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# Flavored Gauge-Mediated Supersymmetry Breaking Models with Discrete Non-Abelian Symmetry

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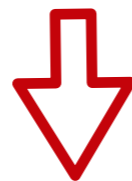
# Gauge mediated SUSY breaking (GMSB)

SUSY is a **broken** symmetry in the vacuum state

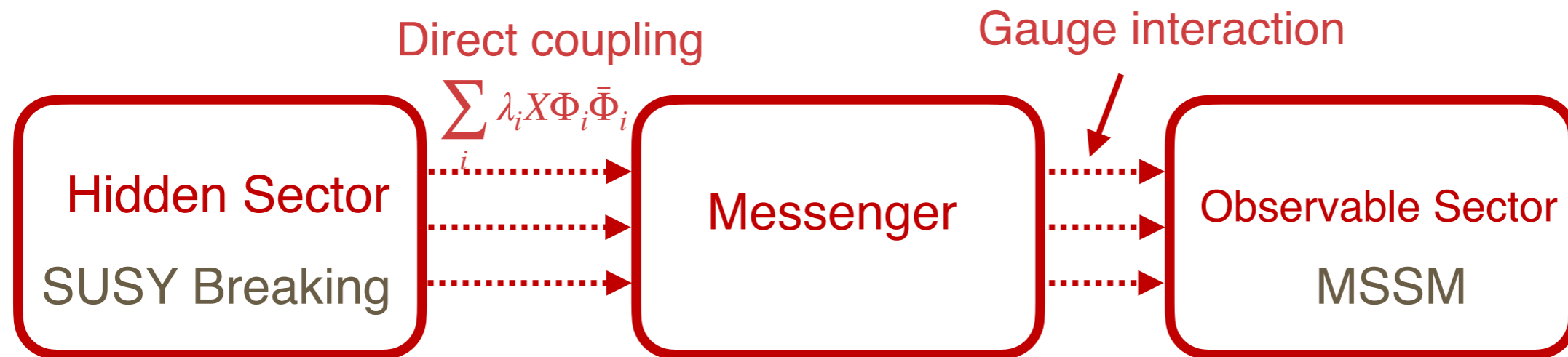
SUSY cannot be broken spontaneously at tree-level (renormalizable)

Supertrace theorem:  $\mathcal{S}\text{Tr}(m^2) = \text{Tr}(m_S^2) - 2\text{Tr}(m_f^2) + 3\text{Tr}(m_V^2) = 0$

Dimopoulos, Georgi (1981)



Require loop effects (non-renormalizable)



SM Singlet  $X$   
SUSY  $\langle F_X \rangle$

$\Phi_i, \bar{\Phi}_i$   
(has SM quantum number)

Dine, Fischler, Srednicki (1981)  
Dine, Nelson (1993)  
Dine, Nelson, Shirman (1996)  
Giudice, Rattazzi (1999)

# Flavored Gauge Mediation (FGM)

## Motivation:

**Minimal Gauge mediation** : Higgs mass of  $\sim 125$  GeV requires heavy stops/ maximal mixing

$$\Delta m_h^2 = \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left( \ln \frac{\tilde{m}_{t1}\tilde{m}_{t2}}{m_t^2} + \frac{X_t^2}{\tilde{m}_{t1}\tilde{m}_{t2}} \left( 1 - \frac{X_t^2}{12\tilde{m}_{t1}\tilde{m}_{t2}} \right) \right)$$

$$X_t = A_t - \mu \cot \beta$$

A terms are zero, **stops must have masses**  $> \mathcal{O}(10 \text{ TeV})$

## One possible extension: Flavored gauge mediation

- Idea:  $SU(2)_L$  doublet messengers mix with MSSM Higgs  $H_{u,d}$
- New messenger Yukawa superpotential coupling terms eg.  $Y_u Q\bar{u}H_u + Y'_u Q\bar{u}M_u$
- This Higgs-messenger mixing is governed by an imposed symmetry eg.  $U(1)$  benchmark model by Ierushalmi et al. (2016)

# FGM with discrete non-Abelian symmetry $\mathcal{S}_3$

- More constraining and thus more predictive
- $\mathcal{S}_3$  is often used in generation of fermion masses

Perez, Ramond, Zhang (2012)

Extend PRZ'12 work for 2-family scenario to 3 families:

- $\mathcal{S}_3$ : Higgs-messenger symmetry + part of family symmetry

	$\mathcal{H}_u^{(2)}$	$\mathcal{H}_u^{(1)}$	$\mathcal{H}_d^{(2)}$	$\mathcal{H}_d^{(1)}$	$Q_2$	$Q_1$	$\bar{u}_2$	$\bar{u}_1$	$\bar{d}_2$	$\bar{d}_1$	$L_2$	$L_1$	$\bar{e}_2$	$\bar{e}_1$	$X_H$
$\mathcal{S}_3$	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>

- Extension of Higgs-messenger sector:  $\mu$  and  $B\mu$  can be tuned separately
- In the basis

$$Q = (Q_2, Q_1)^T = ((Q_2)_1, (Q_2)_2, Q_1)^T, \quad \bar{u} = (\bar{u}_2, \bar{u}_1)^T = ((\bar{u}_2)_1, (\bar{u}_2)_2, \bar{u}_1)^T$$

Superpotential (eg. up quarks)

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u} \mathcal{H}_u^{(1)} & \beta_{2u} \mathcal{H}_{u2}^{(2)} \\ \beta_{1u} \mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u} \mathcal{H}_{u1}^{(2)} \\ \beta_{3u} \mathcal{H}_{u2}^{(2)} & \beta_{3u} \mathcal{H}_{u1}^{(2)} & \beta_{4u} \mathcal{H}_u^{(1)} \end{pmatrix} \bar{u}$$

# FGM with $\mathcal{S}_3$

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u} \mathcal{H}_u^{(1)} & \beta_{2u} \mathcal{H}_{u2}^{(2)} \\ \beta_{1u} \mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u} \mathcal{H}_{u1}^{(2)} \\ \beta_{3u} \mathcal{H}_{u2}^{(2)} & \beta_{3u} \mathcal{H}_{u1}^{(2)} & \beta_{4u} \mathcal{H}_u^{(1)} \end{pmatrix} \bar{u} \quad Y_u = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & \beta_{1u} & \beta_{2u} \\ \beta_{1u} & 1 & \beta_{2u} \\ \beta_{3u} & \beta_{3u} & \beta_{4u} \end{pmatrix}$$

