# Underdamped Axionic Blue Isocurvature Perturbations

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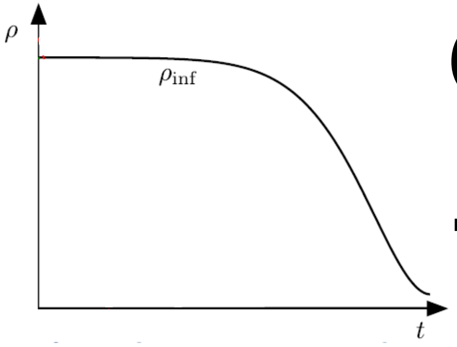
# Layout of the talk

1. Isocurvature fluctuations
2. Axion model and blue power spectrum
3. Underdamped axionic system
4. Key results
5. Prospects



# Our story begins during inflation…

The simplest model: a single scalar “inflaton” field

Quantum fluctuations

**Adiabatic**

$$\Phi(k,\eta\to 0) = \Phi^i(k)$$
$$\delta_S(k,\eta\to 0) = 0$$

$$\delta\phi \to \delta n_\gamma$$
$$\delta\phi \to \delta n_\chi$$

**Isocurvature**

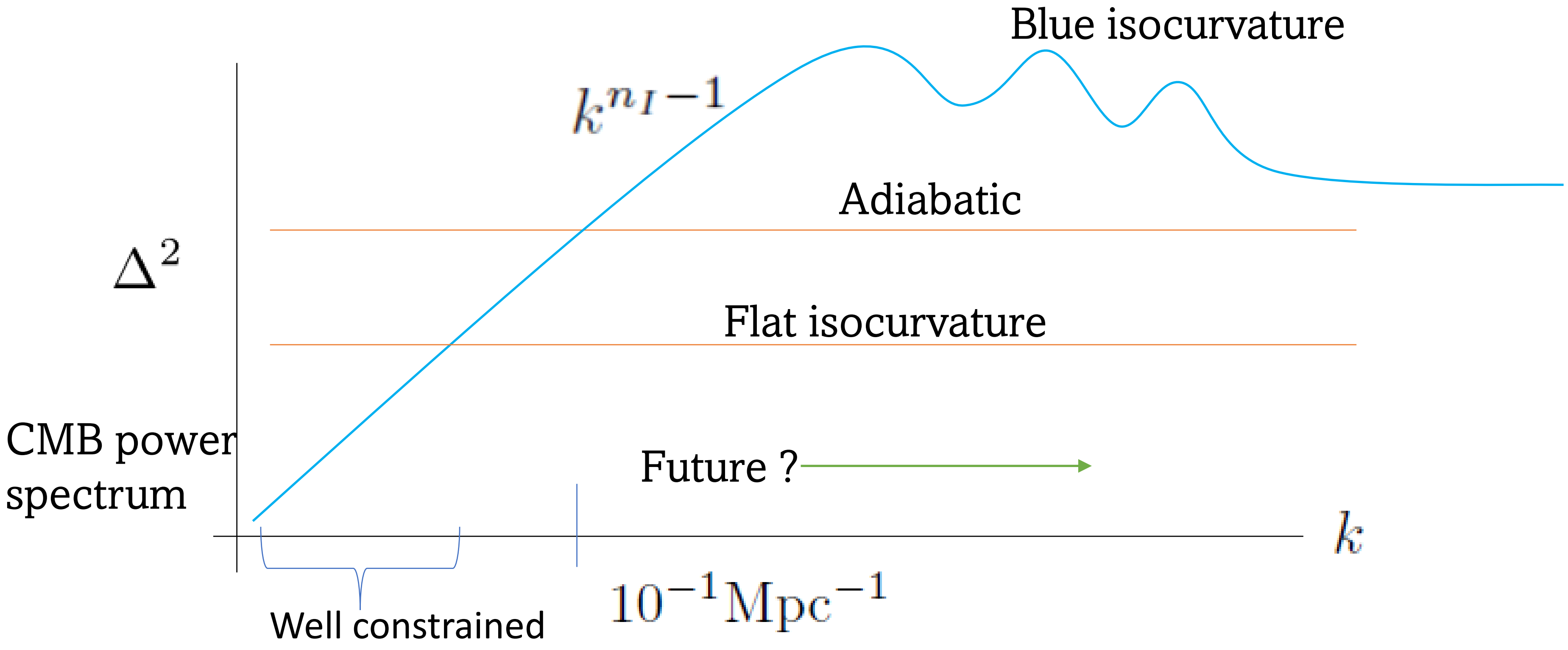
$$\Phi(k,\eta\to 0) = 0$$
$$\delta_S(k,\eta\to 0) = \delta_S^i$$

$$S_{XY} = \frac{\delta n_X}{n_X} - \frac{\delta n_Y}{n_Y}$$

Baumman notes, Dodelson, Kolb-Turner, Weinberg etc

# Current bounds on IC perturbations

scale invariant isocurvature perturbations are observationally constrained to be less than 2% on large (CMB) scales at k=0.05/Mpc. [1807.06211]



# Axions

1. The PQ solution to Strong CP problem (by Peccei-Quinn) -> elevate θ to a dynamical field associated with a U(1) symmetry.

