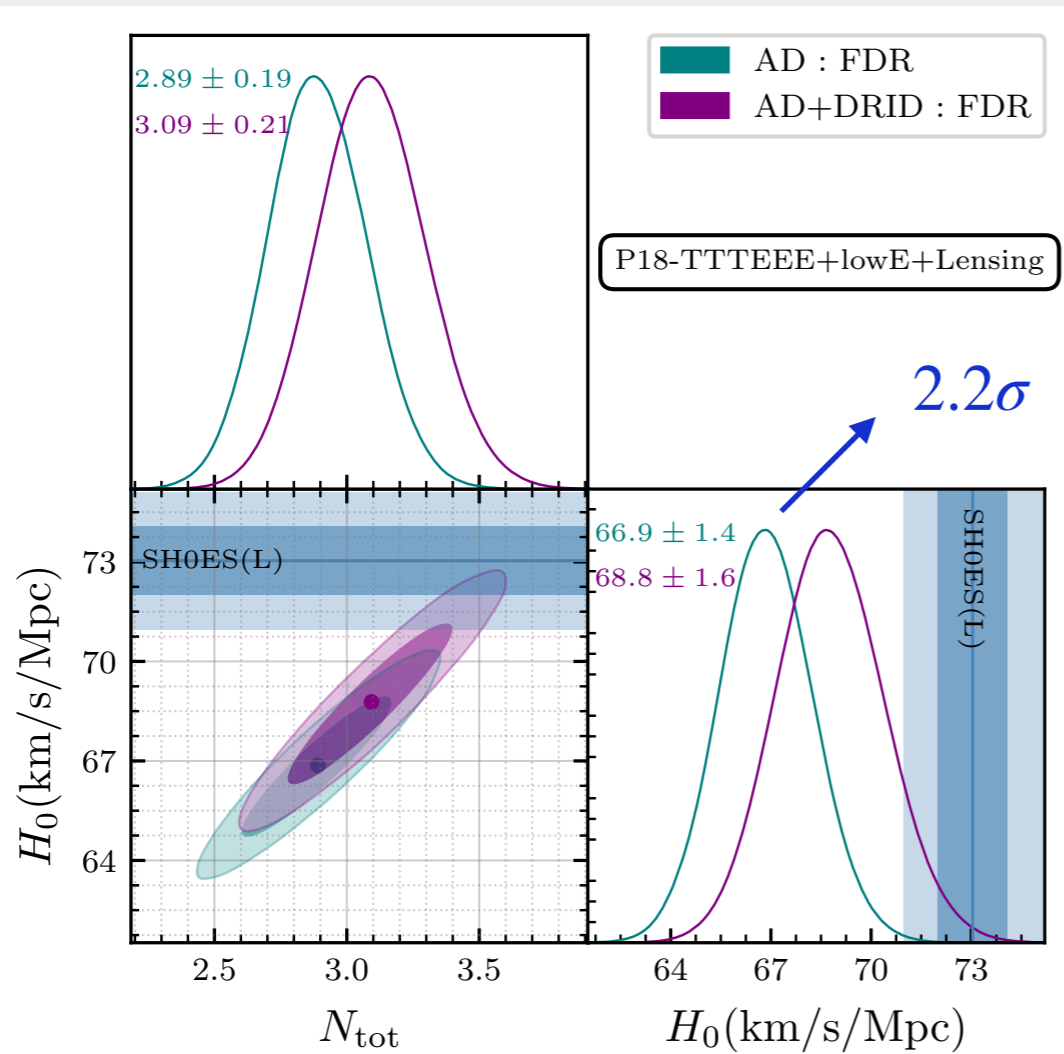
$$H_0 = 69.69^{+0.82}_{-1.3} \text{ km/s/Mpc}$$

$$H_0 = 69.57^{+0.88}_{-1.5} \text{ km/s/Mpc}$$

$2.5\sigma$

$2.5\sigma$

VN



$$H_0 [\text{SH0ES(L)}] = 73.04 \pm 1.04 \text{ km/s/Mpc}$$



Isocurvature accommodates larger  $N_{\text{eff}} \rightarrow$  larger  $H_0$  : Planck  
+BAO+SH0ES(L) Data

