II -Machine-Learning quantum entanglement with top quark pair-production at the LHC

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Based on ongoing work with Z.Dong, D.Gonçalves and K.C. Kong

Outline

- Entanglement in $t\bar{t}$ system
- Experimental Observables
- Results
- Summary

Top quark production as a two qubit system

For a system composed of two spin-1/2 particles the density matrix can be written as

$$\rho = \frac{I_4 + B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}$$

Where $B_i^+ = \langle \sigma^i \otimes I_2 \rangle$, $B_i^- = \langle I_2 \otimes \sigma^i \rangle$ are the spin polarizations of the particles, and $C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$ represents their spin correlations.

In general, for a system composed of two subsystems A and B, a quantum state is said to be separable if the mixed system is described by a density matrix

$$\rho = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

An entangled state is defined as a non-separable state.

Entanglement criterion

The Peres-Horodecki criterion provided a necessary and sufficient condition for entanglement:

Take the partial transpose of the original density matrix with respect to the second subsystem, i.e.,

$$\rho^{T_2} = \sum_n p_n \rho_n^A \otimes (\rho_n^B)^T$$

If ρ^{T_2} has at least one negative eigenvalue, then ρ corresponds to an entangled state.

arXiv:quant-ph/9604005, A. Peres

arXiv:quant-ph/9703004, P.Horodecki

For a system composed of two spin-1/2 particles, a sufficient condition for a negative eigenvalue of ρ^{T_2} is

$$|C_{11} + C_{22}| - C_{33} > 1$$

2003.02280, Y. Afik, J. R. M. de Nova

2203.05582, Y. Afik, J. R. M. de Nova

2110.10112. C. Severi et al.

There is no need to fully reconstruct the density matrix. Only the diagonal elements of the correlation matrix are needed to test entanglement.

Reconstruction of the correlation matrix

We consider top quark pair-production at the LHC, with each top decaying leptonically

$$t\bar{t} \to bW^+\bar{b}W^- \to b\,\bar{b}l^-l^+\nu\bar{\nu}$$

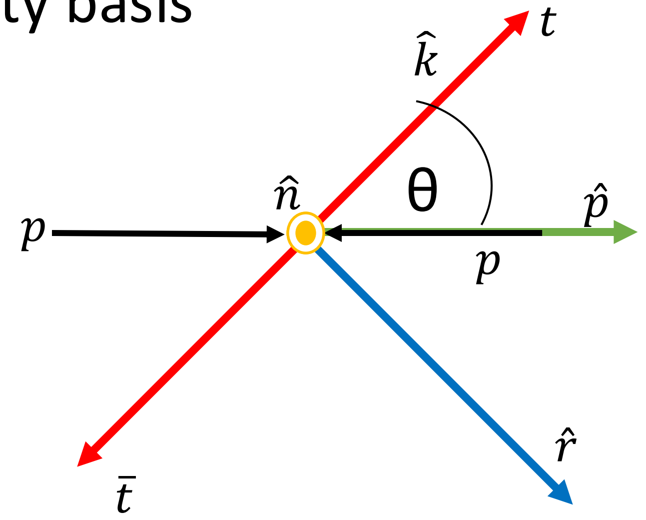
The elements of the correlation matrix are extracted from the angular distributions

$$\frac{1}{\sigma}\frac{d\sigma}{d\xi_{ij}} = \frac{C_{ij}\xi_{ij} - 1}{2}\log|\xi_{ij}|$$

