Altered Axion Abundance from a Dynamical Peccei-Quinn Scale

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Based on:
I.J.A., Y. Lyu, M.P. Hertzberg 2203.15817

1. Motivation
2. Dynamical PQ Scale Mechanism
3. Applications

The QCD Axion

The QCD axion solves the Strong CP problem by driving $\Delta\mathcal{L}_{QCD} \propto \theta G\tilde{G}$ dynamically to zero (Peccei + Quinn 1977)

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}|\partial\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - f_a^2)^2 - V(\theta, T)\right] \quad (1)$$

$$V(\theta, T) = \Lambda(T)^4(1-\cos\theta) = m_a(T)^2 f_a^2(1-\cos\theta) \quad (2)$$

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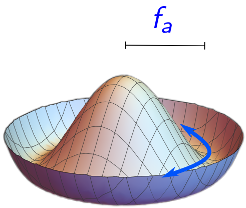

$U(1)_{PQ}$ spontaneously broken, leaving axion θ (Wilczek 1978, Weinberg 1978)

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}f_a^2(\partial\theta)^2 - V(\theta, T)\right] \quad (3)$$

f_a is the Peccei-Quinn (PQ) symmetry-breaking scale

Standard Axion Abundance

The axion makes a natural dark matter (DM) candidate.

(Preskill et al 1983, Abbott et al 1983, Dine et al 1983)

$$\ddot{\theta} + 3H\dot{\theta} + \frac{\Lambda(T)^4}{f_a^2}\theta = 0 \tag{4}$$

- After $3H \sim m_a$, the axion oscillates and behaves as DM

$$\Omega_a \sim \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6} \langle\theta_i^2\rangle = \left(\frac{5.7\,\mu\text{eV}}{m_a}\right)^{7/6} \langle\theta_i^2\rangle \tag{5}$$

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- Abundance parameter $\xi \equiv \rho/n_\gamma(t_0)$ given by

$$\xi_{a,std} \approx 2.9\text{ eV}\left(\frac{f_a}{2\times 10^{11}\text{ GeV}}\right)^{7/6} \quad (6)$$

- where 2.9 eV is the observed value for cold dark matter
- Certain f_a preferred by observations or fundamental physics

Dynamical Peccei-Quinn Scale

Time-varying f_a allows a wider range of (late-time) values to be compatible with DM

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Consider the new Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}|\partial\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - f(\chi)^2)^2 - V(\theta, T) + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2\right] \quad (7)$$

The PQ scale is now controlled by the new scalar χ through

$$f(\chi)^2 = f_a^2\,\alpha(\chi) = f_a^2\,\left(1 + \chi^2/S^2\right)^{\pm 1} \quad (8)$$

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Two Models

$-1 \rightarrow$ Increasing PQ Scale (IPQ) Model
$+1 \rightarrow$ Decreasing PQ Scale (DPQ) Model

Equations of Motion

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}f(\chi)^2(\partial\theta)^2 - V(\theta, T) + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2\right] \quad (9)$$

The corresponding classical equations of motion for the fields are

$$\ddot{\theta} + \left(3H + 2\frac{f'(\chi)}{f(\chi)}\dot{\chi}\right)\dot{\theta} + \frac{\Lambda(T)^4}{f(\chi)^2}\sin\theta = 0 \quad (10)$$

$$\ddot{\chi} + 3H\dot{\chi} + m_\chi^2\chi - f'(\chi)f(\chi)\dot{\theta}^2 = 0 \quad (11)$$

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The corresponding classical equations of motion for the fields are

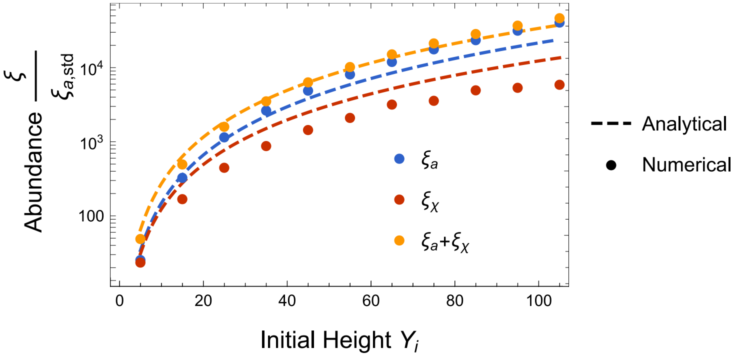
$$\theta_{\tau\tau} + \left(\frac{3}{2\tau} + \frac{\alpha'(Y)}{\alpha(Y)}Y_\tau\right)\theta_\tau + \frac{\lambda(T)}{\alpha(Y)}\theta = 0 \quad (10)$$

