

Detecting High-Frequency Gravitational Waves with Microwave Cavities

Jan Schütte-Engel

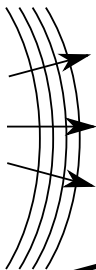
based on: *arxiv:2112.11465*

in collaboration with A. Berlin, D. Blas, R. Tito D'Agnolo, S. A.R. Ellis, R. Harnik, Y. Kahn

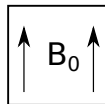
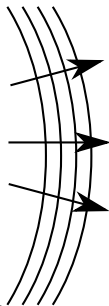
09.05.2022



High Frequency GW Sources

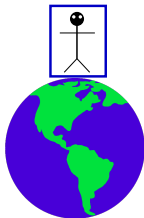
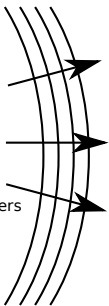


GW detection with cavities

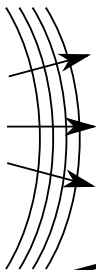
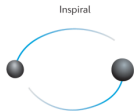


Proper detector frame

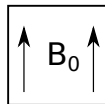
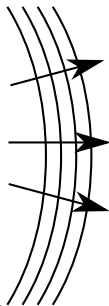
- Sensitivity to direction and polarization
- Why using the proper detector frame matters



High Frequency GW Sources

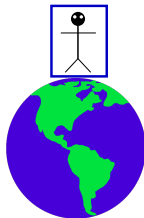


GW detection with cavities

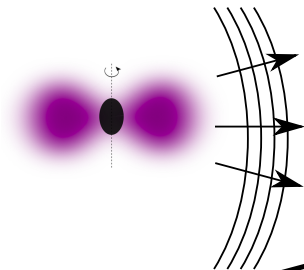


Proper detector frame

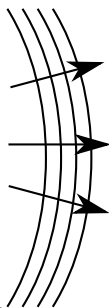
- Sensitivity to direction and polarization
- Why using the proper detector frame matters



High Frequency GW Sources

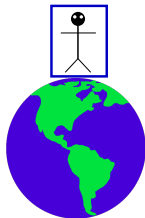
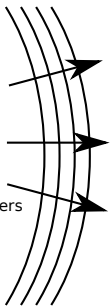


GW detection with cavities



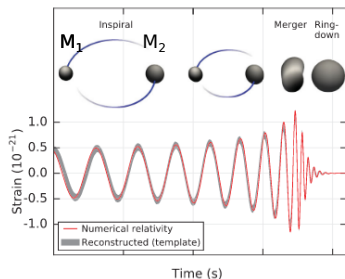
Proper detector frame

- Sensitivity to direction and polarization
- Why using the proper detector frame matters



High Frequency GW Sources

Mergers of sub-solar mass objects



[1602.03837]

innermost stable circular orbit (ISCO)

$$r_{\text{ISCO}} = 0.02 m \frac{M_b}{10^{-6} M_{\odot}}$$

$$f_{\text{ISCO}} = 1.1 \text{ GHz} \left(\frac{10^{-6} M_{\odot}}{M_b} \right)$$

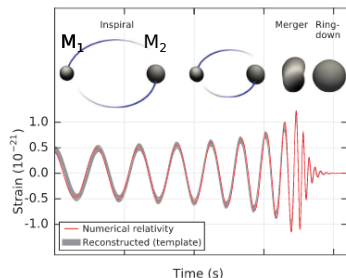
$$\omega_g \simeq 14 \text{ GHz} \times \frac{10^{-6} M_{\odot}}{M_b} \left(\frac{r_{\text{ISCO}}}{r_b} \right)^{2/3}$$

$$M_1 = M_b = M_2$$

$$\mathcal{N}_{\text{cyc}} \geq Q \Rightarrow M_b \leq 10^{-11} M_{\odot} \left(\frac{10^5}{Q} \right)^{3/6} \left(\frac{1 \text{ GHz}}{\omega_g} \right)$$

with $Q = \frac{f}{\Delta f}$.

Mergers of sub-solar mass objects



[1602.03837]

Expected strain

$$h_0 \approx 10^{-29} \times \left(\frac{1 \text{ pc}}{D} \right) \left(\frac{M_b}{10^{-11} M_\odot} \right)^{5/3} \left(\frac{\omega_g}{1 \text{ GHz}} \right)^{2/3}$$

D is distance to Binary merger.

innermost stable circular orbit (ISCO)

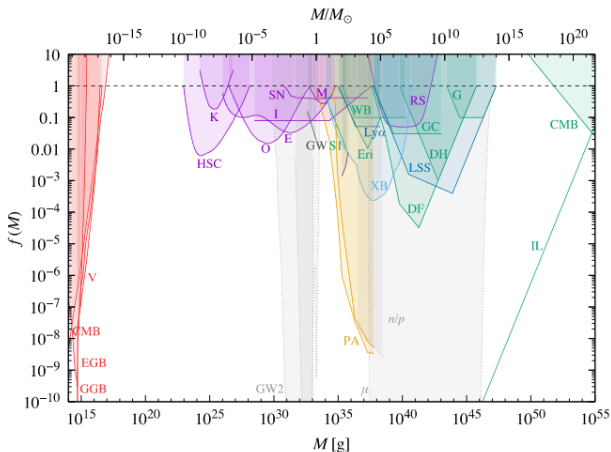
$$r_{\text{ISCO}} = 0.02 \text{ m} \frac{M_b}{10^{-6} M_\odot}$$

