# Electroweak bosons as partons of the $\mu$

## #Pheno22, University of Pittsburgh

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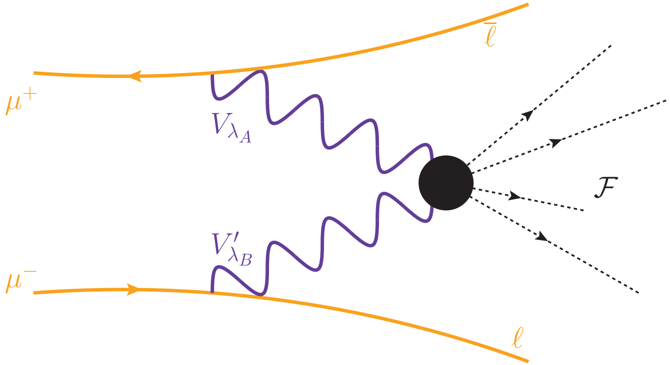
May 10 2022

[1] w/ A. Costantini, F. Maltoni, O. Mattelaer, et al [2005.10289; 2111.02442]

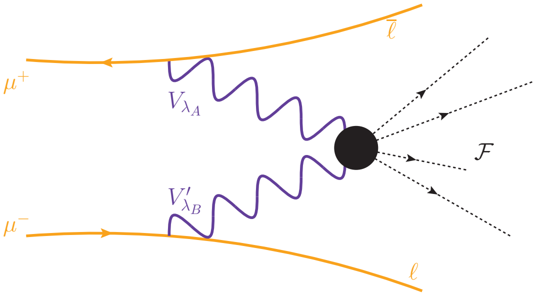
**the big picture**

## What is life like at a multi-TeV $\mu^+\mu^-$ collider?




**Note**: for this talk, no substantial difference between $e^+e^-$ and $\mu^+\mu^-$, only collider energy $\sqrt{s}$

## **Why?**[2] Situation where scattering formalism is **theoretically interesting**




**Partonic** collisions at $Q \sim \mathcal{O}(10)$ TeV explore when **electroweak (EW)** symmetry is nearly restored, i.e., $(M^2_{W/Z/H}/Q^2) \to 0$

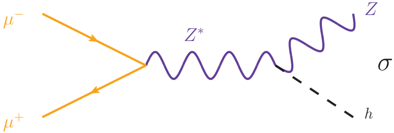
See C. Bauer, et al ('16,'17,'18); T. Han, et al ('16,'20,'21); A. Manohar, et al ('14,'18) + others

When momentum transfers reach $Q \sim \mathcal{O}(10)$ TeV, vector boson scattering (**VBS/VBF**) **acts a bit... funny**

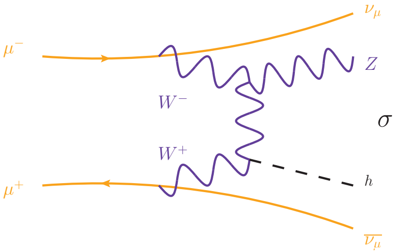
w/ A. Costantini, et al [2005.10289]

[2] Many motivations, e.g., Al Ali, et al. [2103.14043]; European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass (on-going)

# $s$-channel annihilation vs VBF/S




$$\sigma^{s-channel} \sim \frac{g^4}{s}$$




$$\sigma^{VBF} \sim \frac{g^8}{M_{WW}^2} \log^2\left(\frac{M_{WW}^2}{M_W^2}\right)$$

More legs $\implies$ more propagators $\implies$ $\int dk^2/(k^2 - M_W^2) \sim \log(\Lambda^2/M_W^2)$

Larger $s$ $\implies$ larger $(M_{WW}^2/M_W^2)$ $\implies$ collinear $V$ compensate for $g$

Historically, **one approach** to studying the EW theory at high energies is to treat it like massless QCD

- Electroweak boson PDFs (← rich literature!)
- + EW DGLAP evolution
- Electroweak parton showers
- Electroweak Sudakov resummation (← just super cool !)
- ...

