Detecting the Stochastic Gravitational Wave Background from Massive Gravity with Pulsar Timing Arrays

—— Presentation for Pheno 2022

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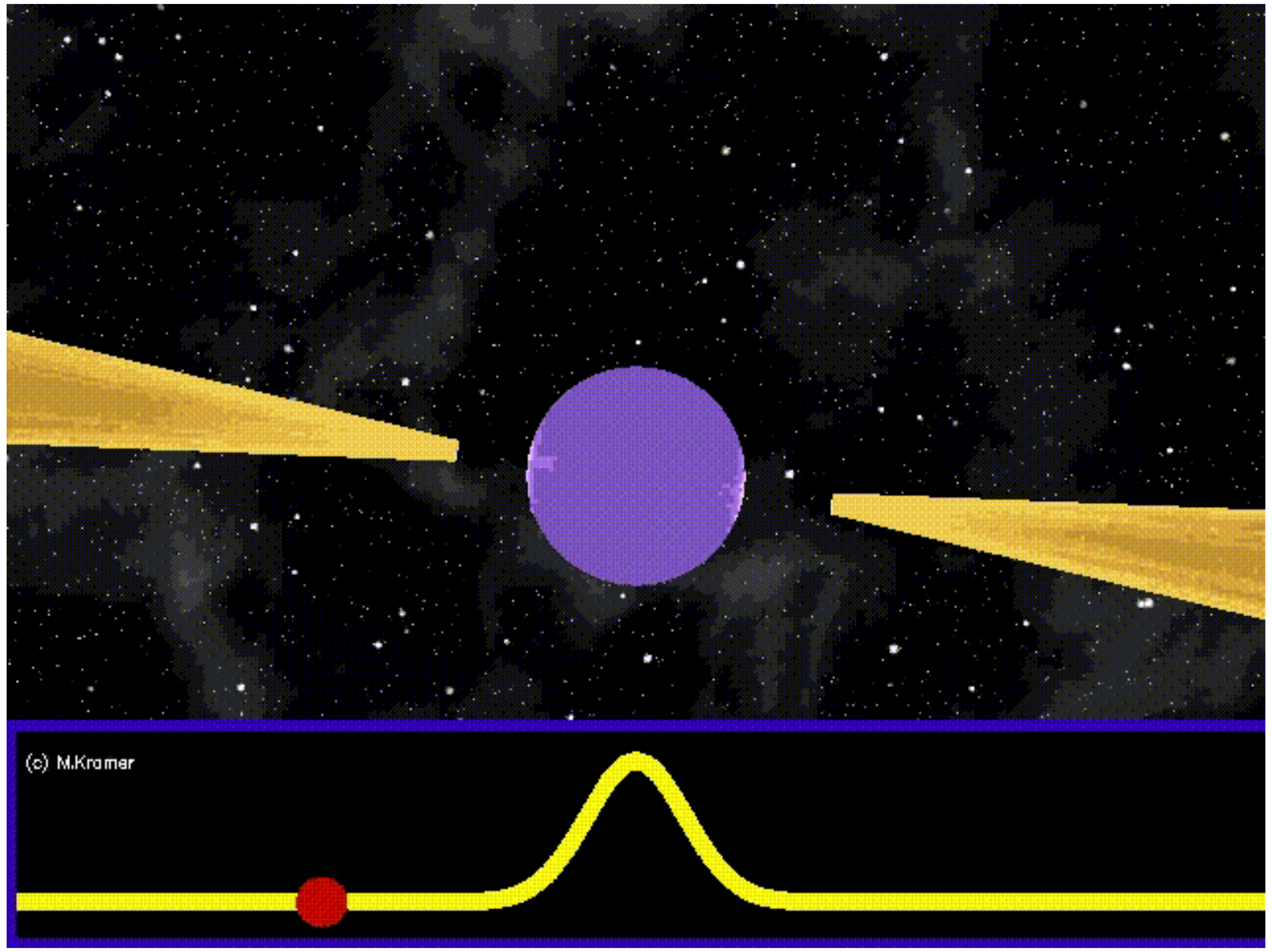
Penn Arts & Sciences

Content

- Introduction to Pulsar Timing Array (PTA) system
    - Underlying physics of PTA
    - Scientific goal: Stochastic Gravitational Wave Background (SGWB)
    - Current on-going program: NANOGrav, PPTA, EPTA, & IPTA
- Distinguished feature between Massive gravity and General Relativity
- Future possibility