- Our goal: Achieve realistic quark mass hierarchy at leading order



Need extra structures—relations among  $\beta_{iu}$



Classification: Different paths to hierarchy



## Case 1:

- Singlet-dominated limit

$$\beta_{1u} = 1, \quad \beta_{2u} \beta_{3u} = \beta_{4u}$$

- Democratic limit

$$\text{All } \beta_{iu} = 1$$

## Case 2:

- Doublet-dominated limit

$$|\beta_{1u}| \gg \beta_{2u,3u} \gg \beta_{4u} = 0$$

Two different orderings:

$$\beta_{3u} > \beta_{2u}$$

$$\beta_{2u} > \beta_{3u}$$

## Case 1: Democratic limit

- All coefficients are equal:  $\beta_{1i} = \beta_{2i} = \beta_{3i} = \beta_{4i} = 1$
- MSSM Yukawa matrix: 
$$Y_i = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$
 Everett, Garon (2018)
- At leading order: 2 vanishing eigenvalues + an  $\mathcal{O}(1)$  eigenvalue (3rd gen.)
- Flavor democratic mass matrix with  $\mathcal{S}_{3L} \times \mathcal{S}_{3R}$  symmetry Eu, Everett, Garon, Leonard (2021)

Generate non-zero 1st and 2nd gen. fermion masses:

$$Y_i^{(\text{corr})} = \frac{\tilde{y}_i \epsilon_i}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\tilde{y}_i \sigma_i}{\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

Xing (1996)

Fritzsch, Xing (2000)

$$\mathcal{S}_{3L} \times \mathcal{S}_{3R} \Rightarrow \mathcal{S}_{2L} \times \mathcal{S}_{2R} \Rightarrow \mathcal{S}_{1L} \times \mathcal{S}_{1R}$$

3 non-vanishing eigenvalues

These terms can be generated via renormalizable & non-renormalizable superpotential couplings

## Estimation of relative strength of $\epsilon_{u,d,e}$ and $\sigma_{u,d,e}$

- Diagonalizing MSSM Yukawa using biunitary diagonalization:

$$(Y_u)^2 = \begin{pmatrix} y_u^2 & 0 & 0 \\ 0 & y_c^2 & 0 \\ 0 & 0 & y_t^2 \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 & 0 \\ 0 & -\frac{2\epsilon_u}{3\sqrt{3}} - \frac{8\epsilon_u^2}{27\sqrt{3}} + \frac{56\epsilon_u^3}{243\sqrt{3}} - \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 \\ 0 & 0 & \sqrt{3} + \frac{5\epsilon_u}{3\sqrt{3}} + \frac{8\epsilon_u^2}{27\sqrt{3}} - \frac{56\epsilon_u^3}{243\sqrt{3}} \end{pmatrix}$$

Keep to appropriate subleading orders

- Quark masses: Yukawa couplings multiplied by the appropriate Higgs VEV

$$m_t = \frac{y_t v_u}{\sqrt{2}} = \frac{y_t v \sin \beta}{\sqrt{2}} \quad m_b = \frac{y_b v_d}{\sqrt{2}} = \frac{y_b v \cos \beta}{\sqrt{2}}$$

where  $v_u^2 + v_d^2 = v^2 = (246 \text{ GeV})^2$ ,  $\tan \beta = \frac{v_u}{v_d}$

- Use the known fermion masses to find the relative strength of the parameters and examine their effects on sparticle spectra

## Estimation of CKM matrix elements

$$\epsilon_u \approx 3 \times 10^{-2}, \sigma_u \approx 10^{-3}, \epsilon_d \approx 0.1, \sigma_d \approx 9 \times 10^{-3}, \epsilon_e \approx 0.3, \sigma_e \approx 8 \times 10^{-3}$$

- Using the approximation of unitary matrices up to order  $\epsilon^4 \sigma^2$

$$|U_{\text{CKM}}| \approx \begin{pmatrix} 0.99 & 0.17 & 0 \\ 0.17 & 0.99 & 0.02 \\ 0.01 & 0.02 & 1 \end{pmatrix}$$

- Reasonable estimate compared to experimental data:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{bmatrix}$$

PDG (2020)