| | $\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{1+\bar{z}^2}{z}\right)$ | | $\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z}{2}\right)$ |
|---|---|---|---|
| | $\to\ V_T\ f_s^{(\prime)}$ | $[BW]_T^0\ f_s$ | $H^{0(*)}\ f_{-s}$ or $\phi^{\pm}\ f'_{-s}$ |
| $f_{s=L,R}$ | $g_V^2 (Q^V_{f_s})^2$ | $g_1 g_2 Y_{f_s} T^3_{f_s}$ | $y^2_{f_R^{(\prime)}}$ |

[Han, et al ('16)]

Historically, success of **approach** unclear since computations are difficult to produce and prescriptions varied

**This is not necessarily the case today due to new technology**

## The Effective $W/Z$ Approximation (EWA)[3]

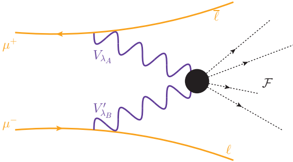
## a.k.a. weak boson parton distribution functions

[3] Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

**Idea**: one can write the following scattering formula

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$

$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dPS_{\mathcal{F}}}_{\text{sum over all configurations / phase space integral}}$$




$$\times \left[ \underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f) \; f_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f)}_{W^+_\lambda/W^-_\lambda/Z_\lambda/\gamma_\lambda \text{ PDFs at LO}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{"hard scattering" at LO}}$$

**Idea**: one can write the following scattering formula

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$

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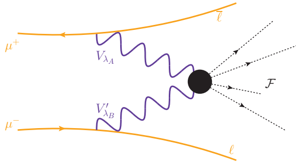



$$\times \left[ \underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f)\ f_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f)}_{W^+_\lambda/W^-_\lambda/Z_\lambda/\gamma_\lambda \text{ PDFs at LO}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{"hard scattering" at LO}}$$

$$+ \underbrace{\mathcal{O}\left(\frac{M^2_{V_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\frac{p^2_{T,V_k}}{M^2_{VV'}}\right)}_{\text{perturbative power-law corrections}} \quad \leftarrow \quad \text{(arise from expanding } \mu_\lambda \to V_\lambda \text{/ matrix elements)}$$

$$+ \underbrace{\mathcal{O}\left(\log\frac{\mu_f^2}{M_V^2}\right)}_{\text{log corrections}} \quad \leftarrow \quad \text{(due to working with LO/Bare PDFs)}$$

We studied the **red** terms

w/ Antonio Costantini, Fabio Maltoni, Olivier Mattelaer [2111.02442]

## LO $W/Z/\gamma$ PDFs from $e^\pm, \mu^\pm$ now supported in MadGraph5_aMC@NLO

- publicly released in v3.3.0 ← milestone for lepton colliders; see Frixione, et al [2108.10261]

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

```
c     /* ************************************************************ *
c     EVA (1/6) for f_L > v_+
      double precision function eva_fL_to_vp(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo              ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

c      print*,'gg2,gL2,mv2,x,mu2,ievo',gg2 !3,gL2,mv2,x,mu2,ievo
      coup2 = gg2*gL2/fourPiSq
      split = (1.d0-x)**2 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vp = coup2*split*xxlog
      return
      end
c     /* ************************************************************ *
c     EVA (2/6) for f_L > v_-
      double precision function eva_fL_to_vm(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo              ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

      coup2 = gg2*gL2/fourPiSq
      split = 1.d0 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vm = coup2*split*xxlog
      return
      end
```