2. Axial direction remains flat giving rise to PNG boson: **axions**

3. Can be a natural non-thermal dark matter candidate due to high decay constant $f_a$ (PQ scale) and hence weak coupling to SM

$$\mathcal{L} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}, \qquad \mathcal{L} = \frac{g_s^2}{32\pi^2} \frac{a}{F_a} G^a_{\mu\nu} \tilde{G}^{\mu\nu a},$$

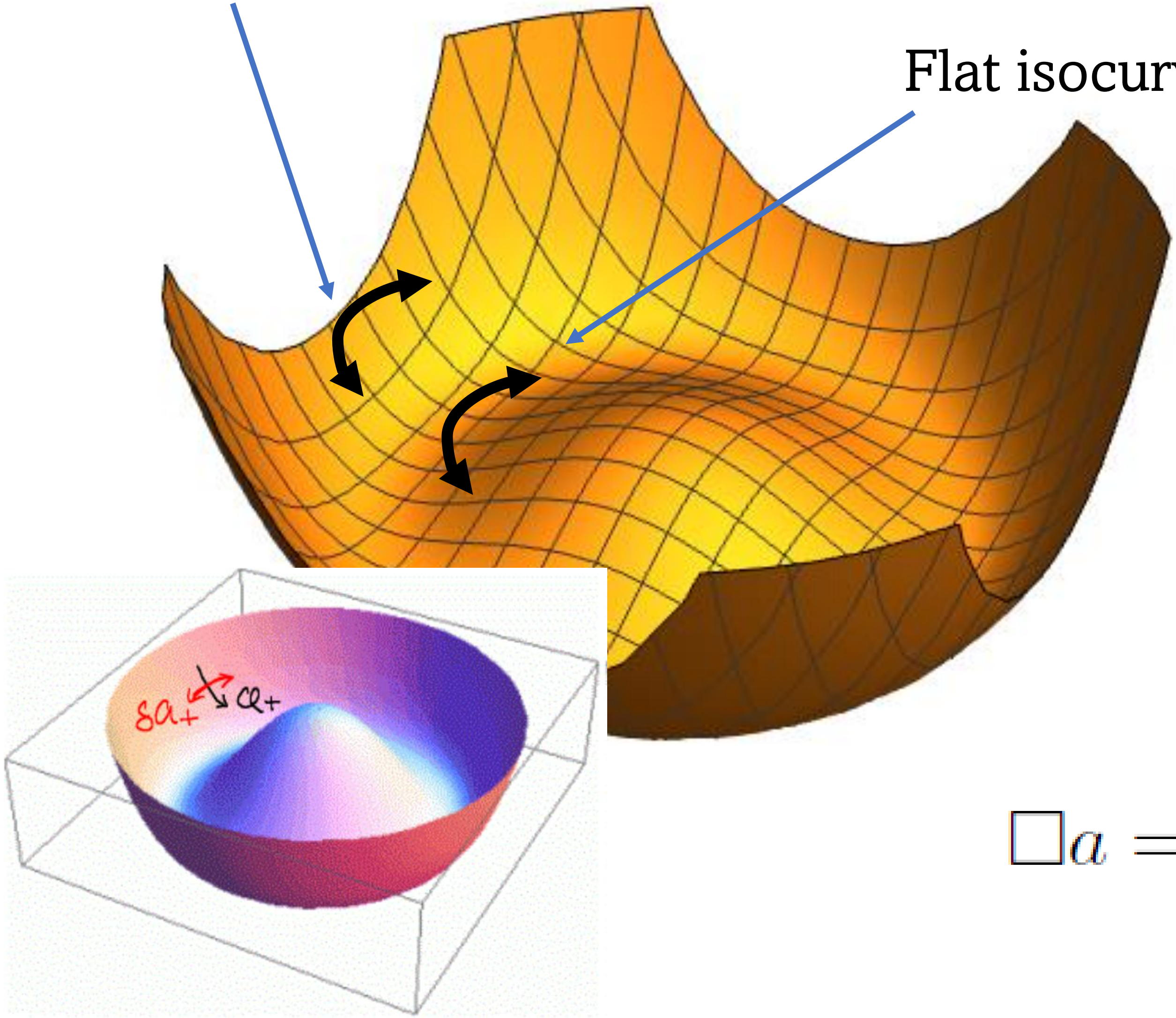
# How to generate blue-tilted spectra ?

S. Kasuya, M. Kawasaki [0904.3800]

$$\phi = |\phi| e^{i\theta_a} = |\phi| e^{ia/\eta}.$$

Blue isocurvature

Flat isocurvature



$$\frac{\delta a}{a} \sim \frac{H/2\pi}{\Phi\theta}$$

$$\Phi : O(M_{\rm Pl}) \to f_{\rm PQ}$$

$$\Box a = 0 \xrightarrow{\text{1501.05618}} \Big(\Box - \overbrace{\frac{\Box\varphi_+}{\varphi_+}}^{\text{Dynamical non-equilib mass}}\Big) a = 0$$

# Radial potential $\phi_\pm$

Renormalizable Superpotential $W_{PQ} = h\left(\Phi_+\Phi_- - F_a^2\right)\Phi_0$

