Mass and Width of Unstable Particles

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What is mass?

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What is the physics principle?

What is width?

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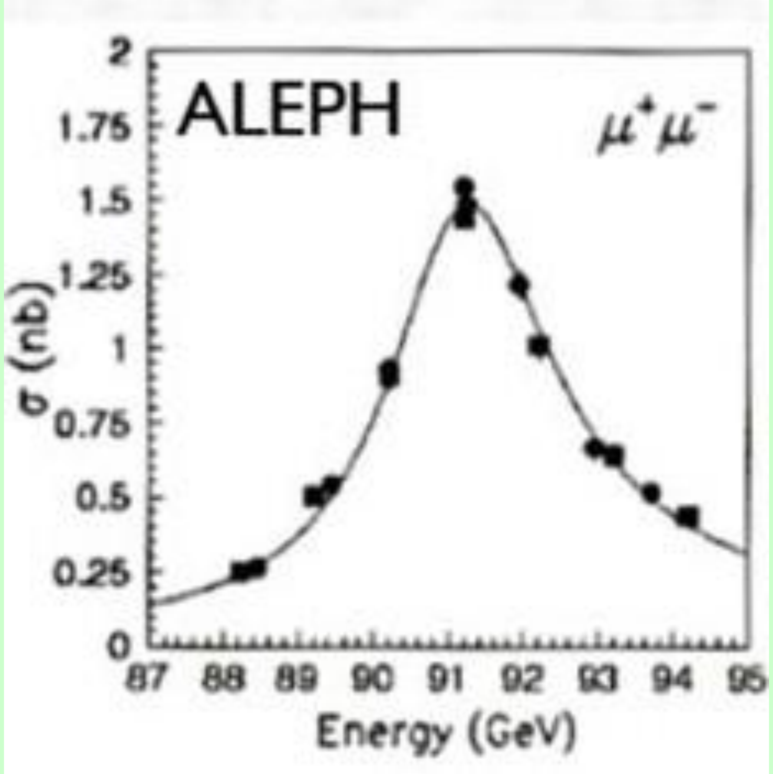
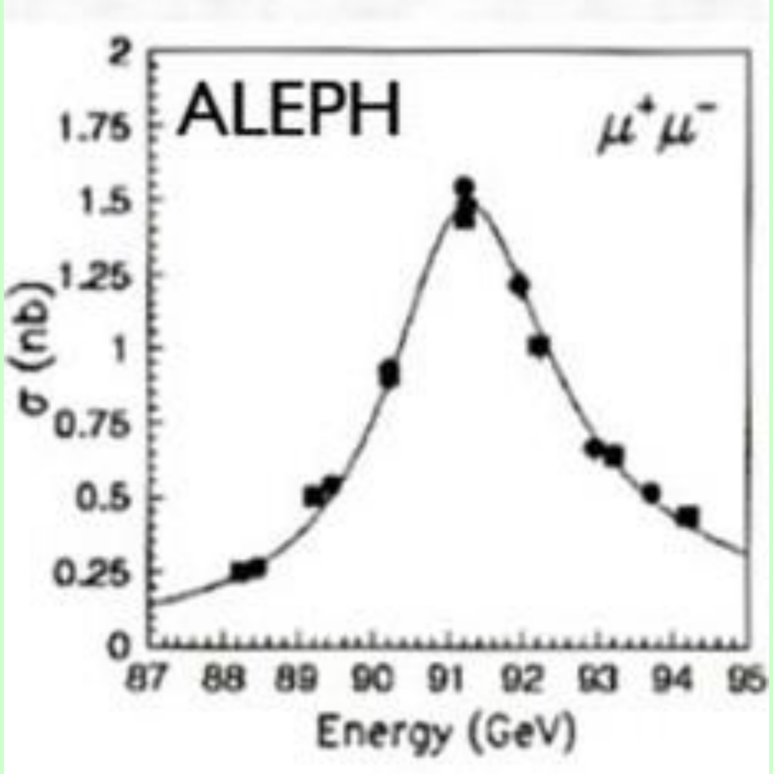


What is width?



$$\Gamma = 1/\tau$$

Time dependence

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In general $$\frac{1}{p^2 - \mu^2}$$ **Unstable particle**

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In general

$$\frac{1}{p^2 - \mu^2}$$

Unstable particle

Rest frame

$$\frac{1}{p_0^2 - \mu^2} = \frac{1}{(p_0 - \mu)(p_0 + \mu)}$$

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$$\frac{1}{p_0^2 - \mu^2} = \frac{1}{(p_0 - \mu)(p_0 + \mu)}$$