FDR

CDR

FN

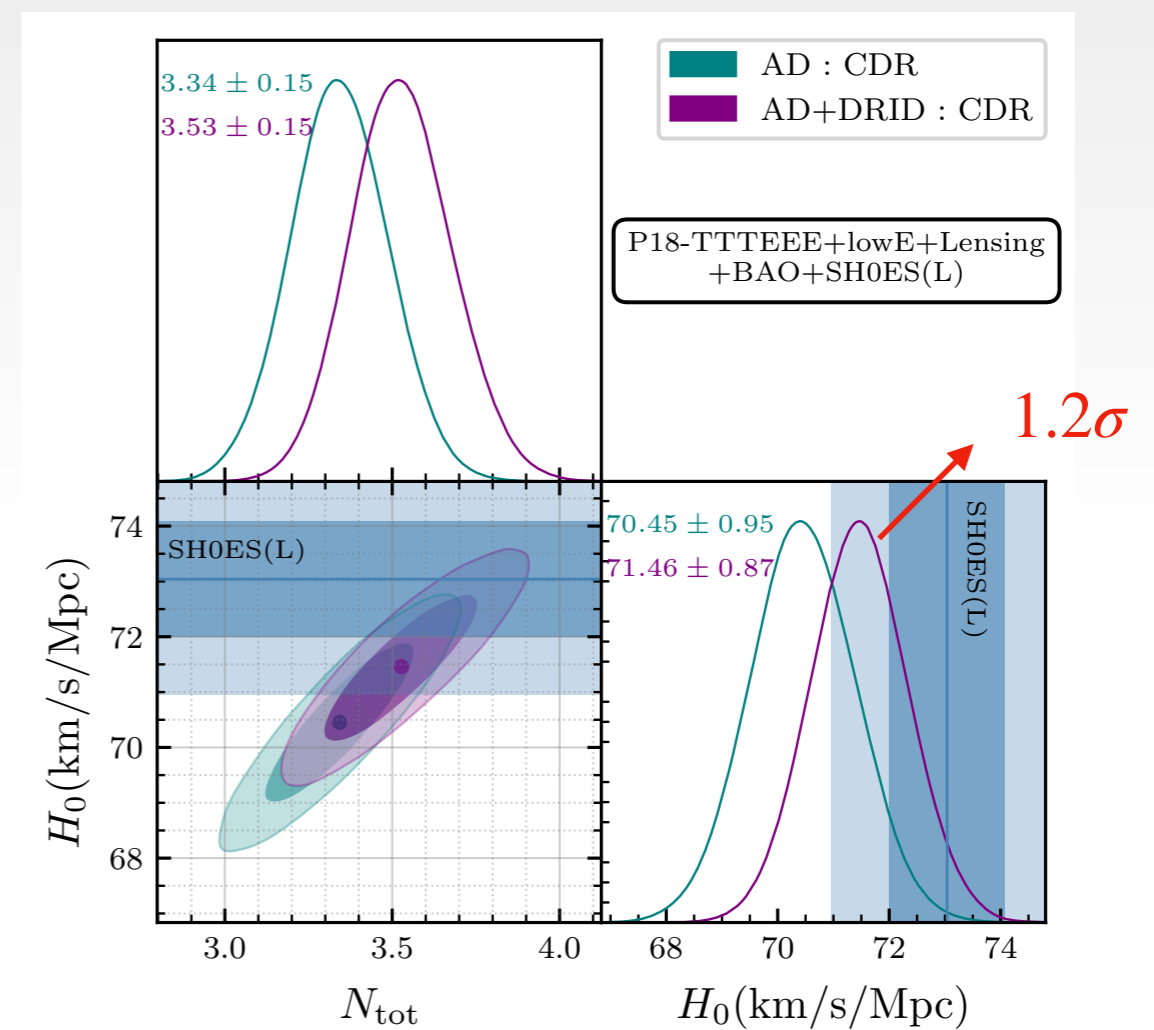