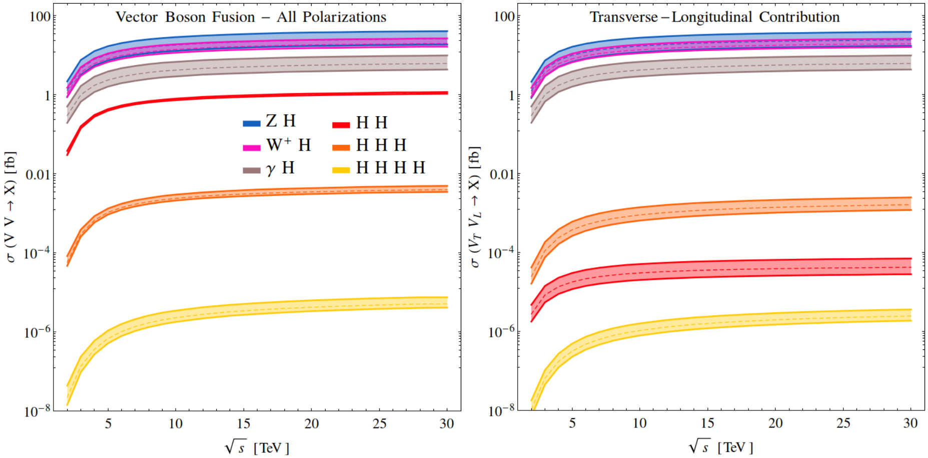
## some results on $V_\lambda V'_{\lambda'} \to X$ in $\mu^+\mu^-$ collisions [4]

[4] w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2111.02442]

# Higgs Production in EWA

## We had fun looking into *many* processes

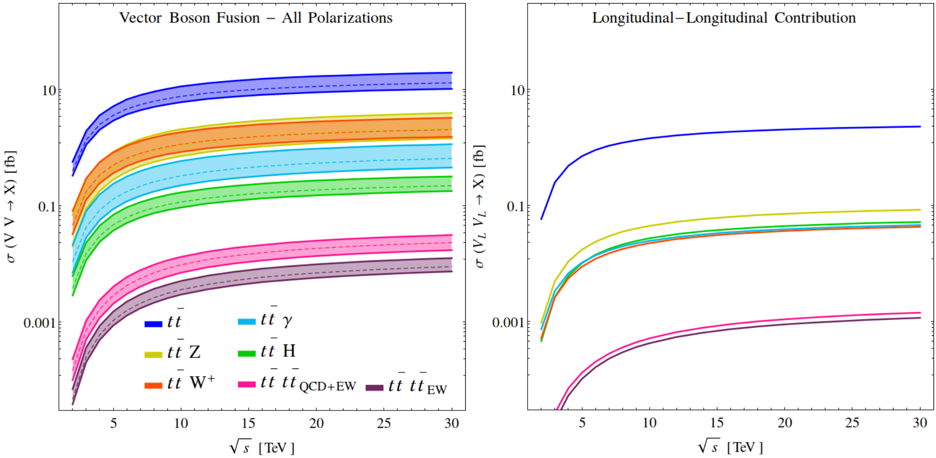
$$\text{(L)} \sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to HX \qquad \text{(R)}\ V_T V_0 \to HX$$



# Top Production in EWA

**... *many* processes**

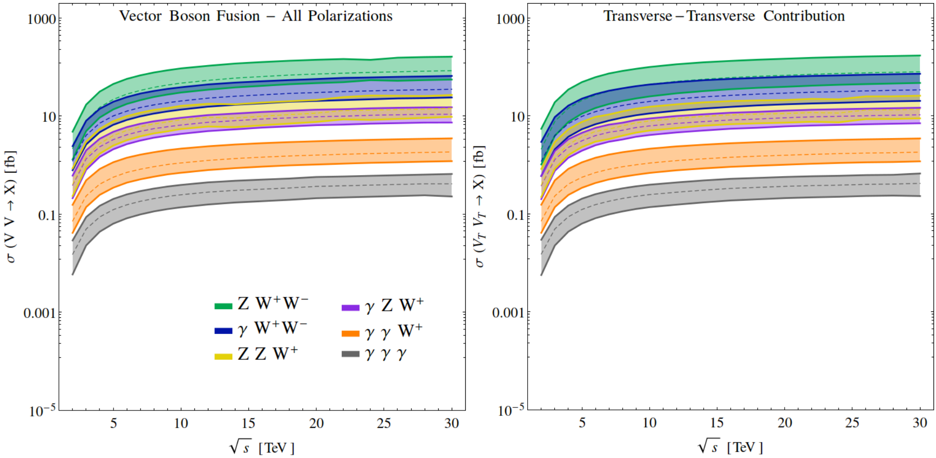
(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to t\bar{t}X$ (R) $V_0 V_0 \to t\bar{t}X$