$$Y_{\tau\tau} + \frac{3}{2\tau}Y_\tau + \mu^2 Y - \frac{F^2}{2}\alpha'(Y)\theta_\tau^2 = 0 \quad (11)$$

$$\tau \equiv m_a t, \quad Y \equiv \frac{\chi}{S}, \quad F \equiv \frac{f_a}{S}, \quad \lambda(T) \equiv \frac{\Lambda(T)^4}{\Lambda_0^4}, \quad \mu \equiv \frac{m_\chi}{m_a} \quad (12)$$

Altered Abundance

$$\frac{\xi_a}{\xi_{a,std}} = \left(\frac{f_i}{f_a}\right)^{\frac{13}{6}} = \alpha(Y_i)^{\frac{13}{12}} \quad (13) \qquad \frac{\xi_\chi}{\xi_{a,std}} \propto \frac{\sqrt{\mu}\,\langle Y_i^2\rangle}{F^2\langle\theta_i^2\rangle} \quad (14)$$



1 Motivation

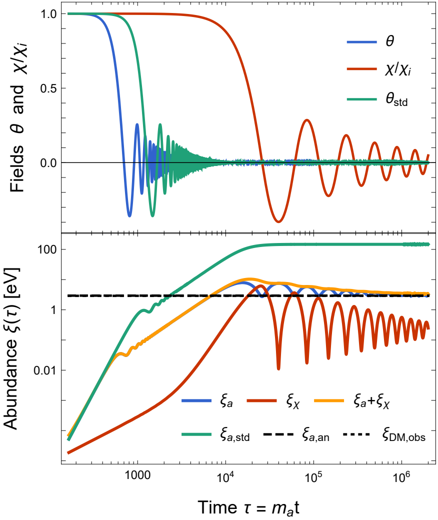
2 Dynamical PQ Scale Mechanism

3 Applications

Increasing PQ Scale Model

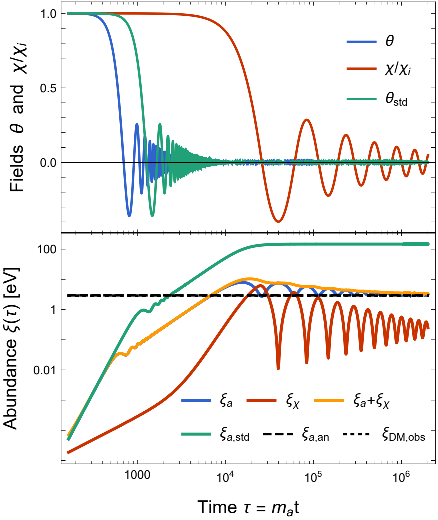
- $m_a = 10^{-6}$ eV
- Standard abundance overcloses universe

Increasing PQ Scale Model



- $m_a = 10^{-6}$ eV
- Standard abundance overcloses universe
- Suppress abundance to DM level in IPQ:
    - $m_\chi = 5 \times 10^{-12}$ eV
    - $F = 0.5$
    - $Y_i = 6$

Increasing PQ Scale Model



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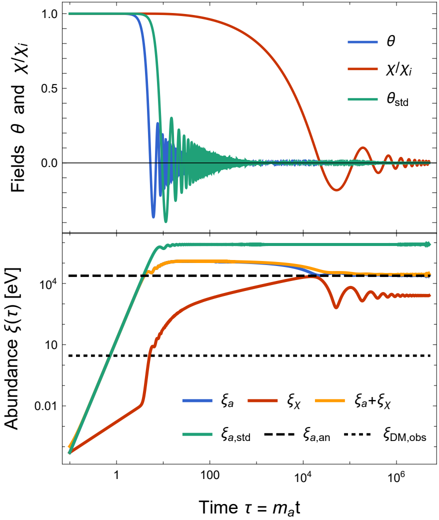
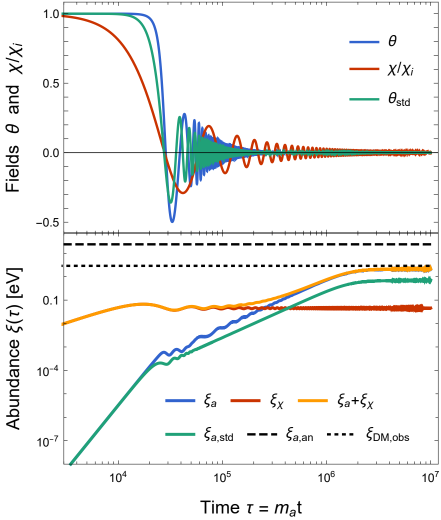
  - $Y_i = 6$
- Target mass in ADMX experiment