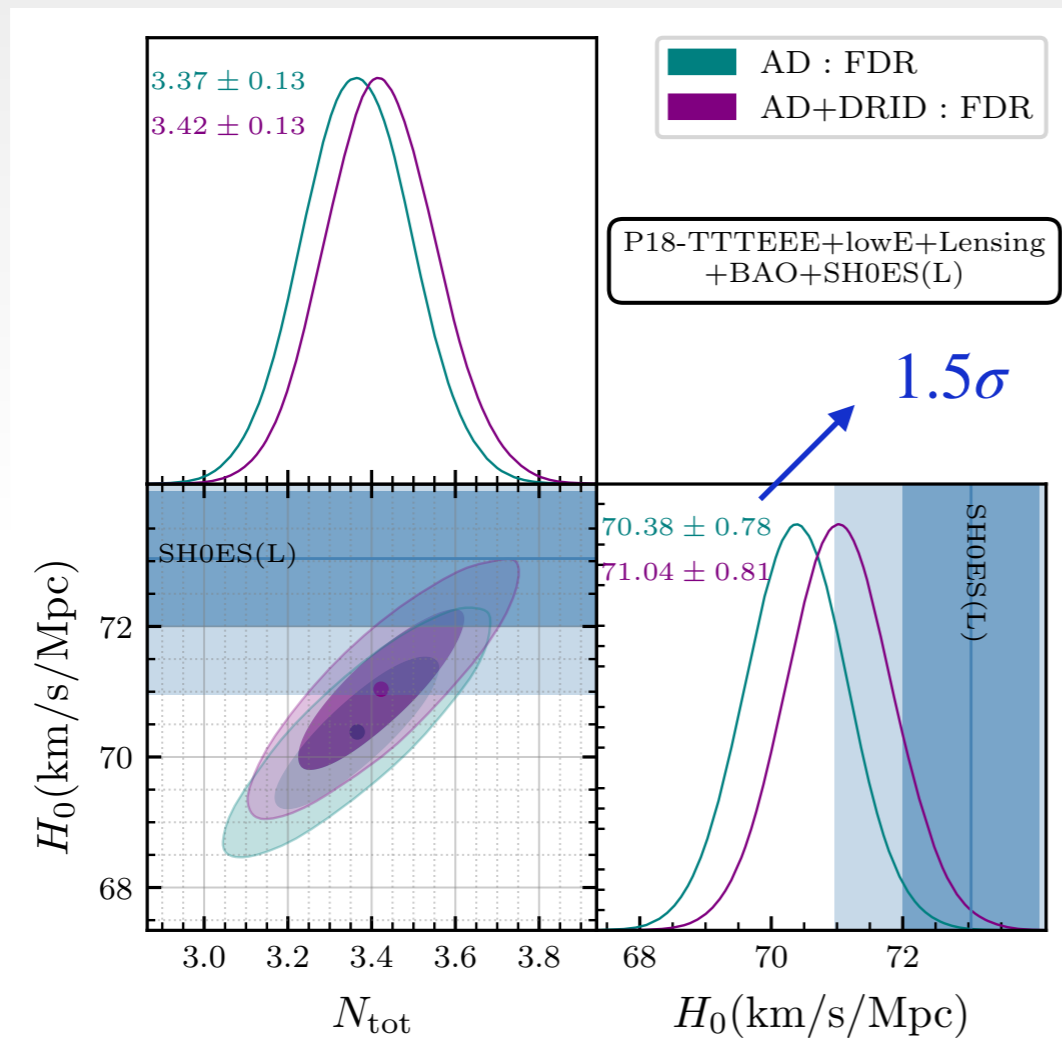
$$H_0 = 70.94 \pm 0.80 \text{ km/s/Mpc}$$