$$f_{\text{ISCO}} = 1.1 \text{ GHz} \left(\frac{10^{-6} M_\odot}{M_b} \right)$$

$$\omega_g \simeq 14 \text{ GHz} \times \frac{10^{-6} M_\odot}{M_b} \left(\frac{r_{\text{ISCO}}}{r_b} \right)^{3/2}$$

$$M_1 = M_b = M_2$$

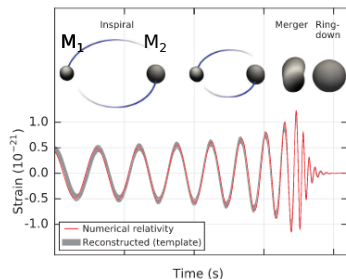
PBHs as dark matter: constraints



[Carr et al. 21]

Evaporation (red), lensing (magenta), dynamical effects (green), gravitational waves (black), accretion (light blue), CMB distortions (orange), large-scale structure (dark blue) and background effects (grey).

Mergers of sub-solar mass objects



[1602.03837]

If PBHs are 100% DM

$$D \approx 10^{-3} \text{ pc} \left(\frac{M_b}{10^{-11} M_\odot} \right)^{\frac{1}{3}} \Rightarrow \text{best case scenario} \Rightarrow h_0 \approx 10^{-26}$$

Study taking into account merger rate: [2205.02153]

innermost stable circular orbit (ISCO)

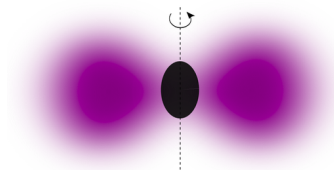
$$r_{\text{ISCO}} = 0.02 \text{ m} \frac{M_b}{10^{-6} M_\odot}$$

$$f_{\text{ISCO}} = 1.1 \text{ GHz} \left(\frac{10^{-6} M_\odot}{M_b} \right)$$

$$\omega_g \simeq 14 \text{ GHz} \times \frac{10^{-6} M_\odot}{M_b} \left(\frac{r_{\text{ISCO}}}{r_b} \right)^{\frac{3}{2}}$$

$$M_1 = M_b = M_2$$

Boson clouds from PBH superradiance



[kipac.stanford.edu]

- Annihilation of bosons

$$m_a \approx \mu\text{eV} \times (10^{-4} M_\odot / M_{\text{PBH}})$$

$$\omega_g = 2m_a \approx \text{GHz} \times (m_a / \mu\text{eV})$$

- GW waveform is monochromatic and coherent over very long timescales

Expected strain

$$h_0 \approx 10^{-27} \left(\frac{\alpha/\ell}{0.5} \right) \left(\frac{\epsilon}{10^{-3}} \right) \times \left(\frac{10 \text{ kpc}}{D} \right) \left(\frac{M_{\text{PBH}}}{10^{-4} M_\odot} \right)$$

$\alpha = GM_{\text{PBH}} m_a$, ℓ is orbital quantum number, ϵ is fraction of PBH mass the axion cloud carries. [Arvanitaki et al.12]

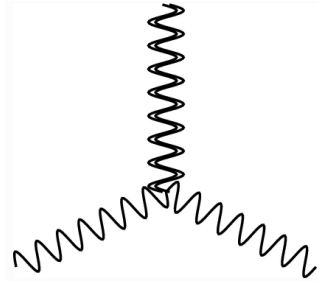
If $10^{-4} M_\odot$ PBHs are 1% of DM:

$$D \approx 1 \text{ pc} \Rightarrow \text{best case scenario} \Rightarrow h_0 \approx 10^{-23}$$

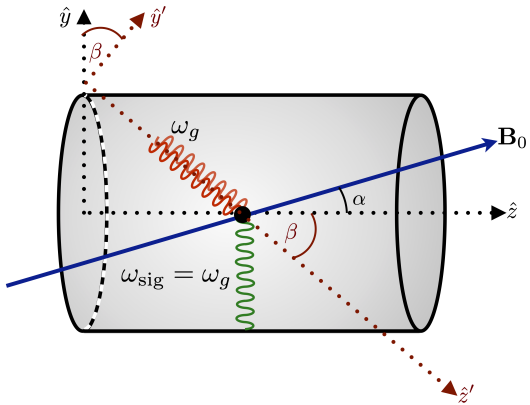
GW detection with cavities

$$\mathcal{L} \supset -\frac{1}{4} \eta^{\mu\alpha} h^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}$$

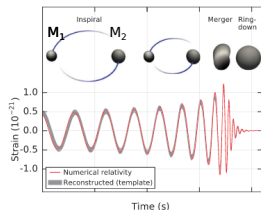
$$(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})$$



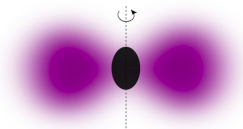
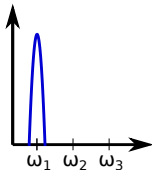
[Gertsenshtein 62]



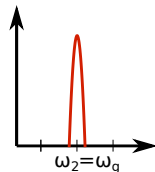
Signal in the two benchmark scenarios



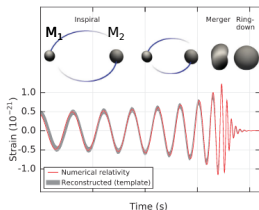
- Keep cavity fixed
- Sweep through all cavity modes



