A New Horizon in the Randall-Sundrum Phase Transition

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(*Work in Progress*)

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Extra-dimensions and RS models

Theoretical issue with Standard Model

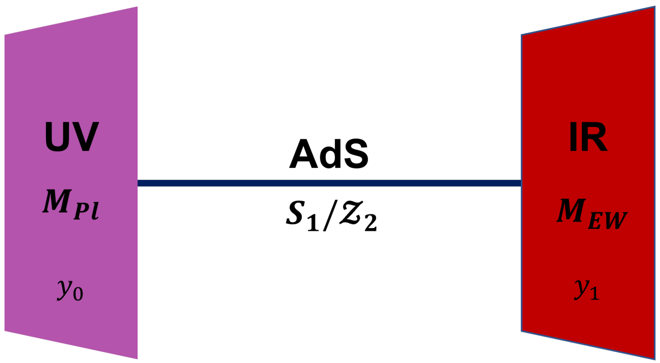
Hierarchy/ naturalness problem ($M_{\mathrm{EW}} \sim 10^3$ GeV, $M_{\mathrm{Pl}} \sim 10^{19}$ GeV); Smallness of Higgs mass (126 GeV)

- Some solutions: SUSY, Composite Higgs, etc.
- Randall-Sundrum models- Elegant geometric solution

Spontaneously broken CFT
on boundary

$$ds^2 = e^{-2A(y)} dx_4^2 - dy^2$$

- $A(y) = k\,y \equiv$ pure AdS. k is inverse curvature
- KK mode scale, $f \sim ke^{-k(y_1 - y_0)}$



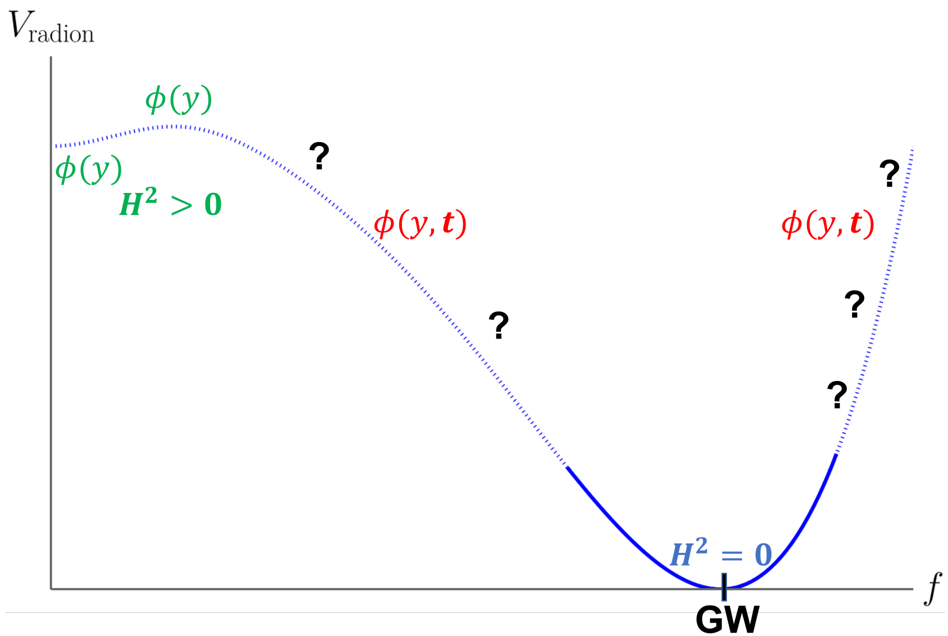
Stabilizing the geometry

- Currently, $y_1 - y_0$ is arbitrary
- Phenomenology requires $M_{\rm Pl}/M_{\rm EW}$ hierarchy
- *Radion*: Relative motion of branes d.o.f $\equiv$ Exact Goldstone of broken CFT

~~stabilization~~	**stabilization**
extreme fine-tuning Radion flat direction	fixes $y_1 - y_0$ Correct low energy pheno, early universe cosmology

- Goldberger-Wise stabilization mechanism- introduce ϕ that gains $\langle\phi\rangle(y)$, deforms AdS geometry (coupling RG evolution)
- Creates effective Radion (pseudo-Goldstone) potential
- Remaining fine-tuning fixes effective 4D cosmological constant (CC)