$$H_0 = 71.28 \pm 0.85 \text{ km/s/Mpc}$$

$1.6\sigma$

$1.3\sigma$

VN



$$H_0 \text{ [SHOES(L)]} = 73.04 \pm 1.04 \text{ km/s/Mpc}$$

# Model Comparison

Planck

Planck+BAO+SHoEs

Scenario	$H_0(\text{km/s/Mpc})$	GT	$\sqrt{\Delta\chi^2}$	$\Delta\text{AIC}$
FN, FDR	$69.69^{+0.82}_{-1.3}$	$2.5\sigma$	4.0	-9.4
FN, CDR	$69.57^{+0.88}_{-1.5}$	$2.5\sigma$	4.1	-5.1
VN, FDR	$68.8^{+1.6}_{-1.6}$	$2.2\sigma$	4.4	-6.1
VN, CDR	$69.2^{+1.6}_{-1.8}$	$2.0\sigma$	4.1	-3.7

$$\text{GT} \equiv \frac{H_{0,\mathcal{D}} - H_{0,\text{SHoES}}}{\sqrt{\sigma_{\mathcal{D}}^2 + \sigma_{\text{SHoES}}^2}}$$

$$\Delta\chi^2 \equiv \chi_{\min,\mathcal{D}+\text{BAO}+\text{SHoES}}^2 - \chi_{\min,\mathcal{D}}^2,$$

$$\Delta\text{AIC} = \chi_{\min,\text{M}}^2 - \chi_{\min,\Lambda\text{CDM}}^2 + 2(N_{\text{M}} - N_{\Lambda\text{CDM}}).$$

# Parameter values: FDR DRID

FDR (FN & NC)	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES(L)
$10^2\omega_b$	$2.264^{+0.018}_{-0.021}$	$2.281 \pm 0.015$
$\omega_{cdm}$	$0.1217^{+0.0017}_{-0.0025}$	$0.1245 \pm 0.0025$
$100\theta_s$	$1.04219 \pm 0.00045$	$1.04193 \pm 0.00047$
$\tau_{reio}$	$0.0563^{+0.0070}_{-0.0079}$	$0.0561 \pm 0.0072$
$10^{10}\mathcal{P}_{RR}^{(1)}$	$23.11 \pm 0.49$	$22.83 \pm 0.47$
$10^{10}\mathcal{P}_{RR}^{(2)}$	$20.55^{+0.35}_{-0.41}$	$20.72 \pm 0.36$
$10^{10}N_{dr}^2\mathcal{P}_{II}^{(1)}$	$< 17.9$	$14.8^{+2.1}_{-15}$
$10^{10}N_{dr}^2\mathcal{P}_{II}^{(2)}$	$106^{+40}_{-70}$	$129^{+50}_{-60}$
$N_{dr}$	$< 0.216$	$0.36 \pm 0.13$
$H_0(\text{km/s/Mpc})$	$69.69^{+0.82}_{-1.3}$	$70.94 \pm 0.80$
$\sigma_8$	$0.8249^{+0.0075}_{-0.0087}$	$0.8313 \pm 0.0085$
$10^9 A_s$	$2.098^{+0.031}_{-0.035}$	$2.107 \pm 0.032$
$n_s$	$0.9700^{+0.0062}_{-0.0074}$	$0.9752 \pm 0.0062$
$n_{iso}$	$1.54^{+0.34}_{-0.29}$	$1.61^{+0.32}_{-0.27}$
$f_{iso}$	$< 18.7$	$< 6.52$
$N_{tot}$	$< 3.26$	$3.41 \pm 0.13$
$f_{dr}$	$0.052^{+0.022}_{-0.047}$	$0.104^{+0.036}_{-0.032}$
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	$-1.94$	$-15.4$