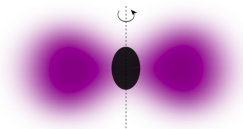
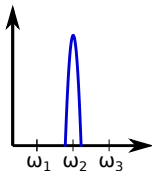
- Change cavity to scan different resonance frequencies
- GW frequency from superradiant cloud fixed



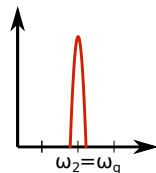
Signal in the two benchmark scenarios



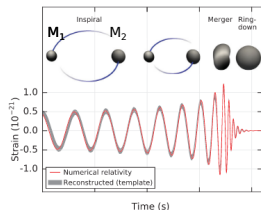
- Keep cavity fixed
- Sweep through all cavity modes



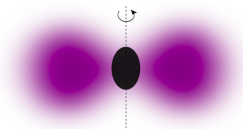
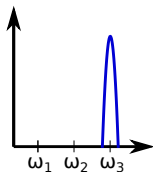
- Change cavity to scan different resonance frequencies
- GW frequency from superradiant cloud fixed



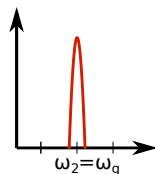
Signal in the two benchmark scenarios



- Keep cavity fixed
- Sweep through all cavity modes



- Change cavity to scan different resonance frequencies
- GW frequency from superradiant cloud fixed



Maxwell equations on curved spacetime

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho_{\text{eff}} + \rho, \\ \nabla \times \mathbf{B} - \partial_t \mathbf{E} &= \mathbf{j}_{\text{eff}} + \mathbf{j},\end{aligned}$$

$$j_{\text{eff}}^\mu \equiv \partial_\nu \left(\frac{1}{2} h F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right)$$

Maxwell equations on curved spacetime

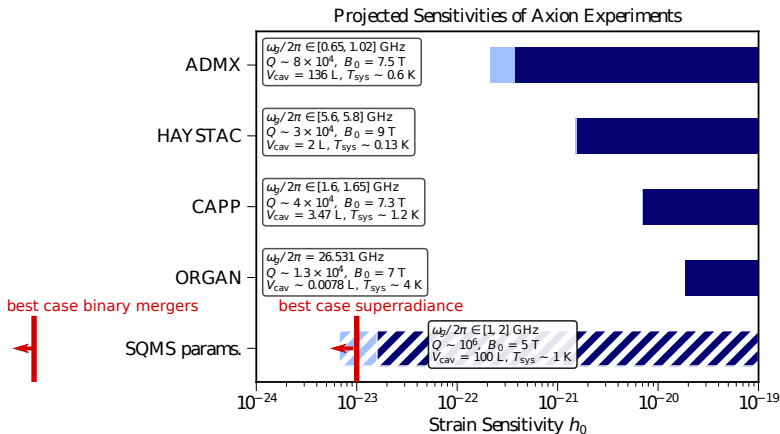
$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho_{\text{eff}} + \rho, \\ \nabla \times \mathbf{B} - \partial_t \mathbf{E} &= \mathbf{j}_{\text{eff}} + \mathbf{j},\end{aligned}$$

$$j_{\text{eff}}^\mu \equiv \partial_\nu \left(\frac{1}{2} h F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right)$$

$$\begin{aligned}h_0 \gtrsim & 3 \times 10^{-22} \times \left(\frac{1 \text{ GHz}}{\omega_g/2\pi} \right)^{3/2} \left(\frac{0.1}{\eta_n} \right) \left(\frac{8 \text{ T}}{B_0} \right) \left(\frac{0.1 \text{ m}^3}{V_{\text{cav}}} \right)^{5/6} \times \\ & \times \left(\frac{10^5}{Q} \right)^{1/2} \left(\frac{T_{\text{sys}}}{1 \text{ K}} \right)^{1/2} \left(\frac{\Delta\nu}{10 \text{ kHz}} \right)^{1/4} \left(\frac{1 \text{ min}}{t_{\text{int}}} \right)^{1/4}\end{aligned}$$

$$\eta_n \equiv \frac{\left| \int_{V_{\text{cav}}} d^3\mathbf{x} \mathbf{E}_n^* \cdot \mathbf{j}_{\text{eff}} \right|}{h_0 B_0 \omega_g^2 V_{\text{cav}}^{5/6} \left(\int_{V_{\text{cav}}} d^3\mathbf{x} |\mathbf{E}_n|^2 \right)^{1/2}}$$

Sensitivity of existing axion experiments

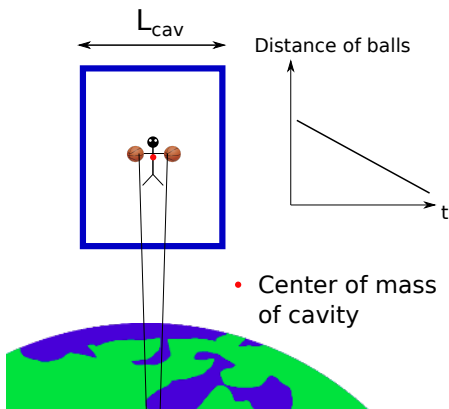


Existing axion experiments only need to reanalyze their data!