# Effects of perturbations

$$\sigma_{u,d,e} : \Delta m_{\tilde{d}_{1,2}} = 0.01 \rightarrow 1\text{GeV}$$

$$\sigma_{u,d,e} : \Delta m_{\tilde{u}_{4,5}} = 0.01 \rightarrow 1\text{GeV}$$

$$\epsilon_u : \Delta m_{\tilde{u}_{1,2}} = 0.02 \rightarrow 70\text{GeV}$$

$$\epsilon_u : \Delta m_{\tilde{d}_{1,2}} = 0.02 \rightarrow 25\text{GeV}$$

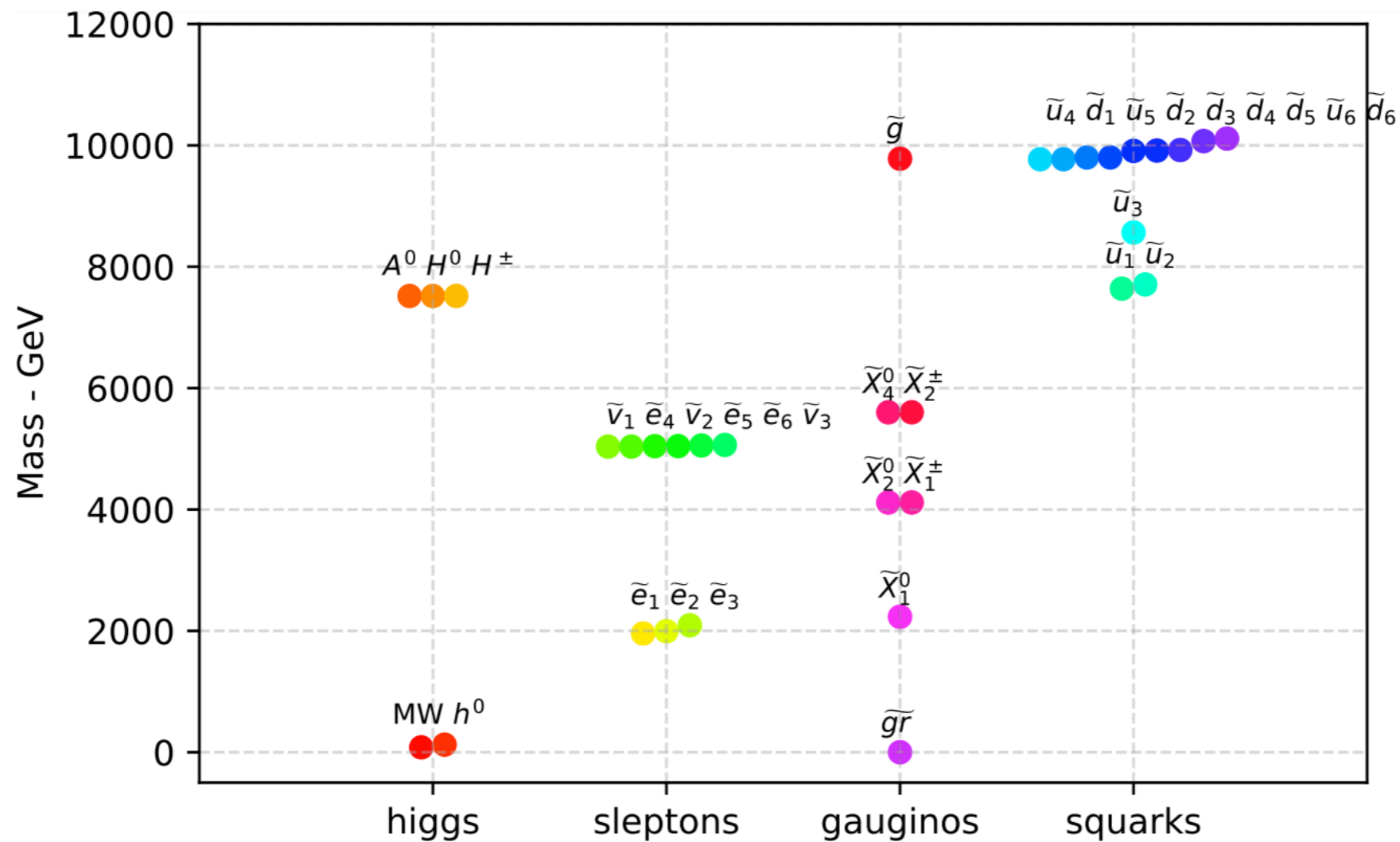
[GeV]	No perturbation	Nonzero $\sigma_{u,d,e}$	Nonzero $\sigma_{u,d,e}, \epsilon_u$	Nonzero $\sigma_{u,d,e}, \epsilon_{u,d,e}$
$M_{\text{mess}}$	$10^{12}$	$10^{12}$	$10^{12}$	$10^{12}$
$\Lambda$	$7.7 \times 10^5$	$7.7 \times 10^5$	$7.7 \times 10^5$	$7.7 \times 10^5$
$\tan \beta$	10	10	10	10
$h^0$	125.14	125.14	125.20	124.97
$H^0$	7175.6	7175.6	7176.6	7176.2
$A^0$	7175.6	7175.5	7176.6	7176.1
$g$	9334.0	9334.0	9334.0	9334.0
$\tilde{\chi}_1^0$	2120.9	2120.9	2120.9	2120.9
$\tilde{\chi}_2^0$	3914.7	3914.7	3914.7	3914.7
$\tilde{\chi}_3^0$	-5353.3	-5353.3	-5354.8	-5354.8
$\tilde{\chi}_4^0$	5356.0	5355.8	5357.3	5357.3
$\tilde{\chi}_1^\pm$	3914.9	3914.9	3914.9	3914.9
$\tilde{\chi}_2^\pm$	5356.1	5356.0	5357.6	5357.6
$\tilde{e}_1$	1876.8	1873.9	1873.9	1858.0
$\tilde{e}_2$	1876.9	1879.8	1879.7	1893.7
$\tilde{e}_3$	1985.7	1985.8	1985.7	1986.7
$\tilde{e}_4$	4799.3	4798.8	4798.8	4795.1
$\tilde{e}_5$	4799.3	4799.9	4799.9	4802.6
$\tilde{e}_6$	4812.0	4812.0	4812.0	4812.7
$\tilde{\nu}_1$	4798.3	4797.8	4797.8	4794.7
$\tilde{\nu}_2$	4798.4	4798.9	4798.9	4801.6
$\tilde{\nu}_3$	4820.8	4820.8	4820.8	4820.9
$\tilde{u}_1$	7334.8	7334.8	7299.9	7299.9
$\tilde{u}_2$	7334.8	7334.8	7365.9	7365.9
$\tilde{u}_3$	8164.4	8164.4	8166.5	8166.5
$\tilde{u}_4$	9338.3	9337.8	9324.3	9323.0
$\tilde{u}_5$	9338.3	9338.8	9350.2	9351.2
$\tilde{u}_6$	9601.3	9601.3	9602.8	9602.8
$\tilde{d}_1$	9338.5	9338.0	9324.8	9323.4
$\tilde{d}_2$	9338.5	9339.0	9350.5	9351.4
$\tilde{d}_3$	9456.6	9445.5	9445.4	9453.2
$\tilde{d}_4$	9456.6	9457.6	9457.5	9458.6
$\tilde{d}_5$	9466.4	9466.5	9466.4	9466.7
$\tilde{d}_6$	9641.4	9641.4	9642.5	9642.5

