Bound on Squeezed State Gravitational Wave Fluctuations from LIGO

Jacob Litterer

Institute of Cosmology, Tufts University

May 9, 2022

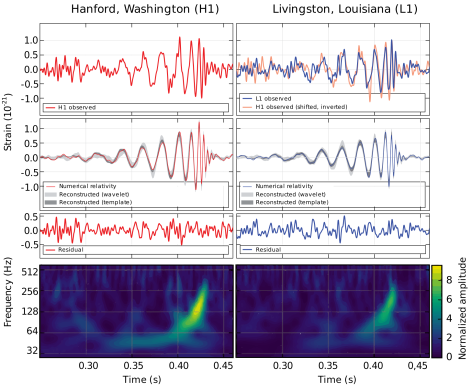
M.P. Hertzberg, J.L. [arXiv:2112.12159]

Aren't quantum fluctuations incredibly tiny?

- Observed gravitational waves are well-described by classical GR, with quantum corrections typically Planck suppressed.
- Indeed if gravitational waves are assumed to be a **coherent** superposition of many gravitons, quantum fluctuations are then very small: $\sigma \sim h_c \, 10^{-19}$.

With these assumptions, one concludes there is nothing (quantum) to see here.

e.g. Dyson (2013)



Quantum gravitational noise enhanced by squeezing

New interest in quantum fluctuations in gravitational waves observed at detectors inspired by recent work by Parikh, Wilczek, and Zahariade ([2005.07211], [2010.08205], [2010.08208]).

Studying test particles in the presence of a graviton background $\longrightarrow$ geodesic deviation modified by the quantum noise of gravitons,

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{N}_{\Psi} + \ddot{h} - \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2\right)\xi.$$

Quantum gravitational noise enhanced by squeezing

New interest in quantum fluctuations in gravitational waves observed at detectors inspired by recent work by Parikh, Wilczek, and Zahariade ([2005.07211], [2010.08205], [2010.08208]).

Studying test particles in the presence of a graviton background $\longrightarrow$ geodesic deviation modified by the quantum noise of gravitons,

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{N}_\Psi + \ddot{h} - \cancel{\frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2}\right)\xi.$$

Neglecting radiation reaction, $\langle \xi \rangle = \left(1 + \frac{h}{2}\right)\xi_0$ and $\sigma = \frac{\xi_0^2}{4}\langle N_\Psi^2 \rangle$. In the vacuum or coherent state,

$$\sigma \sim \xi_0 \ell_{\text{Pl}} \frac{\omega_c}{c}.$$

Quantum gravitational noise enhanced by squeezing

New interest in quantum fluctuations in gravitational waves observed at detectors inspired by recent work by Parikh, Wilczek, and Zahariade ([2005.07211], [2010.08205], [2010.08208]).

Studying test particles in the presence of a graviton background $\longrightarrow$ geodesic deviation modified by the quantum noise of gravitons,

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{N}_\Psi + \ddot{h} - \cancel{\frac{m_0 G}{c^5} \frac{d^5}{dt^5} \xi^2} \right) \xi .$$

Neglecting radiation reaction, $\langle \xi \rangle = \left(1 + \frac{h}{2}\right) \xi_0$ and $\sigma = \frac{\xi_0^2}{4} \langle N_\Psi^2 \rangle$. In the vacuum or coherent state,

$$\sigma \sim \xi_0 \ell_{\mathrm{Pl}} \frac{\omega_c}{c} .$$

Amplified if the gravitational wave is in a **squeezed** state: $\boxed{\sigma \propto e^\zeta}$.

See also Kanno, Soda, Tokuda [2007.09838], [2103.17053].

Squeezed state two-point function

$$\psi_s(h,t) \propto \prod_{a=1,2} \prod_{\mathbf{k}} \prod_{p} \exp\left[i\epsilon_{a\mathbf{k}p} + \frac{i}{2\hbar}\pi_{0a\mathbf{k}p}(t) h_{a\mathbf{k}p} - \frac{k\, S_{a\mathbf{k}p}(t)}{64\pi V G\hbar} \left(h_{a\mathbf{k}p} - h_{0a\mathbf{k}p}(t)\right)^2 \right]$$

$h_{\mathbf{k}p} = \frac{1}{\sqrt{2}}\left(h_{1\mathbf{k}p} + i\, h_{2\mathbf{k}p}\right)$, classical background $h_{0\mathbf{k}p}(t)$.