Radion Potential

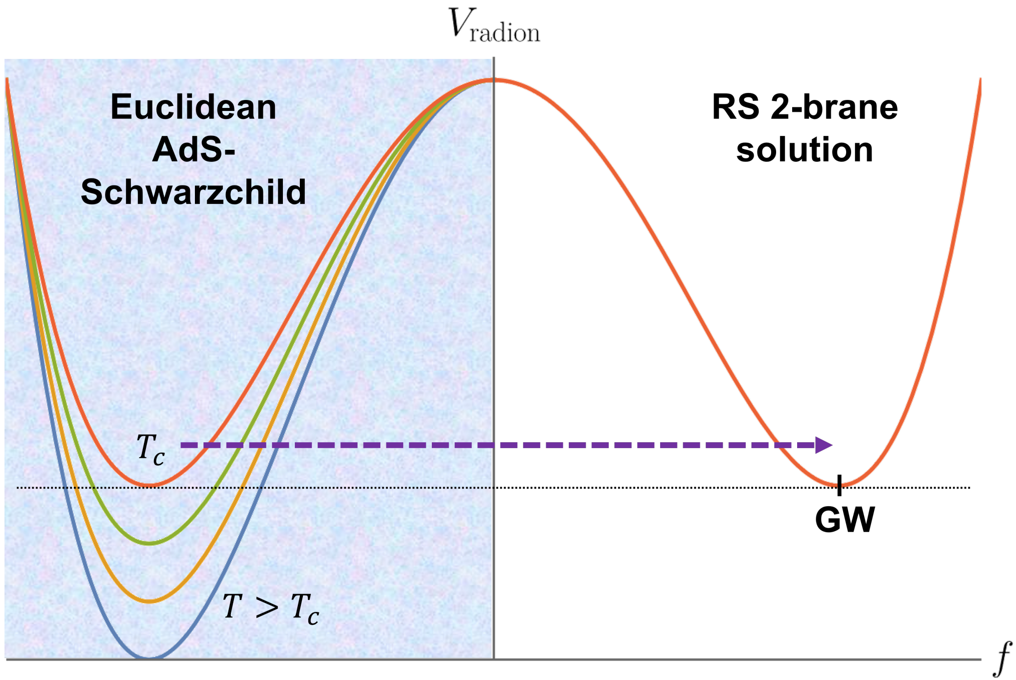


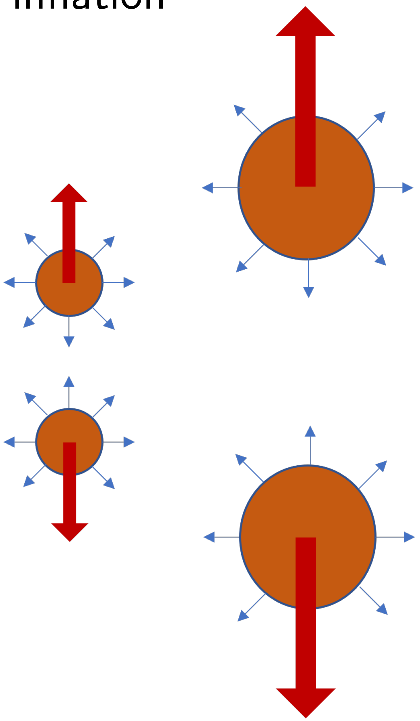
Goal

Map out stationary points of Radion potential and try to address dynamics connecting them (Phase transition?)

Finite Temperature RS Phase Transition

Strongly first order [1,2]. Stuck in Eternal inflation





Is there a non-equilibrium solution?