Proper detector frame

(Fermi Normal coordinates)

Proper detector frame

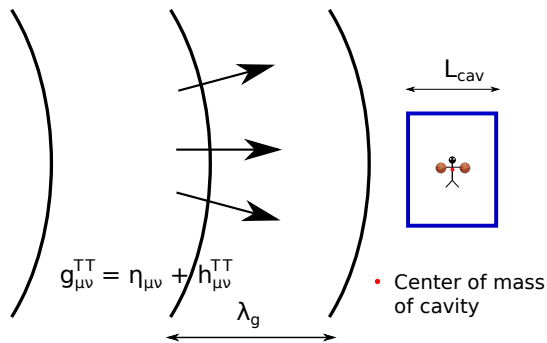


- Cavity freely falling towards Earth
- Coordinate system attached to the center of mass is proper detector frame
- Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\text{cav}}}{K}\right)$$

K is scale on which the gravitational field varies.

Proper detector frame



$$h_{\mu\nu}^{\text{TT}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega_g(t-z)}$$

TT: transverse traceless

Metric in proper detector frame:

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\text{cav}}}{\lambda_g}\right)$$

λ_g wavelength of GW

Proper detector frame

Resonant excitation of cavity:

$$\lambda_g \simeq L_{\text{cav}}$$

$$g_{00} = \eta_{00} + h_{00} = -1 - 2 \sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n,k_1,\dots,k_r} x^m x^n x^{k_1} \dots x^{k_r}$$

(Similar eq. for g_{0i} and g_{ij})

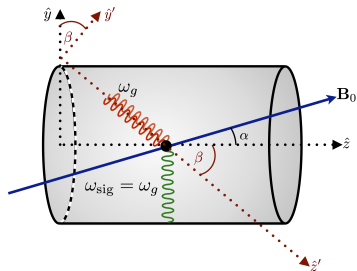
[Marzlin 94, Rakhmanov 14]

- Riemann tensor invariant under infinitesimal coordinate transformations. Evaluate therefore in TT-frame and plug in series expansion above.
- GW propagating in z-direction:

$$R_{ijkl} \sim e^{i\omega_g(t-z)}$$

$$h_{00} = -\omega_g^2 h_{ab}^{\text{TT}} x^a x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

Why the frame matters



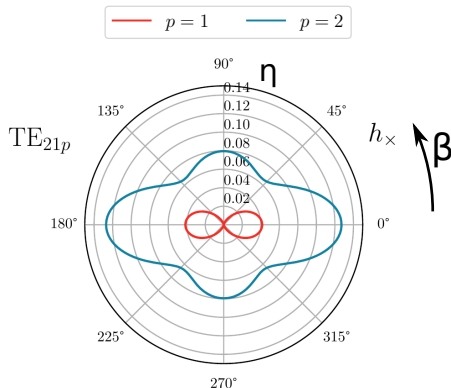
TT frame is not the right frame! Use proper detector frame.

$$\alpha = 0, \beta = 0$$

If we use TT metric $\mathbf{j}_{\text{eff}} = 0$. NO SIGNAL!

However this is **WRONG**. Use proper detector metric.

Proper detector frame result ($\alpha = 0$)



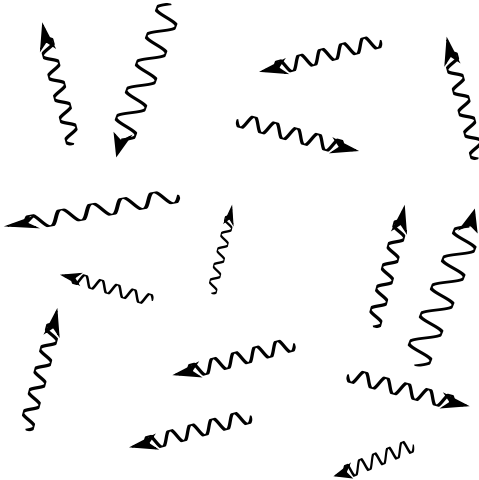
Conclusions

- PBH inspirals and PBH superradiance can generate GWs in GHz regime
- Difficult to probe PBH inspirals but PBH superradiance best case scenario can be probed
- Signal calculation in cavity: Use proper detector frame metric resummed to all orders
- Existing axion experiments only need to reanalyze data

Thank you for your attention

Backup

High Frequency GW Sources



Stochastic GWs: constraints

$$\Omega_{\text{GW}}^{(0)} = \frac{1}{\rho_c^{(0)}} \frac{d\rho_{\text{GW}}^{(0)}}{d \ln f}, \quad \text{with } \rho_c^{(0)} = \frac{3H_0^2}{8\pi G}$$

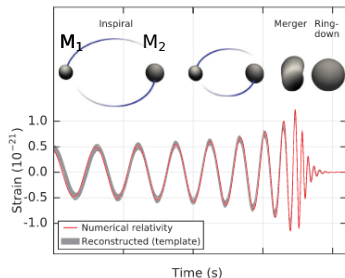
$$\frac{\rho_{\text{GW}}^{(0)}}{\rho_c^{(0)}} = \int_0^\infty \frac{df}{f} \Omega_{\text{GW}}^{(0)} \approx \Omega_{\text{GW}}^{(0)}(f_{\text{max}}^{(0)})$$

$$h^2 \frac{\rho_{\text{GW}}^{(0)}}{\rho_c^{(0)}} \leq h^2 \frac{\Delta \rho_{\text{rad}}^{(0)}}{\rho_c^{(0)}} = h^2 \Omega_\gamma \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} \Delta N_{\text{eff}} = 5.6 \times 10^{-6} \Delta N_{\text{eff}}$$

$$h^2 \Omega_{\text{GW}}^{(0)}(f_{\text{max}}^{(0)}) \leq 1.68 \times 10^{-6} \left(\frac{\Delta N_{\text{eff}}}{0.3} \right)$$

$$g_{*s}(T_0) = 3.90, \quad g_{*s}(T) = 10.75, \quad T_0 = 2.72 \text{ K}, \quad h^2 \Omega_\gamma = 2.47 \times 10^{-5}$$