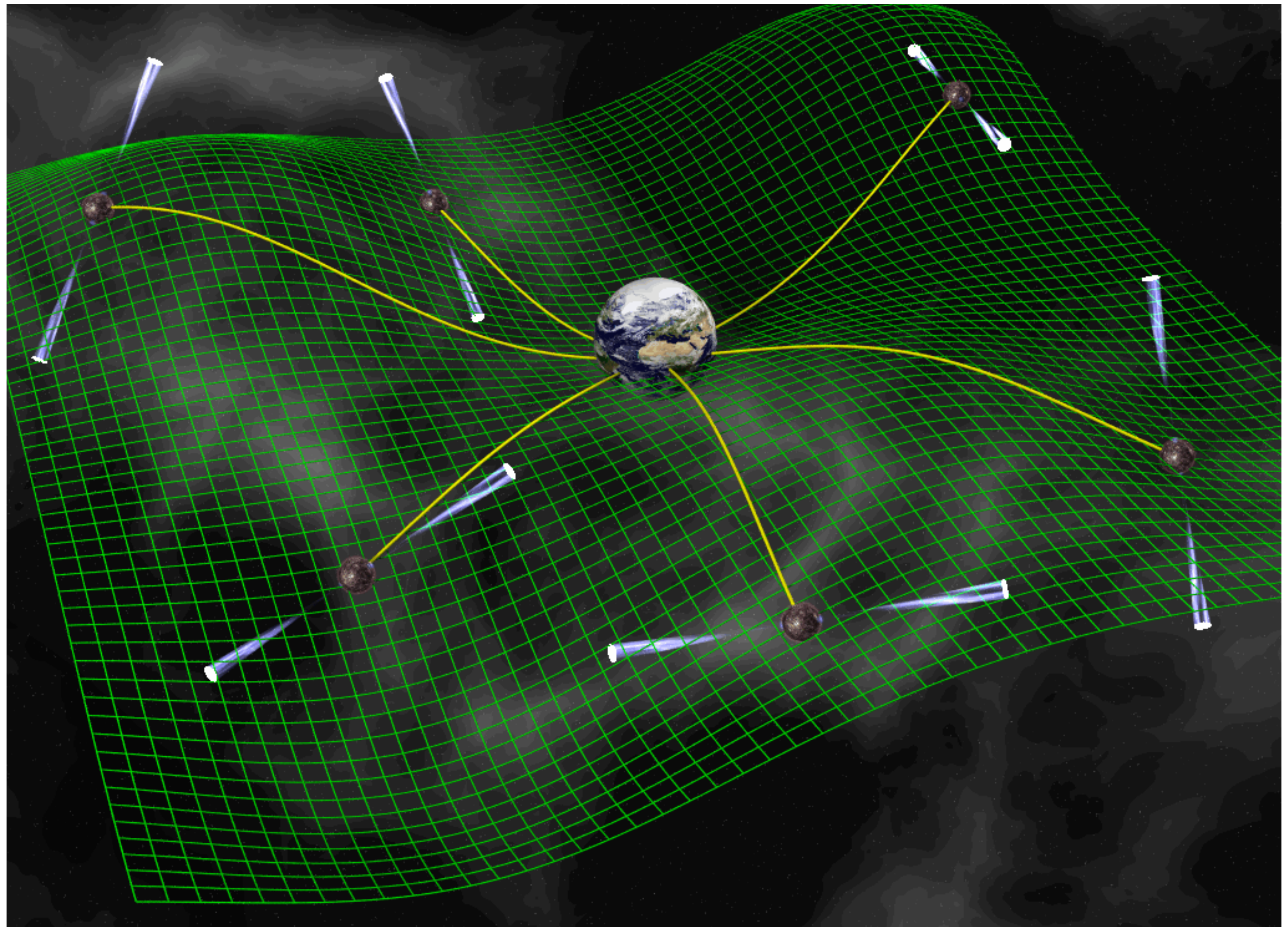
Pulsar

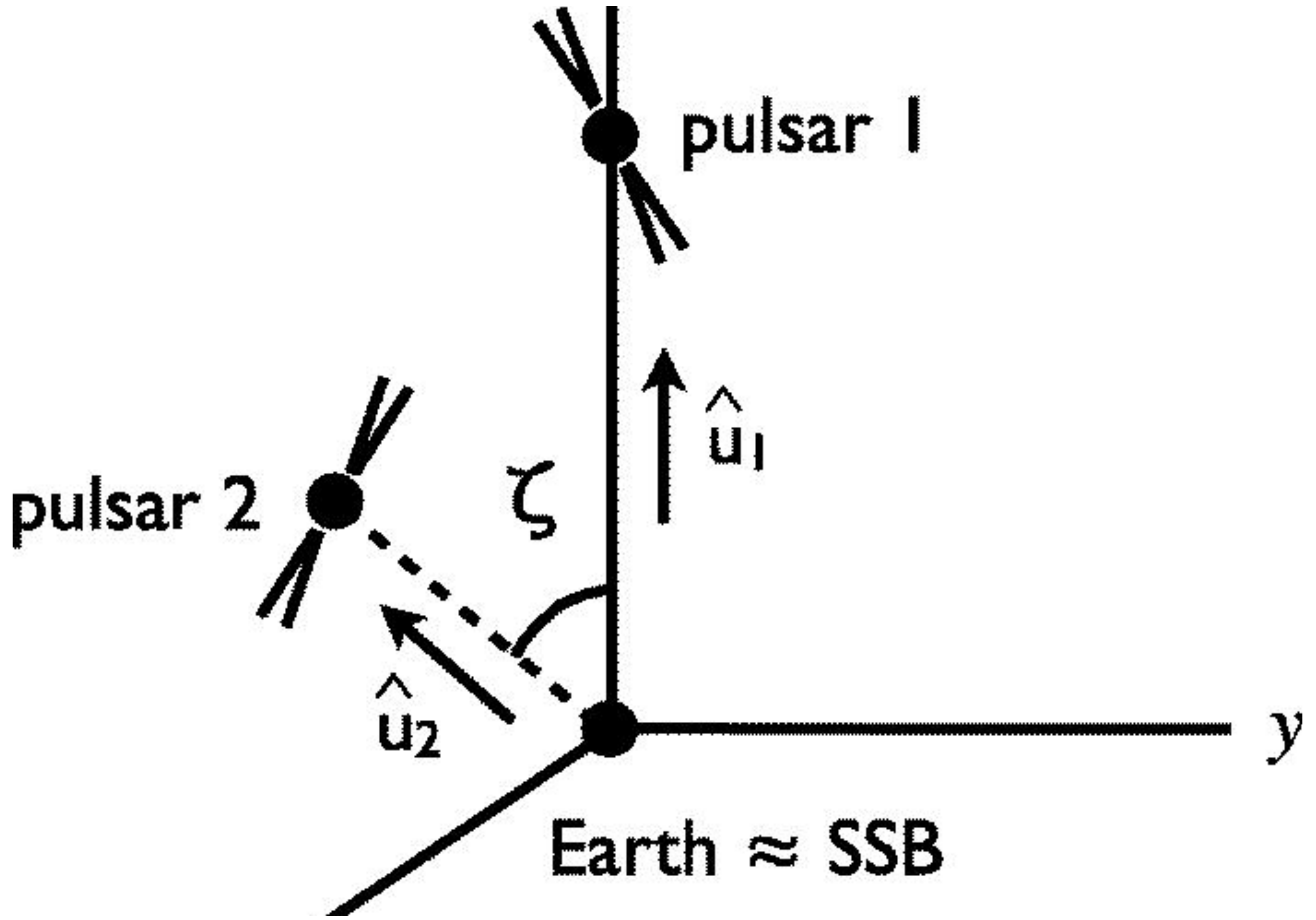
- A pulsar is a highly magnetized rotating compact star that emits narrow beams of electromagnetic radiation out of its magnetic poles.



Pulsar Timing Array (PTA)

- A pulsar timing array (PTA) is a set of pulsars which is analyzed to search for correlated signatures in the pulse arrival times.





Pulsar Timing Array (PTA)

- Goal: to detect the stochastic gravitational wave background.
- Source: Supermassive Black Hole Binaries; Primordial Gravitational Wave; Cosmic String ···
- Current on-going program: North American NanoHertz Observatory for Gravitational Waves (NANOGrav), the European Pulsar Timing Array (EPTA), and the Parkes Pulsar Timing Array (PPTA), International Pulsar Timing Array (IPTA)
- Future: Five-hundred-meter Aperture Spherical Telescope(FAST); Square Kilometer Array (SKA)

Observable

- The observable is the anomalous residue in the pulse arrival time:

$$R(t) \equiv \int_0^t dt' \left(\frac{\nu_0 - \nu(t')}{\nu_0}\right) \qquad \langle R^2(t)\rangle = \frac{1}{T}\int_0^T R^2(t)dt \ ,$$

- pulse frequencies redshift $z \equiv \dfrac{\nu_0 - \nu(t)}{\nu_0}$
- In GR, the metric perturbation only has two polarization modes: $h_+, h_\times$ and we can express $h_{\mu\nu} = \sum_{A=+,\times} e^A_{\mu\nu} h_A$. For each mode, we define a receiving function to denote the influence on the redshift:

$$\tilde{z}(f,\hat{\Omega}) = \left(e^{-i2\pi f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right)\sum_A h_A(f,\hat{\Omega}) F^A(\hat{\Omega}) \qquad F^A(\hat{\Omega}) \equiv e^A_{ij}(\hat{\Omega})\frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}.$$

Correlation function

- One can separate the two-point correlation function in power spectrum $\Omega_{\rm gw}$ and the overlap reduction function $\Gamma(|f|)$ assuming the isotropic SGWB

$$\langle \tilde{z}_1^*(f)\tilde{z}_2(f')\rangle = \frac{3H_0^2}{32\pi^3}\frac{1}{\beta}\delta(f-f')|f|^{-3}\Omega_{\rm gw}(|f|)\Gamma(|f|),$$

- Overlap reduction function:

$$\Gamma(|f|) = \beta \sum_A \int_{S^2} d\hat{\Omega} \left(e^{i2\pi f L_1(1+\hat{\Omega}\cdot\hat{p}_1)} - 1\right) \times \left(e^{-i2\pi f L_2(1+\hat{\Omega}\cdot\hat{p}_2)} - 1\right) F_1^A(\hat{\Omega})F_2^A(\hat{\Omega}),$$

- Exponential factor!

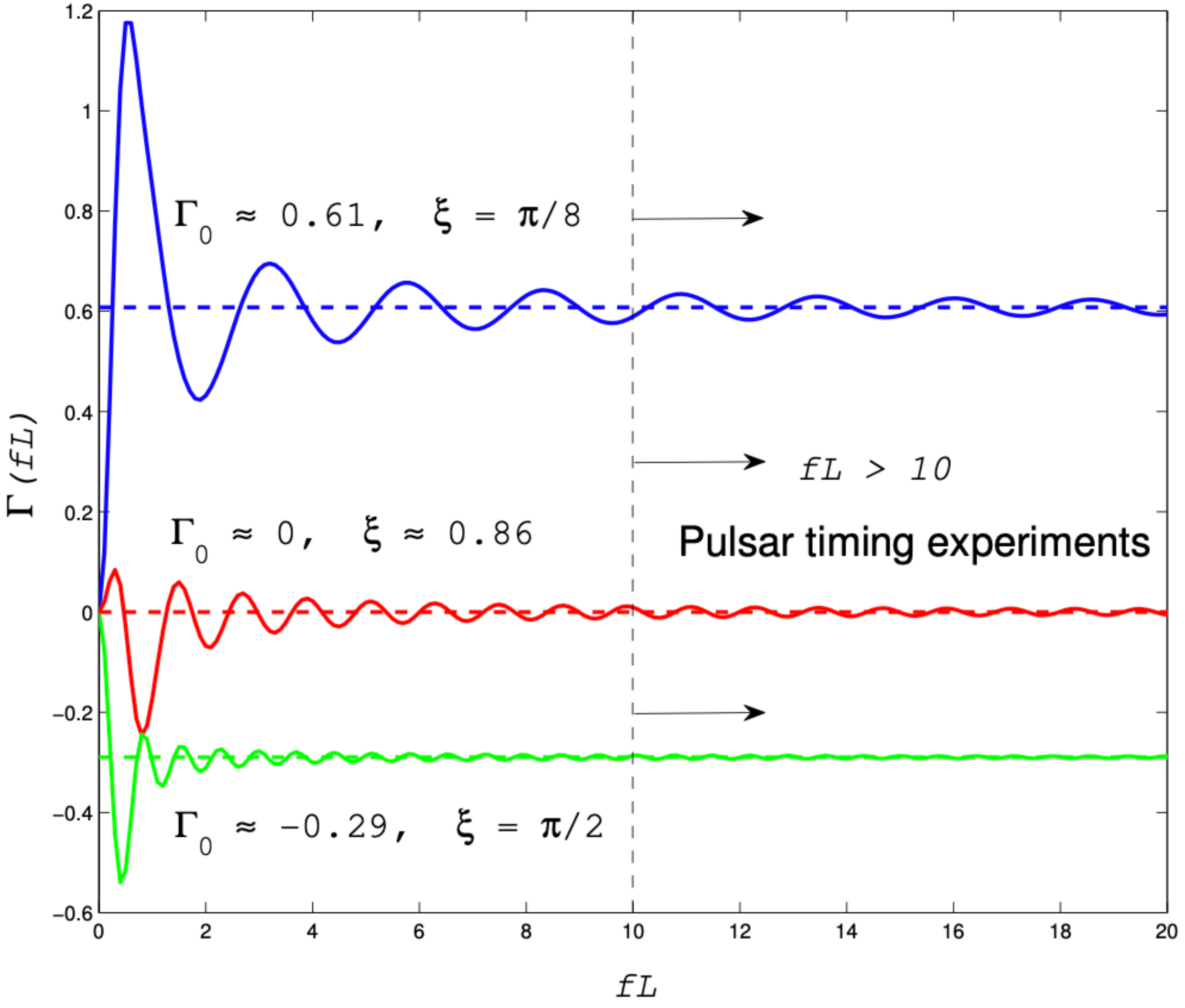
Hellings-Downs curve

- Overlap reduction function:

$$\Gamma(|f|) = \beta \sum_A \int_{S^2} d\hat{\Omega} \left(e^{i2\pi f L_1(1+\hat{\Omega}\cdot\hat{p}_1)} - 1\right) \times$$
$$\times \left(e^{-i2\pi f L_2(1+\hat{\Omega}\cdot\hat{p}_2)} - 1\right) F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}),$$

- Hellings-Downs curve:

$$\Gamma_0 \equiv \frac{3}{4\pi} \sum_A \int_{S^2} d\hat{\Omega}\, F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega})$$
$$= 3\left\{\frac{1}{3} + \frac{1-\cos\xi}{2}\left[\ln\left(\frac{1-\cos\xi}{2}\right) - \frac{1}{6}\right]\right\},$$