Increasing PQ Scale Model

- $m_a = 6 \times 10^{-10}$ eV
- Corresponds to $f_a \sim$ GUT scale
  - $m_\chi = 6 \times 10^{-15}$ eV
  - $F = 0.5$
  - $Y_i = 5$
- Parameters can be tuned to suppress abundance further



Decreasing PQ Scale Model

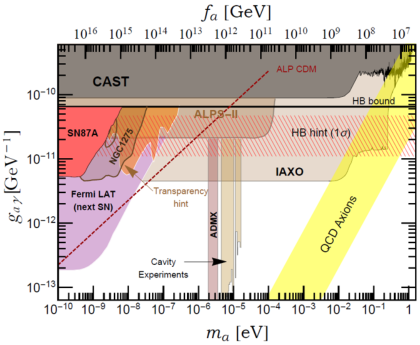


- $m_a = 10^{-4}$ eV
- Standard abundance is a fraction of DM
- Enhance abundance to DM level in DPQ:
  - $m_\chi = 10^{-8}$ eV
  - $F = 0.1$
  - $Y_i = 5$
- Target mass in MADMAX experiment

Decreasing PQ Scale Model

- $m_a = 0.1$ - 1 eV, corresponds to axion-photon couplings that can explain anomalous HB cooling

$$\mathcal{L}_{int} = \frac{1}{4} g_{a\gamma} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \quad (15)$$



(Giannotti 2016)

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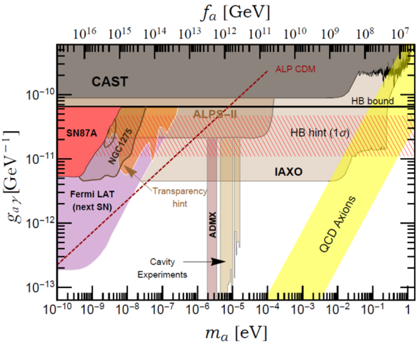
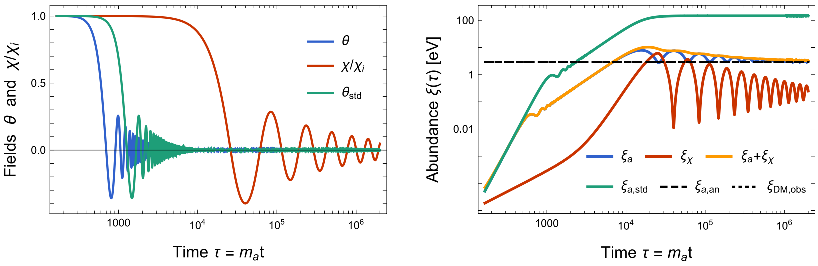
- Enhance abundance to DM level in DPQ:
  - $m_a = 1$ eV
  - $m_\chi < 10^{-7}$ eV
  - $F = 0.1$
  - $Y_i = 280$
- Tuning unexplained, but phenomenology is viable



(Giannotti 2016)

Summary

- QCD Axion abundance is controlled by the PQ scale f_a
- Larger and smaller than standard f_a can be accommodated if f varies in time
- This can expand the window of viable DM axion masses



- Introduce the complex PQ field $\Phi = \rho\, e^{i\theta}$ which carries a new global $U(1)$ symmetry
- The axion θ is a (pseudo-)Goldstone boson when the symmetry is spontaneously broken

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}|\partial\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - f_a^2)^2 - \Lambda(T)^4(1-\cos\theta)\right] \tag{16}$$

- cosine factor from the dilute instanton approximation, appropriate in finite temperature regime

$$\Lambda(T)^4 = \begin{cases} \Lambda_0^4 (T_{\rm QCD}/T)^8, & T \gg T_{\rm QCD} \\ \Lambda_0^4, & T \ll T_{\rm QCD} \end{cases} \quad (17)$$

- the temperature of the QCD phase transition is of order of the QCD scale $T_{\rm QCD} \sim 200\,\text{MeV}$
- The power of 8 is not necessarily precise, as indicated by some lattice simulations
- The low temperature value is given by

$$\Lambda_0^2 = \frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi \, m_\pi \quad (18)$$

- which evaluates to $\Lambda_0 \approx 90\,\text{MeV}$.