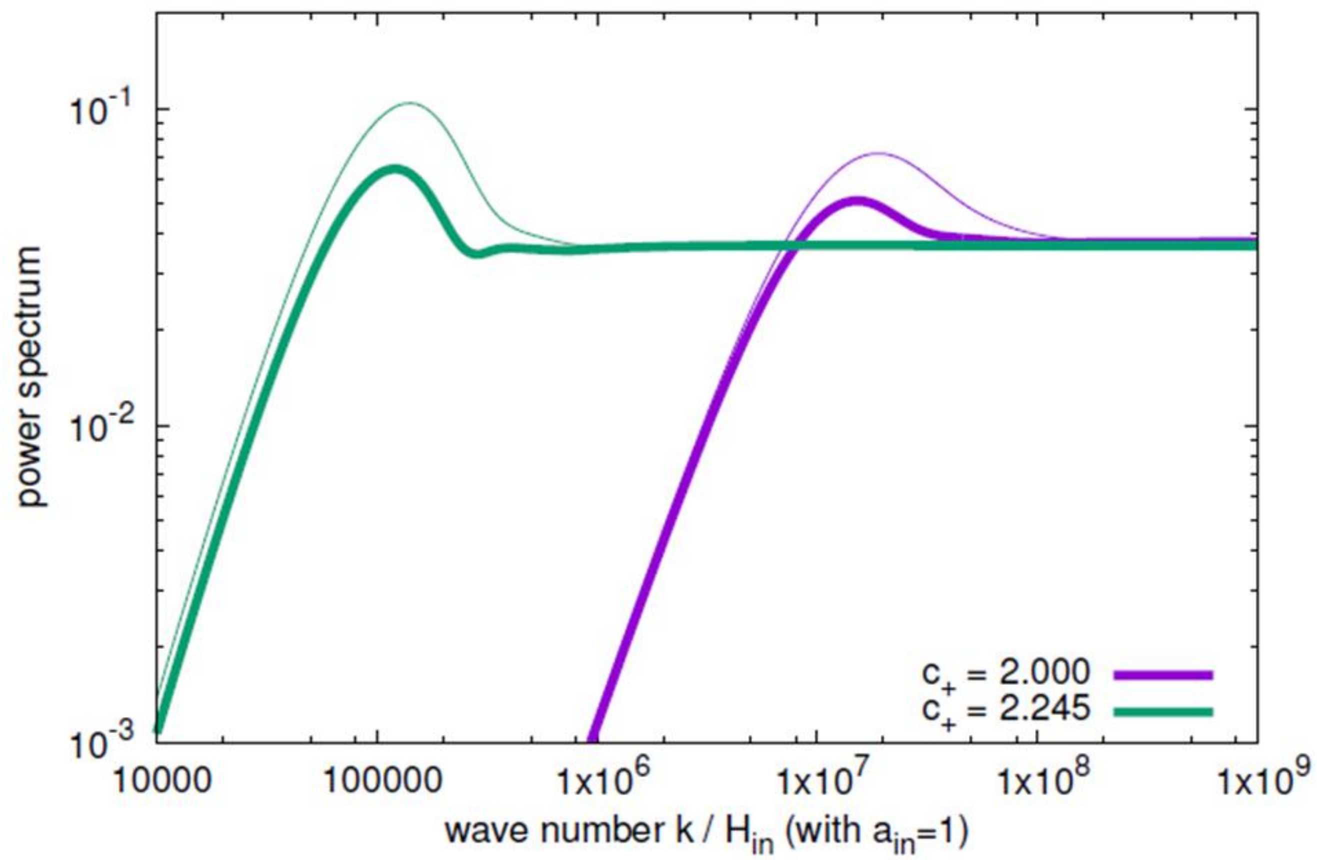
$$\phi = |\phi|e^{i\theta_a} = |\phi|e^{ia/\eta}.$$

$$V = \frac{1}{2}c_+H^2\left|\Phi_+\right|^2 + \frac{1}{2}c_-H^2\left|\Phi_-\right|^2 + \frac{1}{2}\left|\Phi_+\Phi_- - F_a\right|^2$$

c+ > 9/4 is the underdamped situation

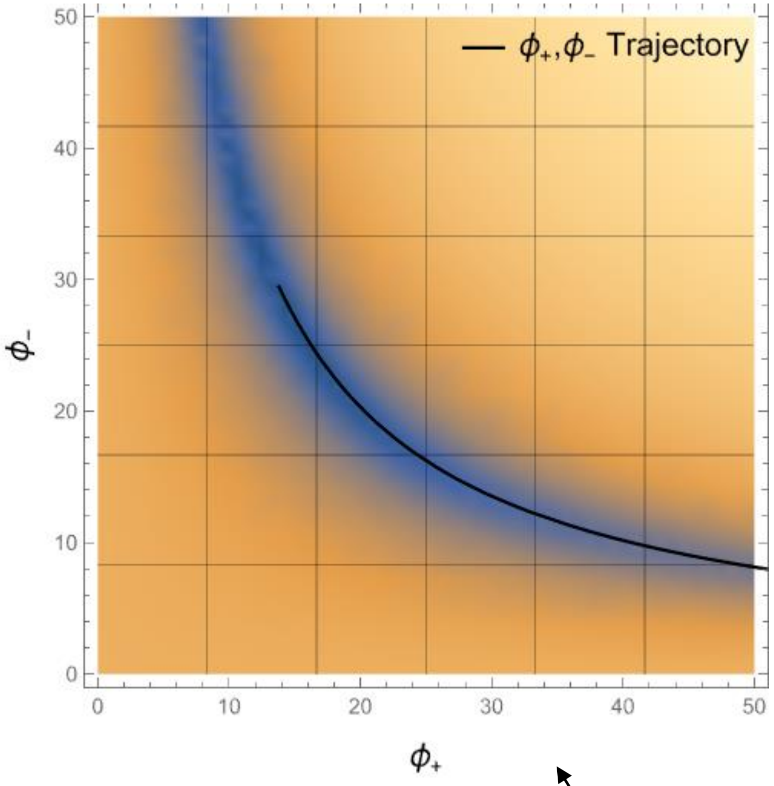
<u>Cubic non-linearity</u>
Complicated dynamical behaviour: highly non-trivial

