On the interference of ggh and $c\bar{c}H$ Higgs production mechanisms and the determination of charm Yukawa coupling y_c

Base on arXiv:2102.04242 with Wojciech Bizoń and Kirill Melnikov

Jérémie Quarroz | May 2022 | Pheno 2022

HL-LHC: a Higgs factory

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- The high luminosity LHC will produce $\mathcal{O}\left(10^8\right)$ Higgs per experiment
- Yukawa couplings of heavy quarks (b,t) and leptons (τ,μ) have been measured to **about 20% to 50% precision.** *Chatrchyan et al. 2013; Aaboud et al. 2018, 2019; Sirunyan et al. 2019a,b; Aad et al. 2021*
- Light quark Yukawas (y_u,y_d,y_s) $\rightarrow$ **only if strongly deviate from SM**
- The charm Yukawa is a **borderline case** ($m_c = 1.3$ GeV).
- y_c has **very recently** been measured from LHC data. *Walker and Krauss 2022*

$\longrightarrow y_c/y_c^{SM}$ can, hopefully, be constrained.

How to constrain charm Yukawa ?

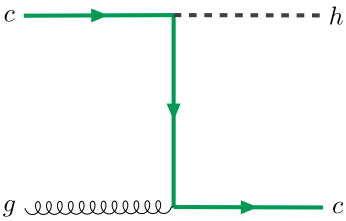
- H $\rightarrow$ J/ψ γ with $J/\psi \rightarrow \mu^+\mu^-$ *Bodwin et al. 2013*
  - \+ Clear signature
  - – $\mathcal{O}(10)$ event/experiment at 14 TeV with 3 ab^{-1} of integrated luminosity

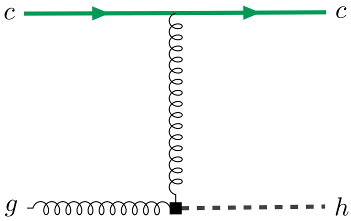
- H $\rightarrow$ c$\bar{\text{c}}$
  - \+ $\mathcal{O}(1000)$ event/experiment at 14 TeV with 3 ab^{-1} of integrated luminosity
  - – QCD background

- pp $\rightarrow$ H+jet$_c$ *Brivio et al. 2015*
  - \+ $\mathcal{O}(1000)$ events at 14 TeV with 3 ab^{-1} of integrated luminosity ($H \rightarrow \gamma\gamma$)
  - \+ cut on jet transverse momentum
  - \+ would benefit from higher order QCD corrections.
  - – experimental issues (low charm-tagging efficiency)

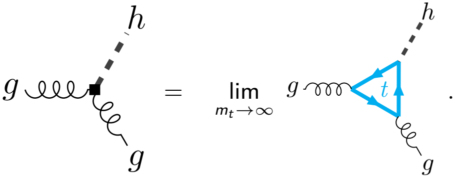
Two Higgs production mechanisms

There are two ways to produce $H + \text{jet}_c$





where *Deutschmann et al. 2017; Mondini et al. 2020*



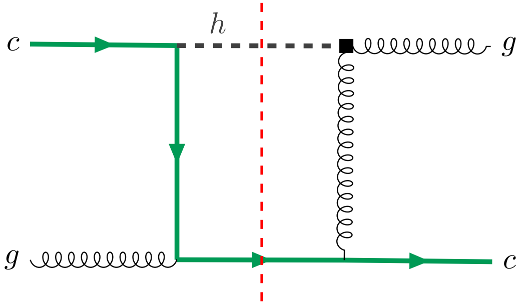
Total cross-section

The cross section for the process $cg \to cgH$ reads then

$$\sigma_{Hc} \propto g_{yuk}^2 \hat{\sigma}_{yuk} + g_{ggh}^2 \hat{\sigma}_{ggh} + g_{yuk} g_{ggh} \hat{\sigma}_{int} \, .$$

However, for massless quark, we have

$\hat{\sigma}_{\text{int}} =$



$= 0 \, .$

Yukawa interactions flip the helicity of the charm quark, whereas $c\bar{c}g$ conserves it.

The interference term

- The interference term **may be not negligible** as

$$\sigma_{int} \propto \sqrt{\sigma_{ggh} \cdot \sigma_{yuk}}$$

  Indeed, the LO calculation leads to

$$\sigma_{\mathrm{ggh}}^{\mathrm{LO}} = 176.6^{+47.6}_{-36.5}\ \mathrm{fb}\,, \qquad \sigma_{\mathrm{yuk}}^{\mathrm{LO}} = 21.22^{+1.47}_{-1.67}\ \mathrm{fb}\,.$$

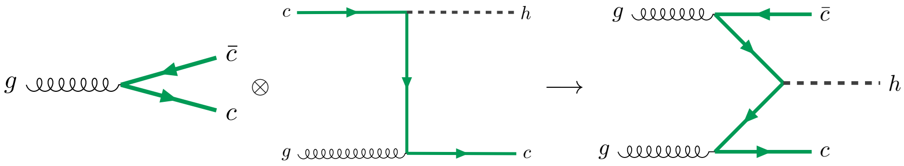
- The interference **has to** be evaluated using **massive quarks**

- Massive quarks in the initial state are **not used** in pQCD calculations.

What do we do ?

What are the different options ?

- Ignore the interference term. $\rightarrow$ Not a good option.
- Consider the following process as the **LO process** in 3 flavours scheme where the charm-quark is provided by gluon splitting.