- At low energies, the radial field ρ gets frozen-in at its minimum energy value $\rho = f_a$

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}f_a^2(\partial\theta)^2 - \Lambda(T)^4(1-\cos\theta)\right] \quad (19)$$

- For an FRW universe, in co-moving co-ordinates, the metric is $g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$. The prefactor in the Lagrangian is $\sqrt{-g} = a^3$.
- The classical field equation of motion for θ (ignoring metric perturbations)

$$\ddot{\theta} + 3H\dot{\theta} - \frac{\nabla^2\theta}{a^2} + \frac{\Lambda(T)^4}{f_a^2}\sin\theta = 0 \quad (20)$$

- If we ignore the spatial variation in θ, and for small angles

$$\ddot{\theta} + 3H\dot{\theta} + \frac{\Lambda(T)^4}{f_a^2}\theta = 0 \quad (21)$$

Hubble Evolution in Rad. Domination

- the Hubble rate $H = \dot{a}/a$ evolves according to

$$H^2 = \rho/(3M_{\text{Pl}}^2) \tag{22}$$

- where $M_{\text{Pl}} = 1/\sqrt{8\pi G} \approx 2.4 \times 10^{18}\,\text{GeV}$ is the (reduced) Planck mass.
- In a radiation dominated universe (roughly the first 70,000 years)

$$\rho = \frac{\pi^2}{30} g\, T^4 \tag{23}$$

- where g is the number of active relativistic degrees of freedom
- temperature redshifts as $T \propto 1/a$, leads to Hubble changing with time as

$$H = \frac{1}{2\,t}, \tag{24}$$

Standard Relic Abundance

- the axion is friction dominated and almost frozen in its potential at some initial angle θ_i
- Then it starts to roll at a temperature T_* when

$$3H(T_*) \approx m(T_*) = \Lambda(T_*)^2/f_a \tag{25}$$

$$T_* = \left(\frac{\sqrt{10}\, M_{\rm Pl}\, T_{\rm QCD}^4 \Lambda_0^2}{f_a \sqrt{g_*}\, \pi} \right)^{1/6} \tag{26}$$

- The corresponding number density of axions at this time is

$$n(T_*) = \frac{\rho_a(T_*)}{m(T_*)} \tag{27}$$

- where the energy density of axions is

$$\rho_a(T_*) \approx \frac{1}{2}\Lambda(T_*)^4 \langle \theta_i^2 \rangle \tag{28}$$

Abundance Parameter ξ

- useful way to represent abundance is through the ratio of axion energy density to photon number density at late times

$$\xi_a \equiv \frac{\rho_a(T_0)}{n_\gamma(T_0)} \tag{29}$$

$$\xi_a = \frac{m_a}{m(T_*)} \frac{\rho_a(T_*)}{n_\gamma(T_0)} \frac{s(T_0)}{s(T_*)} \tag{30}$$

- where $s(T)$ is the total entropy density and $n_\gamma(T)$ is the photon number density

$$s(T) = \frac{2\pi^2}{45} g_s\, T^3, \quad n_\gamma(T) = \frac{2\zeta(3)}{\pi^2} T^3 \tag{31}$$

$$\xi_{a,std} \approx 2\frac{g_*^{7/12} g_{s0}}{g_{s*}} \frac{\Lambda_0^{5/3} f_a^{7/6}}{T_{\rm QCD}^{2/3} M_{\rm Pl}^{7/6}} \langle \theta_i^2 \rangle \tag{32}$$