Mergers of sub-solar mass objects



[1602.03837]

innermost stable circular orbit (ISCO)

$$r_{\text{ISCO}} = 0.02 \text{ m} \frac{M_b}{10^{-6} M_{\odot}}$$

$$f_{\text{ISCO}} = 1.1 \text{ GHz} \left(\frac{10^{-6} M_{\odot}}{M_b} \right)$$

$$\omega_g \simeq 14 \text{ GHz} \times \frac{10^{-6} M_{\odot}}{M_b} \left(\frac{r_{\text{ISCO}}}{r_b} \right)^{2/3}$$

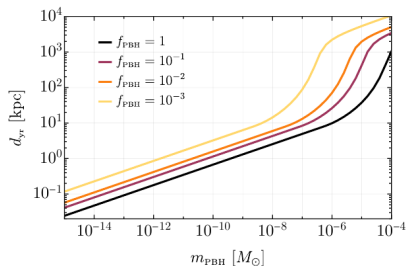
$$M_1 = M_b = M_2$$

- primordial black holes (PBHs) [Hawking 71]
- boson and fermion stars
[Palenzuela et al. 07, Giudice et al. 16, Palenzuela et al. 17, Helfer et al. 18]
- gravitino stars [Narain et al. 06] and gravistars [Mazur et al. 04]
- dark matter blobs [Diamond et al. 21]

Sensitivity to stochastic GW background

- $S_{\text{sig}}(\omega) = \frac{\omega_n}{Q} \frac{(\omega\omega_n)^2}{(\omega^2 - \omega_n^2)^2 + (\omega\omega_n/Q)^2} |\eta|^2 B_0^2 V_{\text{cav}} S_h(\omega)$
- Thermal noise $S_{\text{noise}}(\omega) = \frac{4\pi T(\omega\omega_n/Q)^2}{(\omega^2 - \omega_n^2)^2 + (\omega\omega_n/Q)^2}$
- Non-coherent signal appears as an additional noise source in the detector
- $\text{SNR} = \frac{S_{\text{sig}}}{S_{\text{noise}}} = \frac{\omega_n Q}{4\pi T} |\eta|^2 B_0^2 V_{\text{cav}} S_h(\omega_g)$
- $\Omega_{\text{GW}}(\omega) = \frac{2}{3} \frac{\omega^3}{H_0^2} S_h(\omega)$
- $\Omega_{\text{GW}} = 8 \times 10^{10} \times \left(\frac{(0.2)^2}{|\eta|^2} \right) \left(\frac{10 \text{ T}}{B_0} \right)^2 \left(\frac{\omega_n}{1 \text{ GHz}} \right)^2 \left(\frac{1 \text{ m}^3}{V_{\text{cav}}} \right) \left(\frac{10^{12}}{Q} \right) \left(\frac{T_{\text{sys}}}{10 \text{ mK}} \right)$
- Cosmologically produced GW backgrounds $\Omega_{\text{GW}} < 10^{-6}$
- Without tricks the detection prospects are not great for stochastic GW backgrounds.

Alternative estimate for PBH case



$$d_{\text{yr}} = 0.21 \text{ kpc} \left(\frac{m_{\text{PBH}}}{10^{-11} M_{\odot}} \right)^{\frac{1}{3}}$$

[2205.02153]

$$h_0 = 10^{-31} \left(\frac{m_{\text{PBH}}}{10^{-11} M_{\odot}} \right)^{\frac{4}{3}} \left(\frac{\omega_g}{1 \text{ GHz}} \right)^{\frac{2}{3}}$$

Detector sensitivity:

$$h_0 > 8.3 \times 10^{-20} \left(\frac{1 \text{ GHz}}{\omega_g/2\pi} \right)^{\frac{1}{2}} \left(\frac{0.1}{\eta} \right) \left(\frac{8 \text{ T}}{B_0} \right) \left(\frac{0.1 \text{ m}^3}{V_{\text{cav}}} \right)^{\frac{5}{6}} \left(\frac{T_{\text{sys}}}{1 \text{ K}} \right)^{\frac{1}{2}} \left(\frac{10^{-11} M_{\odot}}{m_{\text{PBH}}} \right)^{\frac{6}{5}}$$

Mode decomposition

$$\nabla \times \nabla \times \mathbf{E} + \partial_t^2 \mathbf{E} = -\partial_t \mathbf{j}_{\text{eff}} - \partial_t \mathbf{j}$$