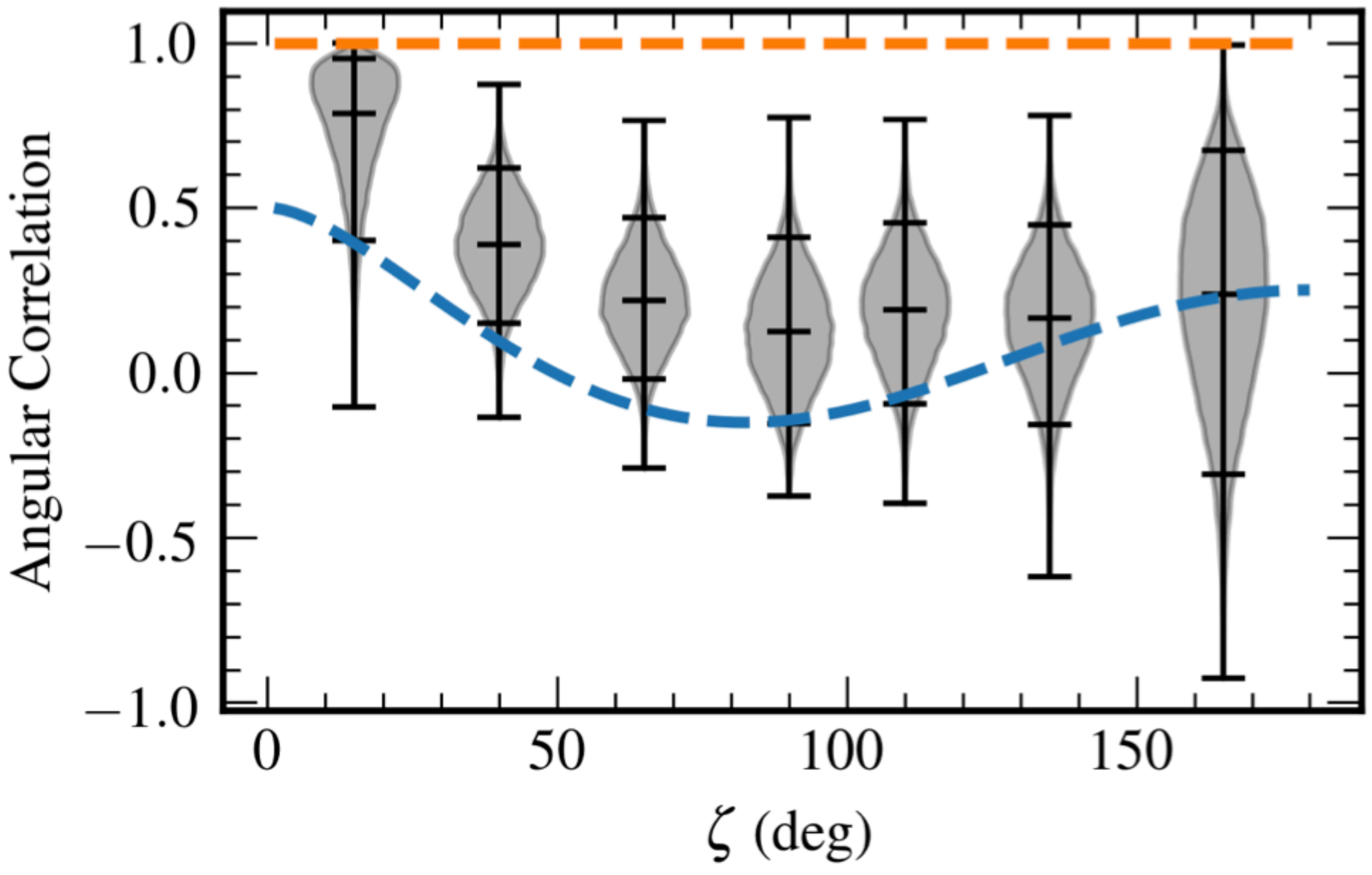
NANOGrav: power spectrum

- The collaborations claim a strong evidence for a power-law like power spectrum: $\Omega_{\rm gw} \sim f^{-\gamma}$, the PPTA collaboration finds $\gamma \in (1.5, 5.5)$ and NANOGrav collaboration finds $\gamma \in (3.76, 6.78)$, EPTA: $\gamma \in (3.11, 4.65)$,
IPTA: $\gamma \in (3.1, 4.9)$

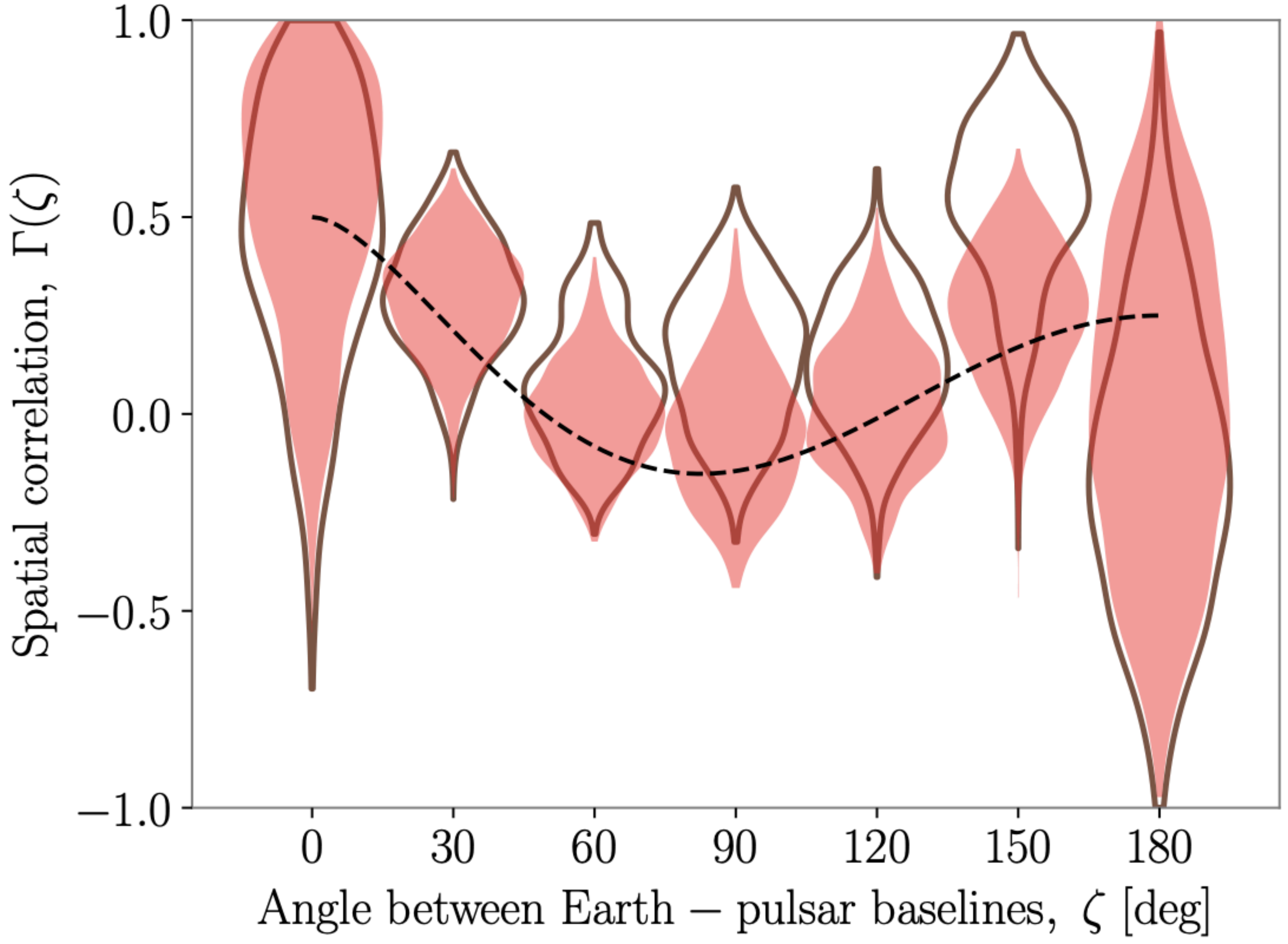
- supermassive black hole binary systems ($\gamma \sim 13/3$); primordial gravitational waves ($\gamma \sim 5$); and networks of cosmic strings ($\gamma \sim 16/3$)

NANOGrav: HD curve

- Error bar on the overlap reduction function is too large to give statistical evidence for GR prediction(HD curve)!



NANOGrav



PPTA

What does this imply?

- Noise
- The assumption of isotropic SGWB
- Modified gravity theories

Massive Gravity

- Action:

$$S = \int d^4x \left[\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} + \partial_\mu h^{\mu\nu}\partial_\nu h - \frac{1}{2}\partial_\lambda h \partial^\lambda h + \frac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

- 5 Polarization modes: 2 tensor modes + 2 vector + 1 scalar mode

- Plane wave,

$$h_{\mu\nu}(x) = \frac{1}{2\pi}\int d^4k \frac{2\delta(|\boldsymbol{k}|^2 - (k_0^2 - m^2))}{|\boldsymbol{k}|} e^{ikx} h_{\mu\nu}(k) = \int_{-\infty}^{\infty} df \int_{\rm sky} d^2\hat{\boldsymbol{\Omega}}\, e^{i2\pi f\left(t - \frac{|\boldsymbol{k}|}{k_0}\hat{\boldsymbol{\Omega}}\cdot\boldsymbol{x}\right)} h_{\mu\nu}\left(f, \frac{|\boldsymbol{k}|}{k_0}\hat{\boldsymbol{\Omega}}\right)$$

Massive Gravity

- Receiving function:

$$F^{(i)}(\hat{\mathbf{\Omega}}) \equiv -\frac{\hat{p}^\mu \hat{p}^\nu}{2\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}\right)}\epsilon^{(i)}_{\mu\nu} + \frac{\hat{p}^\mu}{2}\epsilon^{(i)}_{0\mu} ,$$