# Triboson EWA

☺☺☺

(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to 3V$ (R) $V_T V_T \to 3V$



# Diboson and multi-boson in EWA

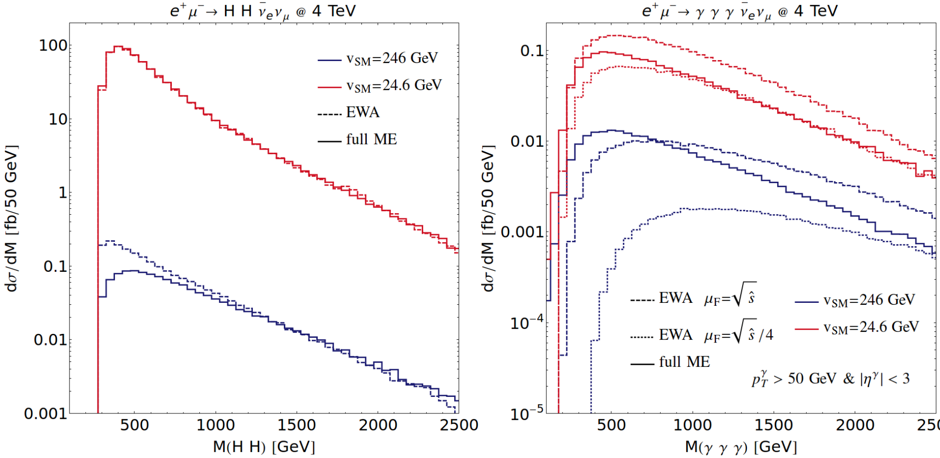
## (4 polarization plots + 1 table) × each class of processes