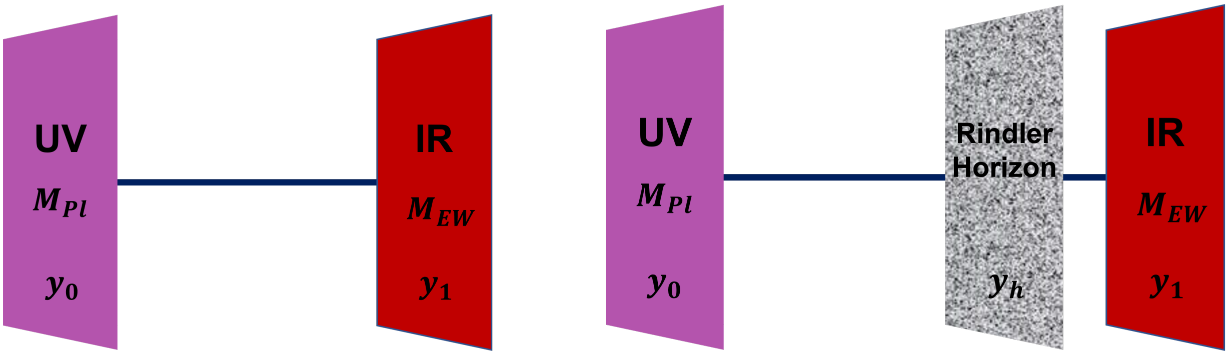
[1] D. Bunk, J. Hubisz, and B. Jain, A Perturbative RS I Cosmological Phase Transition, Eur. Phys. J. C 78, (2018)

[2] B. von Harling and G. Servant, QCD-Induced Electroweak Phase Transition, J. High Energ. Phys. 2018, (2018)

Set-up

$$ds^2 = e^{-2y}\left(dt^2 - e^{2Ht}dx_3^2\right) - \frac{dy^2}{G^2(y)}$$

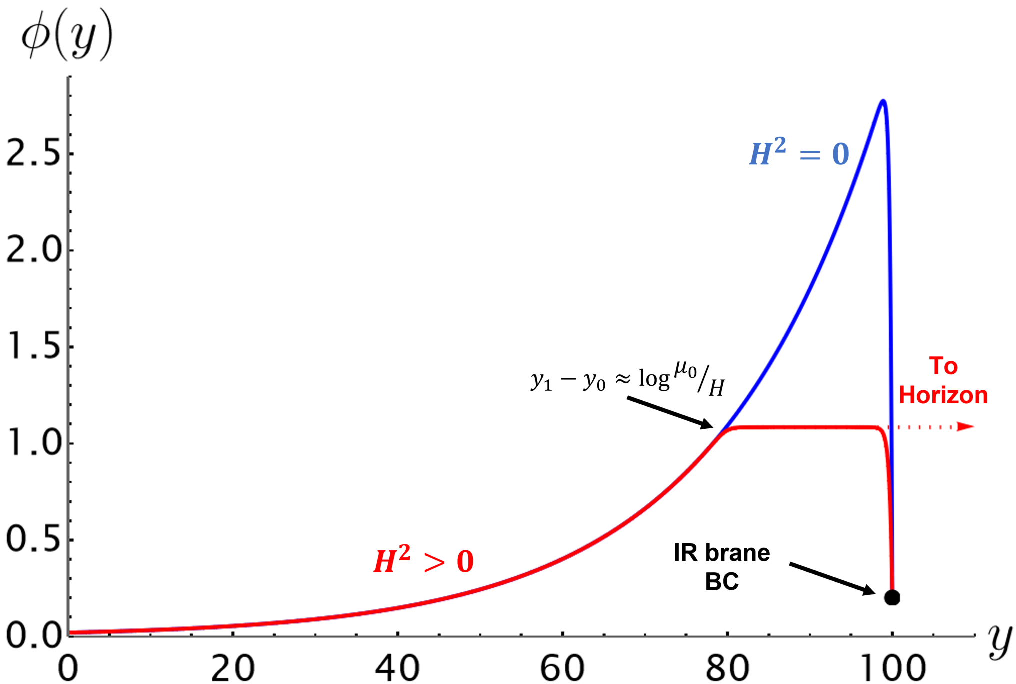
- Two classes of solutions:



$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}(\partial_M\phi)^2 - V(\phi) - \frac{1}{2\kappa^2}R\right] + S_{\rm branes}$$