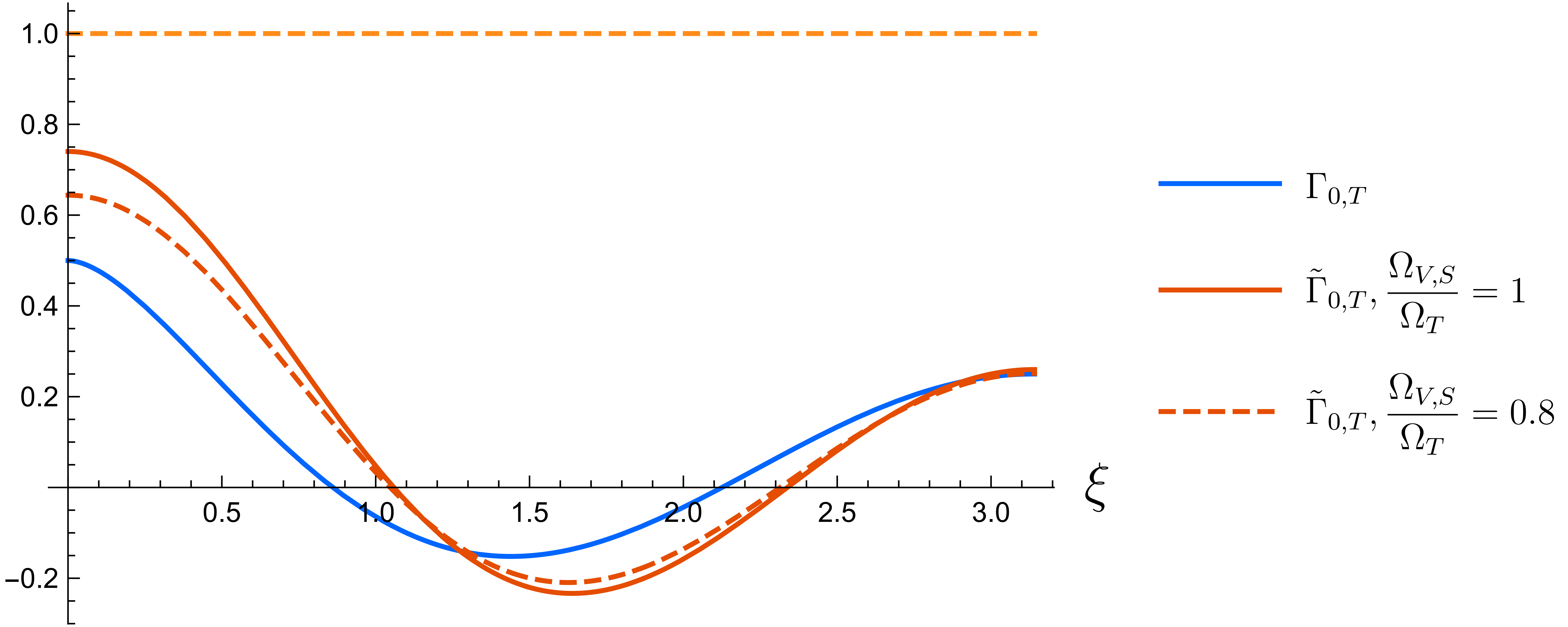
- Overlap reduction function for each mode:

$$\Gamma_I(|f|) = \beta_I \sum_i \int_{S^2} d^2\hat{\mathbf{\Omega}} \left(e^{i2\pi f L_1\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}_1\right)} - 1\right)\left(e^{-i2\pi f L_2\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}_2\right)} - 1\right) F_1^{(i)}(\hat{\mathbf{\Omega}}) F_2^{(i)}(\hat{\mathbf{\Omega}})$$

- Combined effect on the 2-point correlation function

- $$\langle \tilde{z}^2 \rangle \propto \left(\frac{\Omega_T}{\beta_T}\Gamma_T + \frac{\Omega_V}{\beta_V}\Gamma_V + \frac{\Omega_S}{\beta_S}\Gamma_S\right) = \frac{\Omega_T}{\beta_T}\Gamma_T\left(1 + \frac{\Gamma_V}{\Gamma_T}\frac{\Omega_V}{\Omega_T}\frac{\beta_T}{\beta_V} + \frac{\Gamma_S}{\Gamma_T}\frac{\Omega_S}{\Omega_T}\frac{\beta_T}{\beta_S}\right).$$

Combined effective overlap reduction function



Conclusion and Discussion

- We compute the overlap reduction function of massive gravity theory
- For some parameter space, it's possible to distinguish massive gravity in future PTA data release.
- Spin 2 Dark matter: The stochastic fluctuation of the spin 2 dark matter halo will effectively change the metric as massive gravity
- How to generate such signals from massive gravity theory or spin2 Dark Matter interaction?
- What may happen if we go to non-linear regime to analyze the source?

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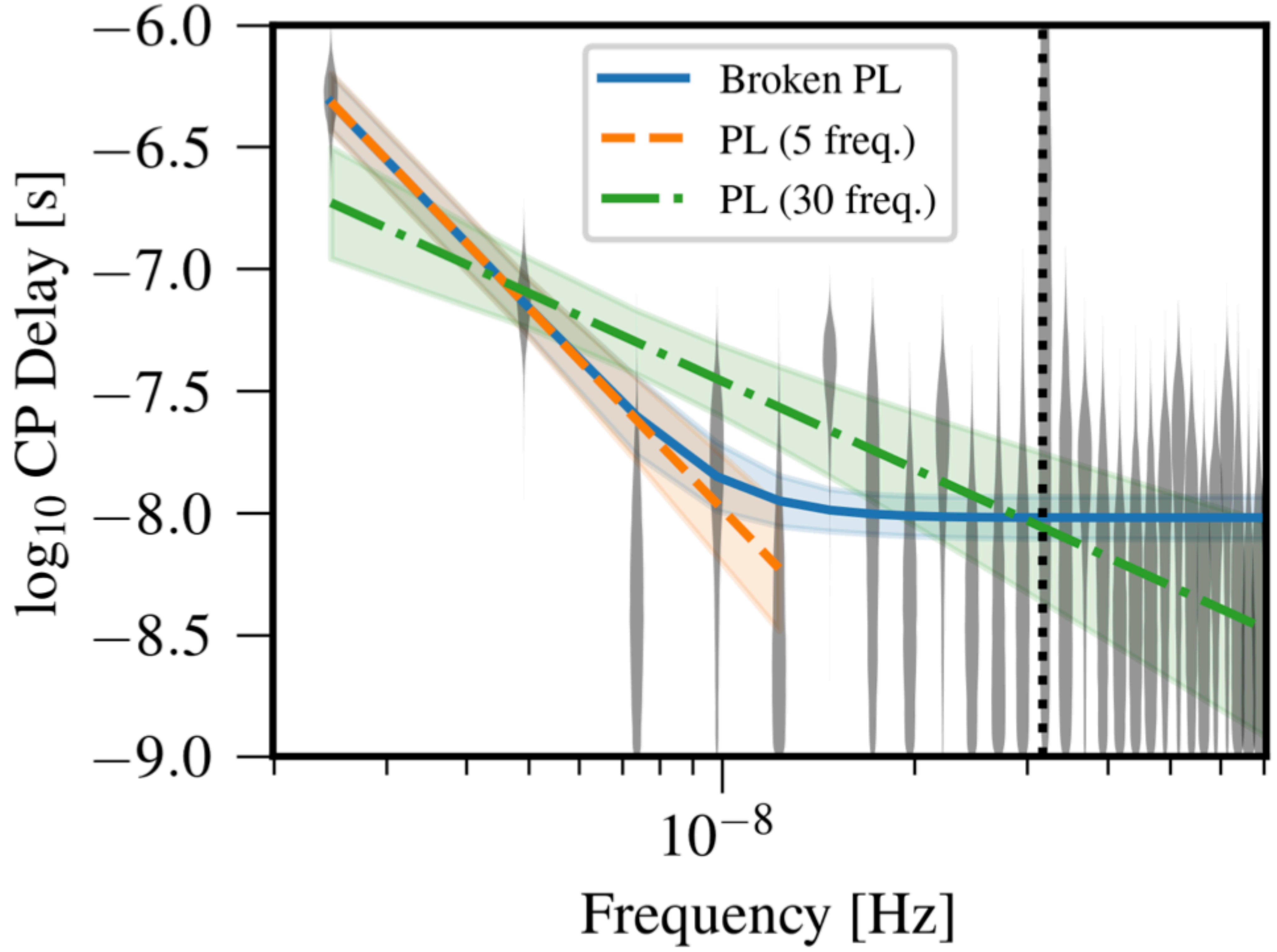
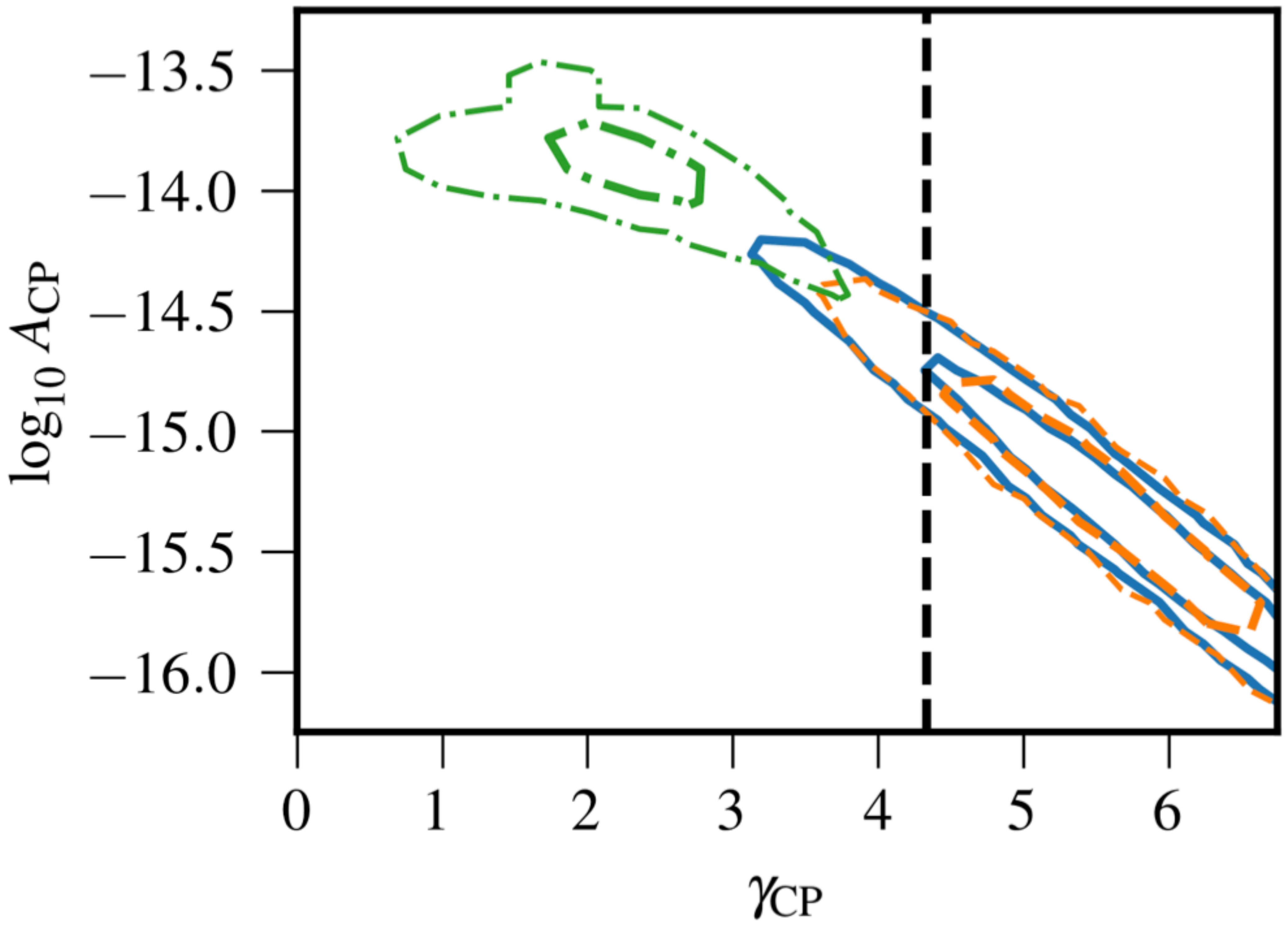
Thanks for your attention!