1508.05271. W. Bernreuther et al

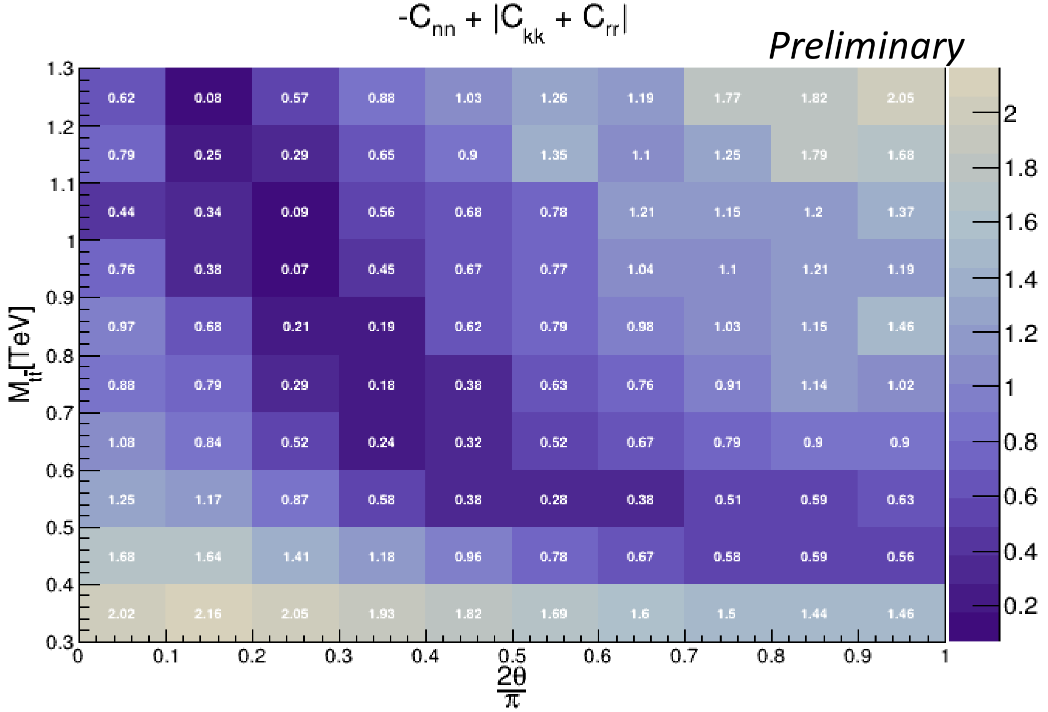
Here $\xi_{ij} = \cos\theta_i \cos\bar{\theta}_j$ and θ_i ($\bar{\theta}_j$) is the angle between l^+ (l^-) and some *i*-th (*j*-th) axis in the t ($\bar{t}$) rest frame. The axes are chosen in the so-called helicity basis

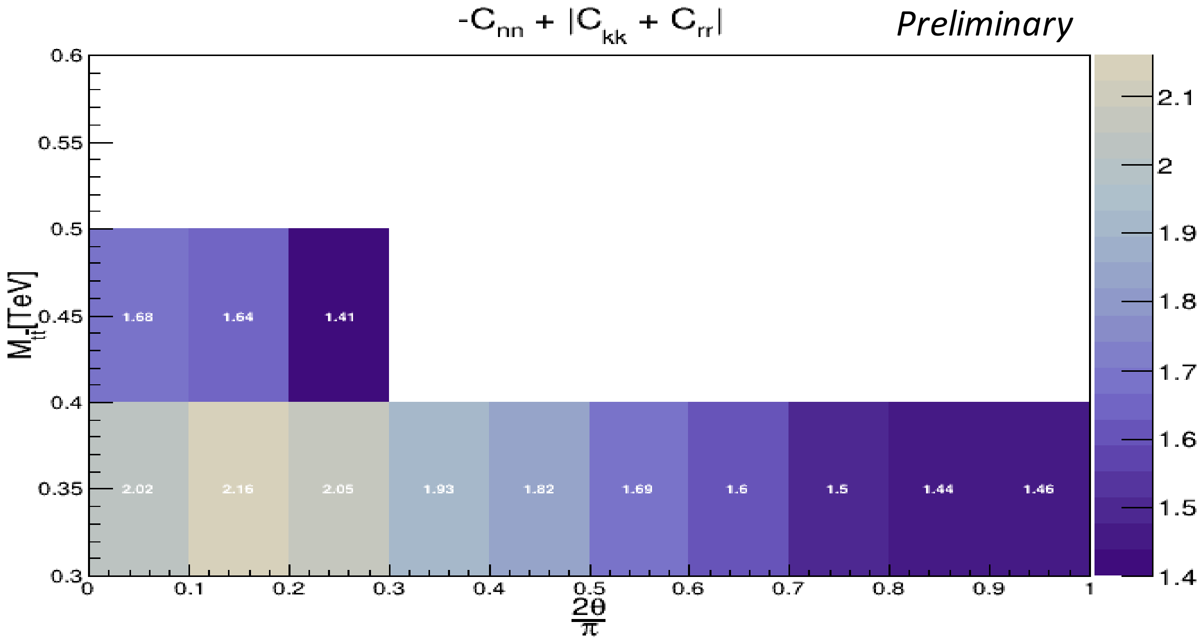
$$\hat{k} = \text{top direction}, \qquad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \qquad \hat{n} = \hat{k} \times \hat{r}, \qquad \hat{p} = (0,0,1)$$



Top momentum reconstruction is crucial.