Fourier transform

$$\int_{-\infty}^{\infty} dp_0 \frac{e^{-ip_0 t}}{(p_0 - \mu)(p_0 + \mu)}$$

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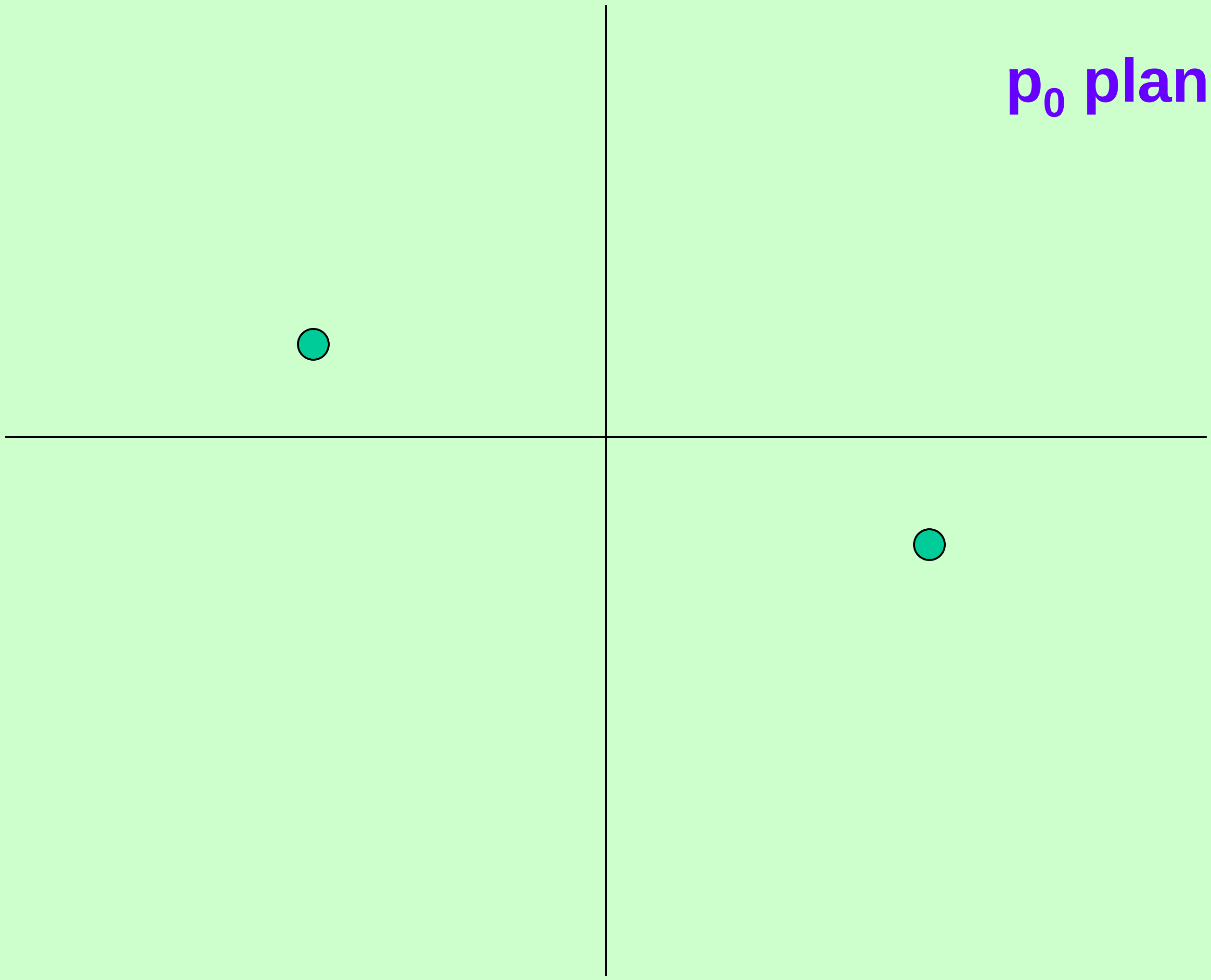
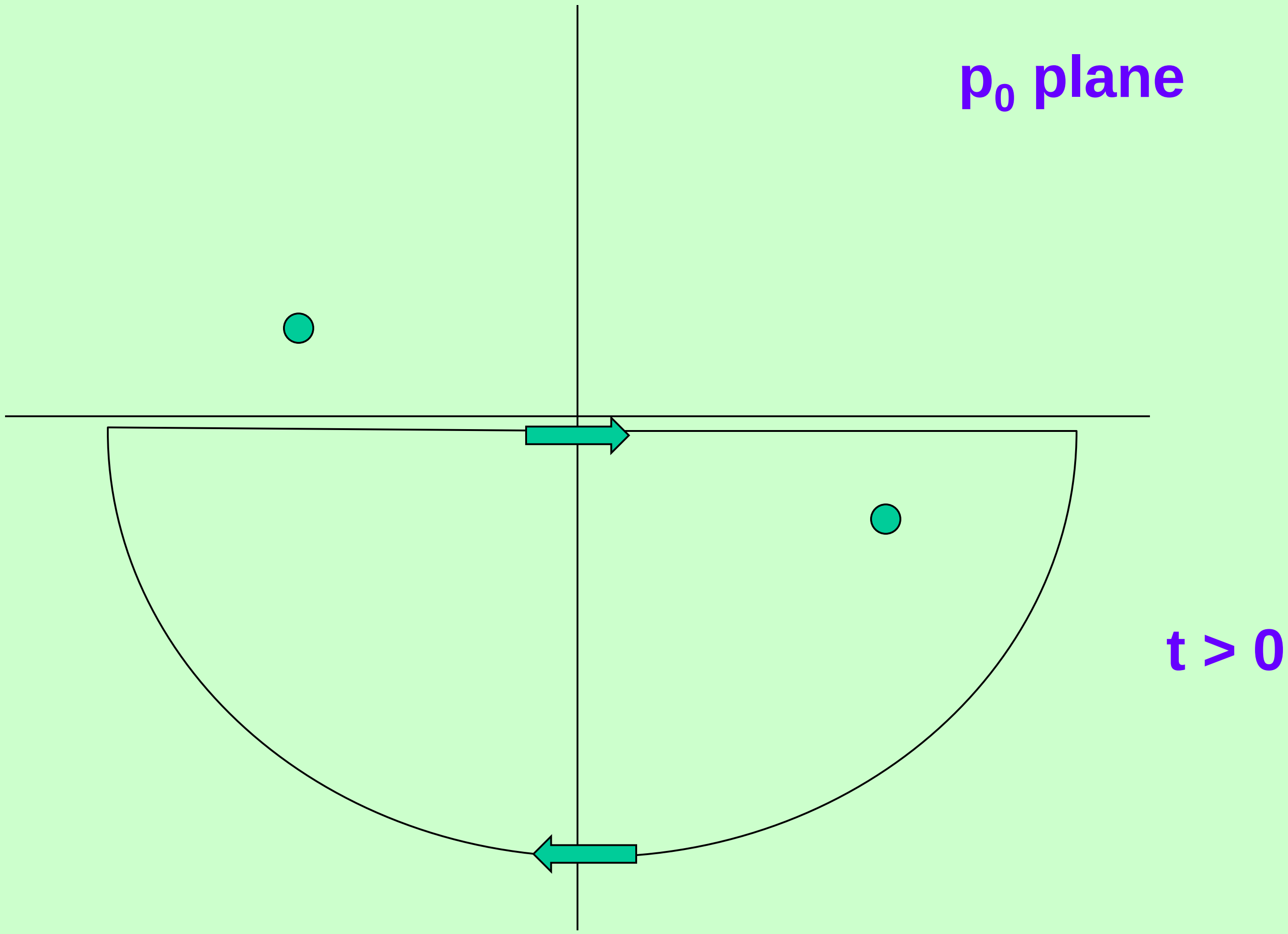