NANOGrav: power spectrum

- NANOGrav (2009.04496) & PPTA (2107.12112)&EPTA (2110.13184) & IPTA (2201.03980)





NANOGrav 2009.04496

Pulsar Timing Array (PTA)

- NANOGrav (2009.04496) & PPTA (2107.12112)&EPTA (2110.13184) & IPTA (2201.03980)
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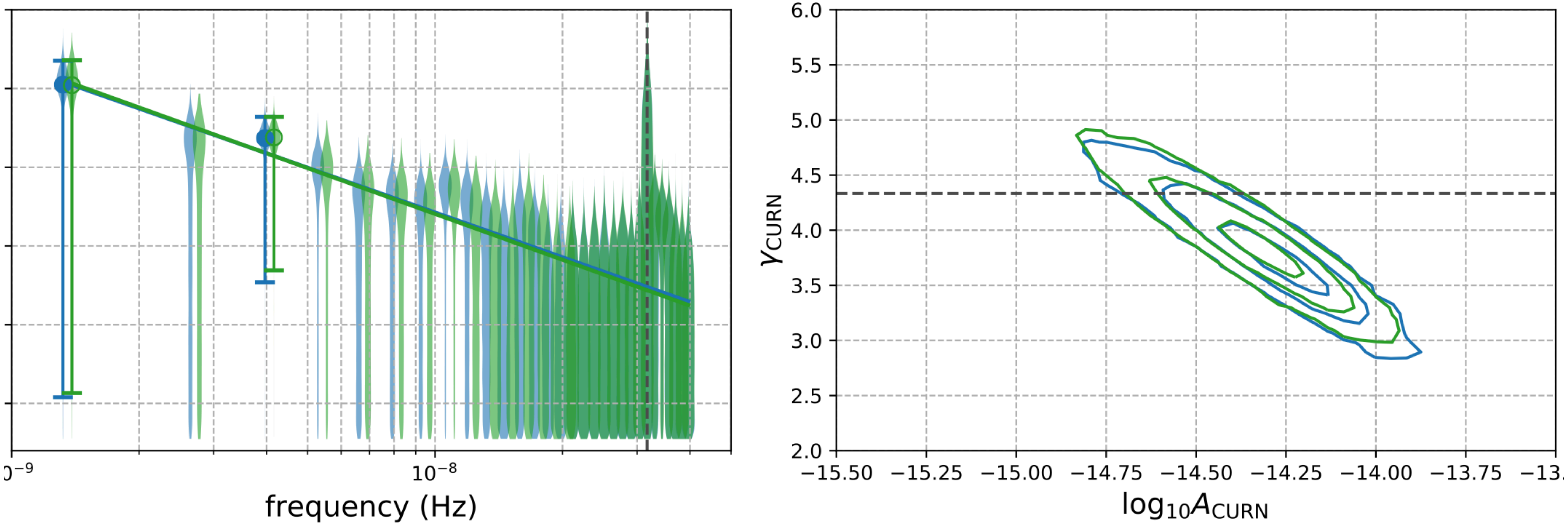
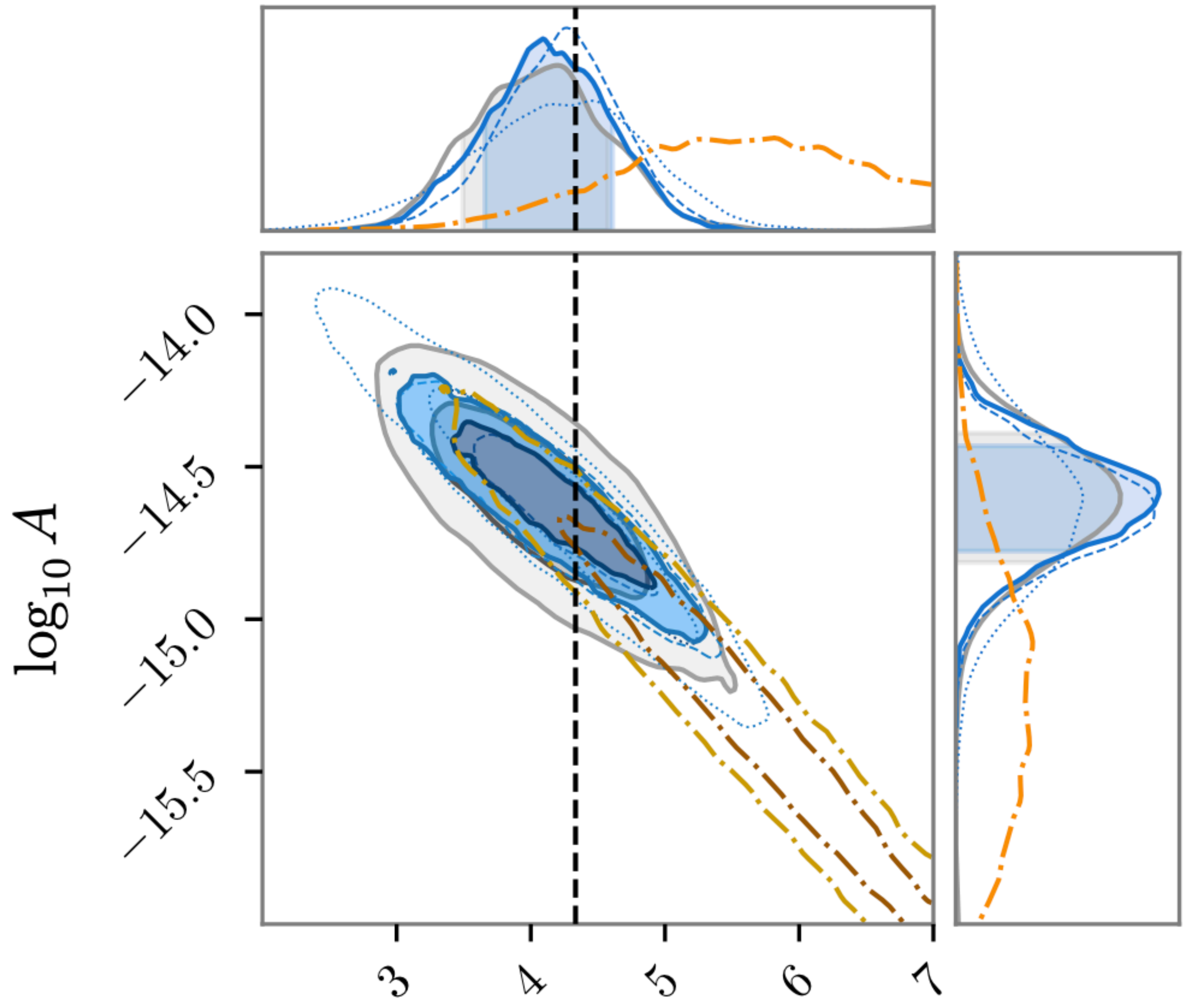
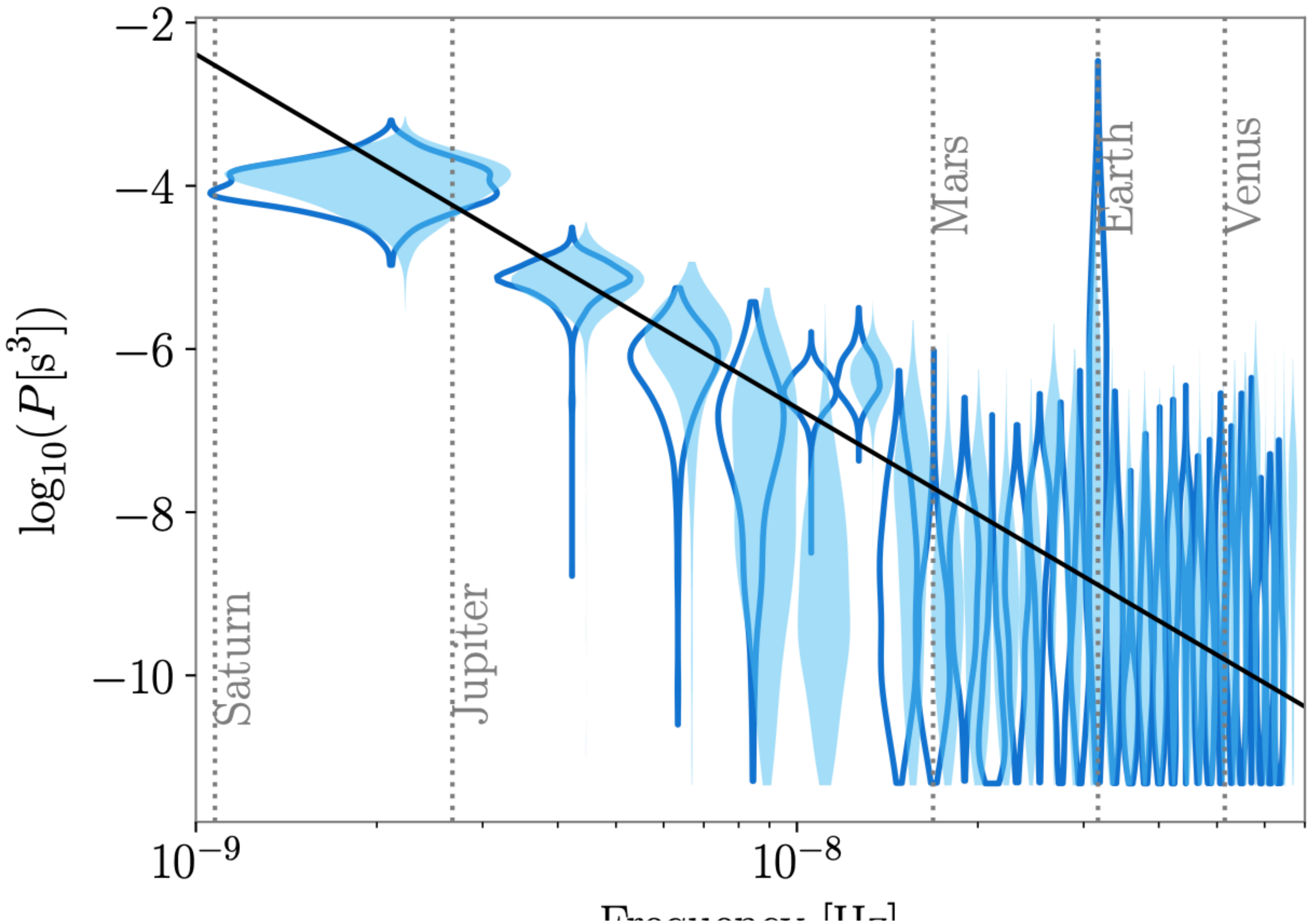