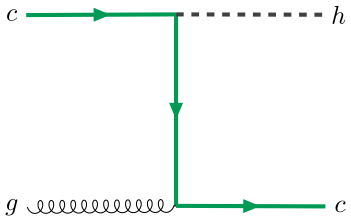
There is no issue with the mass as the charm quarks are exclusively in the final states, but **large logarithms** $\ln(m_H/m_c) \approx 4.6$ **are not resummed**, not even into a PDF. *Dittmaier et al. 2004; Dawson et al. 2006*

What are the different options ?

- ✗ Ignore the interference term.
- ✗ Consider the LO process in 3 flavors scheme with charm from gluon splitting.

✓ **Alternatively:**

Start with this process as LO in 4 flavours scheme, resummed $\ln(m_c)$ into PDF and try to carefully take the limit $m_c \to 0$.



It works **at leading order**, almost trivially, for the helicity flip interference.

Unconventional perturbative QCD

The LO calculation leads to

$$\sigma_{ggh}^{\rm LO} = 176.6^{+47.6}_{-36.5}\ \text{fb}\,, \qquad \sigma_{yuk}^{\rm LO} = 21.22^{+1.47}_{-1.67}\ \text{fb}\,, \qquad \sigma_{int}^{\rm LO} = -2.21^{+0.29}_{-0.31}\ \text{fb}\,.$$

The interference might be **perturbatively unstable**:

$$\sigma_{int} = \sigma_{int}^{\rm LO}\left[1 + \frac{\alpha_s}{2\pi}\,\mathcal{F}\left\{\underbrace{\ln\left(\frac{m_h}{m_c}\right)}_{\approx 4.6},\ \underbrace{\ln^2\left(\frac{m_h}{m_c}\right)}_{\approx 21}\right\} + \mathcal{O}(\alpha_s^2)\right]\,.$$

In standard pQCD calculation, $\ln(m_c)$ and $\ln^2(m_c)$ disappear for infrared-safe observable.

The necessity of a helicity flip on the charm-quark line makes QCD calculations **non-standard**.

We encounter the following **problems** beyond LO:

- ➥ **Unconventional collinear limits**
- ➥ **Soft-quark singularities**

Origin of unconventional collinear limits

Consider now the interference amplitude $\mathcal{A}(1_c, 2_g; 3_c, 4_g)$. In the collinear limit $p_1 \cdot p_4 \to 0$, we find

$$\lim_{p_1 \cdot p_4 \to 0} \mathrm{Int}|\mathcal{A}(1_c, 2_g; 3_c, 4_g)|^2 =$$

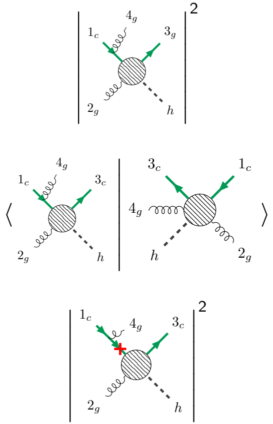
Standard collinear limit

$$g_s^2 \left[\left(\frac{P_{qq}(z)}{(p_1 \cdot p_4)} - \frac{C_F m_c^2 z}{(p_1 \cdot p_4)^2} \right) \mathrm{Int} \left[\frac{|\mathcal{A}(z \cdot 1_c, 2_g; 3_c)|^2}{z} \right] \right.$$

$$+ g_s C_F \frac{(1-z) m_c}{2(p_1 \cdot p_4)} \mathrm{Int} \left[\mathrm{Tr} \left[\hat{p}_1 \mathcal{A}^{i_c}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \times \hat{\mathcal{A}}^{i_c,\dagger}_{\mathrm{fin}}(\tilde{1}_c, 2_g; \tilde{3}_c, (1-z) \cdot \tilde{1}_g) \hat{\epsilon}_4 + \mathrm{h.c.} \right] \right]$$

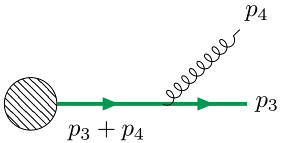
$\mathcal{O}\left(m_c \ln(m_c), m_c \ln^2(m_c)\right)$

$$\left. - \frac{C_F m_c (1-z)}{z(p_1 \cdot p_4)} \mathrm{Int} \left[\mathrm{Tr} \left[\hat{\mathcal{A}}^{i_c}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \times \hat{\mathcal{A}}^{i_c,\dagger}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \right] \right] \right]$$



Origin of soft quark singularities

$u(p_3) \sim \sqrt{E_3}$ for massless quark



$$\sim \bar{u}(p_3)\frac{\hat{\epsilon}_4^*(\hat{p}_3 + \hat{p}_4 + m_c)}{2p_3 \cdot p_4}\mathcal{A}(1_c, 2_g; 4_c) \sim E_3^{-1}\bar{u}(p_3)$$

Because the helicity flip can occur on the external spin line

$$\bar{u}(p_3) \sim \sqrt{m_c}$$

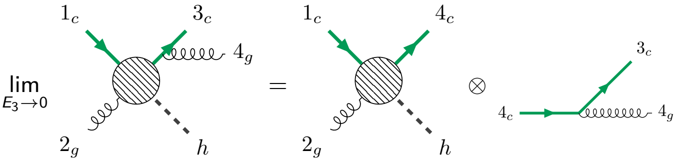
Therefore

$$\mathrm{Int}|\mathcal{A}|^2 \sim \mathrm{Tr}\left[(\hat{p}_3 + m_c)\mathcal{A}_{ggh}\mathcal{A}^*_{yuk}\right] \sim \frac{m_c}{E_3^2} \xrightarrow[\text{PS integration}]{\text{E3} \to 0} \mathcal{O}(m_c \log^2 m_c)$$

Mass suppressed but **logarithmically enhanced** when integrated over quark phase space.