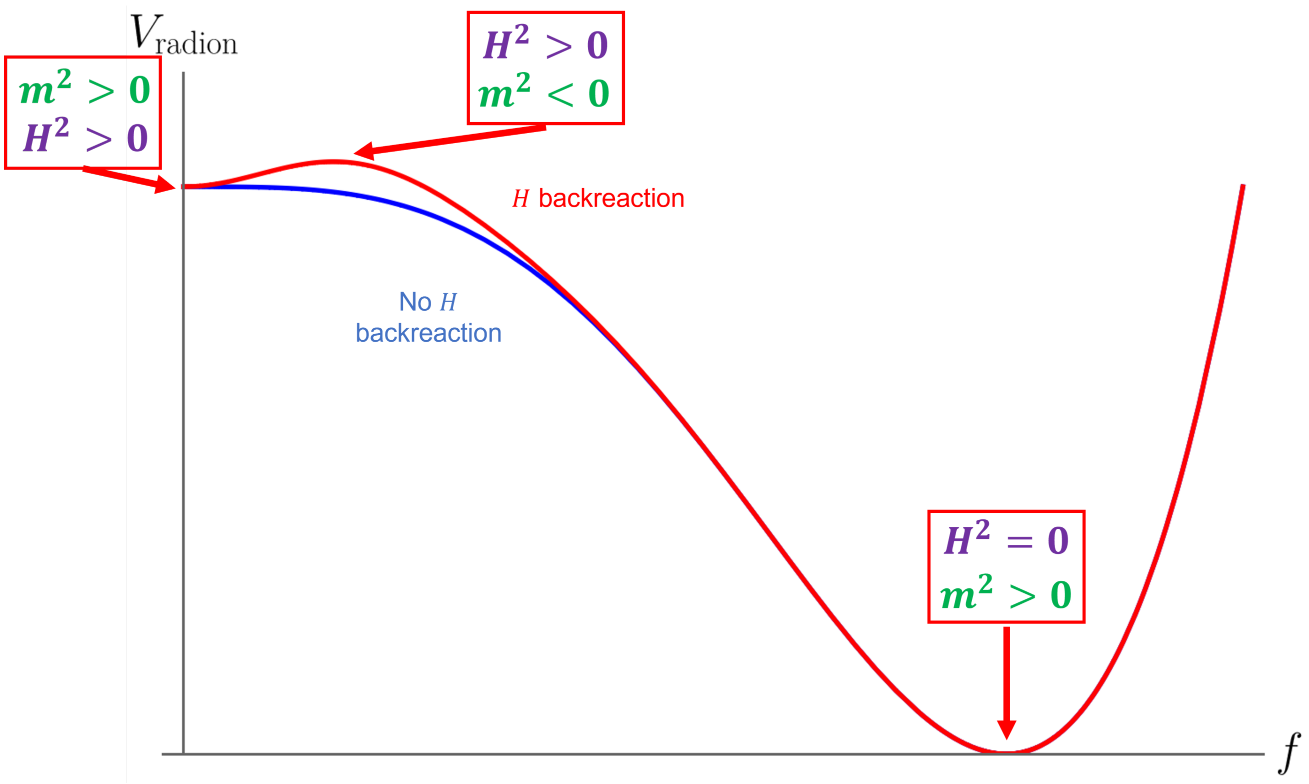
$$S_{0,1} = -\int d^4x\sqrt{g_{0,1}}\lambda_{0,1}(\phi) \quad \text{(Stiff-wall potential)}$$

Scalar solutions



- H keeps scalar small (RG evolution scale can't be larger than the universe!)
- Under perturbative control

Cartoon of potential



Spectra of modes[5] (horizon case)

Mode	EoM in Schrödinger-form[3,4]	Mass gap
Radion	$-\tilde{F}'' + \left(\frac{9}{4}A'^2 - 6H^2 + \frac{5}{2}A'' - A'\frac{\phi''}{\phi'} + 2\left(\frac{\phi''}{\phi'}\right)^2 - \frac{\phi'''}{\phi'}\right)\tilde{F} = m^2\tilde{F}$	$\frac{3}{2}H$
Scalar	$-\tilde{\phi}'' + H^2\left[\frac{9}{4}\coth^2(Hz) + \frac{3+2m^2}{2\sinh^2(Hz)}\right]\tilde{\phi} = \mu^2\tilde{\phi}$	$\frac{3}{2}H$
4D Vector	$-\tilde{\psi}'' - \frac{H^2}{4}\left[\coth^2(Hz) + \frac{2}{\sinh^2(Hz)}\right]\tilde{\psi} = \mu^2\tilde{\psi}$	$\frac{1}{2}H^\dagger$
Fermions	$-\tilde{f}'' + \left[m^2 e^{-2A} - mA'e^{-A}\right]\tilde{f} = \mu^2\tilde{f}$	0
Graviton	$-h'' + H^2\left[\frac{9}{4}\coth^2(hz) + \frac{3}{2\sinh^2(Hz)}\right] = \mu^2 H^2$	$\frac{3}{2}H$