EPTA 2110.13184

Pulsar Timing Array (PTA)

- NANOGrav (2009.04496) & PPTA (2107.12112)&EPTA (2110.13184) & IPTA (2201.03980)
- 



PPTA 2107.12112

Pulsar Timing Array (PTA)

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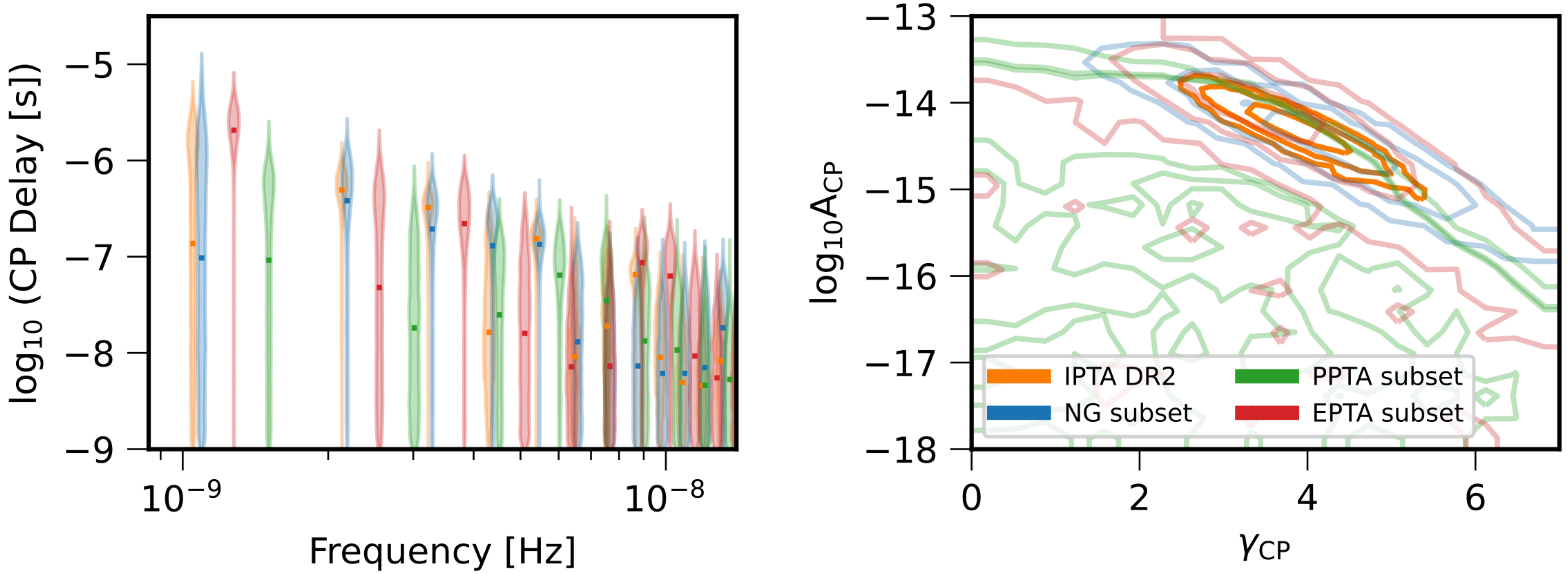
- 

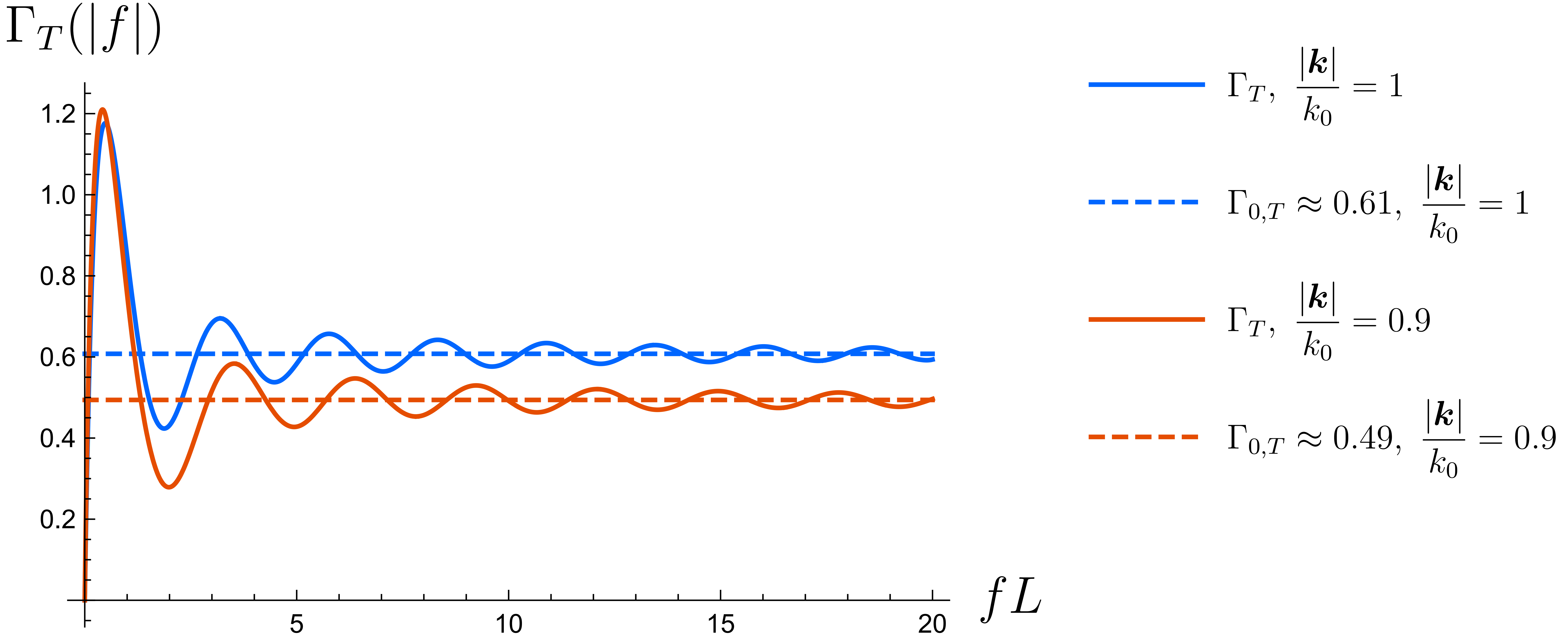
IPTA 2201.03980

Future Direction

- Anisotropy: search the anisotropy 1904.05348 ; search circular polarization mode 2111.05867 ; ···
- Dark matter: axion 1810.03227 ; axion-photon coupling 2201.03422; dark photon 2009.13909 ; ···
- Primordial Gravitational Wave: Joint analysis with Cosmic Microwave Background, Large Scale Structure, and Interferometer detection (LIGO/VIRGO/LISA)···