Factorisation of soft quark

Analogously to soft gluon factorisation, **the amplitude factorises** into a lower order amplitude and a vector current in the soft quark limit



$$= \mathcal{A}^{i_c i_1}(1_c, 2_g; 4_c^{\lambda_q}) \otimes i T^{a_4}_{i_3 i_c} \frac{\bar{u}_3 \hat{\epsilon}_4 u_4^{\lambda_q}}{2 p_3 \cdot p_4}$$

Considering all possible contributing diagrams, we find the following factorisation of the interference

$$S_3 \,\mathrm{Int} \sum_{\mathrm{hel}} |\mathcal{A}(1_c, 2_g; 3_c, 4_g)|^2 = 2 m_c\, g_s^4\, g_{yuk}\, g_{ggh}\, N_c\, C_F$$

$$\otimes \left[\frac{(2C_F - C_A) p_2 \cdot p_4}{p_2 \cdot p_3\, p_3 \cdot p_4} \left(\frac{s^3 + u^3}{s + u} \right) + \frac{C_A\, p_1 \cdot p_4}{(p_1 \cdot p_3 - m_c^2)\, p_3 \cdot p_4} \left(\frac{u^3 + m_h^6}{s\, t\, u} \right) + \frac{C_A\, p_1 \cdot p_2}{(p_1 \cdot p_3 - m_c^2)\, p_3 \cdot p_2} \left(\frac{s^3 + m_h^6}{s\, t\, u} \right) \right]$$

LO and NLO interference

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The LO calculation leads to

$$\sigma^{\mathrm{LO}}_{\mathrm{ggh}} = 176.6^{+47.6}_{-36.5}\ \mathrm{fb}\,, \qquad \sigma^{\mathrm{LO}}_{\mathrm{yuk}} = 21.22^{+1.47}_{-1.67}\ \mathrm{fb}\,, \qquad \sigma^{\mathrm{LO}}_{\mathrm{int}} = -2.21^{+0.29}_{-0.31}\ \mathrm{fb}\,.$$

The interference at NLO reads

$$\sigma^{\mathrm{NLO}}_{\mathrm{int}} = -1.024(5)^{+0.224}_{-0.144}\ fb.$$

Good news! It does not increase dramatically.

$\Delta\sigma^{\mathrm{NLO}}$ [fb]	cg	cq	gg	cc	$c\bar{c}$	PDF	sum
const	−1.63	0.13	2.33	0.01	−0.01	0.11	0.94
$\ln(m_H/m_c)$	2.23	–	−6.33	−0.04	0.01	1.66	−2.47
$\ln^2(m_H/m_c)$	−0.06	–	2.66	0.01	−0.08	–	2.52
total	0.54	0.13	−1.34	−0.02	−0.08	1.76	1.00

Table: Strong cancellation between $\ln^2\left(\frac{m_H}{m_c}\right)$ and $\ln\left(\frac{m_H}{m_c}\right)$

Differential cross-section at NLO

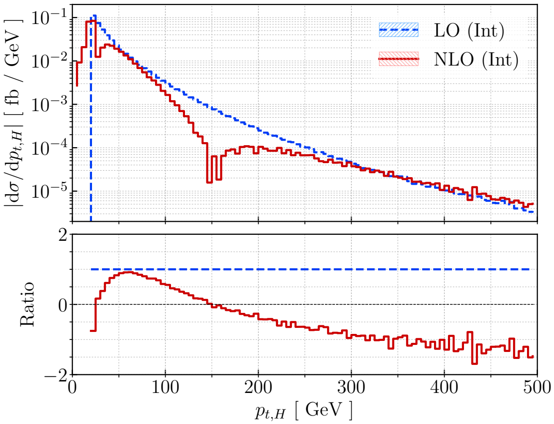

Figure: Higgs-boson transverse momentum

- NLO correction highly depends on the transverse momentum of the Higgs
- Correction changes sign at $p_{t,H} = 150\,\mathrm{GeV}$.

Conclusions

- The interference term between EFT and Yukawa-like Higgs production mechanism **vanishes** in standard perturbative QCD for massless quarks.
- A mass insertion (additional helicity flip) has been introduced by considering **massive charm quark**.
- Helicity flip on external quark line leads to **unconventional perturbative QCD** calculation since factorisation/limits change.
- Quasi-collinear terms that appear in these limits and their dependences on $\ln(m_c)$ have been identified.
- The NLO corrections are large but still acceptable due to **strong cancellation between** $\ln^2(m_c)$ and $\ln(m_c)$ terms.

Thank you for your attention !

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Resummation of the large $\ln(m_c)$ in the PDF

As an example, we consider the NLO corrections to $c + \bar{c} \to H$.

To understand the necessity of PDF redefinition, we will compare calculation of the partonic cross-section

- in the massless case: $\mathrm{d}\hat{\sigma}_{\overline{\mathrm{MS}}}$, regulated by dim.reg. $d = 4 - 2\epsilon$
- in the massive case: $\mathrm{d}\hat{\sigma}_m$, regulated by $m_c \neq 0$

How do we regulate singularity ?