c+ < 9/4 is the overdamped situation

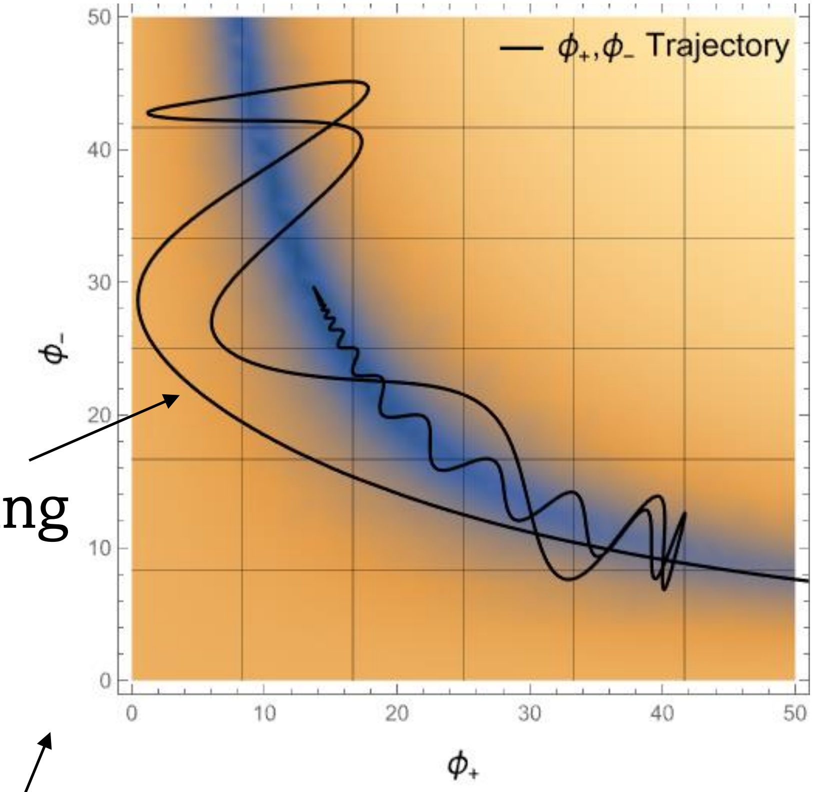


**Overdamped isocurvature power spectrum**

Overdamped
c+ = 9/4 – 0.1



Underdamped
c+ = 9/4 + 0.1



overshooting

$$\phi_+ \sim \exp\left(-\frac{3}{2}T + T\sqrt{\frac{9}{4} - \frac{m_a^2}{H^2}}\right)$$

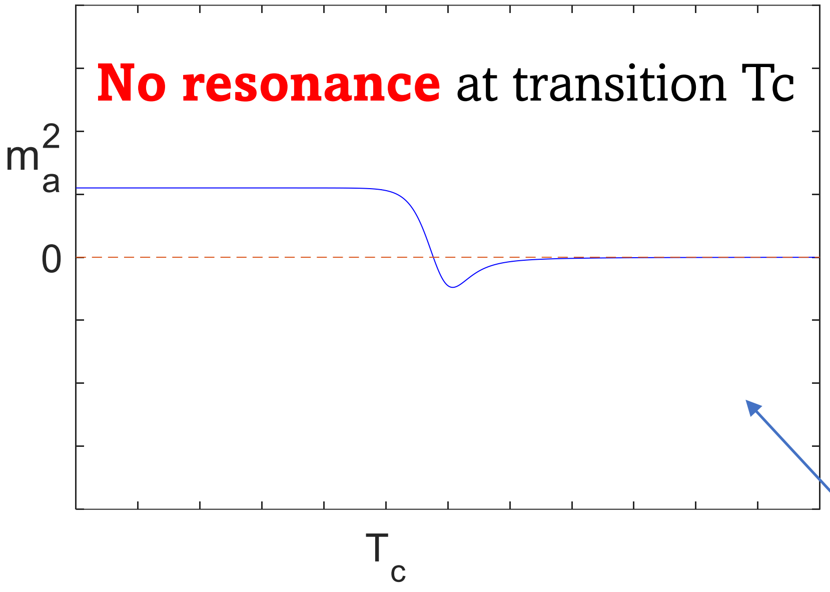
Gravity driven or Potential driven

$$\phi_+ \sim \exp\left(-\frac{3}{2}T\right)\cos\left(T\sqrt{\frac{m_a^2}{H^2} - \frac{9}{4}} + \varphi\right)$$