Squeezed state two-point function

$$\psi_s(h,t) \propto \prod_{a=1,2}\prod_{\mathbf{k}}\prod_{p} \exp\left[i\epsilon_{a\mathbf{k}p} + \frac{i}{2\hbar}\pi_{0a\mathbf{k}p}(t) h_{a\mathbf{k}p} - \frac{k\, S_{a\mathbf{k}p}(t)}{64\pi V G\hbar}\left(h_{a\mathbf{k}p} - h_{0a\mathbf{k}p}(t)\right)^2 \right]$$

$h_{\mathbf{k}p} = \frac{1}{\sqrt{2}}\left(h_{1\mathbf{k}p} + i\, h_{2\mathbf{k}p}\right)$, classical background $h_{0\mathbf{k}p}(t)$.
Now $S_{a\mathbf{k}p}(t) = \tanh\left(\tanh^{-1}(\beta_{a\mathbf{k}p}) + ikt\right)$, with $\beta = \beta_{a\mathbf{k}p}$.

Squeezed state two-point function

$$\psi_s(h,t) \propto \prod_{a=1,2}\prod_{\mathbf{k}}\prod_{p} \exp\left[i\epsilon_{a\mathbf{k}p} + \frac{i}{2\hbar}\pi_{0a\mathbf{k}p}(t)h_{a\mathbf{k}p} - \frac{k\,S_{a\mathbf{k}p}(t)}{64\pi VG\hbar}\left(h_{a\mathbf{k}p} - h_{0a\mathbf{k}p}(t)\right)^2\right]$$

$h_{\mathbf{k}p} = \frac{1}{\sqrt{2}}\left(h_{1\mathbf{k}p} + i\,h_{2\mathbf{k}p}\right)$, classical background $h_{0\mathbf{k}p}(t)$.
Now $S_{a\mathbf{k}p}(t) = \tanh\left(\tanh^{-1}(\beta_{a\mathbf{k}p}) + ikt\right)$, with $\beta = \beta_{a\mathbf{k}p}$.
Ultimately want $\sigma = \sqrt{\langle(\delta h_p)^2\rangle}$

Squeezed state two-point function

$$\psi_s(h,t) \propto \prod_{a=1,2}\prod_{\mathbf{k}}\prod_{p} \exp\left[i\epsilon_{a\mathbf{k}p} + \frac{i}{2\hbar}\pi_{0a\mathbf{k}p}(t)h_{a\mathbf{k}p} - \frac{k\,S_{a\mathbf{k}p}(t)}{64\pi VG\hbar}\left(h_{a\mathbf{k}p} - h_{0a\mathbf{k}p}(t)\right)^2\right]$$

$h_{\mathbf{k}p} = \frac{1}{\sqrt{2}}\left(h_{1\mathbf{k}p} + i\,h_{2\mathbf{k}p}\right)$, classical background $h_{0\mathbf{k}p}(t)$.
Now $S_{a\mathbf{k}p}(t) = \tanh\left(\tanh^{-1}(\beta_{a\mathbf{k}p}) + ikt\right)$, with $\beta = \beta_{a\mathbf{k}p}$.
Ultimately want $\sigma = \sqrt{\langle(\delta h_p)^2\rangle}$, so we compute

$$\xi_p(\mathbf{x},\mathbf{x},'\,t,t') = \delta_{pp'}\int \frac{d^3k}{(2\pi)^3} Q_p(\mathbf{k},t,t')e^{i\,\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

with power spectrum

$$Q_p(\mathbf{k},t,t') = \frac{8\pi G\hbar}{k}\left[\beta_{\mathbf{k}p}^{-1}\cos(kt)\cos(kt') + \beta_{\mathbf{k}p}\sin(kt)\sin(kt') + i\sin(k(t'-t))\right]$$

Which modes to squeeze

Now need to decide at least on the **k** dependence of the squeezing.