We use a subtraction scheme to organise the real corrections calculations

$$2s \cdot \mathrm{d}\hat{\sigma}_{c\bar{c}\to H+g} = \int [\mathrm{d}g_3] F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g) \equiv \langle F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g)\rangle$$

$$= \underbrace{\langle (I - C_{31} - C_{32})(I - S_3) F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g)\rangle}_{m_c, \epsilon \to 0} + \langle S_3 F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g)\rangle + \langle (C_{31} + C_{32})(I - S_3) F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g)\rangle ,$$

where

$$F_{\mathrm{LM}}(1_c, 2_{\bar{c}}; 3_g) = \mathrm{dLips}_H \, |\mathcal{M}(1_c, 2_{\bar{c}}; 3_g, H)|^2 \underbrace{\mathcal{F}_{\mathrm{kin}}(1_c, 2_{\bar{c}}; 3_g, H)}_{\text{IR-safe observable}} .$$

We define the soft- S_i and the collinear-projection operator C_{ij}

$$S_i \equiv \lim_{E_i \to 0} \qquad C_{ij} \equiv \lim_{\theta_{ij} \to 0}$$

Partonic cross section

Partonic cross section $\mathrm{d}\hat{\sigma}_m$ regulated by $m_c \neq 0$

$$2s\cdot\mathrm{d}\hat{\sigma}_m = \overbrace{\langle (I - C_{31} - C_{32})(I - S_3)F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}; 3_g)\rangle}^{m_c \to 0} + C_F \frac{\alpha_s(\mu)}{2\pi}\left(\frac{2\pi^2}{3} + 5 - 3\ln\frac{m_H^2}{\mu^2}\right)\left\langle F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}})\right\rangle + \frac{\alpha_s(\mu)}{2\pi}\sum_{i=1}^{2}\int_0^1 \mathrm{d}z\,\left\{P_{qq}^{AP}(z)\ln\frac{\mu^2}{m_c^2} + P_{qq}^{\mathrm{fin}}(z)\right\}\left\langle \frac{F_{\mathrm{LM}}^{(i)}(\tilde{1}_c, \tilde{2}_{\bar{c}}; z)}{z}\right\rangle$$

Partonic cross section $\mathrm{d}\hat{\sigma}_{\overline{MS}}$ regulated by $d = 4 - 2\epsilon$

$$2s\cdot\mathrm{d}\hat{\sigma}_{\overline{MS}} = \overbrace{\langle (I - C_{31} - C_{32})(I - S_3)F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}; 3_g)\rangle}^{\epsilon \to 0} + C_F \frac{\alpha_s(\mu)}{2\pi}\left(\frac{2\pi^2}{3} + 5 - 3\ln\frac{m_H^2}{\mu^2}\right)\langle F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}})\rangle + \frac{\alpha_s(\mu)}{2\pi}\sum_{i=1}^{2}\int_0^1 \mathrm{d}z\, P_{qq}^{(\epsilon)}(z)\left\langle \frac{F_{\mathrm{LM}}^{(i)}(\tilde{1}_c, \tilde{2}_{\bar{c}}; z)}{z}\right\rangle$$

Physical cross section **does not depend on the regulator**.

$$f_m^c \otimes d\hat{\sigma}_m \otimes f_m^{\bar{c}} \overset{!}{=} \sigma_{c\bar{c}\to h} \overset{!}{=} f_{\overline{MS}}^c \otimes d\hat{\sigma}_{\overline{MS}} \otimes f_{\overline{MS}}^{\bar{c}}$$

where f_m^i are PDF of parton i in the massive charm-quark scheme.

Matching PDF

We suppose a relation between the two PDF scheme

$$f_m^a = \hat{O}^{ab} \otimes f_{\overline{MS}}^b$$

and an expansion in the coupling for $\hat{O}$

$$\hat{O}^{ab}(z) = \delta^{ab}\delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right) G^{ab}(z) + \mathcal{O}(\alpha_s^2)$$

Therefore, as the cross-section should not depend on the regulator, one can write

$$\begin{aligned}
\sigma_{c\bar{c}\to h} &= f_m^c \otimes \mathrm{d}\hat{\sigma}_m^{NLO} \otimes f_m^{\bar{c}} \\
&= f_{\overline{MS}}^c \otimes \mathrm{d}\hat{\sigma}_m^{NLO} \otimes f_{\overline{MS}}^{\bar{c}} + \left(\frac{\alpha_s}{2\pi} G^{cc} \otimes f_{\overline{MS}}^c\right) \otimes \mathrm{d}\hat{\sigma}_m^{LO} \otimes f_{\overline{MS}}^{\bar{c}} + f_{\overline{MS}}^c \otimes \mathrm{d}\hat{\sigma}_m^{LO} \otimes \left(\frac{\alpha_s}{2\pi} G^{cc} \otimes f_{\overline{MS}}^{\bar{c}}\right) + \mathcal{O}(\alpha^2) \\
&\stackrel{!}{=} f_{\overline{MS}}^c \otimes \mathrm{d}\hat{\sigma}_{\overline{MS}}^{NLO} \otimes f_{\overline{MS}}^{\bar{c}}
\end{aligned}$$

From this equation and the two partonic cross-section previously shown, the value of the coefficient G^{cc} is determined

$$G^{cc}(z) = -\ln\left(\frac{\mu^2}{m_c^2}\right) P_{qq}^{AP}(z) + C_F \left[\frac{1+z^2}{1-z}(1+2\ln(1-z))\right]_+$$