† 5-component of vector mode has no physical massive modes or zero modes

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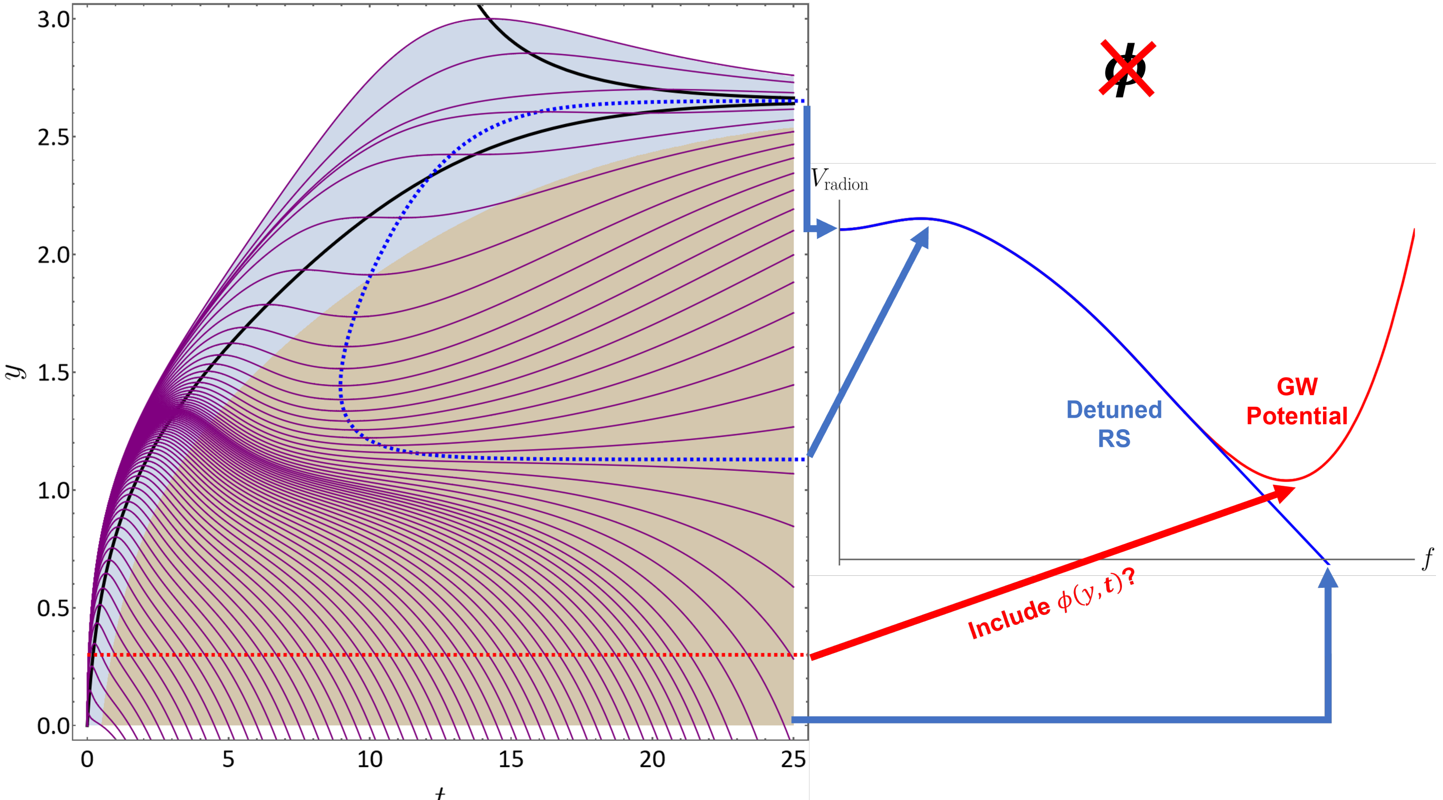
[3] A. Falkowski and M. Pérez-Victoria, Holographic Unparticle Higgs Boson, Phys. Rev. D 79, (2009)

[4] A. Falkowski and M. Pérez-Victoria, Electroweak Breaking on a Soft Wall, J. High Energy Phys., 107 (2008)