# Example mass spectrum

$$M_{\text{Mess}} = 10^{12} \text{GeV}, \Lambda = 8.1 \times 10^5 \text{GeV}, \tan \beta = 10$$

$$\epsilon_u = 0.033, \epsilon_d = 0.108, \epsilon_e = 0.281$$

$$\sigma_u = 0.001, \sigma_d = 0.009, \sigma_e = 0.008$$

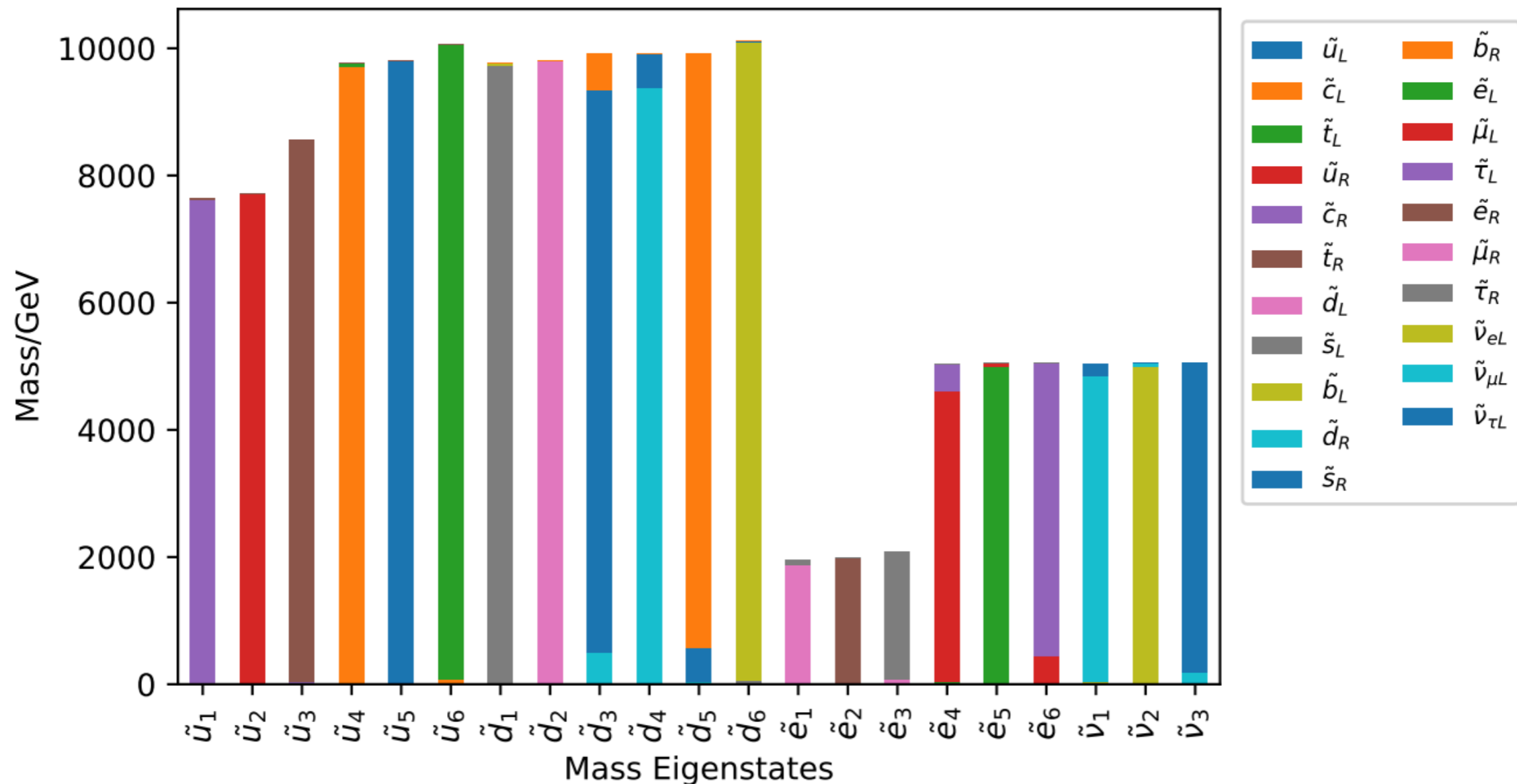


## Example mass spectrum (continued)

$$M_{\text{Mess}} = 10^{12} \text{GeV}, \Lambda = 8.1 \times 10^5 \text{GeV}, \tan \beta = 10$$

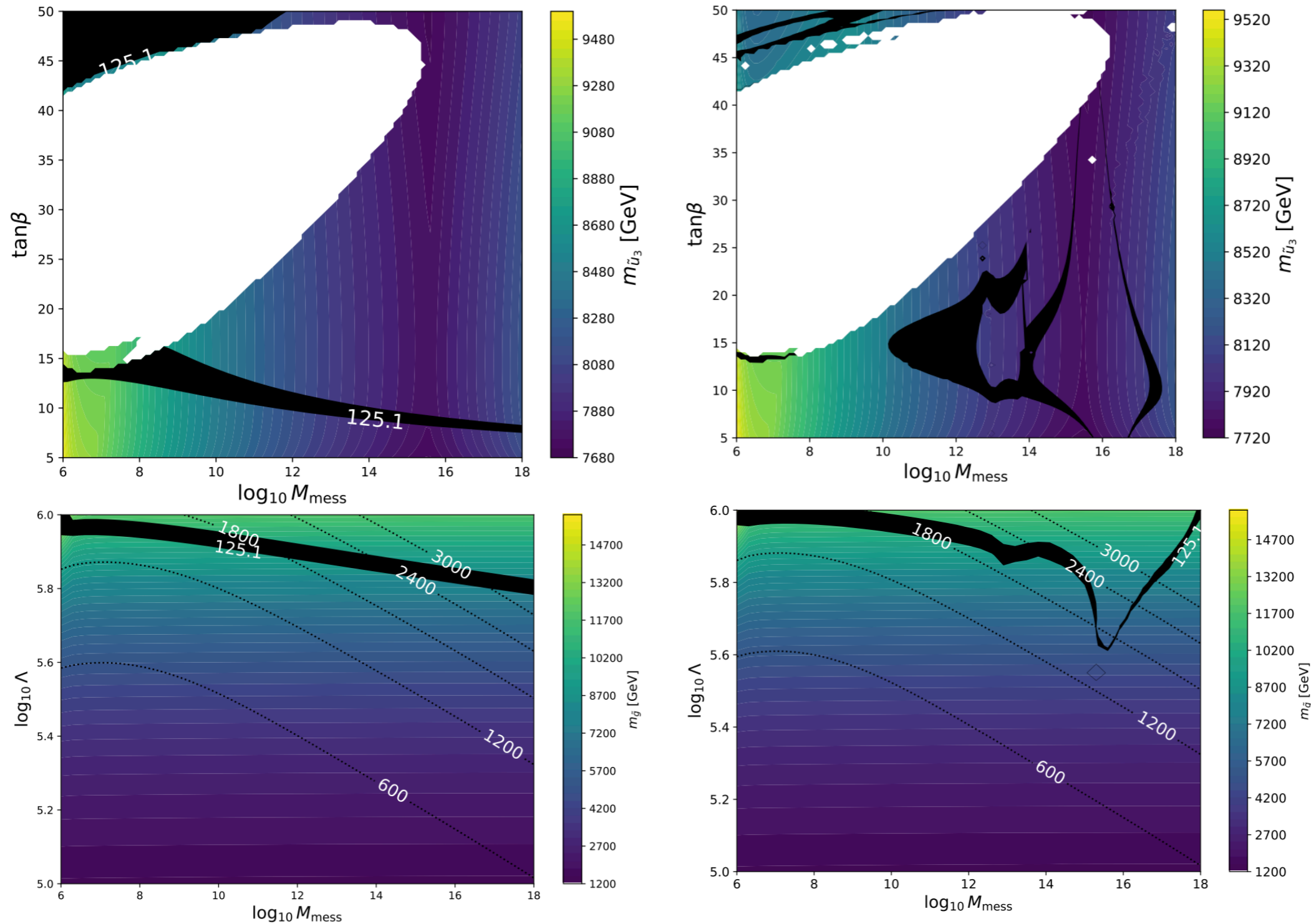
$$\epsilon_u = 0.033, \epsilon_d = 0.108, \epsilon_e = 0.281$$