FDR (VN & NC)	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES(L)
$10^2\omega_b$	$2.252 \pm 0.025$	$2.282 \pm 0.015$
$\omega_{cdm}$	$0.1200 \pm 0.0031$	$0.1248 \pm 0.0025$
$100\theta_s$	$1.04241 \pm 0.00052$	$1.04189 \pm 0.00047$
$\tau_{reio}$	$0.0554 \pm 0.0077$	$0.0560 \pm 0.0072$
$10^{10}\mathcal{P}_{RR}^{(1)}$	$23.32 \pm 0.55$	$22.80 \pm 0.46$
$10^{10}\mathcal{P}_{RR}^{(2)}$	$20.37 \pm 0.44$	$20.73 \pm 0.36$
$10^{10}N_{dr}^2\mathcal{P}_{II}^{(1)}$	$< 13.0$	$< 13.3$
$10^{10}N_{dr}^2\mathcal{P}_{II}^{(2)}$	$74^{+20}_{-60}$	$107^{+40}_{-70}$
$N_{ur}$	$2.06^{+1.0}_{-0.50}$	$2.29^{+1.1}_{-0.49}$
$N_{dr}$	$< 1.32$	$< 1.44$
$H_0(\text{km/s/Mpc})$	$68.8 \pm 1.6$	$71.04 \pm 0.81$
$\sigma_8$	$0.820 \pm 0.010$	$0.8318 \pm 0.0086$
$10^9 A_s$	$2.086 \pm 0.037$	$2.108 \pm 0.032$
$n_s$	$0.9655 \pm 0.0090$	$0.9756 \pm 0.0062$
$n_{iso}$	$1.51^{+0.34}_{-0.30}$	$1.62^{+0.32}_{-0.28}$
$f_{iso}$	$14.8^{+7.8}_{-14}$	$15.0^{+7.1}_{-14}$
$N_{tot}$	$3.09 \pm 0.21$	$3.42 \pm 0.13$
$f_{dr}$	$0.33^{+0.14}_{-0.32}$	$0.33^{+0.14}_{-0.32}$
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	$-3.92$	$-14.08$

# Parameter values: CDR DRID

CDR (FN & NC)	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES(L)
$10^2 \omega_b$	$2.262^{+0.019}_{-0.024}$	$2.286 \pm 0.016$
$\omega_{cdm}$	$0.1228^{+0.0018}_{-0.0030}$	$0.1268 \pm 0.0028$
$100\theta_s$	$1.04230^{+0.00034}_{-0.00038}$	$1.04260 \pm 0.00034$
$\tau_{reio}$	$0.0562^{+0.0070}_{-0.0080}$	$0.0568^{+0.0066}_{-0.0075}$
$10^{10} \mathcal{P}_{RR}^{(1)}$	$23.32 \pm 0.47$	$23.19 \pm 0.47$
$10^{10} \mathcal{P}_{RR}^{(2)}$	$20.33 \pm 0.35$	$20.22 \pm 0.36$
$10^{10} N_{dr}^2 \mathcal{P}_{II}^{(1)}$	$< 20.6$	$< 15.8$
$10^{10} N_{dr}^2 \mathcal{P}_{II}^{(2)}$	$241^{+70}_{-200}$	$339^{+100}_{-300}$
$N_{dr}$	$< 0.244$	$0.43 \pm 0.13$
$H_0(\text{km/s/Mpc})$	$69.57^{+0.88}_{-1.5}$	$71.28 \pm 0.85$
$\sigma_8$	$0.8237 \pm 0.0069$	$0.8270 \pm 0.0069$
$10^9 A_s$	$2.083 \pm 0.032$	$2.072 \pm 0.032$
$n_s$	$0.9649^{+0.0062}_{-0.0056}$	$0.9650^{+0.0071}_{-0.0055}$
$n_{tot}$	$1.69^{+0.40}_{-0.34}$	$1.86^{+0.42}_{-0.31}$
$f_{tot}$	$< 22.4$	$7.1^{+1.7}_{-3.2}$
$N_{tot}$	$< 3.29$	$3.48 \pm 0.13$
$f_{dr}$	$0.059^{+0.024}_{-0.052}$	$0.123 \pm 0.034$
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	1.34	-11.06