[5] Kumar, S., Sundrum, R. Seeing higher-dimensional grand unification in primordial non-gaussianities. J. High Energ. Phys., 120 (2019)

Cosmological History of the IR brane

$$ds^2 = \left(\frac{\dot{a}}{\dot{a}_0}\right)^2 (y,t)dt^2 - a^2(y,t)dx_3^2 - dy^2; \quad H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{4}{a_0^4} + \delta_0(2+\delta_0)$$



Conclusions

- Studied out of equilibrium dynamics ≡ conformal PT
- Identified and characterized stationary points of Radion potential with 4D inflationary metric ansatz
- Solutions with UV, IR branes; UV brane, IR horizon (finite L)
- Studied continuous spectra of modes in horizon case and identified mass gaps
- Considered background time dynamics, obtained numerical results for IR brane motion in dynamic background
- Ongoing work: Obtain approximate solutions to scalar equation and construct toy model for GW stabilizing mechanism, consider order of early universe PT

Thank you ☺

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Solutions

- Solution to the Einstein Equations:

$$G^2 = \frac{-\frac{\kappa^2}{6}V(\phi) + e^{2y}H^2}{1 - \frac{\kappa^2}{12}\phi'^2} \approx \sqrt{1 + e^{2y}H^2}$$

- Scalar equation of motion

$$\phi'' = \left(4 - \frac{G'}{G}\right)\phi' + \frac{1}{G^2}\frac{\partial V}{\partial \phi}$$

$$\phi_\pm = e^{(2\pm\nu)y}\frac{(G \pm \nu)}{(G+1)^{\pm\nu}}$$

- $y_h = \infty$ since $\lim_{y\to\infty}\frac{1}{G(y)} = 0$, but $L = \int \frac{dy}{G(y)} \approx \log\frac{2}{H} - y_0$

Solutions with UV and IR branes

- Stiff-Wall potentials: $\lambda_{0,1}(\phi) = T_{0,1} + \gamma_{0,1}(\phi^2 - v_{0,1}^2)^2$, $\gamma_{0,1} \to \infty$
- Small backreaction limit: $T_{0,1} = \frac{6}{\kappa^2}(\pm 1 + \delta_{0,1})$

$$V_{\text{eff}} = e^{-4y_0}\left[\lambda_0(\phi) - \frac{6}{\kappa^2}G_0\right] + e^{-4y_1}\left[\lambda_1(\phi) + \frac{6}{\kappa^2}G_1\right]$$

$$H^2 = \frac{\kappa^6}{432}T_0^2 e^{-2y_0}\left[\left(\bar{\phi}_0'\right)^2 - \left(\phi_0'\right)^2\right] = \frac{\kappa^6}{432}T_1^2 e^{-2y_1}\left[\left(\bar{\phi}_1'\right)^2 - \left(\phi_1'\right)^2\right]$$

- Example parameters: $y_1 - y_0 \approx 36.8$, $\kappa = \frac{1}{3}$, $v_0 = \frac{1}{50}$, $v_1 = \frac{1}{5}$, $m^2 = -0.1975$ (ΔCFT by marginally relevant operator)
- Yields additional solution: $y_1 \approx 52.7$, $H_{max} = 5.8 \times 10^{-35}$

Solutions with UV brane and IR horizon

- $y_h \to \infty$ in these co-ordinates
- IR BC $\longrightarrow$ finiteness of ϕ at horizon

$$\phi = c_- \left[e^{(2-\nu)y} \frac{(G-\nu)}{(G+1^{-\nu})} - H^{2\nu} e^{(2+\nu)y} \frac{G+\nu}{(G+1)^{+\nu}} \right]$$

- No IR junction condition

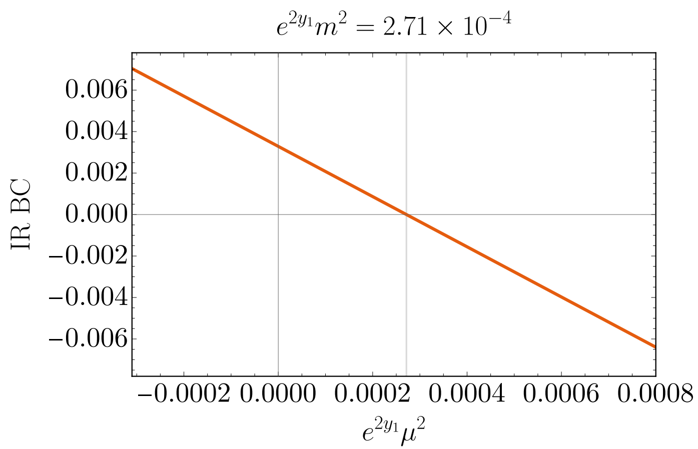
$$H^2 = \frac{\kappa^6}{432} T_0 e^{-2y_0} \left[\left(\bar{\phi}_0'\right)^2 - \left(\phi_0'\right)^2 \right]$$