$$\sigma_u = 0.001, \sigma_d = 0.009, \sigma_e = 0.008$$



- No mixing of flavor eigenstates when there is no perturbation added
- Generational mixing & small left-right mixing

# Full Parameter Space Scans



Higgs mass  
(black shading)

Increase error  
bar — same  
pattern

# Flavor Constraints (SUSY Flavor problem)

- Flavor changing mixing in sfermion mass matrices  $\Rightarrow$  FCNC

Mass Insertion Approximation (MIA):

$$(\delta_Q^{IJ})_{XY} = \frac{(\Delta_Q^{IJ})_{XY}}{(m_{QI})_{XX}(m_{QJ})_{YY}}$$

eg. Super-CKM squark mass squared matrix

$$(M_{\tilde{U}}^2)_{LL} = \begin{pmatrix} (m_{U1}^2)_{LL} & (\Delta_U^{12})_{LL} & (\Delta_U^{13})_{LL} \\ (\Delta_U^{21})_{LL} & (m_{U2}^2)_{LL} & (\Delta_U^{23})_{LL} \\ (\Delta_U^{31})_{LL} & (\Delta_U^{32})_{LL} & (m_{U3}^2)_{LL} \end{pmatrix}$$

$I, J$  : quark flavor

$Q$  : up/ down quark superfield sector

$X, Y$ : superfield chirality

- Non-degenerate squark masses but not strongly hierarchical  $\Rightarrow$  MIA 

- $|(\delta_Q^{IJ})_{XY}|$  predicted in our models are well bounded

Loose bounds since the constraints scale with squark masses (heavy squarks )

Mass insertion is proportional to mass difference between squarks which are small

# Summary

So far we have ...

- Built models with **3 massive quarks** consistent with SM quark masses, with enhanced  $\mathcal{S}_{3L} \times \mathcal{S}_{3R}$  symmetry.
- Achieved reasonable **estimation of CKM** in Case 1 democratic model
- Explored SUSY parameter space in Case 1 democratic model
- For Case 1 democratic model, spectra remain heavy
- Related SUSY breaking and flavor symmetry breaking with the same symmetry group  $\mathcal{S}_3$
- Shown that our models with flavor mixing **satisfy FCNC constraints**
- Gravitino can be plausible dark matter candidate



An aerial photograph of a city waterfront at sunset. The sun is low on the horizon, casting a golden glow over the scene. The water is a deep blue, and numerous sailboats are scattered across it. The city buildings and a forested hillside are visible in the background.

Thank you!

Questions?



## Backup: S3 symmetry

- Permutation Group on three objects
- Three irreducible representations:  
 The singlet  $\mathbf{1}$ , a one-dimensional representation  $\mathbf{1}'$ , a doublet  $\mathbf{2}$
- Tensor products:  
 $\mathbf{1} \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{1}' \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$
- Possible singlet representation obtained:

$$\begin{aligned}
 (\mathbf{2} \otimes \mathbf{2})_{\mathbf{1}} &= \left[ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right]_{\mathbf{1}} = a_1 b_2 + a_2 b_1. \\
 (\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2})_{\mathbf{1}} &= \left[ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \right]_{\mathbf{1}} = a_1 b_1 c_1 + a_2 b_2 c_2
 \end{aligned}$$



## Backup: Higgs Messenger Fields and Rotational Matrix

- Higgs-Messenger field with  $\mathcal{S}_3$  quantum number

$$\mathcal{H}_u \equiv \begin{pmatrix} (\mathcal{H}_u^{(2)})_1 \\ (\mathcal{H}_u^{(2)})_2 \\ \mathcal{H}_u^{(1)} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{H}_{u1}^{(2)} \\ \mathcal{H}_{u2}^{(2)} \\ \mathcal{H}_u^{(1)} \end{pmatrix} = \mathcal{R}_u \begin{pmatrix} H_u \\ M_{u1} \\ M_{u2} \end{pmatrix}$$

$$\mathcal{H}_d \equiv \begin{pmatrix} (\mathcal{H}_d^{(2)})_1 \\ (\mathcal{H}_d^{(2)})_2 \\ \mathcal{H}_d^{(1)} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{H}_{d1}^{(2)} \\ \mathcal{H}_{d2}^{(2)} \\ \mathcal{H}_d^{(1)} \end{pmatrix} = \mathcal{R}_d \begin{pmatrix} H_d \\ M_{d1} \\ M_{d2} \end{pmatrix}$$