Partonic cross-sections

In the massive case, we find

$$
\begin{aligned}
2s \cdot \mathrm{d}\hat{\sigma}_{\mathrm{NLO}} = &\langle (I - C_{31} - C_{32})(I - S_3) F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}; 3_g) \rangle \\
&+ C_F \frac{\alpha_s(\mu)}{2\pi} \left(\frac{2\pi^2}{3} + 5 - 3 \ln \frac{m_H^2}{\mu^2} \right) \left\langle F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}) \right\rangle \\
&+ \frac{\alpha_s(\mu)}{2\pi} \sum_{i=1}^{2} \int_0^1 \mathrm{d}z \left\{ P_{qq}^{AP}(z) \ln \frac{\mu^2}{m_c^2} + P_{qq}^{\mathrm{fin}}(z) \right\} \left\langle \frac{F_{\mathrm{LM}}^{(i)}(\tilde{1}_c, \tilde{2}_{\bar{c}}; z)}{z} \right\rangle ,
\end{aligned}
$$

whereas in the massless case, we have

$$
\begin{aligned}
2s \cdot \mathrm{d}\hat{\sigma}_{\mathrm{NLO}}^{m_c=0} = &\langle (I - C_{31} - C_{32})(I - S_3) F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}; 3_g) \rangle \\
&+ C_F \frac{\alpha_s(\mu)}{2\pi} \left(\frac{2\pi^2}{3} + 5 - 3 \ln \frac{m_H^2}{\mu^2} \right) \langle F_{\mathrm{LM}}(\tilde{1}_c, \tilde{2}_{\bar{c}}) \rangle \\
&+ \frac{\alpha_s(\mu)}{2\pi} \sum_{i=1}^{2} \int_0^1 \mathrm{d}z \, P_{qq}^{(\epsilon)}(z) \left\langle \frac{F_{\mathrm{LM}}^{(i)}(\tilde{1}_c, \tilde{2}_{\bar{c}}; z)}{z} \right\rangle .
\end{aligned}
$$

Standard massless calculation

In the massless case, we have a true collinear singularity that is regulated by *dimensional regularization* ($d = 4 - 2\epsilon$) and produce $1/\epsilon$ pole.

The collinear limit reads

$$C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c) = g_s^2 S_\epsilon \mu^{2\epsilon} \frac{P_{qg}^{[0]}(z) + \epsilon P_{qg}^{[\epsilon]}(z)}{p_2 \cdot p_3} \frac{F_{\mathrm{LM}}(1_c, z \cdot 2_{\bar{c}})}{z} + \mathcal{O}(\epsilon^2)$$

The operator C_{32} also acts on the phase space

$$\begin{aligned} C_{32} \mathrm{d}\phi_{(1_c 2_g \to 3_c h)} &= \frac{\mathrm{d}\Omega_3}{2(2\pi)^{3-2\epsilon}} \mathrm{d}E_3 E_3^{1-2\epsilon} [\mathrm{d}p_H] (2\pi)^d \delta^{(d)}(p_1 + p_2 - \bar{p}_3 - p_H) \\ &= \frac{\mathrm{d}\Omega_3}{2(2\pi)^{3-2\epsilon}} E_2^{2-2\epsilon} (1-z)^{1-2\epsilon} \mathrm{d}z \underbrace{\mathrm{d}\phi_{(1_c (z \cdot 2_{\bar{c}}) \to h)}}_{\equiv \mathrm{d}\phi_z} \end{aligned}$$

where $\bar{p}_3 = E_3(1, \beta_3 \vec{n}_2)$ and $z = (1 - E_3/E_2)$.

After angular integration, the integrated collinear term reads

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle = -\frac{g_s^2}{2(2\pi)^2}\left(\frac{\mu^2}{E_2^2}\right)^{\epsilon}\int_0^1 \mathrm{d}z\,\mathrm{d}\phi_z \frac{(1-z)^{1-2\epsilon}}{1-z}$$
$$\left[P_{qg}^{[0]}(z) + \epsilon P_{qg}^{[\epsilon]}\right]\left(\frac{1}{\epsilon} - 2\ln(2)\right)\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

The pole is cancelled by the PDF renormalization

$$\langle \delta\hat{\sigma}_{PDF}\rangle = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\int_0^1 \mathrm{d}z\,\mathrm{d}\phi_z P_{qg}^{[0]}(z) f^c_{\overline{MS}}(x_1) f^g_{\overline{MS}}(x_2)\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

The integrated collinear term finally reads

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle + \langle \delta\hat{\sigma}_{PDF}\rangle = -\frac{\alpha_s}{2\pi}\int_0^1 \mathrm{d}z\mathrm{d}\phi_z$$
$$\left[P_{qg}^{[0]}(z)\left(\ln\left(\frac{\mu^2}{4E_2^2}\right) - 2\ln(1-z)\right) + P_{qg}^{[\epsilon]}(z)\right]\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

Massive cross-section

The quasi-collinear limit $p_2//p_3$ reads

$$C_{32}F_{\mathrm{LM}}(1_c,2_g,3_c)=g_s^2\left[\frac{P_{qg}^{[0](z)}}{p_2\cdot p_3}+T_F\frac{m_c^2\,z}{(p_2\cdot p_3)^2}\right]\frac{F_{\mathrm{LM}}(1_c,z\cdot 2_{\bar{c}},3_c)}{z}$$

The phase space in this limit reads

$$\begin{aligned}C_{32}\mathrm{d}\phi_{1_c2_g\to 3_ch}&=[\mathrm{d}p_3][\mathrm{d}p_h](2\pi)^4\delta^{(4)}(p_1+p_2-\bar{p}_3-p_h)\\&=\frac{1}{(2\pi)^2}\frac{\mathrm{d}\Omega_3}{4\pi}E_2^2(1-z)\mathrm{d}z\mathrm{d}\phi_z\end{aligned}$$

The massless limit is taken everywhere *except in the singular propagator*. This is allowed as there is no soft divergence as $E_3\to 0$.