| | mg5amc syntax | $\sigma$ [fb] $\sqrt{s}=3$ TeV | $\sqrt{s}=14$ TeV | $\sqrt{s}=30$ TeV |
|---|---|---|---|---|
| $\sum V_{\lambda_A} V'_{\lambda_B} \to W^+W^-$ | `vp vm > w+ w-` | $2.2\cdot10^{2}\ {}^{+98\%}_{-35\%}$ | $7.0\cdot10^{2}\ {}^{+91\%}_{-33\%}$ | $8.6\cdot10^{2}\ {}^{+88\%}_{-32\%}$ |
| $V_T V'_T \to W^+W^-$ | `vp{T} vm{T} > w+ w-` | $2.0\cdot10^{2}\ {}^{+99\%}_{-35\%}$ | $6.6\cdot10^{2}\ {}^{+93\%}_{-34\%}$ | $8.0\cdot10^{2}\ {}^{+92\%}_{-33\%}$ |
| $V_0 V'_T \to W^+W^-$ | `vp{0} vm{T} > w+ w-` | $1.2\cdot10^{1}\ {}^{+54\%}_{-27\%}$ | $4.4\cdot10^{1}\ {}^{+50\%}_{-25\%}$ | $5.2\cdot10^{1}\ {}^{+49\%}_{-24\%}$ |
| $V_0 V'_0 \to W^+W^-$ | `vp{0} vm{0} > w+ w-` | $4.2\cdot10^{-1}$ | $1.7\cdot10^{0}$ | $2.0\cdot10^{0}$ |
| $\sum V_{\lambda_A} V'_{\lambda_B} \to W^+Z$ | `vp vm > w+ z` | $5.3\cdot10^{1}\ {}^{+105\%}_{-40\%}$ | $1.8\cdot10^{2}\ {}^{+97\%}_{-37\%}$ | $2.2\cdot10^{2}\ {}^{+95\%}_{-37\%}$ |
| $V_T V'_T \to W^+Z$ | `vp{T} vm{T} > w+ z` | $5.0\cdot10^{1}\ {}^{+111\%}_{-42\%}$ | $1.6\cdot10^{2}\ {}^{+103\%}_{-39\%}$ | $2.0\cdot10^{2}\ {}^{+100\%}_{-38\%}$ |
| $V_0 V'_T \to W^+Z$ | `vp{0} vm{T} > w+ z` | $3.4\cdot10^{0}\ {}^{+36\%}_{-18\%}$ | $1.4\cdot10^{1}\ {}^{+34\%}_{-17\%}$ | $1.7\cdot10^{1}\ {}^{+34\%}_{-17\%}$ |
| $V_0 V'_0 \to W^+Z$ | `vp{0} vm{0} > w+ z` | $3.9\cdot10^{-2}$ | $2.1\cdot10^{-1}$ | $2.6\cdot10^{-1}$ |
| $\sum V_{\lambda_A} V'_{\lambda_B} \to ZZ$ | `vp vm > z z` | $4.4\cdot10^{1}\ {}^{+164\%}_{-52\%}$ | $1.6\cdot10^{2}\ {}^{+144\%}_{-48\%}$ | $1.9\cdot10^{2}\ {}^{+143\%}_{-48\%}$ |
| $V_T V'_T \to ZZ$ | `vp{T} vm{T} > z z` | $4.0\cdot10^{1}\ {}^{+171\%}_{-54\%}$ | $1.4\cdot10^{2}\ {}^{+153\%}_{-50\%}$ | $1.7\cdot10^{2}\ {}^{+150\%}_{-49\%}$ |
| $V_0 V'_T \to ZZ$ | `vp{0} vm{T} > z z` | $4.2\cdot10^{0}\ {}^{+66\%}_{-33\%}$ | $1.8\cdot10^{1}\ {}^{+61\%}_{-30\%}$ | $2.2\cdot10^{1}\ {}^{+60\%}_{-30\%}$ |
| $V_0 V'_0 \to ZZ$ | `vp{0} vm{0} > z z` | $1.1\cdot10^{-1}$ | $6.0\cdot10^{-1}$ | $7.2\cdot10^{-1}$ |
| $\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma Z$ | `vp vm > a z` | $1.9\cdot10^{1}\ {}^{+169\%}_{-53\%}$ | $7.1\cdot10^{1}\ {}^{+149\%}_{-49\%}$ | $8.8\cdot10^{1}\ {}^{+145\%}_{-48\%}$ |
| $V_T V'_T \to \gamma Z$ | `vp{T} vm{T} > a z` | $1.8\cdot10^{1}\ {}^{+172\%}_{-54\%}$ | $6.8\cdot10^{1}\ {}^{+153\%}_{-50\%}$ | $8.4\cdot10^{1}\ {}^{+149\%}_{-49\%}$ |
| $V_0 V'_T \to \gamma Z$ | `vp{0} vm{T} > a z` | $9.5\cdot10^{-1}\ {}^{+67\%}_{-33\%}$ | $4.4\cdot10^{0}\ {}^{+61\%}_{-30\%}$ | $5.5\cdot10^{0}\ {}^{+60\%}_{-30\%}$ |
| $V_0 V'_0 \to \gamma Z$ | `vp{0} vm{0} > a z` | $5.6\cdot10^{-4}$ | $4.5\cdot10^{-3}$ | $6.5\cdot10^{-3}$ |
| $\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma W^+$ | `vp vm > a w+` | $1.1\cdot10^{1}\ {}^{+111\%}_{-42\%}$ | $4.0\cdot10^{1}\ {}^{+101\%}_{-39\%}$ | $4.9\cdot10^{1}\ {}^{+99\%}_{-38\%}$ |
| $V_T V'_T \to \gamma W^+$ | `vp{T} vm{T} > a w+` | $1.1\cdot10^{1}\ {}^{+111\%}_{-42\%}$ | $3.9\cdot10^{1}\ {}^{+102\%}_{-39\%}$ | $4.8\cdot10^{1}\ {}^{+100\%}_{-38\%}$ |
| $V_0 V'_T \to \gamma W^+$ | `vp{0} vm{T} > a w+` | $1.6\cdot10^{-2}\ {}^{+62\%}_{-31\%}$ | $7.3\cdot10^{-1}\ {}^{+56\%}_{-28\%}$ | $9.2\cdot10^{-1}\ {}^{+54\%}_{-27\%}$ |
| $V_0 V'_0 \to \gamma W^+$ | `vp{0} vm{0} > a w+` | $1.5\cdot10^{-4}$ | $1.2\cdot10^{-3}$ | $1.7\cdot10^{-3}$ |
| $\sum V_{\lambda_A} V'_{\lambda_B} \to \gamma\gamma$ | `vp vm > a a` | $2.1\cdot10^{0}\ {}^{+172\%}_{-54\%}$ | $8.5\cdot10^{0}\ {}^{+152\%}_{-50\%}$ | $1.1\cdot10^{1}\ {}^{+147\%}_{-48\%}$ |
| $V_T V'_T \to \gamma\gamma$ | `vp{T} vm{T} > a a` | $2.1\cdot10^{0}\ {}^{+172\%}_{-54\%}$ | $8.5\cdot10^{0}\ {}^{+152\%}_{-50\%}$ | $1.1\cdot10^{1}\ {}^{+147\%}_{-48\%}$ |
| $V_0 V'_T \to \gamma\gamma$ | `vp{0} vm{T} > a a` | $7.8\cdot10^{-4}\ {}^{+70\%}_{-35\%}$ | $3.4\cdot10^{-3}\ {}^{+67\%}_{-34\%}$ | $4.2\cdot10^{-3}\ {}^{+67\%}_{-33\%}$ |
| $V_0 V'_0 \to \gamma\gamma$ | `vp{0} vm{0} > a a` | $5.8\cdot10^{-4}$ | $4.7\cdot10^{-3}$ | $6.8\cdot10^{-3}$ |