- The unitary rotational matrices: 
$$\mathcal{R}_{u,d} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \mp \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) & \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) & -\frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

- Higgs-messenger fields in terms of MSSM Higgs doublets and messenger doublets:

$$\begin{pmatrix} \mathcal{H}_{u1}^{(2)} \\ \mathcal{H}_{u2}^{(2)} \\ \mathcal{H}_u^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} H_u - \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) M_{u1} + \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) M_{u2} \\ \frac{1}{\sqrt{3}} H_u + \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) M_{u1} - \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) M_{u2} \\ \frac{1}{\sqrt{3}} (H_u + M_{u1} + M_{u2}) \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{H}_{d1}^{(2)} \\ \mathcal{H}_{d2}^{(2)} \\ \mathcal{H}_d^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} H_d + \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) M_{d1} + \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) M_{d2} \\ \frac{1}{\sqrt{3}} H_d - \frac{1}{2} \left(1 - \frac{1}{\sqrt{3}}\right) M_{d1} - \frac{1}{2} \left(1 + \frac{1}{\sqrt{3}}\right) M_{d2} \\ \frac{1}{\sqrt{3}} (H_d - M_{d1} + M_{d2}) \end{pmatrix}$$

## Backup: Addressing $\mu/b$ problem

$$\begin{aligned}
 W_H &= \mathcal{H}_u^\top \mathbb{M} \mathcal{H}_d + \theta^2 \mathcal{H}_u^\top \mathbb{F} \mathcal{H}_d \\
 &= \lambda (X_H \mathcal{H}_u^{(2)} \mathcal{H}_d^{(2)}) + \lambda' (X_H \mathcal{H}_u^{(1)} \mathcal{H}_d^{(2)}) + \lambda'' (X_H \mathcal{H}_u^{(2)} \mathcal{H}_d^{(1)}) + \kappa M (\mathcal{H}_u^{(2)} \mathcal{H}_d^{(2)}) + \kappa' M (\mathcal{H}_u^{(1)} \mathcal{H}_d^{(1)})
 \end{aligned}$$

Use the expression for Higgs-messenger fields and:

$$\langle \lambda X_H \rangle = M \begin{pmatrix} \sin \phi \\ \cos \phi \end{pmatrix} + \theta^2 F \begin{pmatrix} \sin \xi \\ \cos \xi \end{pmatrix}$$

We can get:

$$\mathbb{M} = M \begin{pmatrix} \sin \phi & \kappa & \epsilon' \cos \phi \\ \kappa & \cos \phi & \epsilon' \sin \phi \\ \epsilon'' \cos \phi & \epsilon'' \sin \phi & \kappa' \end{pmatrix}, \quad \mathbb{F} = F \begin{pmatrix} \sin \xi & 0 & \epsilon' \cos \xi \\ 0 & \cos \xi & \epsilon' \sin \xi \\ \epsilon'' \cos \xi & \epsilon'' \sin \xi & 0 \end{pmatrix}.$$

For simplicity, choose  $\epsilon'' = \epsilon$ ,  $\epsilon' = 1$ :

$$[\mathbb{M}, \mathbb{F}] = 0 \Rightarrow \kappa' = \kappa = \frac{\sin(\phi - \xi)}{\cos \xi - \sin \xi}$$

$$\mathbb{M} = M \cos \phi \begin{pmatrix} \tan \phi & \frac{\tan \phi - \tan \xi}{1 - \tan \xi} & 1 \\ \frac{\tan \phi - \tan \xi}{1 - \tan \xi} & 1 & \tan \phi \\ 1 & \tan \phi & \frac{\tan \phi - \tan \xi}{1 - \tan \xi} \end{pmatrix}, \quad \mathbb{F} = F \cos \xi \begin{pmatrix} \tan \xi & 0 & 1 \\ 0 & 1 & \tan \xi \\ 1 & \tan \xi & 0 \end{pmatrix}$$

## Backup: Addressing $\mu/b$ problem (cont.)

Eigenvalues of  $\mathbb{M}$  and  $\mathbb{F}$ :

$$F_1 = F(\cos \xi + \sin \xi), \quad F_{2,3} = \mp F \sqrt{1 - \sin \xi \cos \xi}.$$

$$M_1 = M \left( \frac{\cos(\xi + \phi) - 2 \sin(\xi - \phi)}{\cos \xi - \sin \xi} \right), \quad M_{2,3} = \mp M \left( \frac{\cos \phi - \sin \phi}{\cos \xi - \sin \xi} \right) \sqrt{1 - \sin \xi \cos \xi}.$$

Build in eigenvalues hierarchy:

$$F_1 \equiv b \ll F_{2,3} \quad \text{and} \quad M_1 \equiv \mu \ll M_{2,3}$$

Choosing

$$\xi = -\pi/4 + \eta, \quad \phi = \xi + \rho$$

We get:

$$\frac{b}{F} \equiv F_1 = \sqrt{2}\eta + O(\eta^2), \quad \frac{F_{2,3}}{F} = \sqrt{\frac{3}{2}} + O(\eta^2)$$

$$\frac{\mu}{M} \simeq \sqrt{2}\eta + \frac{3}{\sqrt{2}}\rho, \quad \frac{M_{2,3}}{M} \simeq \sqrt{\frac{3}{2}}$$

$$B_\mu = \frac{b}{\mu} = \frac{F}{M} \frac{2\eta}{3\rho} \sim \frac{1}{16\pi^2} \frac{F}{M} \sim m_{\text{soft}}$$

# Backup: Classification of models

- Yukawa matrix:
 
$$Y_i = \frac{y_i}{\sqrt{3}} \begin{pmatrix} 1 & \beta_{1i} & \beta_{2i} \\ \beta_{1i} & 1 & \beta_{2i} \\ \beta_{3i} & \beta_{3i} & \beta_{4i} \end{pmatrix}$$