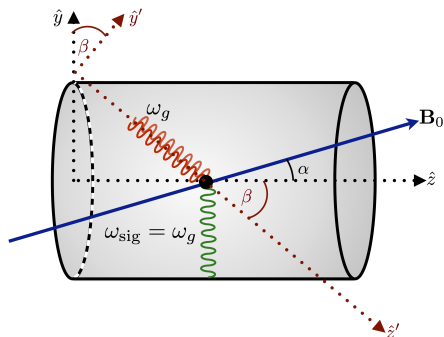
Cavity mode decomposition:

$$\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t) \mathbf{E}_n(\mathbf{x}),$$

$$P_{\text{sig}} = \frac{1}{2} Q \omega_g^3 V_{\text{cav}}^{5/3} (\eta_n h_0 B_0)^2$$

$$\eta_n \equiv \frac{\left| \int_{V_{\text{cav}}} d^3 \mathbf{x} \mathbf{E}_n^* \cdot \mathbf{j}_{\text{eff}} \right|}{h_0 B_0 \omega_g^2 V_{\text{cav}}^{5/6} \left(\int_{V_{\text{cav}}} d^3 \mathbf{x} |\mathbf{E}_n|^2 \right)^{1/2}}$$

Sensitivity estimate

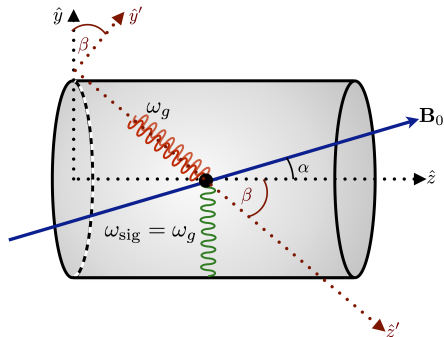


$$\text{SNR} \simeq \frac{P_{\text{sig}}}{T_{\text{sys}}} \sqrt{\frac{t_{\text{int}}}{\Delta\nu}}$$

$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1 \text{ GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8 \text{ T}}{B_0}\right) \left(\frac{0.1 \text{ m}^3}{V_{\text{cav}}}\right)^{5/6} \times \\ \times \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\text{sys}}}{1 \text{ K}}\right)^{1/2} \left(\frac{\Delta\nu}{10 \text{ kHz}}\right)^{1/4} \left(\frac{1 \text{ min}}{t_{\text{int}}}\right)^{1/4}$$

$$\text{Bandwidth } \Delta\nu = \frac{\omega_g}{2\pi Q}.$$

Sensitivity estimate

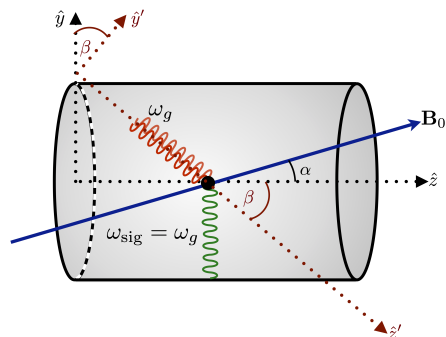


$$\text{SNR} \simeq \frac{P_{\text{sig}}}{T_{\text{sys}}} \sqrt{\frac{t_{\text{int}}}{\Delta\nu}}$$

$$h_0 \gtrsim 1 \times 10^{-23} \times \left(\frac{1 \text{ GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8 \text{ T}}{B_0}\right) \left(\frac{0.1 \text{ m}^3}{V_{\text{cav}}}\right)^{5/6} \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\text{sys}}}{1 \text{ K}}\right)^{1/2} \left(\frac{1 \text{ min}}{t_{\text{int}}}\right)^{1/2}$$

$$\text{Bandwidth } \Delta\nu = \frac{1}{t_{\text{int}}}$$

Sensitivity estimate



$$\text{SNR} \approx \frac{P_{\text{sig}}}{T_{\text{sys}}} \sqrt{\frac{t_{\text{int}}}{\Delta\nu}}$$

$$h_0 \gtrsim 1 \times 10^{-23} \times \left(\frac{1 \text{ GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8 \text{ T}}{B_0}\right) \left(\frac{0.1 \text{ m}^3}{V_{\text{cav}}}\right)^{5/6} \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\text{sys}}}{1 \text{ K}}\right)^{1/2} \left(\frac{1 \text{ min}}{t_{\text{int}}}\right)^{1/2}$$

$$\text{Bandwidth } \Delta\nu = \frac{1}{t_{\text{int}}}$$

-	Superradiance	Binary mergers
t_{int}	1 min (single scan time)	10^{-4} s (no scan)
best case h_0	10^{-23}	10^{-26}

Coordinate transformations in linearized theory

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Coordinate transformation

$$x'^{\mu\nu} = x^\mu + \xi^\mu$$

$$g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma} \stackrel{!}{=} \eta_{\mu\nu} + h'_{\mu\nu}$$

with

$$h'_{\mu\nu}(x') = h_{\mu\nu}(x(x')) - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$$

In linearized theory we consider only the coordinate transformations that leave $h_{\mu\nu} \ll 1$.
Therefore

$$\partial_\mu \xi_\nu \ll 1$$

Coordinate transformations in linearized theory

Example: First rank tensor that is separable in zeroth, first \dots order pieces

$$f_{\mu} = \left(f^{(0)} + f^{(1)} + \dots \right)_{\mu}$$

Transformation:

$$f'_{\mu}(x') = \frac{\partial x^{\rho}}{\partial x'^{\mu}} f_{\rho} = \left(\delta_{\mu}^{\rho} + \frac{\partial \xi^{\rho}}{\partial x'^{\mu}} \right) \left(f^{(0)} + f^{(1)} + \dots \right)_{\rho} = f_{\mu}^{(0)} + \frac{\partial \xi^{\rho}}{\partial x'^{\mu}} f_{\rho}^{(0)} + f_{\mu}^{(1)} + \dots$$

$$f'^{(1)} = \frac{\partial \xi^{\rho}}{\partial x'^{\mu}} f_{\rho}^{(0)} + f_{\mu}^{(1)}$$