**Large kinetic energy** (due to cos function) makes a difference at transition

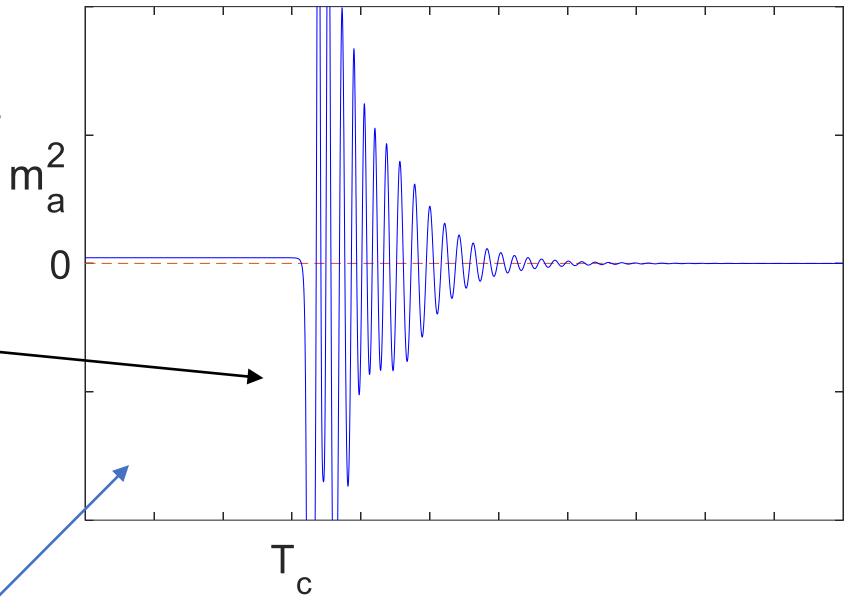
# New resonant behavior

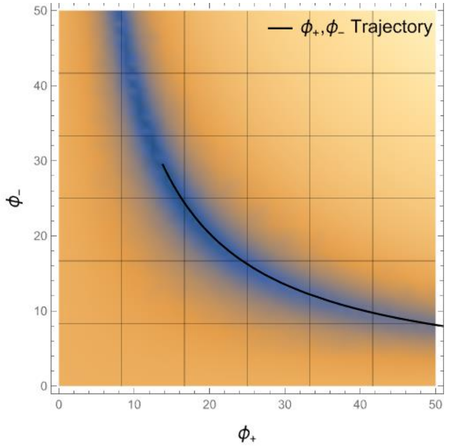
Overdamped
c+ = 9/4 – 0.1

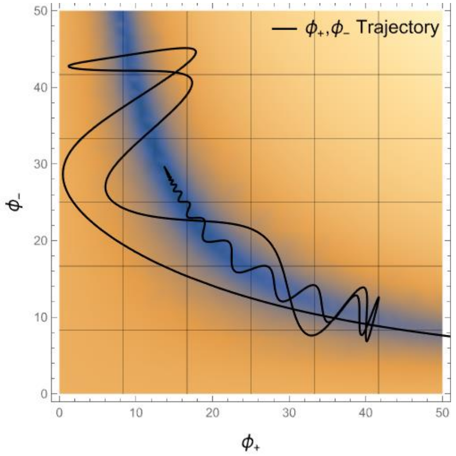

**No resonance** at transition Tc

Significant deviation from flat-direction results in **resonant behavior** at transition Tc

Underdamped
c+ = 9/4 + 0.1







$$\left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right) a = 0$$

$$\chi_{\rm HM}\left(l_1^2,l_2^2\right)\approx\frac{1}{l_2^2-l_1^2}\left(1+2\sqrt{-l_1^2}+\frac{8/3}{\sqrt{-l_1^2}}\left(-1+e^{-\sqrt{-l_1^2}}\cos\left[l_2\right]\right)\right)$$

$$\begin{bmatrix} y_1 \\ \dot{y}_1 \end{bmatrix}_{T_N^-} = \prod_{j=0}^{N-1} S(T_j,T_{j+1}) \begin{bmatrix} y_1 \\ \dot{y}_1 \end{bmatrix}_{T=T_0} .$$

