Axion Mass from Magnetic Monopole Loops

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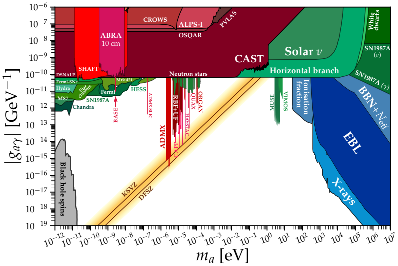
Motivation
How does the axion get a mass?

- Axions coupled to non-abelian gauge fields get a potential from instantons. (Ex: QCD Axion) It can be computed with:
    - Dilute instanton gas at high temp
    - Chiral lagrangian mass terms at low temp

- But what about coupling to abelian gauge fields? Many experiments search for the $aF\tilde{F}$ coupling.
- What is the mass of an ALP that only couples to abelian gauge fields?



[https://github.com/cajohare/AxionLimits]

A New Contribution

When coupled to ***abelian*** gauge fields, the axion gets a potential from loops of magnetic monopoles

Why do we expect monopole states in our theory?

- The “completeness hypothesis” [Polchinski: hep-th/0304042]
- The lattice of charged dyons should be completely filled in quantum gravity.

The Short Version

The Witten effect gives a θ dependent mass contribution to heavy dyons.

This mass term couples the axion θ to dyons, so axions get a potential when we integrate out the dyons.

A Distinction from Previous Work

This is ***different*** from the potential that is generated by the monopole and anti-monopole plasma.

[Fischler & Preskill `83, Kawasaki et al: 1511.05030 & 1708.06047, Nomura et al: 1511.06347]

In that case, physical monopoles are generated in the early universe, for example through the Kibble Zurek mechanism.

Our potential is from ***virtual*** monopole/dyon loops.

Outline

1. Review: Monopoles, Dyons, and the Witten Effect
2. The Calculation
3. Phenomenology and Future Directions

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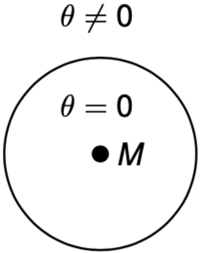
The Witten Effect

Witten Effect: Monopoles in the presence of a non-zero θ acquire electric charge



- Imagine a magnetic monopole inside a $\theta = 0$ region, surrounded by $\theta \neq 0$
- Magnetic field from the monopole induces an electric field when θ changes: $\nabla \cdot E = -\frac{\alpha}{\pi}(\nabla\theta \cdot B)$
- Electric charge $Q_E = -\frac{e\theta}{2\pi}$ is independent of the size of the $\theta = 0$ region, can take the limit where its size $\rightarrow 0$

[See David Tong's gauge theory lectures]

Dyons and their Energy Spectrum

- In 4d, classical magnetic monopoles are solitons that depend only on a radial coordinate. Ex: 't Hooft Polyakov monopole in the Georgi-Glashow model:

$$\mathscr{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu,a} + \frac{1}{2}(D_\mu \phi^a)(D_\mu \phi^a) - V(\phi)$$

Admits a Monopole Soln for ϕ^a and A_i^a

- To quantize around the classical solution, consider fluctuations of the collective coordinate σ which restores the unbroken U(1)

$$\mathscr{L} = \frac{1}{2} l_\sigma \dot{\sigma}^2 + \frac{\theta}{2\pi}\dot{\sigma} \xrightarrow[\text{Transform}]{\text{Legendre}} E_n = \frac{1}{2l_\sigma}\left(n - \frac{\theta}{2\pi}\right)^2$$

Coefficient of $\frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma$ in $S_{worldline}$

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The Calculation

The Effective Potential is

$$V_{\text{eff}}(\theta) = -\lim_{V\to\infty} \frac{1}{V} \log Z(\theta)$$

Path Integral

Spacetime Volume

After summing over all disconnected vacuum paths that are topologically a circle:

$$V_{\text{eff}}(\theta) = -\lim_{V\to\infty} \frac{1}{V} Z_{S_1}(\theta)$$

transition amplitude for trajectories that return to the same point they started at

Two equivalent ways to get $Z_{S_1}(\theta)$:

1. Coleman-Weinberg like calculation with the axion as the background field where we integrate out the dyon tower
2. Path integral over winding of the monopole around its collective coordinate

The Calculation: More Details

Using the free particle path integral, we compute

$$Z_{S_1} = \int_0^\infty \frac{d\tau}{2\tau} \frac{1}{2(2\pi\tau)^2} \exp\Big(-\frac{m^2\tau}{2}\Big)$$

$\downarrow$

m = Dyon mass

$$m^2 = m_M^2 + m_\Delta^2 (n - \frac{\theta}{2\pi})^2$$

$$Z_{S_1} = \sum_{n\in\mathbb{Z}} \int_0^\infty \frac{d\tau}{2\tau} \frac{1}{2(2\pi\tau)^2} \exp\Big(-\frac{m_M^2\tau}{2} - \frac{m_\Delta^2\tau}{2}(n - \frac{\theta}{2\pi})^2\Big)$$

$\downarrow$

Plug In + Poisson Resum:

$$\sum_{n\in\mathbb{Z}} e^{-\frac{1}{2}m_\Delta^2\tau(n-\frac{\theta}{2\pi})^2} = \sqrt{\frac{2\pi}{m_\Delta^2\tau}} \sum_{\ell\in\mathbb{Z}} e^{-2\pi^2\ell^2/m_\Delta^2\tau + i\ell\theta}$$