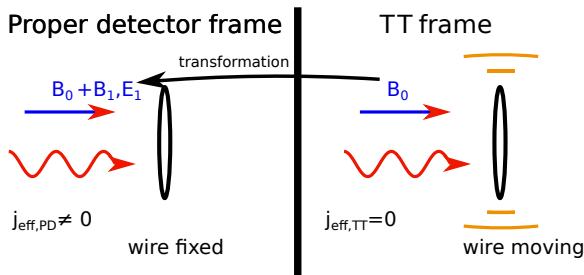
This is what we mean when we say that a tensor is not invariant in linearized theory.

Example: Riemann tensor in linearized theory:

$$R_{\mu\nu\rho\sigma} = \frac{1}{2} (\partial_{\nu}\partial_{\rho}h_{\mu\sigma} + \partial_{\mu}\partial_{\sigma}h_{\nu\rho} - \partial_{\mu}\partial_{\rho}h_{\nu\sigma} - \partial_{\nu}\partial_{\sigma}h_{\mu\rho})$$

is invariant under infinitesimal coordinate transformations

Toy example



$$t_{\text{TT}} \simeq t - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t}, \quad x_{\text{TT}} \simeq x - \frac{1}{2} x (1 - i\omega_g z) h_+ e^{i\omega_g t}.$$

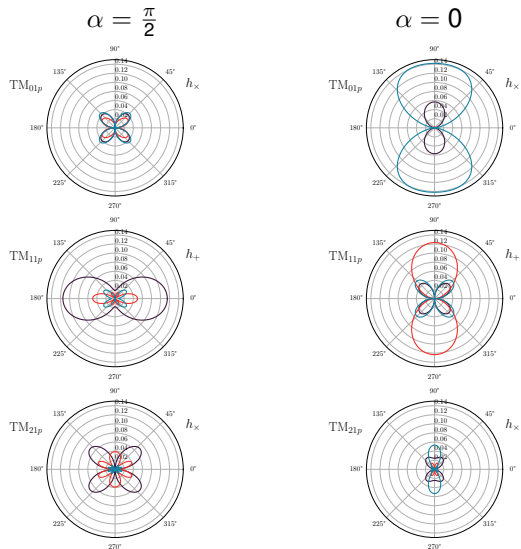
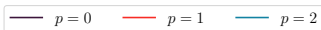
$$y_{\text{TT}} \simeq y + \frac{1}{2} y (1 - i\omega_g z) h_+ e^{i\omega_g t}, \quad z_{\text{TT}} \simeq z - \frac{i}{4} \omega_g (x^2 - y^2) h_+ e^{i\omega_g t}$$

- Wire $U_\mu = (1, 0, 0, 0)$
- Signal: E^1 induces current in wire
- Wire moves $U_{\text{TT},\mu} \neq (1, 0, 0, 0)$
- Signal: moving wire in static B-field induces current in wire

$$\mathbf{E} \simeq \frac{i}{2} B_0 \omega_g h_+ e^{i\omega_g t} (y, x, 0), \quad \mathbf{J}_{\text{sig,PD}} = \sigma \mathbf{E}$$

$$\mathbf{J}_{\text{sig,TT}}^i \simeq \frac{i}{2} \sigma B_0 \omega_g h_+ e^{i\omega_g t} (y, x, 0)$$

Evaluation of coupling coefficient



- Directional sensitivity
- Sensitivity to polarization

PBHs production mechanism

General relation between PBH mass and production time in early universe:

$$M \sim 10^{-18} M_{\odot} \left(\frac{t}{10^{-23} \text{ s}} \right)$$

Production mechanisms:

- Primordial inhomogeneities
- Collapse in a matter-dominated era
- Collapse from inflationary fluctuations: Superhorizon density fluctuations collapse at horizon reentry to PBHs. Gives PBHs in $10^{-15} - 10^{-11} M_{\odot}$ mass range.
- Collapse from scale-invariant fluctuations

GWs from merger of compact objects

$$h_+(t) = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f_g}{c} \right)^{2/3} \frac{1 + \cos^2 \theta}{2} \cos(2\pi f_g t_{\text{ret}} + 2\phi)$$
$$h_\times(t) = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f_g}{c} \right)^{2/3} \cos \theta \sin(2\pi f_g t_{\text{ret}} + 2\phi)$$

[Maggiore]

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}, \theta \text{ emission angle, } t_{\text{ret}} = t - \frac{r}{c}.$$

Superradiance: Decay of bosons

$$f_g = 2 \left(\frac{m_a}{10^{-9} \text{ eV}} \right) 10^6 \text{ Hz}$$

$$h_0 \sim 10^{-27} \left(\frac{1 \text{ GHz}}{f_g} \right) \left(\frac{M_{\text{BH}}}{10^{-4} M_{\odot}} \right)^{\frac{1}{2}} \left(\frac{10 \text{ kpc}}{D} \right)$$

Superradiance: Signal estimate with arxiv:2010.13157

Consider $\ell = m = 1$

$$f_g = 145 \text{ kHz} \left(\frac{m_a}{3 \times 10^{-10} \text{ eV}} \right) \stackrel{!}{=} 1 \text{ GHz} \Rightarrow m_a = 2 \times 10^{-6} \text{ eV}$$

$$\alpha = 0.2 \left(\frac{M_{\text{BH}}}{M_{\odot}} \right) \left(\frac{m_a}{3 \times 10^{-11} \text{ eV}} \right)$$

Superradiance condition

$$\alpha \leq \frac{1}{2} \frac{a_*}{1 + \sqrt{1 - a_*^2}}$$

for $0 < a_* < 1$. We take $a_* = 1 \Rightarrow \alpha \leq \frac{1}{2}$.