1. Squeeze only one mode:
$$\beta_{a\mathbf{k}p} = 1 + \frac{e^{2\zeta_p}(2\pi)^3 k^{*3}}{2}\left[\delta^3(\mathbf{k}-\mathbf{k}^*) + \delta^3(\mathbf{k}+\mathbf{k}^*)\right]$$

2. Smoothed squeezing:
$$\beta_{a\mathbf{k}p} = 1 + \frac{e^{2\zeta_p}(2\pi)^3 k^{*3}}{2}\delta(k_x)\delta(k_y)\frac{1}{\sqrt{2\pi\kappa^2}}\left[e^{-\frac{(k_z-k^*)^2}{2\kappa^2}} + e^{-\frac{(k_z+k^*)^2}{2\kappa^2}}\right]$$

Which modes to squeeze

Now need to decide at least on the **k** dependence of the squeezing.

1. Squeeze only one mode:
$$\beta_{a\mathbf{k}p} = 1 + \frac{e^{2\zeta_p}(2\pi)^3 k^{*3}}{2}\left[\delta^3(\mathbf{k}-\mathbf{k}^*) + \delta^3(\mathbf{k}+\mathbf{k}^*)\right]$$

2. Smoothed squeezing:
$$\beta_{a\mathbf{k}p} = 1 + \frac{e^{2\zeta_p}(2\pi)^3 k^{*3}}{2}\delta(k_x)\delta(k_y)\frac{1}{\sqrt{2\pi\kappa^2}}\left[e^{-\frac{(k_z-k^*)^2}{2\kappa^2}} + e^{-\frac{(k_z+k^*)^2}{2\kappa^2}}\right]$$

Can also insert a modulating function in ξ so that fluctuations are more realistically localized to the center of the classical wave packet

e.g. for $\mathbf{k} = k\hat{z}$, take $M_p(z,t) = \exp\left[-(x-t-\phi_c^2)/\lambda_c^2\right]$

or keep in mind that we are focusing on the center of the wave packet.

Full two-point function

1. Monochromatic squeezing:

$$\xi_p(\mathbf{x},\mathbf{x}',t,t') = \frac{\delta_{pp'}}{\pi}\frac{4G\hbar}{|\mathbf{x}-\mathbf{x}'|^2-(t-t')^2} + \delta_{pp'}8\pi G\hbar e^{2\zeta_p}k^{*2}\sin(k^*t)\sin(k^*t')\cos(\mathbf{k}^*\cdot(\mathbf{x}-\mathbf{x}'))$$

   Sinusoidal in both t and t', not very realistic squeezing.

2. Smoothed squeezing:

$$\xi_p(\mathbf{x},\mathbf{x}',t,t') = \frac{\delta_{pp'}}{\pi}\frac{4G\hbar}{|\mathbf{x}-\mathbf{x}'|^2-(t-t')^2} + \delta_{pp'}2\pi G\hbar e^{2\zeta_p}k^{*2}\sum_{\pm} e^{-((z-z')\pm(t-t'))\kappa^2/2}\cos\left[k^*((z-z')\pm(t-t'))\right]$$

   More realistically only a function of $t-t'$.

Detector response

$$\xi_p(\mathbf{x}, \mathbf{x}', t, t') = \frac{\delta_{pp'}}{\pi} \frac{4G\hbar}{|\mathbf{x} - \mathbf{x}'|^2 - (t - t')^2} + \delta_{pp'} 2\pi G\hbar e^{2\zeta_p} k^{*2} \sum_{\pm} e^{-((z-z')\pm(t-t'))\kappa^2/2} \cos\left[k^*((z - z') \pm (t - t'))\right]$$

Integrating over all $k \to \infty$, even the coherent part diverges at $\mathbf{x} = \mathbf{x}'$.

Detector response

$$\xi_p(\mathbf{x}, \mathbf{x}', t, t') = \frac{\delta_{pp'}}{\pi} \frac{4G\hbar}{|\mathbf{x} - \mathbf{x}'|^2 - (t - t')^2}$$
$$+ \delta_{pp'} 2\pi G\hbar e^{2\zeta_p} k^{*2} \sum_{\pm} e^{-((z-z')\pm(t-t'))\kappa^2/2} \cos\left[k^*((z - z') \pm (t - t'))\right]$$

Integrating over all $k \to \infty$, even the coherent part diverges at $\mathbf{x} = \mathbf{x}'$.
Unrealistic to expect a detector to see arbitrarily high frequencies.
Introduce a response function to suppress high k modes:

$$\xi_p(\mathbf{x} = \mathbf{x}', t, t')_R = \delta_{pp'} \int \frac{d^3k}{(2\pi)^3} P_p(\mathbf{k}, t) e^{-ik(t-t')} R(k)$$