The observed value of ξ for cold dark matter is

$$\xi_{obs} \approx 2.9\,\mathrm{eV} \tag{33}$$

$$\mathcal{L} = \sqrt{-g}\Big[\frac{1}{2}|\partial\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - f(\chi)^2)^2 - V(\theta, T) + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2\Big] \quad (34)$$

the PQ scale f_a is now effectively dynamical, as it is controlled by the value of the second field χ through the function $f(\chi)$

$$f(\chi)^2 = f_a^2\,\alpha(\chi) \quad (35)$$

where α is a dimensionless function. Some concrete choices are

$$\alpha(\chi) = \left(1 + \chi^2/S^2\right)^{\pm 1} \quad (36)$$

where S is a new scale that controls the size of these effects. The Decreasing PQ Scale (DPQ) model has the exponent $+1$, and the Increasing PQ Scale (IPQ) model has -1 the PQ field is still extremely heavy, once again gets stuck at the value that minimizes its potential in the very early universe

$$\rho \to f(\chi) \tag{37}$$

so its value is controlled by the dynamics of the new light field χ. This leaves a low energy Lagrangian for the angular field (axion) θ and χ as

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}f(\chi)^2(\partial\theta)^2 - V(\theta, T) + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2\right] \tag{38}$$

The corresponding classical equations of motion for the fields are

$$\ddot{\theta} + \left(3H + 2\frac{f'(\chi)}{f(\chi)}\dot{\chi}\right)\dot{\theta} - \frac{\nabla^2\theta}{a^2} + \frac{\Lambda(T)^4}{f(\chi)^2}\sin\theta = 0 \tag{39}$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2\chi}{a^2} + m_\chi^2\chi - f'(\chi)f(\chi)\dot{\theta}^2 = 0 \tag{40}$$

Dimensionless Variables

$$\tau \equiv m_a\, t, \quad Y \equiv \frac{\chi}{S}, \quad F \equiv \frac{f_a}{S}, \quad \lambda(T) \equiv \frac{\Lambda(T)^4}{\Lambda_0^4}, \quad \mu \equiv \frac{m_\chi}{m_a} \tag{41}$$

We also once again ignore spatial variations, assume small angles, and use $H = 1/(2\,t)$ in the radiation era to obtain the dimensionless equations of motion

$$\theta_{\tau\tau} + \left(\frac{3}{2\,\tau} + \frac{\alpha'(Y)}{\alpha(Y)} Y_\tau\right)\theta_\tau + \frac{\lambda(T)}{\alpha(Y)}\theta = 0 \tag{42}$$

$$Y_{\tau\tau} + \frac{3}{2\,\tau} Y_\tau + \mu^2 Y - \frac{F^2}{2}\alpha'(Y)\theta_\tau^2 = 0 \tag{43}$$

Note that we can re-write the temperature dependence in λ in terms of the time variable instead. This becomes

$$\lambda(\tau) = \begin{cases} (\tau/\tau_{\rm QCD})^4, & \tau \ll \tau_{\rm QCD} \\ 1, & \tau \gg \tau_{\rm QCD} \end{cases} \tag{44}$$

where the dimensionless time scale of the QCD phase transition is found to be

$$\tau_{\rm QCD} = \frac{\sqrt{90}\,\Lambda_0^2}{2\pi\sqrt{g_{\rm QCD}}\,T_{\rm QCD}^2}\frac{M_{\rm Pl}}{f_a} \tag{45}$$

Suppose the χ field is initially at the value χ_i; the corresponding dimensionless initial value is $Y_i = \chi_i/S$. At early times the χ field will be effectively frozen at $Y_i = \chi_i/S$

$$f_i = f_a\sqrt{\alpha(Y_i)} \tag{46}$$

Assuming the coupling between the two fields is not too significant, one anticipates that the number density of axions is given by the same formula as before (32), except with this effective PQ-scale f_i instead

$$\frac{\xi_a}{m_a} = \frac{1}{m(T_*(f_i))}\frac{\rho_a(T_*(f_i))}{n_\gamma(T_0)}\frac{s(T_0)}{s(T_*(f_i))} \tag{47}$$

Using eq. (32), but with $f_a \to f_i$ and then multiplying throughout by m_a (the late-time value), we obtain the alteration factor of

$$\frac{\xi_a}{\xi_{a,std}} = \left(\frac{f_i}{f_a}\right)^{13/6} = \alpha(Y_i)^{13/12} \tag{48}$$