Tensor

$$\xi = \frac{\pi}{8}$$



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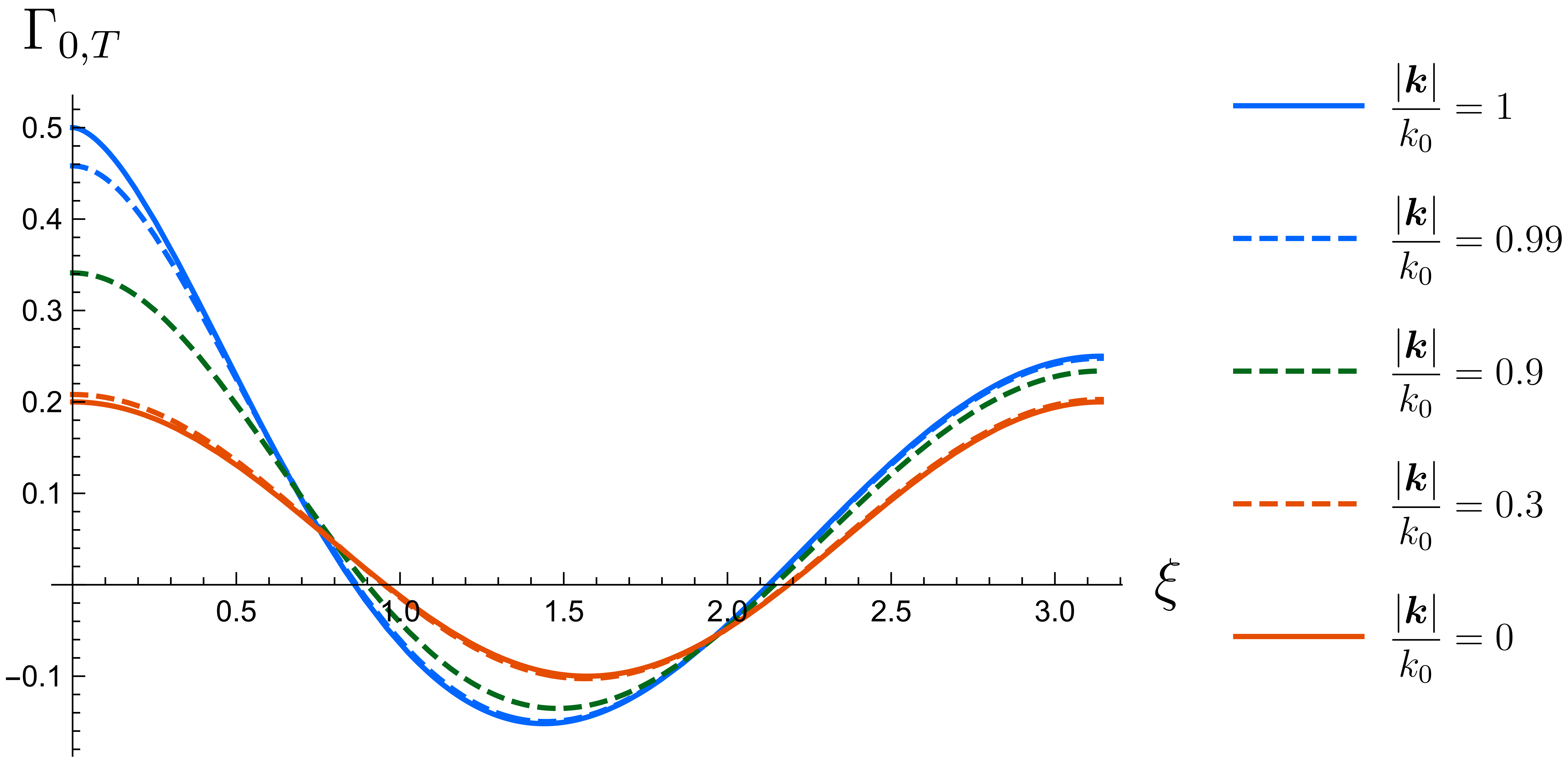
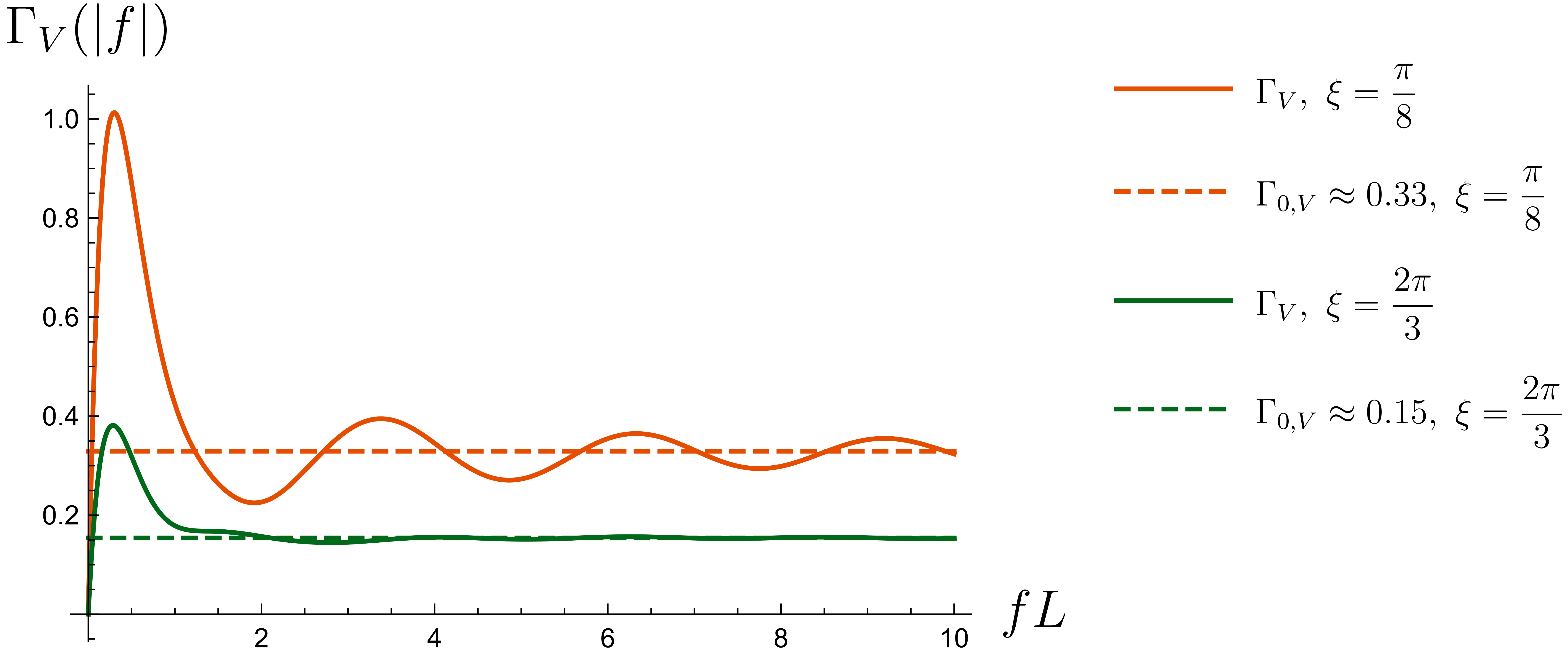
Tensor



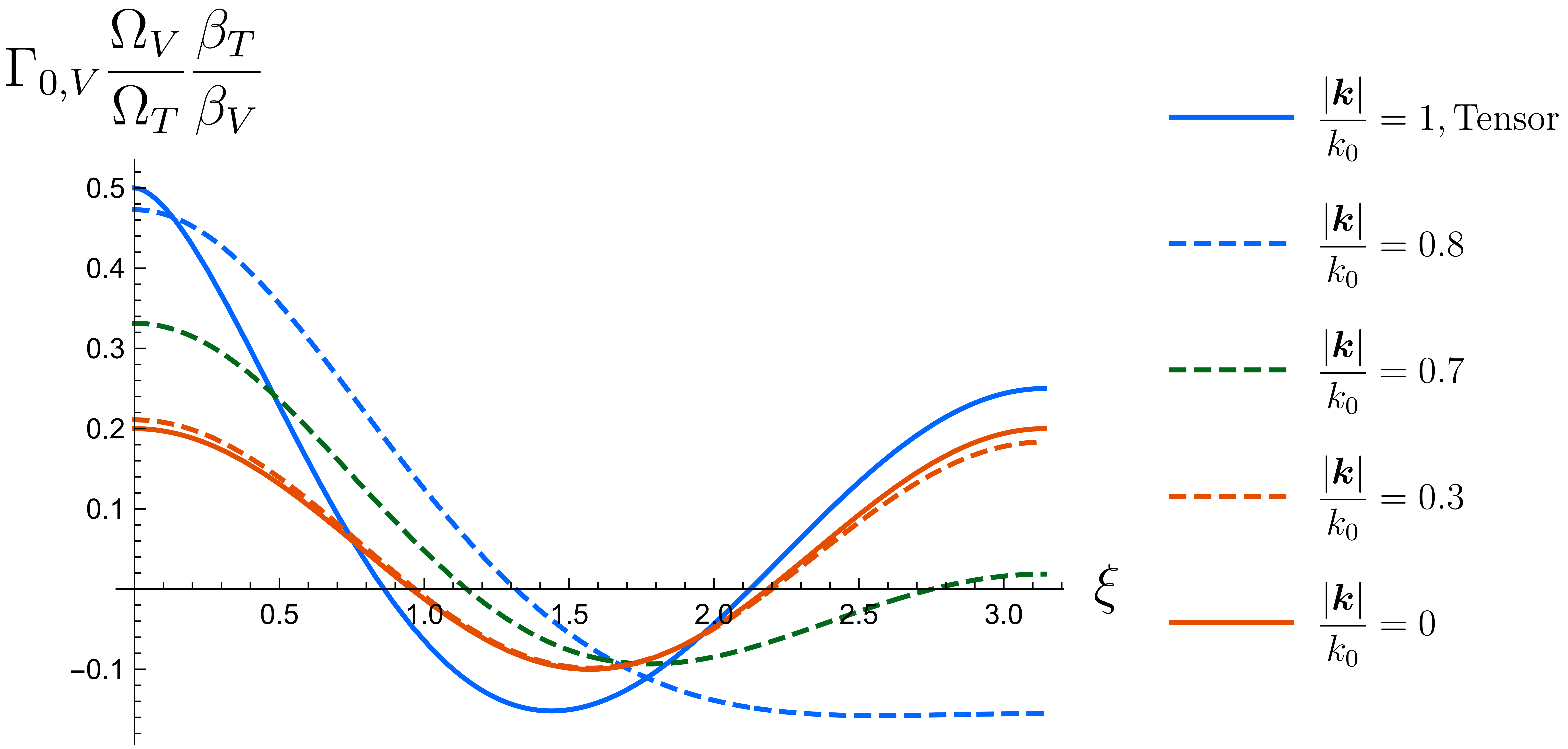
$\Gamma_{0,T}$
0.5
0.4
0.3
0.2
0.1
−0.1
0.5
1.0
1.5
2.0
2.5
3.0
ξ
$\frac{|\boldsymbol{k}|}{k_0} = 1$
$\frac{|\boldsymbol{k}|}{k_0} = 0.99$
$\frac{|\boldsymbol{k}|}{k_0} = 0.9$
$\frac{|\boldsymbol{k}|}{k_0} = 0.3$
$\frac{|\boldsymbol{k}|}{k_0} = 0$