Entanglement





Close to threshold the tops are entangled. At the parton level

Region	$\lvert C_{kk} + C_{rr}\rvert - C_{nn}$ (Parton)
Threshold	1.60 ± 0.02

Different reconstruction methods yield different reconstructed values for the entanglement signature

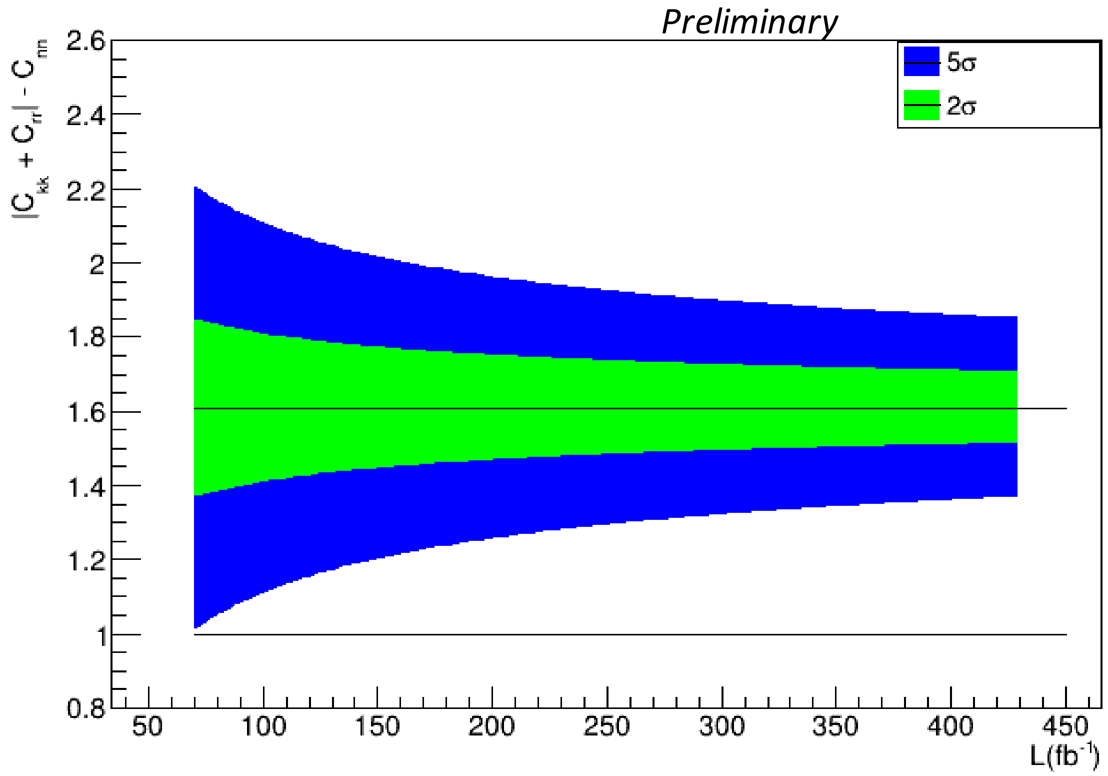
Region	$-C_{nn} + \lvert C_{kk} + C_{rr}\rvert$ (Delphes)				
	Analytic	ML-M2CW	ML-M2CT	χ^2	NN
Threshold	0.73 ± 0.02	1.33 ± 0.02	3.71 ± 0.02	1.00 ± 0.02	2.02 ± 0.02

The ML-M2CW method gives the best result.

Sensitivity

The distributions reconstructed with hybrid (m2cw) are then unfolded and the entanglement signature is computed again.

Region	$\lvert C_{kk} + C_{rr} \rvert - C_{nn}$ (Unfolded)
Threshold	1.61 ± 0.02



Summary

- Top quark pair-production at the LHC provides a window to study the foundations of QM in the high-energy regime.
- The dilepton final state is useful as the top (antitop) and lepton spins are fully correlated.
- Top momentum reconstruction is crucial to accurately probe entanglement.
- The ML-M2CW method performs better than others in the reconstruction of the entanglement signature.