Just like for the axion, the χ field is frozen in its potential until a temperature T_χ of

$$3H(T_\chi) \approx m_\chi \tag{49}$$

Note that in this model, we assume the χ field has a temperature independent mass, and so this relation is relatively simple. Solving for T_χ, we find

$$T_\chi = \left(\frac{\sqrt{10}\, M_{\rm Pl} m_\chi}{\sqrt{g_\chi}\, \pi} \right)^{1/2} \tag{50}$$

The corresponding energy density of χ particles at this time is

$$\rho_\chi(T_\chi) \approx \frac{1}{2} m_\chi^2 \langle \chi_i^2 \rangle \tag{51}$$

The late-time ratio of χ energy density to photon number density is

$$\xi_\chi = \frac{\rho_\chi(T_\chi)}{n_\gamma(T_0)} \frac{s(T_0)}{s(T_*)} \tag{52}$$

This gives

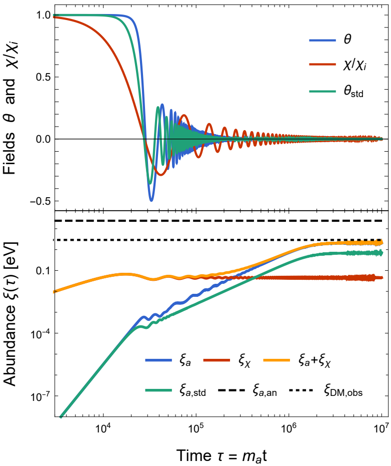
$$\xi_\chi \approx 2 \frac{g_\chi^{3/4} g_{s0}}{g_{s\chi}} \frac{\sqrt{m_\chi}}{M_{\rm Pl}^{3/2}} \langle \chi_i^2 \rangle \tag{53}$$

$$\frac{\xi_\chi}{\xi_{a,std}} \approx \frac{g_\chi^{3/4} g_{s*}\, T_{\rm QCD}^{2/3}}{g_*^{7/12} g_{s\chi}\, \Lambda_0^{2/3}} \frac{f_a^{1/3}}{M_{\rm Pl}^{1/3}} \frac{\sqrt{\mu}\, \langle Y_i^2 \rangle}{F^2 \langle \theta_i^2 \rangle} \tag{54}$$

Comparing eq. 54 and eq. 48, we can see that both fields have a relic abundance related to Y_i. Thus it is also interesting to consider the ratio

$$\frac{\xi_\chi}{\xi_a} \approx \frac{g_\chi^{3/4} g_{s*}\, T_{\rm QCD}^{2/3}}{g_*^{7/12} g_{s\chi}\, \Lambda_0^{2/3}} \frac{f_a^{1/3}}{M_{\rm Pl}^{1/3}} \frac{\sqrt{\mu}\, \langle Y_i^2 \rangle}{F^2 \langle \theta_i^2 \rangle} \alpha(Y_i)^{-13/12} \tag{55}$$

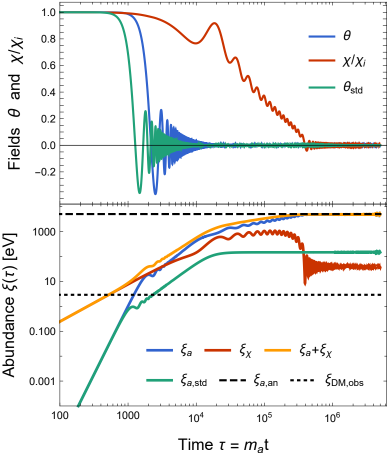
Decreasing PQ Scale Model



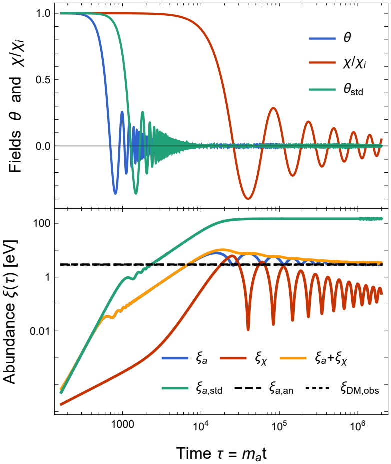
- $m_a = 10^{-4}$ eV
- $m_\chi = 10^{-8}$ eV
- $F = 0.1$
- $Y_i = 5$