Vector

$$\frac{|\boldsymbol{k}|}{k_0} = 0.9, \text{vector}$$

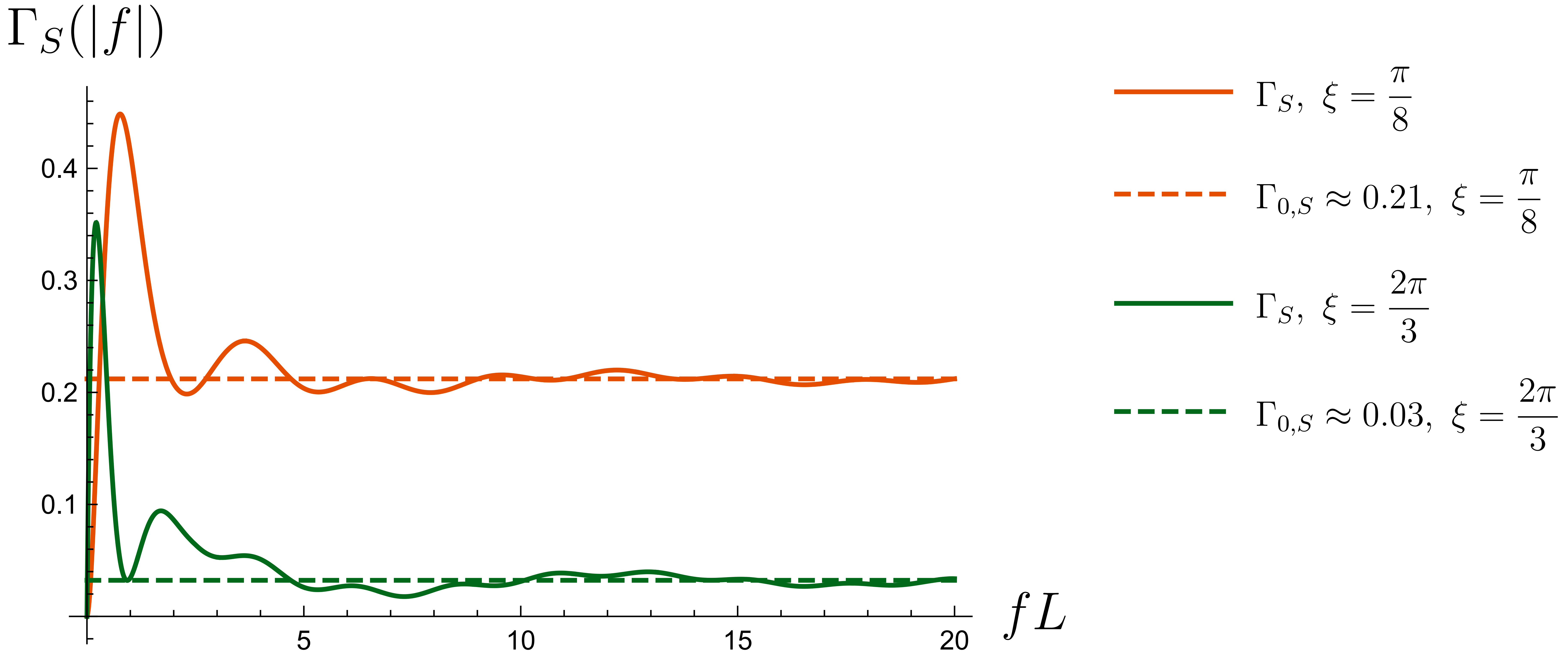


Vector

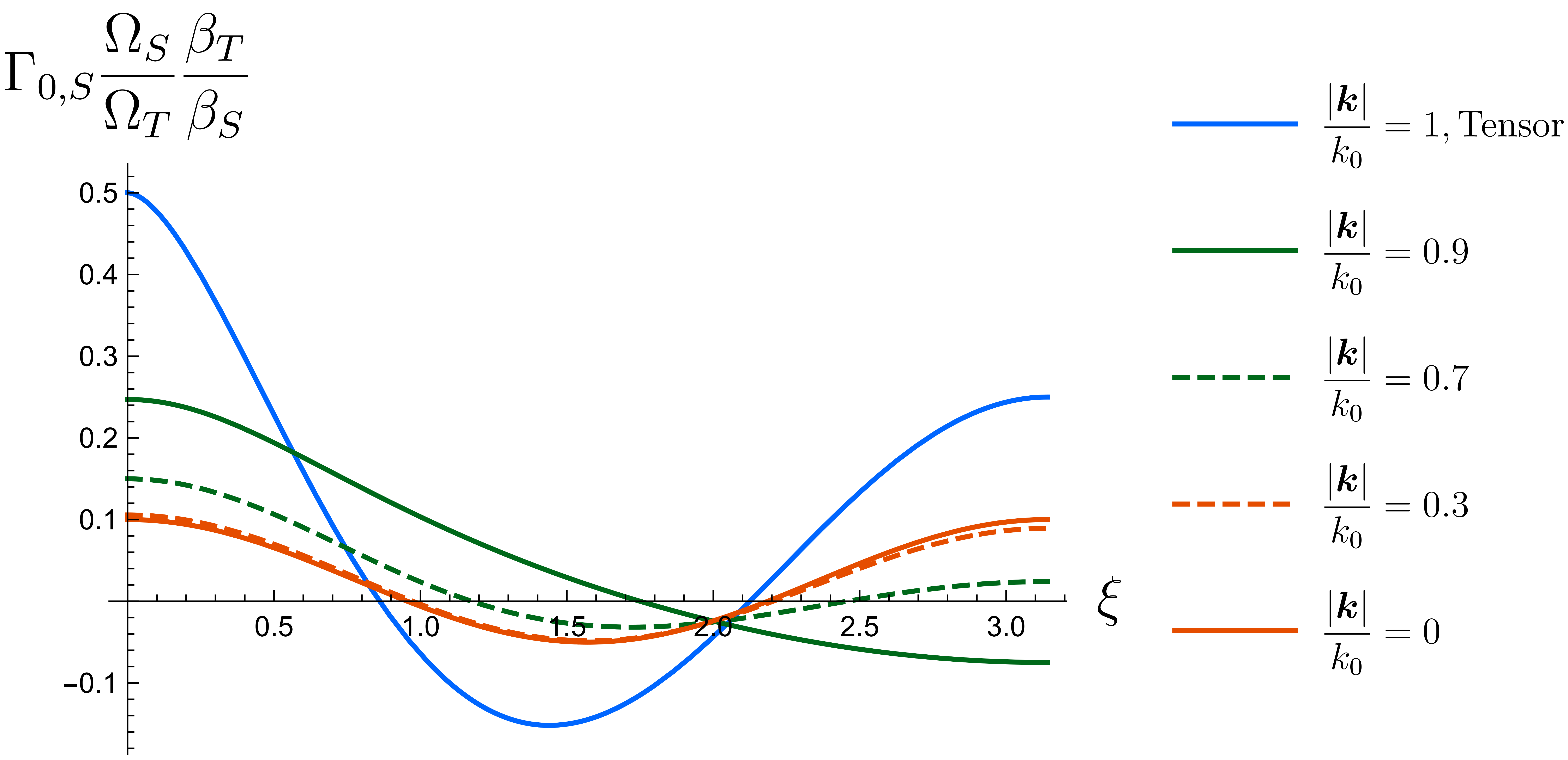

$\Gamma_{0,V} \frac{\Omega_V}{\Omega_T} \frac{\beta_T}{\beta_V}$
0.5
0.4
0.3
0.2
0.1
−0.1
0.5
1.0
1.5
2.0
2.5
3.0
ξ
$\frac{|\boldsymbol{k}|}{k_0} = 1$, Tensor
$\frac{|\boldsymbol{k}|}{k_0} = 0.8$
$\frac{|\boldsymbol{k}|}{k_0} = 0.7$
$\frac{|\boldsymbol{k}|}{k_0} = 0.3$
$\frac{|\boldsymbol{k}|}{k_0} = 0$

Scalar

$$\frac{|\boldsymbol{k}|}{k_0} = 0.9, \text{scalar}$$



Scalar



$\Gamma_{0,S}\frac{\Omega_S}{\Omega_T}\frac{\beta_T}{\beta_S}$
0.5
0.4
0.3
0.2
0.1
−0.1
0.5
1.0
1.5
2.0
2.5
3.0
ξ
$\frac{|\boldsymbol{k}|}{k_0} = 1$, Tensor
$\frac{|\boldsymbol{k}|}{k_0} = 0.9$
$\frac{|\boldsymbol{k}|}{k_0} = 0.7$
$\frac{|\boldsymbol{k}|}{k_0} = 0.3$
$\frac{|\boldsymbol{k}|}{k_0} = 0$

Vector mode

- Massless limit $\Gamma_{0,V} = \frac{\beta_V}{4}\frac{k_0^2}{2m^2}\frac{8\pi}{3}\cos\xi$
- The combination effect of energy density and Hellings-Downs curve analog is zero
-

Scalar mode

- DO NOT have Massless limit -> vDVZ discontinuity!
- Stay in linearized regime and should not go to massless limit