p_0 plane

$$\int_{-\infty}^{\infty} dp_0 \frac{e^{-ip_0 t}}{(p_0 - \mu)(p_0 + \mu)} \sim e^{-i\mu t}$$



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$$\Rightarrow \quad \boxed{\mu = m - \frac{i}{2}\Gamma} \; .$$

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Probability $\sim e^{-\Gamma t}$

Don't we already know this?

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PHYSICAL REVIEW D **71**, 085018 (2005)

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width parameters, the width Γ_R of the relativistic resonance such that the lifetime $\tau = \hbar/\Gamma_R$. This leads to the parametrization $s_R = (M_R - i\Gamma_R/2)^2$ and uniquely defines these (M_R, Γ_R) as the mass and width

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49.2 Properties of resonances

The main characteristics of a resonance is its pole position, s_R, in the complex s–plane that is independent of the reaction studied. The more traditional parameters mass $M_{\rm R}$ and total width $\Gamma_{\rm R}$ may be introduced via the pole parameters

$$\sqrt{s_{\rm R}} = M_{\rm R} - i\Gamma_{\rm R}/2\ . \qquad (49.12)$$

Review of Particle Properties 2020

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Review of Particle Properties 2020

Capture of Slow Neutrons

G. Breit and E. Wigner, *Institute for Advanced Study and Princeton University*

(Received February 15, 1936)

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Precisely $$\frac{1}{p^2 - \mu^2} = \frac{i}{p^2 - m^2 + \Gamma^2/4 + im\Gamma}$$ $\mu = m - (i/2)\Gamma$

Numerics

$$m = M\left(1 - \frac{3}{8}\left(\frac{\Gamma}{M}\right)^2 + \cdots\right)$$

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$m_W = M_W$ – 20 MeV $\Delta M_W = 10$ MeV