Decreasing PQ Scale Model

- $m_a = 10^{-6}$ eV
- $m_\chi = 2 \times 10^{-10}$ eV
- $F = 0.1$
- $Y_i = 5$



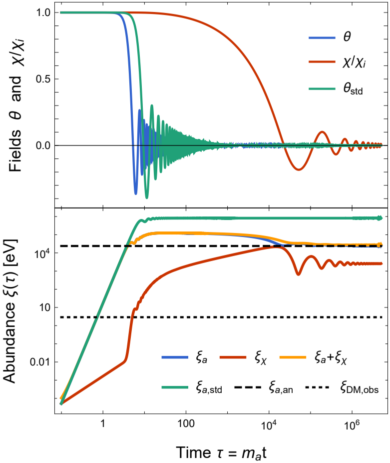
Increasing PQ Scale Model



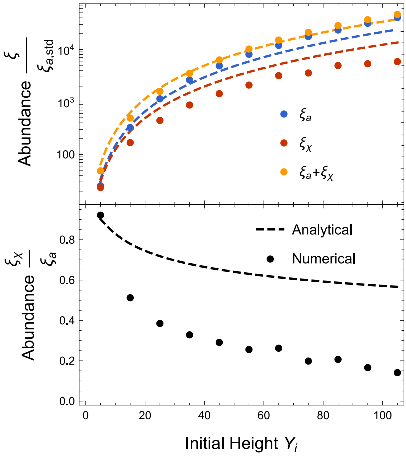
- $m_a = 10^{-6}$ eV
- $m_\chi = 5 \times 10^{-12}$ eV
- $F = 0.5$
- $Y_i = 6$

Increasing PQ Scale Model

- $m_a = 6 \times 10^{-10}$ eV
- $m_\chi = 6 \times 10^{-15}$ eV
- $F = 0.5$
- $Y_i = 5$



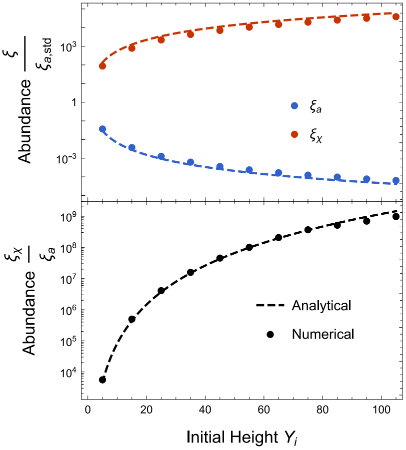
Varying Y_i (DPQ)



- $m_a = 10^{-10}$ eV
- $m_\chi = 5 \times 10^{-4}\, m_a$
- $F = 0.1$

Varying Y_i (IPQ)

- $m_a = 10^{-10}$ eV
- $m_\chi = 10^{-2}\, m_a$
- $F = 0.1$



Varying m_χ

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- $F = 0.1$
- $Y_i = 5$

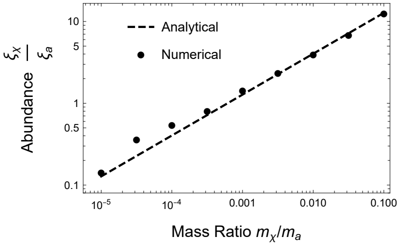

Figure: DPQ

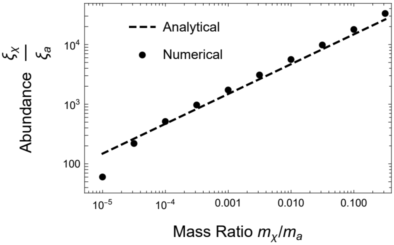

Figure: IPQ

Varying F

- $m_a = 10^{-10}$ eV
- $m_\chi = 10^{-2}\, m_a$
- $Y_i = 5$

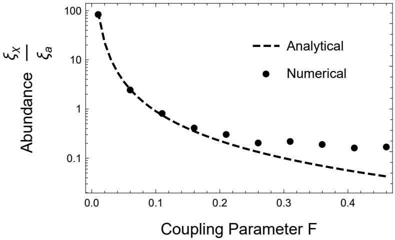

Figure: DPQ

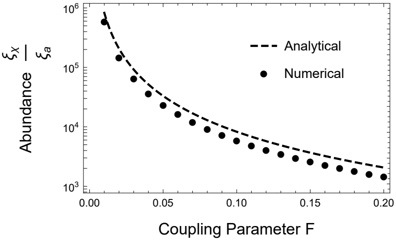

Figure: IPQ