Detector response

$$\xi_p(\mathbf{x},\mathbf{x}',t,t') = \frac{\delta_{pp'}}{\pi}\frac{4G\hbar}{|\mathbf{x}-\mathbf{x}'|^2-(t-t')^2}$$
$$+\,\delta_{pp'}2\pi G\hbar e^{2\zeta_p}k^{*2}\sum_{\pm} e^{-((z-z')\pm(t-t'))\kappa^2/2}\cos\left[k^*((z-z')\pm(t-t'))\right]$$

Integrating over all $k\to\infty$, even the coherent part diverges at $\mathbf{x}=\mathbf{x}'$.
Unrealistic to expect a detector to see arbitrarily high frequencies.
Introduce a response function to suppress high k modes:

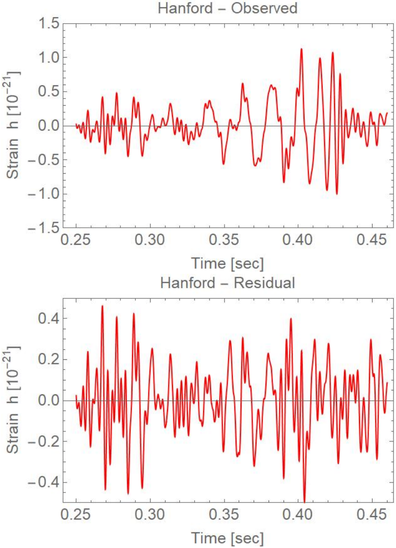
$$\xi_p(\mathbf{x}=\mathbf{x}',t,t')_R = \delta_{pp'}\int\frac{d^3k}{(2\pi)^3}P_p(\mathbf{k},t)e^{-ik(t-t')}\,R(k)$$

e.g. Taking $R(k)=\exp\left(-k/k_{\text{max}}\right) \implies$

$$\xi_p(\mathbf{x}=\mathbf{x}',t,t')_R = \frac{4}{\pi}\left(\frac{k_{\text{max}}}{\omega_{\text{Pl}}}\right)^2\frac{1-k_{\text{max}}^2(t-t')^2}{\left[1+k_{\text{max}}^2(t-t')^2\right]^2} = \text{finite}.$$

Upper bound on ζ_p

- (residual) = (data) − (numerical GR) naively Gaussian around zero with $\sigma \sim 10^{-21}$.
- From GW150914 LIGO data: $\sigma_{\text{obs}} \approx 0.16 \times 10^{-21}$



Upper bound on ζ_p

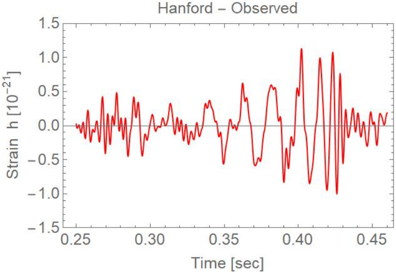
- (residual) = (data) − (numerical GR) naively Gaussian around zero with $\sigma \sim 10^{-21}$.
- From GW150914 LIGO data: $\sigma_{\text{obs}} \approx 0.16 \times 10^{-21}$

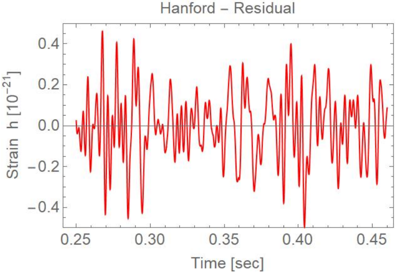
Compare to squeezed state

$$\sigma_S = \sqrt{4\pi} e^{\zeta_p} \left(\frac{k^*}{\omega_{\text{Pl}}} \right)$$

with $k^* = 2\pi f^*$ taken to be the peak frequency in the data, $f^* \sim 200 Hz$:

$$\boxed{\sigma_S < \sigma_{\text{obs}} \implies \zeta_p < 41}$$





Summary and outlook

Direct observational bound on a quantum gravitational effect.

The bound of $\zeta_p < 41$ may be somewhat improved by more detailed analysis including more events.

Astrophysical sources of squeezed gravitational waves?

- Mergers, particularly of black holes?
- Is there a similar mechanism as in the time-dependent background of the early universe to set up waves in a squeezed state?