Analytic methods

Decoupling

Perturbation theory

$$\lambda \leftrightarrow O\left(\sqrt{\frac{\phi_-}{\phi_+}}\right)$$

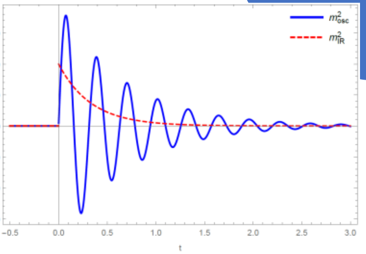
Polynomial fitting

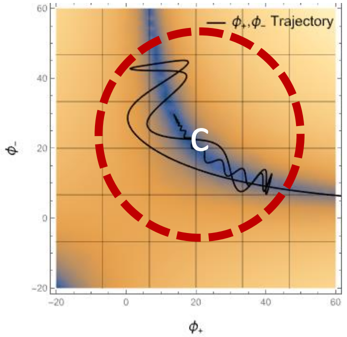
Non-linear field re

$$\xi=\phi_+\phi_- - F^2$$

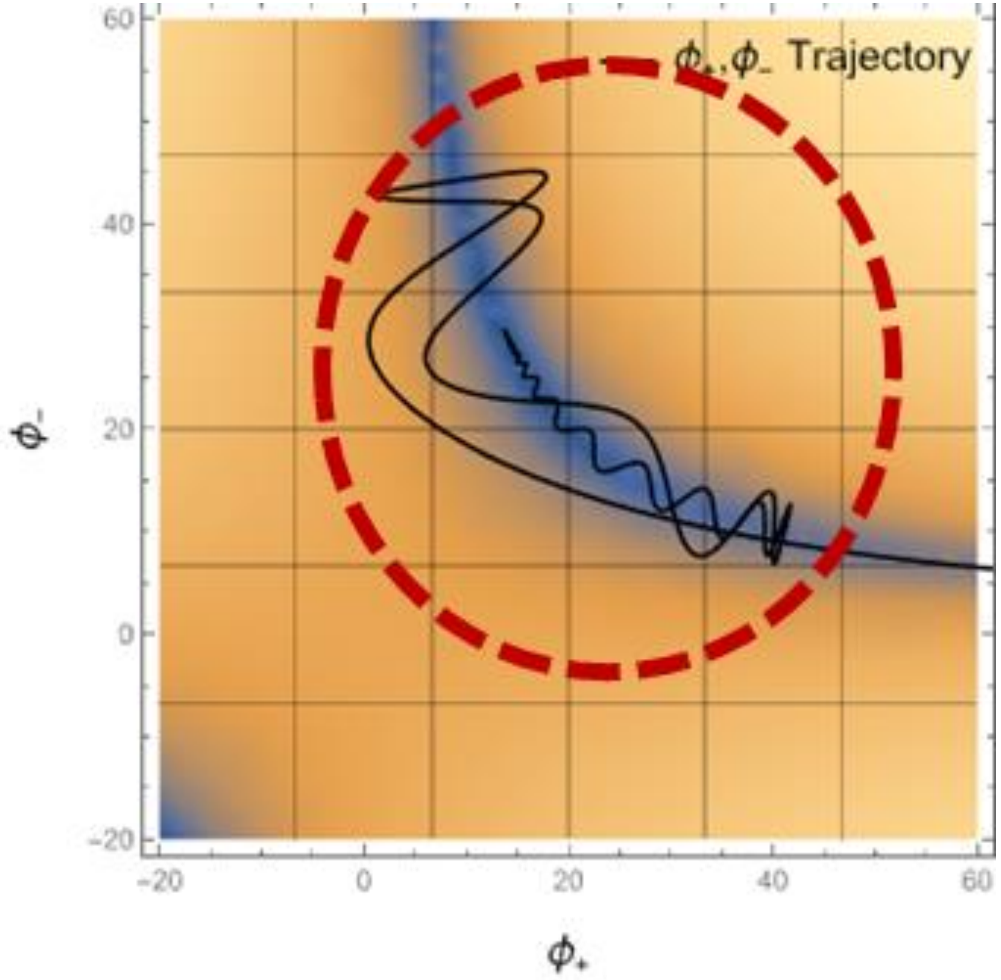
Time-Space effective pot.