**the money plot**

**Plot**: $M_{WW}$ for (L) $W_0 W_0 \to HH$ (R) $W_T W_T \to \gamma\gamma\gamma$

solid (dashed) = full ME (EVA); lower (upper)= $\sqrt{2}\langle\Phi\rangle = v_{\rm EW} \left(\frac{v_{\rm EW}}{10}\right)$




EVA works within uncertainties when $(M_V^2/M_{VV}^2) < 10^{-2}$.

tl;dr: $M_V$ is large $\implies$ $M_{VV}$ must be larger!

Consistent with heavy Q factorization!

When $M^2_{W/Z/H}/Q^2 \to 0$, qualitatively new behavior emerges

**Bluntly,** a $\mathcal{O}(10)$ TeV $\mu^+\mu^-$ collider behaves more like a high-energy hadron collider than a sub-TeV $e^+e^-$ collider

**Take-away:** EWA/EVA can work (EW theory is a gauge theory!) but historical disagreements can be tied to size of power/log corrections

**Outlook:** EWA/EVA in MadGraph is now available and plans underway to merge parallel Snowmass efforts

see Snowmass 21 LoI: SNOWMASS21-TF7_TF0-EF4_EF0-026

**Thank you!**

**EWA/EVA in `MadGraph5_aMC@NLO`**

# Implementing EW boson PDFs in MadGraph5

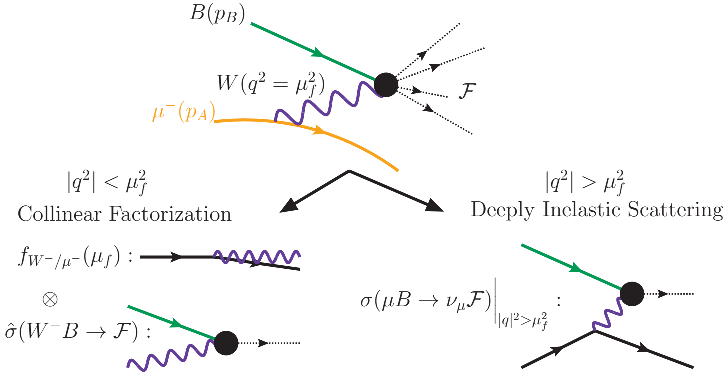
- **NEW: (Polarized) Effective Vector Boson Approx. (EVA)**
  - Bare (LO) PDFs for helicity-polarized $W_\lambda, Z_\lambda, \gamma_\lambda$ from $\ell^\pm_\lambda$
  - Automatically support PDFs for unpolarized $W/Z$ **(EWA)** from $\ell^\pm_\lambda$
- **KEPT: Improved Weizsäcker-Williams approximation (iWWA)**
  - Unpolairzed $\gamma$ PDF + power corrections from $\ell^\pm$ (Frixione, et al [hep-ph/9310350])
- **Technicalities:**
  - $M_W, M_Z$ always nonzero in PDFs and matrix elements!
  - static and dyamic $\mu_f$
  - $n$-point $\mu_f$ variation
  - Choice of $p_T$ and $q$ as evolution variable (this gives extra $\log(1-\xi)$ terms in PDFs!)
  - Also enabled **EVA+DIS** collider configuration
- **Technical appendix** rederiving $W_\lambda, Z_\lambda$ PDFs to provide standard reference and mapping between different approaches in the literature
  - Publicly released in v3.3.0 (Major milestone for lepton colliders; see Frixione, et al [2108.10261])