Superradiance condition is fulfilled by

$$M_{\text{BH}} \leq 3.75 \times 10^{-5} M_{\odot}$$

Characteristic strain:

$$h_0 = 10^{-24} \left(\frac{\Delta a_*}{0.1} \right) \left(\frac{10 \text{ kpc}}{D} \right) \left(\frac{M_{\text{BH}}}{M_{\odot}} \right) \left(\frac{\alpha}{0.2} \right)^7 = 2.2 \times 10^{-26} \left(\frac{\Delta a_*}{0.1} \right) \left(\frac{10 \text{ kpc}}{D} \right)$$

Proper detector frame

$$g_{00} = -1 - 2 \sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n, k_1, \dots, k_r} x^m x^n x^{k_1} \dots x^{k_r}$$

$$g_{0i} = -2 \sum_{r=0}^{\infty} \frac{r+2}{(r+3)!} R_{0nin, k_1, \dots, k_r} x^m x^n x^{k_1} \dots x^{k_r}$$

$$g_{ij} = \delta_{ij} - 2 \sum_{r=0}^{\infty} \frac{r+1}{(r+3)!} R_{ijnj, k_1, \dots, k_r} x^m x^n x^{k_1} \dots x^{k_r}$$

Proper detector frame

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{00} = -\omega_g^2 h_{ab}^{\text{TT}} x^a x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

$$h_{ij} = \omega_g^2 \left[(\delta_{iz} h_{ja}^{\text{TT}} + \delta_{jz} h_{ia}^{\text{TT}}) z x^a - h_{ij}^{\text{TT}} z^2 - \delta_{iz} \delta_{jz} h_{ab}^{\text{TT}} x^a x^b \right] \times \\ \times \left[-\frac{1 + e^{-i\omega_g z}}{(\omega_g z)^2} - 2i \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

$$h_{0i} = -\omega_g^2 \left(h_{ia}^{\text{TT}} z x^a - \delta_{iz} h_{ab}^{\text{TT}} x^a x^b \right) \left[-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

Cross check: Long wavelength limit ($x^i \omega_g \ll 1$)

$$h_{00} = -\omega_g^2 h_{ab}^{\text{TT}} x^a x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

Proper detector frame

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{00} = -\omega_g^2 h_{ab}^{\text{TT}} x^a x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

$$h_{ij} = \omega_g^2 \left[(\delta_{iz} h_{ja}^{\text{TT}} + \delta_{jz} h_{ia}^{\text{TT}}) z x^a - h_{ij}^{\text{TT}} z^2 - \delta_{iz} \delta_{jz} h_{ab}^{\text{TT}} x^a x^b \right] \times \\ \times \left[-\frac{1 + e^{-i\omega_g z}}{(\omega_g z)^2} - 2i \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

$$h_{0i} = -\omega_g^2 \left(h_{ia}^{\text{TT}} z x^a - \delta_{iz} h_{ab}^{\text{TT}} x^a x^b \right) \left[-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

Cross check: Long wavelength limit ($x^i \omega_g \ll 1$)

$$h_{00} = -\omega_g^2 h_{ab}^{\text{TT}} x^a x^b \left[\frac{1}{2} + \mathcal{O}(\omega_g z) \right] \ll h^{\text{TT}}$$

More details on cavities

- Orthogonality and Eigenfunctions

$$\begin{aligned}\nabla^2 \mathbf{E}_n(\mathbf{x}) &= -\omega_n^2 \mathbf{E}_n(\mathbf{x}) , \\ \int_{V_{\text{cav}}} d^3\mathbf{x} \mathbf{E}_n(\mathbf{x}) \cdot \mathbf{E}_m^*(\mathbf{x}) &= \delta_{nm} \int_{V_{\text{cav}}} d^3\mathbf{x} |\mathbf{E}_n(\mathbf{x})|^2 ,\end{aligned}$$

- Resonance frequencies

$$\omega^2 = \frac{x_{nm}^2}{R^2} + \frac{\pi^2 p^2}{L^2}$$

Scaling of coupling coefficient

Naive dimensionless coupling coefficient:

$$\eta_n'^2 = \frac{1}{h_0^2} \frac{|\int dV \mathbf{E}_n^*(\mathbf{x}) \cdot \mathbf{J}_{\text{eff}}(\mathbf{x})|^2}{\int dV |\mathbf{E}_n(\mathbf{x})|^2 B_0^2 \omega^2 V}$$

We see that this scales as p^2 . Therefore we divide by an additional dimensionless factor $(\omega V^{1/3})^2$:

$$\eta_n'^2 = \frac{1}{h_0^2} \frac{|\int dV \mathbf{E}_n^*(\mathbf{x}) \cdot \mathbf{J}_{\text{eff}}(\mathbf{x})|^2}{\int dV |\mathbf{E}_n(\mathbf{x})|^2 B_0^2 \omega^2 V (\omega V^{1/3})^2}$$