Scattering matrices

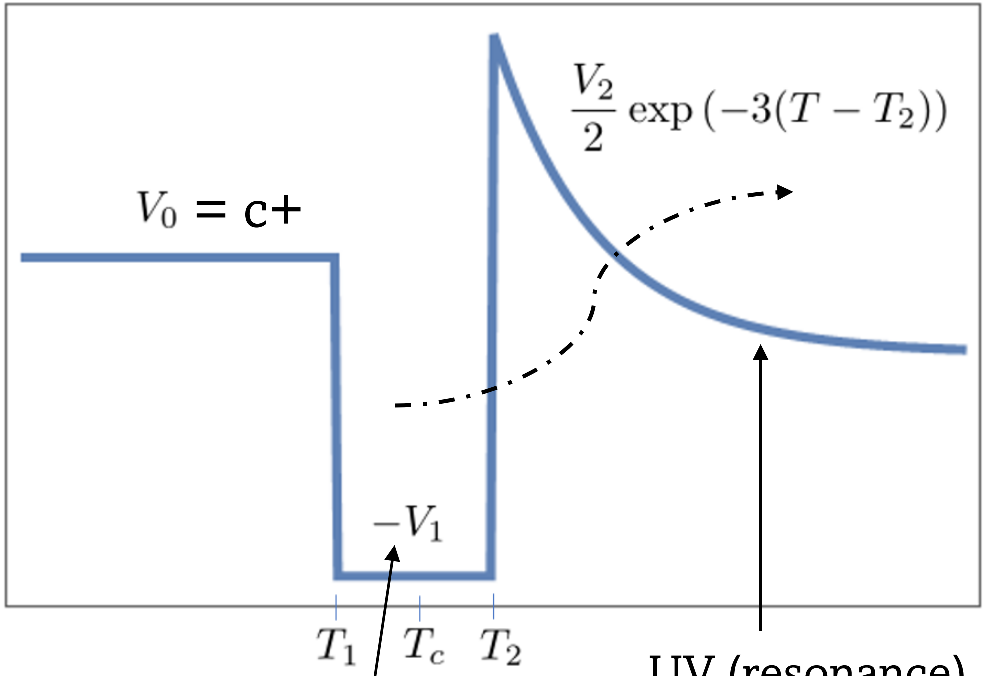




# Key physics: Scattering of tachyonic quantum modes as they exit the well



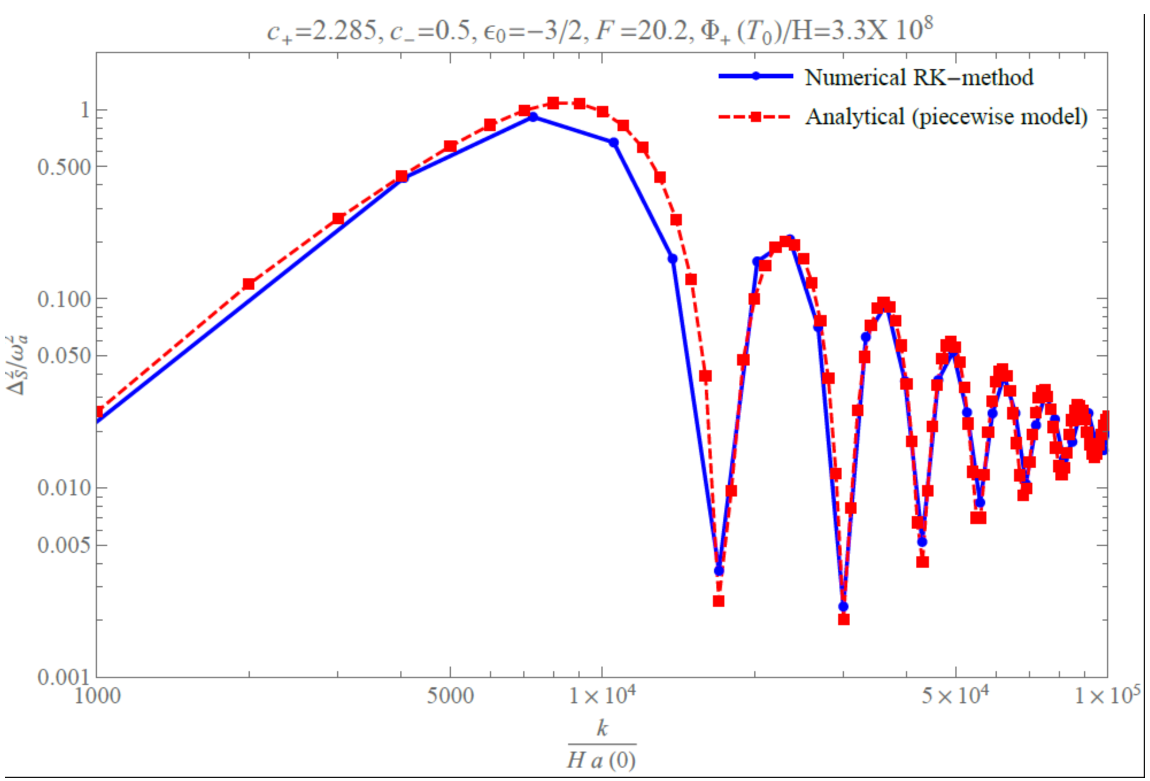
Analytical framework

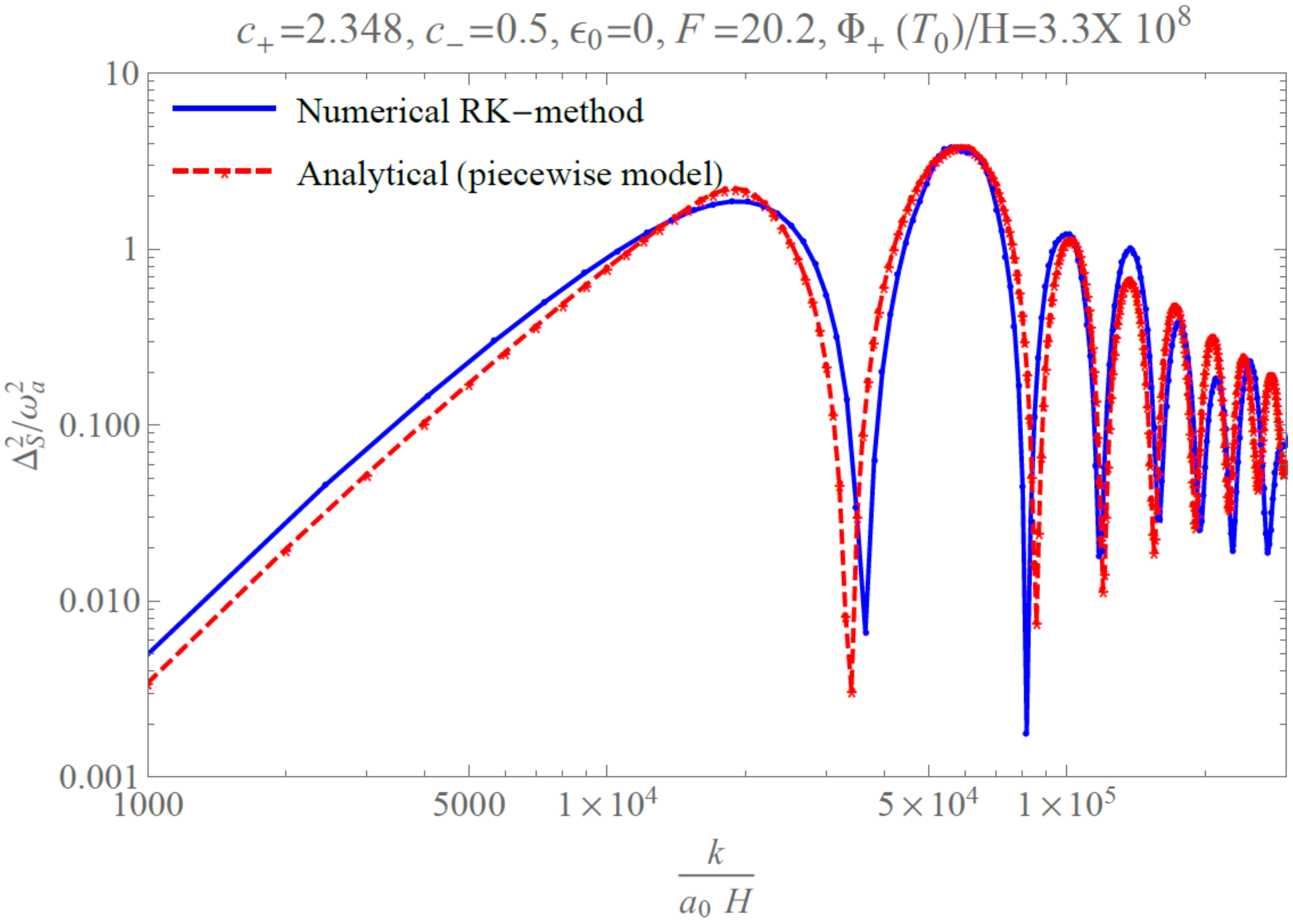


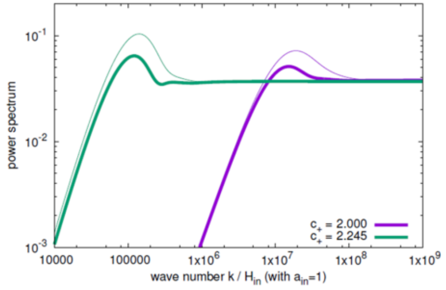
Nonadiabatic mass

UV (resonance) integrated effective IR potential

# Successful in generating key features of the spectrum.







1. Large amplification due to tachyonic mass,
2. and multiple bumps due to quantum scattering of vacuum modes

# Final results are intricate as expected (but stories can be told through them):

For resonant, not too energetic case:

$$\Delta_S^2(K) \approx |f_{\mathrm{correction}}(K)|^2 \times \begin{cases} C_1 K^3 \left| 1 - i\frac{\Gamma(i\omega)\Gamma(i\omega+1)}{\pi(1+i\cot(i\omega\pi))} e^{-2i\omega \ln\left(\frac{-K\tau_1}{2}\right)} \right|^2 & -K\tau_c \ll 1 \\ C_2 \mathfrak{D}^2 \left| H^1_{i\omega}(-K\tau_1) \right|^2 (-K\tau_2) \Big( \sin(-K\tau_2) & \\ + (3/2 + b\tanh[-b\Delta T]) \left( \frac{\cos[-K\tau_2]}{-K\tau_2} - \frac{\sin[-K\tau_2]}{(-K\tau_2)^2} \right) \Big)^2 & 0.5 \lesssim -K\tau_c < 3 \\ C_3 \mathfrak{D}^2 \cosh^2[b\Delta T] \times & \\ \Big| (-ie^{iK\tau_2}) + \tanh[-b\Delta T] \times & \\ \left( \left( \frac{b}{-K\tau_2} \right) \cos[-K\tau_2] + \left( i\frac{-K\tau_2}{b} \right) \sin[-K\tau_2] \right) \Big|^2 & 3 \lesssim -K\tau_c < K_2 \\ C_4 \times 1 & K > K_P \end{cases}$$

