Beyond the Free-Streaming Scale:

The Detailed Shape of the Dark-Matter Velocity Distribution and its Impact on Cosmic Structure

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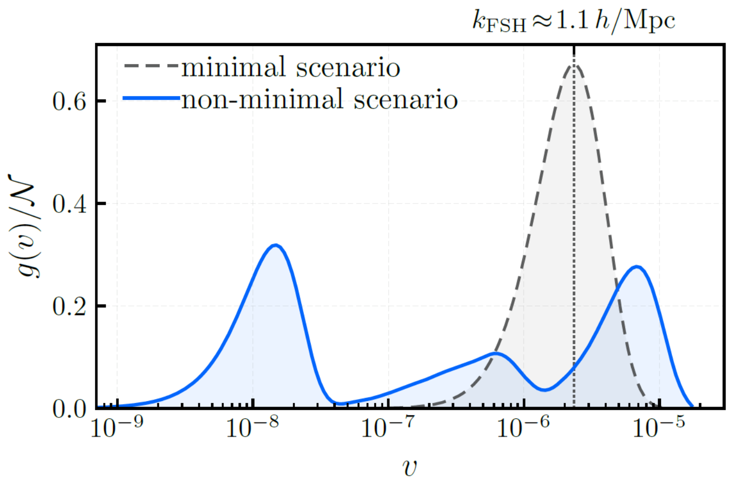

Based on work done in collaboration with:

- **Keith Dienes, Fei Huang, Jeff Kost, Kevin Manogue [arXiv:2101.10337]**
- **Keith Dienes, Fei Huang, Jeff Kost, Hai-Bo Yu [arXiv:2112.09105]**

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The Dark-Matter Velocity Distribution

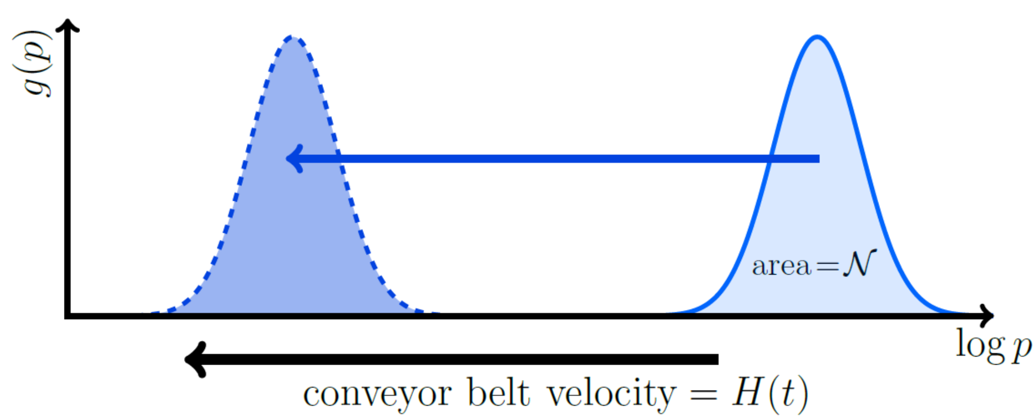
- The ***primordial distribution*** $f(v,t)$ of dark-matter (DM) velocities in the early universe characterizes the distribution of DM-particle speeds in the early (homogeneous, isotropic) universe.
- It's normalized with respect to the comoving number density $N(t)$:

$$N(t) = \frac{g_{\rm int} a^3}{2\pi^2} \int dv\, v^2 f(v,t)$$

- Equivalently, we can define a velocity distribution in $(\log v)$-space:

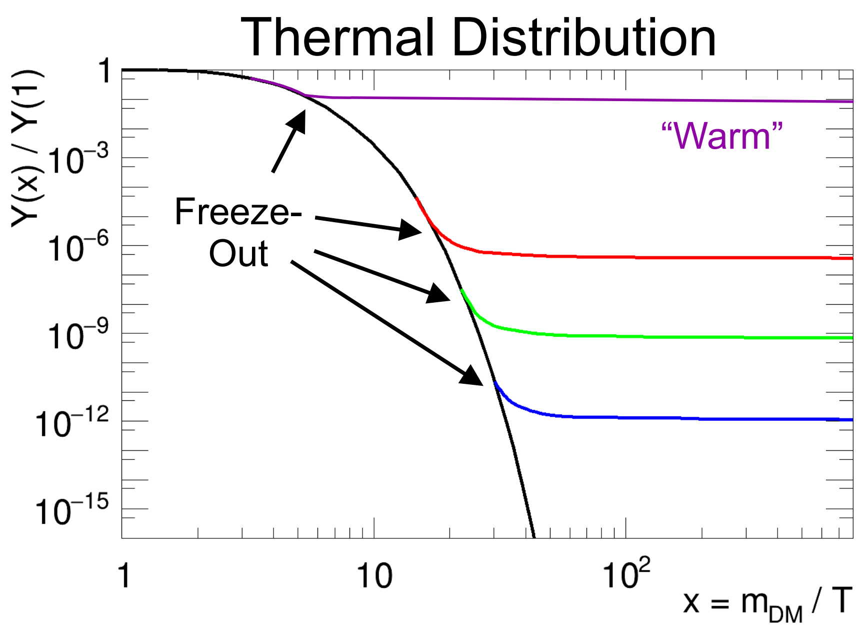
$$g_v(v,t) \equiv (av)^3 f(v,t) ,\quad \text{where} \quad N(t) = \frac{g_{\rm int}}{2\pi^2} \int d\log v\, g_v(v,t)$$

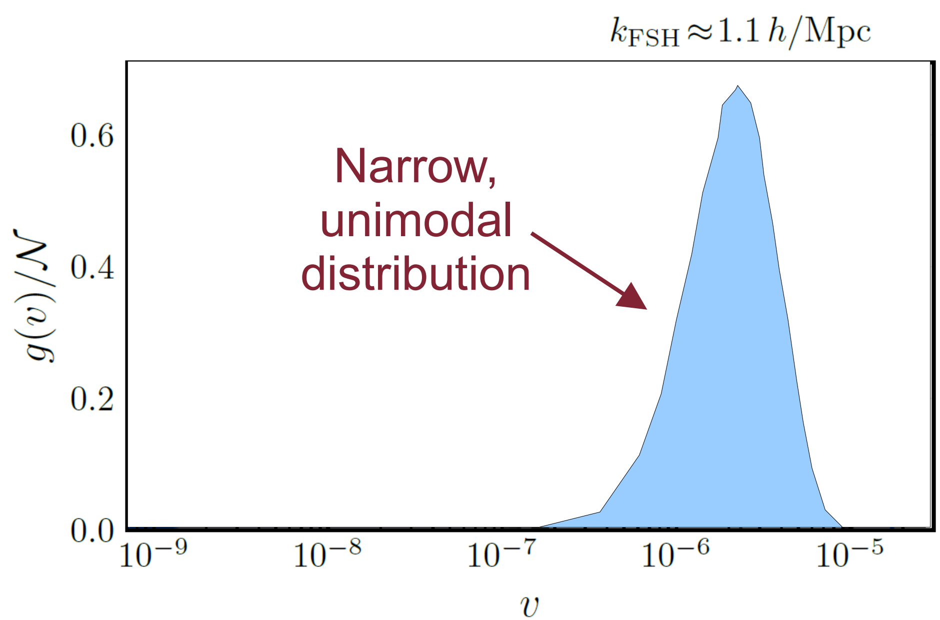
- Convenient, since $g_v(v,t)$ shifts uniformly to lower $\log v$ in the absence of DM production, scattering, and decay. [Dienes, Huang, Kost, Su, BT: 2001.02193]
- We'll also define $g_v(v) \equiv g_v(v,t_{\rm now})$ by extrapolating this distribution to present time (and ingnoring the effect of virialization, etc.).



DM Production and the Shape of $g_v(v)$

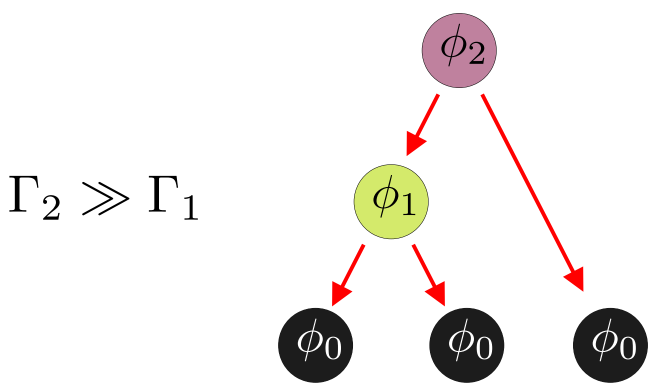
- The reason that the primordial DM velocity distribution is interesting is that it carries information about the processes through which the DM abundance was initially generated.
- If kinetic equilibrium is established across the population of DM particles at any point, all detailed information in $g_v(v)$ about the prior history of the DM is typically washed out.
- As a result, in many DM-production scenarios, including thermal freeze-out, $g_v(v)$ is ***unimodal*** and consists of a relatively ***narrow peak***.