**money plot #2**

# Matching EW PDFs to matrix elements

**Idea**: Total cross section ($\sigma^{\text{total}}$) can be split into "collinear" ($\sigma^{\text{collinear}}$) and "wide-angle" ($\sigma^{\text{wide-angle}}$) bits. Summing *should* recover $\sigma^{\text{total}}$

$$\sigma^{\text{total}} = \sigma^{\text{collinear}} + \sigma^{\text{wide-angle}} + \mathcal{O}\left(\frac{M_W^2}{M_{WW}^2}\right) + \underbrace{\mathcal{O}\left(\frac{p_T^{\nu 2}}{M_{WW}^2}\right)}_{\text{How large?}}$$



Consider $\mu^- \to W^- \nu_\mu$ splitting in $W^+ W^-$ scattering

$$\sigma^{\rm total} = \underbrace{\int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\rm collinear} + \underbrace{\int_{\mu_f}^{\Lambda} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\rm quasi-collinear} + \underbrace{\int_{\Lambda}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\rm hard}$$

The collinear term contains the $W$ PDF:

$$\sigma^{\rm collinear} = \int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\mu_f^2}{M_W^2}\right)}_{\rm PDF} + \underbrace{\text{power corrections (PC1)}}_{\rm neglect}$$

The quasi-collinear term also depends on $\mu_f$:

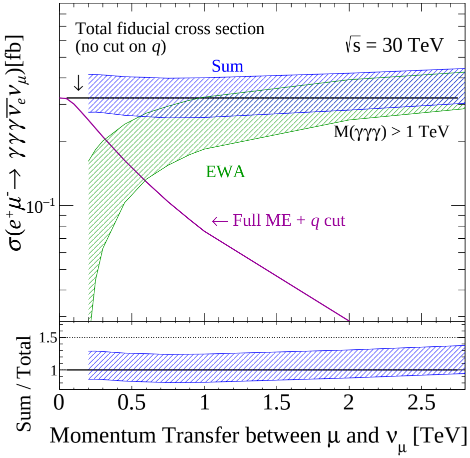
$$\sigma^{\rm quasi-collinear} = \int_{\mu_f}^{\Lambda} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\Lambda^2}{\mu_f^2}\right)}_{\text{same log as in PDF}} + \underbrace{\rm PC2}_{\rm keep}$$

The full matrix element **without** collinear term **(the wide-angle term!)** is

$$\sigma^{\rm wide-angle} = \sigma^{\rm qc} + \sigma^{\rm hard} \sim \log\left(\frac{\Lambda^2}{\mu_f^2}\right) + {\rm PC2} + \sigma^{\rm hard}$$

**ME matching:** $\sigma^{sum} = \sigma^{\rm EWA} + \sigma^{\rm wide-angle}$ is independent of $\mu_f$:

**Plot**: $\sigma(e^+\mu^- \to \gamma\gamma\gamma\overline{\nu_e}\nu_\mu)$ vs matching scale $(\mu_f)$




**Take away:** PDFs high when $\mathcal{O}(p_T^\nu / M_{WW}^2)$ too large! (breakdown of coll. limit)