$\downarrow$

The Calculation: Final Result

$$V_{eff}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^2 m_M^2}{32\pi^4 \ell^3} e^{\overbrace{-2\pi\ell m_M/m_\Delta}^{\text{Instanton Action: } S = 8\pi^2/g^2 \text{ in the tHP case, like YM instantons}}}$$

$$\times \cos(\ell\theta)\left(1 + \frac{3m_\Delta}{2\pi\ell m_M} + \frac{3m_\Delta^2}{(2\pi\ell m_M)^2}\right)$$

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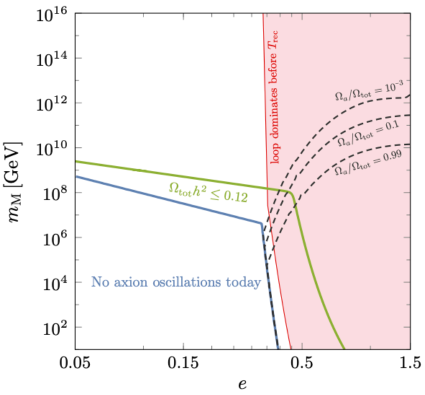
U(1)$_{dark}$ Hidden Sector Model



red region = region of phase space where our contribution dominates

Consider a model with a hidden gauged $U(1)_{dark}$ and a spontaneously broken $U(1)_{PQ}$. Then $m_a(T) = m_a^{loop} + m_a^{plasma}(T)$:

- m_a^{loop} is the contribution from virtual monopole loops (this talk)
- $m_a^{plasma}(T)$ is the temperature dependent contribution from the monopole/anti-monopole plasma (previously known)

Future Directions

- How does the story look with light fermions?
- How robust is the assumption that the potential is dominated by single monopole loops? What about long-range coulomb interactions?
- Can this be formally connected to the instanton potential in the UV?

Summary

1. We point out a new contribution to the axion potential from loops of magnetic monopoles when the axion is coupled to abelian gauge fields.
2. This contribution is distinct from the previously known monopole and anti-monopole plasma potential.
3. Our contribution can be phenomenologically relevant.

Back Up Slides

The Calculation: More Details (1/3)

1) To derive general expression for V_{eff}, use the worldline formalism. The effective potential is:

$$V_{eff}(\theta) = -\lim_{V\to\infty} \frac{1}{V} \log Z(\theta)$$

where $Z(\theta) = \sum_{worldlines} \int D(fields) e^{-S_E[fields, worldlines, \theta]}$ is the Euclidean path integral.

Summing over disconnected vacuum paths that are topologically a circle (the analog of insertion-less torus in closed string theory) gives:

$$Z(\theta) = \sum_{n=0}^{\infty} \frac{1}{n!} (Z_{S_1})^n \Rightarrow V_{eff}(\theta) = -\lim_{V\to\infty} \frac{1}{V} Z_{S_1}(\theta)$$

The Calculation: More Details (2/3)

2) In the Coleman-Weinberg picture, $Z_{S_1} = \lim_{x' \to x} \langle x' | x \rangle_\tau$, where $\langle x' | x \rangle_\tau$ is the gauge fixed transition amplitude. Use the free particle path integral to compute $\langle x' | x \rangle_\tau$:

$$\langle x' | x \rangle = \mathcal{N} \int_{x(0)=x}^{x(1)=x'} Dx^\mu \exp\left(-\frac{1}{2} \int_0^1 dt (\tau^{-1} \dot{x}^2 + \tau m^2) \right)$$

$\mathcal{N}$ is a normalization factor chosen so the propagator is normalized correctly after integrating over τ. After shifting the classical path $x^\mu(t) = (x'^\mu - x^\mu)t + \delta x^\mu(t)$, we find

$$\langle x' | x \rangle = \frac{1}{2(2\pi\tau)^2} \exp\left(-\frac{1}{2\tau}(x - x')^2 - \frac{m^2\tau}{2} \right)$$

The Calculation: More Details (3/3)

3) To compute Z_{S_1} using the path integral over winding, treat the dyon collective coordinate like an extra compact spatial dimension. The worldline action is:

$$S_M = m_M \int_\gamma d\lambda \sqrt{\frac{dx_\mu}{d\lambda}\frac{dx^\mu}{d\lambda} + \frac{l_\sigma}{m_M}\left(\frac{d_A\sigma}{d\lambda}\right)^2} + \frac{\theta}{2\pi}\int_\gamma d_A\sigma$$

which gives transition amplitude

$$\langle x', \sigma' | x, \sigma\rangle_\tau = \frac{1}{2(2\pi\tau)^{5/2}} e^{\left(-\frac{1}{2\tau}(x'-x)^2 - \frac{l_\sigma}{2m_M\tau}(\sigma'-\sigma)^2 - \frac{m_M^2\tau}{2} + \frac{i\theta}{2\pi}(\sigma'-\sigma)\right)}$$

following the same procedure as the Coleman-Weinberg case. Take $x' \to x$ and sum over windings $\sigma' - \sigma = 2\pi l$ to get Z_{S_1} and reproduce the Poisson resummed V_{eff}.