$$\omega \equiv \sqrt{c_+ - 9/4}$$

$$C_1 \approx C\mathfrak{D}^2 \frac{\pi}{8} e^{-\omega\pi} \cosh^2[b\Delta T] \frac{e^{-3T_2}}{3} \left( \frac{3}{2} - b\tanh[-b\Delta T] \right)^2 \left| \frac{1 + i\cot(i\omega\pi)}{\Gamma(i\omega+1)} \right|^2$$

$$C_2 \approx C\frac{\pi}{8} e^{-\omega\pi} \cosh^2[b\Delta T]$$

$$C_3 \approx C\frac{1}{4}$$

$$C_4 \approx \omega_a^2 \frac{h^2}{2\pi^2\theta_+^2 F^2} \left( \frac{r}{1+r^2} \right)$$

$$C = \omega_a^2 \frac{4}{\pi^2} \frac{r(1+r^4)}{(1+r^2)^3} \frac{h^2}{\theta_+^2 F^2}$$

$$r = \sqrt{c_+/c_-}$$

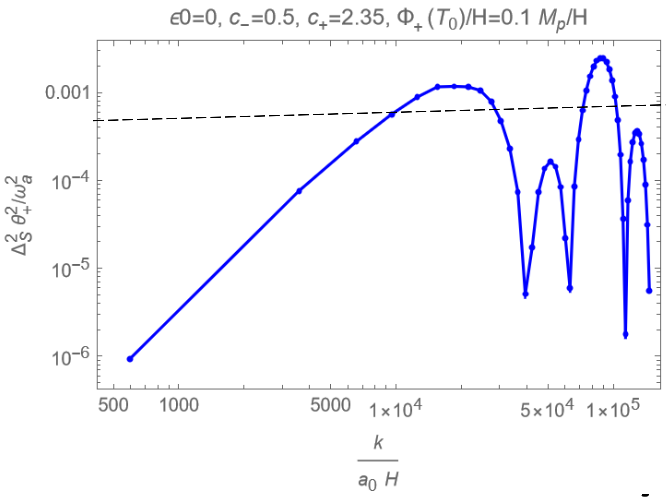
$$V_B \approx c_- + \frac{1}{(T_L - T_2)} \left( \frac{1063}{3072} + \frac{106793 c_-}{393216 c_+} \right)$$

$$\mathfrak{D} \approx \exp\left( \left( -\frac{3}{2} + \sqrt{\frac{9}{4} - V_B} \right) (T_B - \tilde{T}) \right)$$

... more ... → https://pages.physics.wisc.edu/~stadepalli/Blue-Axion-IsoCurvSpec-Underdamped.nb

# **Blues** prospects:

- A **2-sigma** hint from latest evaluations [**1711.06736, 1707.09354**]
- Planck TT,TE,EE+lowE+lensing **[1807.06211]** gives **95 % CL** for a blue index 1.55 < $\mathbf{n_I}$ < 3.67 consistent with the recent findings (C&U, 2017).
- Large room for discovery with future expts like **SKA**, **LSST**, and **Pixie**.
- Bumps in the power spectrum can give rise to peaked **PBH** mass spectrum



Accessible from: **https://pages.physics.wisc.edu/~stadepalli/file.nb**

# Thanks