After angular integration, the integrated collinear term in the massive case reads

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle = \frac{\alpha_s}{2\pi}\int_0^1 \mathrm{d}z\mathrm{d}\phi_z$$
$$\left[\left(2\ln\left(\frac{2E_2}{m_c}\right) + 2\ln(1-z)\right) P_{qg}^{[0]}(z) + 2T_F z(1-z)\right]\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

This has to be compared with massless result

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle + \langle \delta\hat{\sigma}_{PDF}\rangle = \frac{\alpha_s}{2\pi}\int_0^1 \mathrm{d}z\mathrm{d}\phi_z$$
$$\left[\left(2\ln\left(\frac{2E_2}{\mu}\right) + 2\ln(1-z)\right) P_{qg}^{[0]}(z) + P_{qg}^{[\epsilon]}(z)\right]\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

Physical quantities, like σ, should not depend on the regulator.

Eventually, the coefficient $G_{cg}(z)$ can be deduced from the two partonic cross-section

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle = \frac{\alpha_s}{2\pi}\int_0^1 \mathrm{d}z\mathrm{d}\phi_z \left[\left(2\ln\left(\frac{2E_2}{m_c}\right) + 2\ln(1-z)\right) P_{qg}^{[0]}(z) + 2T_F z(1-z)\right]\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

$$\langle C_{32} F_{\mathrm{LM}}(1_c, 2_g, 3_c)\rangle + \langle \delta\hat{\sigma}_{PDF}\rangle = \frac{\alpha_s}{2\pi}\int_0^1 \mathrm{d}z\mathrm{d}\phi_z \left[\left(2\ln\left(\frac{2E_2}{\mu}\right) + 2\ln(1-z)\right) P_{qg}^{[0]}(z) + P_{qg}^{[\epsilon]}(z)\right]\frac{F_{\mathrm{LM}}(1_c, z\cdot 2_{\bar{c}})}{z}$$

To the first order in α_s, one obtains

$$G_{cg}(z) = P_{qg}^{[0]}(z)\ln\left(\frac{m_c^2}{\mu^2}\right)$$

Other coefficients: G_{gc}, G_{cg}

- We have just derived the coefficient G_{cc} from the NLO correction to the $c\bar{c}$ Higgs production.
- The off-diagonal element G_{cg} (G_{gc}) can be computed in a analogous way from the process $c + g \to c + H$ with Yukawa-like (EFT) Higgs production mechanism

$$G_{cg}(z) = -\ln\left(\frac{\mu^2}{m_c^2}\right) P_{qg}^{AP}(z)\,,$$

$$G_{gc}(z) = -\left[\ln\left(\frac{\mu^2}{m_c^2}\right) - 2\ln(z) - 1\right] P_{gq}^{AP}(z)\,.$$

We suppose a relation between the two PDF scheme

$$f_a^m = \hat{O}_{ab} \otimes f_b^{\overline{MS}}$$

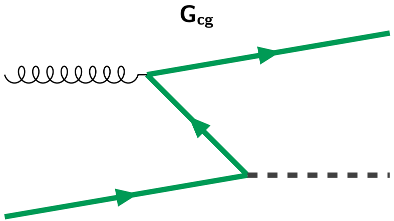
and an expansion in the coupling for $\hat{O}$

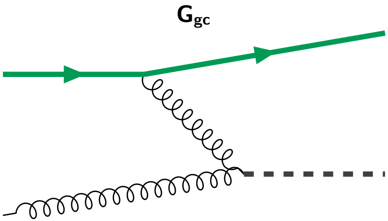
$$\hat{O}_{ab}(z) = \delta_{ab}\delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right) G_{ab}(z) + \mathcal{O}(\alpha_s^2)$$

To the first order in α_s, one obtains

$$G_{cc}(z) = -\ln\left(\frac{\mu^2}{m_c^2}\right) P_{qq}^{AP}(z) + C_F\left[\frac{1+z^2}{1-z}(1+2\ln(1-z))\right]$$

Off-diagonal elements G_{cg} and G_{gc} also need to be computed.





$$G_{cg}(z) = -\ln\left(\frac{\mu^2}{m_c^2}\right) P_{qg}^{AP}(z)$$

$$G_{gc}(z) = -\left[\ln\left(\frac{\mu^2}{m_c^2}\right) - 2\ln(z) - 1\right] P_{gq}^{AP}(z)$$

Let's sum up

- The collinear divergences are regulated thanks to the charm-quark mass.
- The resulting logarithmic dependences get cancelled by the one from PDFs change.
- In standard massive calculations, one could then simply compute NLO QCD corrections from there.

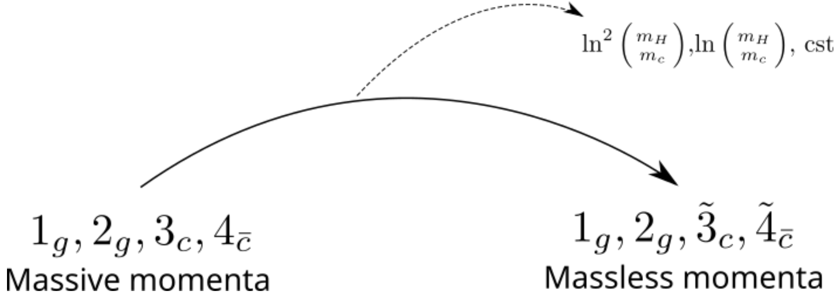
The necessity of helicity flip on the charm-quark line makes perturbative QCD non-standard.