- Biunitary diagonalization of its Hermitian combination:

$$U_{iL}^\dagger Y_i Y_i^\dagger U_{iL} = (Y_i Y_i^\dagger)_{\text{diag}} = (Y_i^{(\text{diag})})^2, \quad U_{iR}^\dagger Y_i^\dagger Y_i U_{iR} = (Y_i^\dagger Y_i)_{\text{diag}} = (Y_i^{(\text{diag})})^2$$

- Eigenvalues found:

$$\lambda_{1u} = \frac{\tilde{y}_u^2}{3} (1 - \beta_{1u})^2, \quad \lambda_{2u,3u} = \frac{\tilde{y}_u^2}{6} \left( (1 + \beta_{1u})^2 + 2(\beta_{2u}^2 + \beta_{3u}^2) + \beta_{4u}^2 \mp \sqrt{\Lambda_u} \right)$$

$$\Lambda_u = (1 + \beta_{1u})^4 + 4(\beta_{2u}^4 + \beta_{3u}^4) + \beta_{4u}^4 + 4((1 + \beta_{1u})^2 + \beta_{4u}^2)(\beta_{2u}^2 + \beta_{3u}^2) - 2(1 + \beta_{1u})^2 \beta_{4u}^2 - 8\beta_{2u}^2 \beta_{3u}^2 + 16(1 + \beta_{1u})\beta_{2u}\beta_{3u}\beta_{4u}$$

**Case 1:**  $\lambda_{1,2} \ll \lambda_3$

$$\beta_1 \rightarrow 1, \quad \beta_4 = \beta_2 \beta_3$$

- democratic limit: All  $\beta_{iu} = 1$
- singlet-dominated:

$$\beta_{1u} = 1, \quad \beta_{2u}\beta_{3u} = \beta_{4u}$$

**Case 2:**  $\lambda_{2,3} \ll \lambda_1$

$$\beta_1 \rightarrow -1, \quad \beta_{i=2,3,4} \ll 1, \quad \Lambda \rightarrow 0$$

- Doublet dominated limit

## Backup: Soft mass terms for democratic limit (no perturbation)

$$\delta m_{Q_{11}}^2 = \delta m_{Q_{22}}^2 = \frac{\Lambda^2}{(4\pi)^4} [6Y_t^4 + 6Y_b^4 + 2Y_b^2 Y_t^2 + Y_b^2 Y_\tau^2 - \tilde{g}_u^2 Y_t^2 - \tilde{g}_d^2 Y_b^2]$$

$$\delta m_{\tilde{u}_{11}}^2 = \delta m_{\tilde{u}_{22}}^2 = \frac{\Lambda^2}{(4\pi)^4} [12Y_t^4 + 2Y_t^2 Y_b^2 - 2\tilde{g}_u^2 Y_t^2]$$

$$\delta m_{\tilde{d}_{11}}^2 = \delta m_{\tilde{d}_{22}}^2 = \frac{\Lambda^2}{(4\pi)^4} [12Y_b^4 + 2Y_t^2 Y_b^2 + 2Y_b^2 Y_\tau^2 - 2\tilde{g}_d^2 Y_b^2]$$

$$\delta m_{L_{11}}^2 = m_{L_{22}}^2 = \frac{\Lambda^2}{(4\pi)^4} [4Y_\tau^4 + 3Y_b^2 Y_\tau^2 - \tilde{g}_e^2 Y_\tau^2]$$

$$\delta m_{\tilde{e}_{11}}^2 = \delta m_{\tilde{e}_{22}}^2 = \frac{\Lambda^2}{(4\pi)^4} [8Y_\tau^4 + 6Y_b^2 Y_\tau^2 - 2\tilde{g}_e^2 Y_\tau^2]$$

$$\delta m_{H_u}^2 = \delta m_{H_d}^2 = 0, \quad \tilde{A}_u = \tilde{A}_d = \tilde{A}_e = 0$$

$$\tilde{g}_u^2 = \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2, \quad \tilde{g}_d^2 = \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2, \quad \tilde{g}_e^2 = 3g_2^2 + \frac{9}{5}g_1^2$$

- We have matrix structures of the following general form:

$$\begin{aligned} \delta m_{\tilde{Q}}^2 &= (\delta m_{\tilde{Q}}^2)_{11} \text{Diag}(1, 1, 0), & \delta m_{\tilde{L}}^2 &= (\delta m_{\tilde{L}}^2)_{11} \text{Diag}(1, 1, 0) \\ \delta m_{\tilde{u}, \tilde{d}}^2 &= (\delta m_{\tilde{u}, \tilde{d}}^2)_{11} \text{Diag}(1, 1, 0), & \delta m_{\tilde{e}}^2 &= (\delta m_{\tilde{e}}^2)_{11} \text{Diag}(1, 1, 0) \\ \delta m_{H_u}^2 &= \delta m_{H_d}^2 = \tilde{A}_u = \tilde{A}_d = \tilde{A}_e = 0. \end{aligned}$$

## Backup: Soft mass terms for democratic limit (with perturbation)

- Difference: nonzero 23 and 32 entries (equal) generated at the first order of  $\epsilon$ :
- eg. (only some soft mass terms with  $\epsilon$  are shown here)

$$\delta m_{Q_{23}}^2 = \delta m_{Q_{32}}^2 = \frac{\Lambda^2}{(4\pi)^4} \left[ \epsilon_u \left( \frac{8\sqrt{2}}{3} y_t^4 - \frac{4\sqrt{2}}{9} \tilde{g}_u^2 y_t^2 + \frac{4\sqrt{2}}{9} y_b^2 y_t^2 \right) + \epsilon_d \left( \frac{8\sqrt{2}}{3} y_b^4 - \frac{4\sqrt{2}}{9} \tilde{g}_d^2 y_b^2 + \frac{4\sqrt{2}}{9} y_b^2 y_t^2 + \frac{4\sqrt{2}}{9} y_b^2 y_\tau^2 \right) + \mathcal{O}(\epsilon^2) \right]$$

$$\delta m_{\bar{u}_{23}}^2 = \delta m_{\bar{u}_{32}}^2 = \frac{\Lambda^2}{(4\pi)^4} \left[ \frac{8\sqrt{2}\epsilon_u}{9} (6y_t^4 + y_b^2 y_t^2 - \tilde{g}_u^2 y_t^2) + \mathcal{O}(\epsilon^2) \right]$$