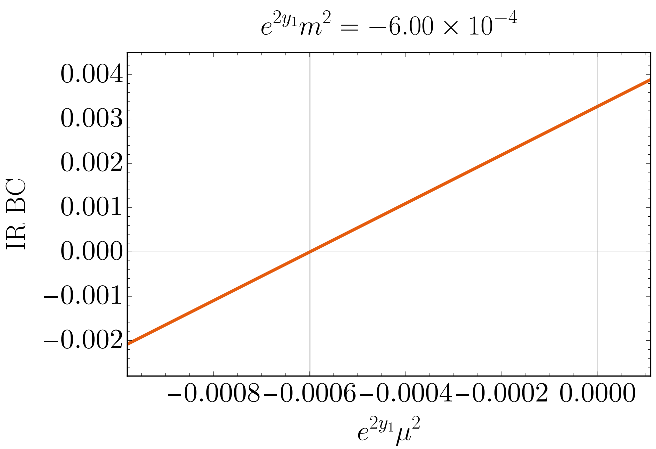
- Yields solution: $L = 79.5,\ H_{\rm hor} \approx H_{\rm max} \left(1 - 10^{-27.5}\right)$

Radion Spectrum

$$\Box_{dS} F = G^2 e^{-2y} \left[F'' - \left(2 + \frac{G'}{G} + 2\frac{\phi''}{\phi'} \right) F' + \left(4\frac{\phi''}{\phi'} + 6H^2 \right) F \right]$$

- Boundary Conditions: $F'_{0,1} = 2F_{0,1}$





Radion Spectrum (Horizon case)

- Use co-ordinates: $e^y = e^{A(z)}$, $\frac{dy}{G(y)} = e^{-A(z)}dz$

$$\Box_{\text{dS}} = F'' - F'\left(3A' - 2\frac{\phi''}{\phi'}\right) + F\left(4A'\frac{\phi''}{\phi'} - A'' + 6H^2\right)$$

- Rescale radion field, $\exp(3/2A)\phi'\tilde{F} = F$

$$-\tilde{F}'' + \left(\frac{9}{4}A'^2 - 6H^2 + \frac{5}{2}A'' - A'\frac{\phi''}{\phi'} + 2\left(\frac{\phi''}{\phi'}\right)^2 - \frac{\phi'''}{\phi'}\right)\tilde{F} = m^2\tilde{F}$$

- In the $y \to \infty$ limit,

$$-\tilde{F}'' + \frac{9}{4}H^2\tilde{F} = m^2\tilde{F}$$

Takeaway

This leads to a gapped continuum with a mass gap of $\mu = \frac{3}{2}H$

Cosmological History of the IR Brane

$$ds^2 = \left(\frac{\dot{a}}{\dot{a}_0}\right)^2 (y,t)dt^2 - a^2(y,t)dx_3^2 - dy^2$$

$$S = \int d^5x\sqrt{g}\left(-\Lambda_5 - \frac{1}{2\kappa^2}R\right) - \int d^4\xi_0\sqrt{g_0}\,T_0 - \int d^4\xi_1\sqrt{g_1}\,T_1$$

$$a^2 = \Lambda_+(t)e^{2y} + \Lambda_-(t)e^{-2y} - \frac{1}{2}\dot{a}_0^2,$$

$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \underbrace{\frac{4}{a_0^4}}_{\text{radiation}} + \underbrace{\delta_0(2+\delta_0)}_{\text{vacuum energy}}$$

$$\gamma\left(\beta\frac{\dot{a}_0}{a} + \frac{a'}{a}\right) = \frac{\kappa^2}{6}T_1, \text{ where } \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{\dot{R}\dot{a}_0}{\dot{a}}$$