Treatment of the gg-initiated channel

Let's consider a gg-initiated process with massive charm-quark

$$g(p_1) + g(p_2) \longrightarrow H(p_H) + c(p_3) + \bar{c}(p_4)$$

This process is free of true singular and soft singularities of finite m_c.
The aim:



We identify the (quasi-)singular regions.

- **quasi-collinear limits:** $1_g//4_{\bar{c}}$, $2_g//4_{\bar{c}}$, $3_c//4_{\bar{c}}$
  (the process is symmetric under $p_1 \leftrightarrow p_2$)
- **soft-quark limit (!):** $E_4 \to 0$
  (the process is symmetric under $p_3 \leftrightarrow p_4$)

To organize the calculation, we use the *nested collinear-soft subtractions scheme*

$$\begin{aligned}\langle F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle = \sum_{i=1}^{3} &\langle (1 - C_{4i})(1 - S_4)\omega_{123}^{(i)} F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle \\ &+ \langle C_{4i}(1 - S_4)\omega_{123}^{(i)} F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle \\ &+ \langle S_4 F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle\end{aligned}$$

F_{LM} denotes the integral of the matrix element with an IR-safe observable.
$\omega_{123}^{(i)}$ is the weight from partitioning the phase space.

Let's focus on the collinear limit $1_c // 4_g$. From Sudakov decomposition, we determine that in this limit

$$p_1 \cdot p_4 \sim m_c^2$$

The matrix element is then obtained by direct inspection of the interference term. It reads

$$C_{41}\mathrm{Int}\left[|M(1_g, 2_g, 3_c, 4_{\bar{c}})|^2\right] = \left[\frac{C_F m_c^2\, z}{(p_1 \cdot p_4)^2} + \frac{P_{qq}(z)}{p_1 \cdot p_4}\right] \frac{\mathrm{Int}\left[|M(\tilde{1}_c, 2_g, \tilde{3}_c)|^2\right]}{z}$$
$$+ \mathcal{A}\left(\tilde{1}_c, 2_g, \tilde{3}_c, z\right) \frac{1}{(p_1 \cdot p_4)(1 - z)}$$

There is an additional term in the collinear limit $1_c // 4_g$!

Integrated term

The regulated collinear term needs to be added back to our calculation. One needs to perform the angular integral in the small mass approximation.

$$\int_{-1}^{1} \frac{\mathrm{d}\cos\theta}{p_1 \cdot p_4} = \ln\left(\frac{2E_1}{m_c}\right) + \ln(1-z)$$

The non-standard integrated soft-regulated term ("$(1-S_4)C_{41}\ldots$") then reads

$$\left[\mathcal{D}_1 + \ln\left(\frac{2E_1}{m_c}\right)\mathcal{D}_0\right]\mathcal{A}(\tilde{1}_c, 2_g, \tilde{3}_c, z)$$

where

$$D_0 = \frac{1}{(1-z)_+} \qquad D_1 = \frac{\ln(1-z)}{(1-z)_+}$$

Soft quark divergence

The soft quark limit is obtain by direct inspection of the matrix element for $E_4 \sim m_c$. It reads

$$S_4 \text{Int}\left[|M|^2(\tilde{1}_c, 2_g, 3_c, 4_{\bar{c}})\right] = \Bigg[(C_F - C_A/2) F_{12}(\tilde{1}_c, 2_g, \tilde{3}_c) \frac{p_1 \cdot p_2}{(p_1 \cdot p_4)(p_2 \cdot p_4)}$$
$$+ C_A F_{13}(\tilde{1}_c, 2_g, \tilde{3}_c) \frac{p_1 \cdot p_3}{(p_1 \cdot p_4)(p_3 \cdot p_4)}$$
$$+ C_A F_{23}(\tilde{1}_c, 2_g, \tilde{3}_c) \frac{p_2 \cdot p_3}{(p_2 \cdot p_4)(p_3 \cdot p_4)}\Bigg] \text{Int}\left[|M^2\left(\tilde{1}_c, 2_g, \tilde{3}_c\right)|\right]$$

Integrated term

The soft term needs to be added back to our calculation. One needs to perform the integral of the eikonal factor in the small mass approximation.

Two massless emitters

$$\int [\mathrm{d}p_4] \frac{p_1 \cdot p_2}{(p_1 \cdot p_4)(p_2 \cdot p_4)} = \frac{1}{(2\pi)^2} \left[\ln^2 \left(\frac{2 s_{12} E_{max}}{m_c} \right) - \frac{\pi^2}{12} + \frac{1}{2} \mathrm{Li}_2(c_{12}) \right]$$

One massive, one massless

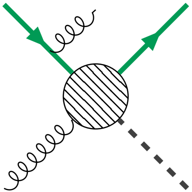
$$\int [\mathrm{d}p_4] \frac{p_1 \cdot p_3}{(p_1 \cdot p_4)(p_3 \cdot p_4)} = \frac{1}{(2\pi)^2} \left[\ln^2 \left(\frac{2 s_{13} E_{max}}{m_c} \right) - \frac{\pi^2}{12} + \frac{1}{2} \mathrm{Li}_2(c_{12}) + \frac{1}{4} \mathrm{Li}_2 \left(- \frac{E_{max}^2}{E_3^2} \right) \right]$$

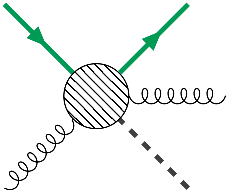
Origin of unconventional collinear limits

In singular regions of the phase space, an amplitude $\mathcal{M}$ can be splitted

$$\mathcal{M} = \mathcal{M}_{-1} + \mathcal{M}_{\text{fin}}$$

where $\mathcal{M}_{-1}$ denotes the singular part and $\mathcal{M}_{\text{fin}}$ the finite part.