$$\tilde{A}_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{2\epsilon_u}{9} (3y_t^3 + y_t y_b^2) & \frac{4\sqrt{2}\epsilon_u}{9} y_t^3 + \frac{4\sqrt{2}\epsilon_d}{9} y_t y_b^2 \\ 0 & \frac{8\sqrt{2}\epsilon_u}{9} y_t^3 & \mathcal{O}(\epsilon^2) \end{pmatrix} \quad \tilde{A}_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{2\epsilon_d}{9} (3y_b^3 + y_t^2 y_b) & \frac{4\sqrt{2}\epsilon_d}{9} y_b^3 + \frac{4\sqrt{2}\epsilon_u}{9} y_t^2 y_b \\ 0 & \frac{8\sqrt{2}\epsilon_d}{9} y_b^3 & \mathcal{O}(\epsilon^2) \end{pmatrix} \quad \tilde{A}_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{2\epsilon_e}{3} y_\tau^3 & \frac{4\sqrt{2}\epsilon_e}{9} y_\tau^3 \\ 0 & \frac{8\sqrt{2}\epsilon_e}{9} y_\tau^3 & \mathcal{O}(\epsilon^2) \end{pmatrix}$$

- In actual codes, take into account relative strength between all perturbation parameters, subleading terms included:

$$\frac{\sigma_u}{\epsilon_u}, \epsilon_u, \epsilon_d, \epsilon_d^2, \frac{\sigma_d}{\epsilon_d}, \sigma_d, \epsilon_e, \epsilon_e^2, \epsilon_e^3, \epsilon_e^4, \sigma_e, \frac{\sigma_e}{\epsilon_e}, \epsilon_u \epsilon_e, \epsilon_d \epsilon_e, \epsilon_d \epsilon_e^2$$

## Backup: Generation of superpotential terms

- eg.  $\epsilon$  perturbation for up quarks can be generated by renormalizable couplings:

$$\epsilon_u y_u [\beta_{2u} Q_2 \bar{u}_1 \mathcal{H}_u^{(2)} + \beta_{3u} Q_1 \bar{u}_2 \mathcal{H}_u^{(2)} + \beta_{4u} Q_1 \bar{u}_1 \mathcal{H}_u^{(1)}]$$

- eg.  $\sigma$  perturbation for up quarks can be generated by non-renormalizable couplings:

$$(\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2})_1 = \left[ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \otimes \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \right]_1 = (a_1 b_2 \pm a_2 b_1)(c_1 d_2 \pm c_2 d_1)$$

Adding two flavons  $\phi$ ,  $\phi_2$  in doublets representation of  $\mathcal{S}_3$ :  $\phi = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$ ,  $\phi_2 = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix}$

$$\sigma_u y_u \left[ \frac{1}{2} \left( (Q_2 \phi \bar{u}_2 \mathcal{H}_u^{(2)})_+ + (Q_2 \phi \bar{u}_2 \mathcal{H}_u^{(2)})_- \right) + \beta_{2u} Q_2 \phi \bar{u}_1 \mathcal{H}_u^{(2)} + \beta_{3u} Q_1 \phi \bar{u}_2 \mathcal{H}_u^{(2)} + \frac{1}{2} \left( (Q_2 \phi_2 \bar{u}_2 \mathcal{H}_u^{(2)})_+ + (Q_2 \phi_2 \bar{u}_2 \mathcal{H}_u^{(2)})_- \right) + \beta_{2u} Q_2 \phi_2 \bar{u}_1 \mathcal{H}_u^{(2)} + \beta_{3u} Q_1 \phi_2 \bar{u}_2 \mathcal{H}_u^{(2)} \right]$$

$$Y_u = \begin{pmatrix} \frac{\sigma \sin(\theta)}{\sqrt{3}} + \frac{\sigma \sin(\theta_2)}{\sqrt{3}} & 0 & \frac{\sigma \cos(\theta)}{\sqrt{3}} + \frac{\sigma \cos(\theta_2)}{\sqrt{3}} & \frac{\sigma \cos(\theta)}{\sqrt{3}} + \frac{\sigma \cos(\theta_2)}{\sqrt{3}} \\ 0 & \frac{\sigma \cos(\theta)}{\sqrt{3}} + \frac{\sigma \cos(\theta_2)}{\sqrt{3}} & \frac{\sigma \sin(\theta)}{\sqrt{3}} + \frac{\sigma \sin(\theta_2)}{\sqrt{3}} & \frac{\sigma \sin(\theta)}{\sqrt{3}} + \frac{\sigma \sin(\theta_2)}{\sqrt{3}} \\ \frac{\sigma \cos(\theta)}{\sqrt{3}} + \frac{\sigma \cos(\theta_2)}{\sqrt{3}} & \frac{\sigma \sin(\theta)}{\sqrt{3}} + \frac{\sigma \sin(\theta_2)}{\sqrt{3}} & 0 & 0 \\ \frac{\sigma \cos(\theta)}{\sqrt{3}} + \frac{\sigma \cos(\theta_2)}{\sqrt{3}} & \frac{\sigma \sin(\theta)}{\sqrt{3}} + \frac{\sigma \sin(\theta_2)}{\sqrt{3}} & 0 & 0 \end{pmatrix}$$

separately tune  $\theta = \frac{\pi}{2}$ ,  $\theta_2 = \pi$

## Backup: Muon $g-2$

- Contribution coming from smuon-sneutrino and sneutrino-chargino loops (significant):

$$\delta a_\mu \simeq \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan \beta \simeq 15 \times 10^{-10} \left( \frac{100 \text{GeV}}{\tilde{m}} \right)^2 \tan \beta$$

Using  $\tilde{m} \approx 1000 \text{GeV}$ ,  $\tan \beta = 50$

$$\delta a_\mu \approx 0.17 \times 10^{-10}$$

- Current experiment bound:

$$\delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$$

- Need detailed analysis including all relevant loop diagrams and other contributions including  $(\chi V H)$ ,  $(\tilde{t} \gamma H)$ ,  $(\tilde{b} \gamma H)$ ...

Stockinger (2006)  
Carena et al (1996)