CDR (VN & NC)	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES(L)
$10^2 \omega_b$	$2.257 \pm 0.026$	$2.287 \pm 0.016$
$\omega_{cdm}$	$0.1220^{+0.0033}_{-0.0038}$	$0.1276^{+0.0028}_{-0.0032}$
$100\theta_s$	$1.04258^{+0.00061}_{-0.00073}$	$1.04226^{+0.00063}_{-0.00081}$
$\tau_{reio}$	$0.0561^{+0.0071}_{-0.0081}$	$0.0565 \pm 0.0072$
$10^{10} \mathcal{P}_{RR}^{(1)}$	$23.45 \pm 0.55$	$23.11 \pm 0.50$
$10^{10} \mathcal{P}_{RR}^{(2)}$	$20.19 \pm 0.46$	$20.35 \pm 0.45$
$10^{10} N_{dr}^2 \mathcal{P}_{II}^{(1)}$	$< 18.9$	$< 17.6$
$10^{10} N_{dr}^2 \mathcal{P}_{II}^{(2)}$	$< 264$	$407^{+200}_{-300}$
$N_{ur}$	$2.94 \pm 0.25$	$3.18^{+0.27}_{-0.22}$
$N_{dr}$	$< 0.304$	$0.35^{+0.15}_{-0.27}$
$H_0(\text{km/s/Mpc})$	$69.2^{+1.6}_{-1.8}$	$71.46 \pm 0.87$
$\sigma_8$	$0.820 \pm 0.011$	$0.8304 \pm 0.0092$
$10^9 A_s$	$2.073 \pm 0.039$	$2.081 \pm 0.038$
$n_s$	$0.9617 \pm 0.0090$	$0.9675^{+0.0086}_{-0.0075}$
$n_{iso}$	$1.66^{+0.43}_{-0.35}$	$1.87^{+0.39}_{-0.28}$
$f_{iso}$	$58^{+22}_{-53}$	$31.7^{+6.7}_{-27}$
$N_{tot}$	$3.18^{+0.21}_{-0.25}$	$3.53 \pm 0.15$
$f_{dr}$	$0.076^{+0.031}_{-0.068}$	$0.098^{+0.041}_{-0.074}$
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	0.8	-11.68

# Constraint on isocurvature parameters

$$\frac{\delta\sigma}{\sigma} \lesssim 2 \times 10^{-4}$$

Isocurvature constraints at 95 % C.L.

	Planck	Planck +BAO+ SHoES
FDR	$\leq 2 \times 10^{-8}$	$\leq 2.2 \times 10^{-8}$
CDR	$\leq 6 \times 10^{-8}$	$\leq 10 \times 10^{-8}$

$$\frac{\delta\sigma}{\sigma} \lesssim 5 \times 10^{-4}$$

95 % C.L. limits of  $N_{\text{dr}}^2 P_{II}^{(2)}$  for  $N_{\text{dr}} = 0.4$

Planck  $\equiv$  TTTEEE+lowE+lensing

$$P_{II}^{(2)} = A_{\text{iso}} (k = 0.1 \text{Mpc}^{-1})$$

# Conclusion

- DR isocurvature is a very generic in multi field inflation models
- In presence of isocurvature perturbation :  
FDR gives more anisotropy than CDR
- First bound on CDR Isocurvature
- Blue tilted isocurvature accommodates a larger Hubble constant
- For CDR isocurvature - the Hubble tension is reduced to  $2.0\sigma$

*THANK YOU*