In standard pQCD calculations, the collinear limit is provided by

$$|\mathcal{M}_{-1}|^2 + \underbrace{(\mathcal{M}^{\dagger}_{-1}\mathcal{M}_{\text{fin}} + h.c.)}_{=0}$$

Origin of unconventional collinear limits

Karlsruhe Institute of Technology

Consider now the interference amplitude $\mathcal{A}$. In the collinear limit $p_1 \cdot p_4 \to 0$, we find

$$\lim_{p_1 \cdot p_4 \to 0} \mathrm{Int}\left[\mathcal{M}^2(1_c, 2_g; 3_c, 4_g)\right] = g_s^2 \Bigg[\left(\frac{P_{qq}(z)}{(p_1 \cdot p_4)} - \frac{C_F m_c^2 z}{(p_1 \cdot p_4)^2} \right) \mathrm{Int}\left[\frac{|\mathcal{M}(z 1_c, 2_g; 3_c)|^2}{z} \right]$$
$$+ g_s C_F \frac{(1-z) m_c}{2(p_1 \cdot p_4)} \mathrm{Int}\left[\mathrm{Tr}\left[\hat{p}_1 \mathcal{A}^{i_c}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \hat{\mathcal{A}}^{i_c,\dagger}_{\mathrm{fin}}(\tilde{1}_c, 2_g; \tilde{3}_c, (1-z)\cdot \tilde{1}_g) \hat{\epsilon}_4 + \mathrm{h.c.} \right]\right]$$
$$- \frac{C_F m_c (1-z)}{z (p_1 \cdot p_4)} \mathrm{Int}\left[\mathrm{Tr}\left[\hat{\mathcal{A}}^{i_c}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \hat{\mathcal{A}}^{i_c,\dagger}(z \cdot \tilde{1}_c, 2_g; \tilde{3}_c) \right]\right] \Bigg]$$

Origin of soft quark singularities

They originate from helicity flip on the external charm-quark line

$$|M|^2 = \mathrm{Tr}((\hat{p}_3 + m_c)A^\dagger A)$$

where

$$M(1_c, 2_g, 3_c, 4g) = \bar{u}(p_3)\mathcal{A}\,,$$

The presence of $\hat{p}_3$ is responsible for the suppression of such soft quarks singularities in standard perturbative QCD.

Soft quark limits have the following structure

$$\lim_{E_4 \to 0} \mathrm{Int}\left[M^2(1_c, 2_g; 3_c, 4_g)\right] = g_s^2 \sum_{i<j} F_{ij}(p_1, p_2, p_3) \frac{p_i \cdot p_j}{(p_i \cdot p_4)(p_j \cdot p_4)}$$

where F_{ij} are finite functions of the other momenta.

In pratice

- At NLO, **5 different channels** contribute to the production process

$$cg,\ gg,\ cq,\ cc \text{ and } c\bar{c}$$

- Quasi-collinear and soft-quark limits are found by **direct inspection** of the expression for the interference

- The calculation is organised using the **nested collinear-soft subtraction scheme**.

Setup and remarks

- The charm mass is renormalized in **on-shell scheme** ($m_c = 1.3$ GeV), whereas the Yukawa coupling is renormalized in the $\overline{\textbf{MS}}$**-scheme** ($\bar{m}_c(m_H) = 0.81$ GeV)

- Anti-$k_\perp$ jet algorithm with $\Delta R = 0.4$. We require $p_{t,j} \geq 20$ GeV and $|\eta_j| \leq 2.5$

- Jet algorithm has **logarithmic sensitivity to $\mathbf{m_c}$**. To circumvent this, we impose
  - an additional cut on the charm momentum inside the charm jet: $p_c/p_j \geq 0.75$
  - that clustered charm-anticharm quarks jets are charm jets (and not gluon jets).

- Virtual corrections are calculated using **Passarino-Veltman reduction**. Each scalar integral is then expanded in the **small mass limit** by hand.

Numerical check

The validity of the approach can be checked. We consider the gg-initiated process

$$g(p_1) + g(p_2) \longrightarrow H(p_H) + c(p_3) + \bar{c}(p_4)$$

This process is free of true collinear and soft singularities of finite m_c.

Similarly to what has been done to find the PDF change, we use a subtraction scheme to extract logarithmic dependences on m_c

$$\begin{aligned}\langle F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle = \sum_{i=1}^{3} & \underbrace{\langle (1 - C_{4i})(1 - S_4)\omega_{123}^{(i)} F_{\mathrm{LM}}(1_g, 2_g; \tilde{3}_c, \tilde{4}_{\bar{c}})\rangle}_{m_c \to 0} \\ & + \langle C_{4i}(1 - S_4)\omega_{123}^{(i)} F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle \\ & + \langle S_4 F_{\mathrm{LM}}(1_g, 2_g; 3_c, 4_{\bar{c}})\rangle\end{aligned}$$

Numerical check

- σ_{real} is the interference term with **massive momenta**.
- σ_{rec} is the reconstructed interference term where $\ln^2(m_c)$, $\ln(m_c)$ and constant terms have been **extracted**.