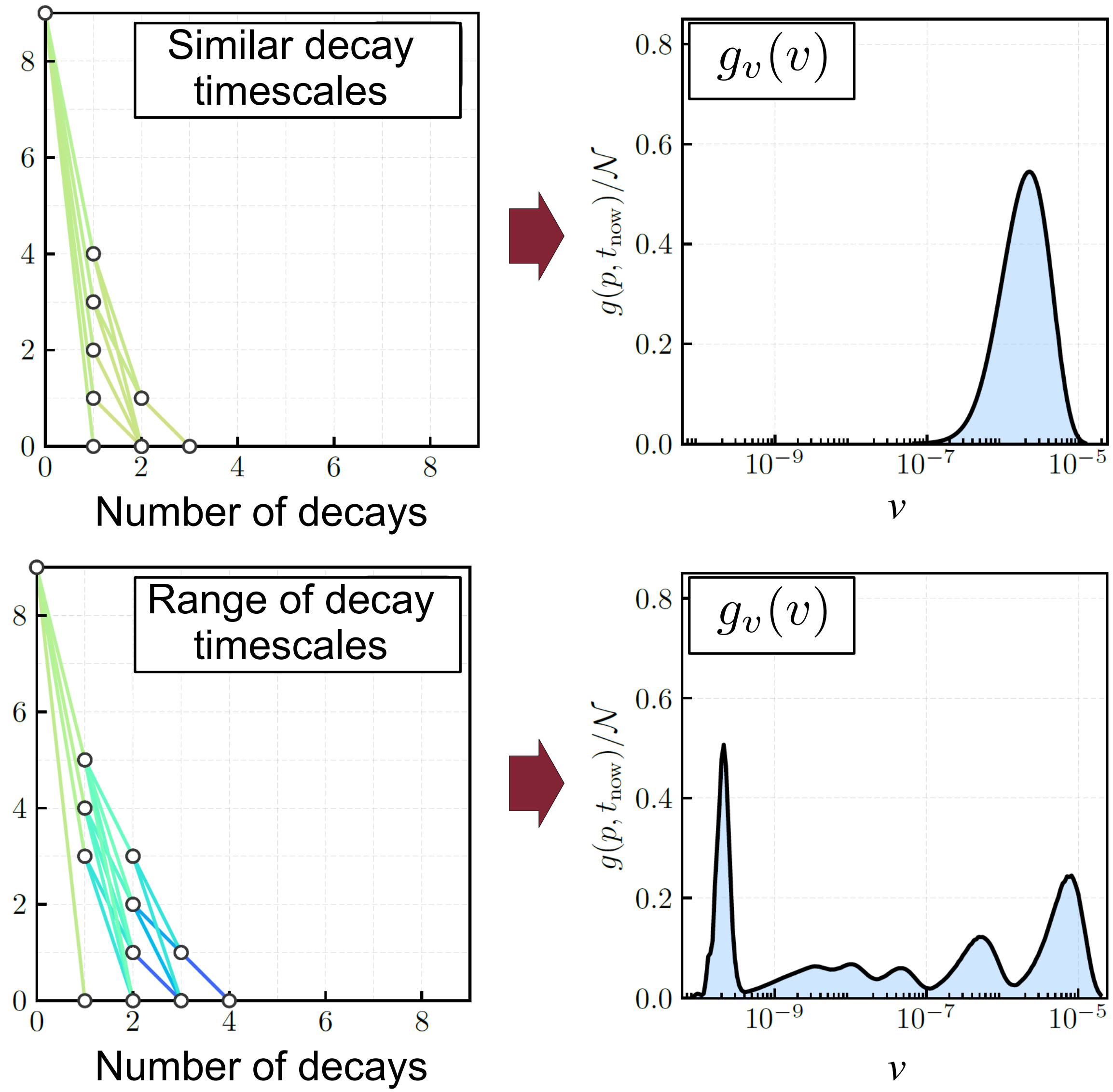




DM Production and the Shape of $g_v(v)$

- By contrast, if DM is produced ***non-thermally*** and ***multiple production processes*** with distinct kinematics contribute to the overall DM abundance, $g_v(v)$ can be highly non-trivial and even multimodal.
- One example: DM production via cascade decays within a non-minimal dark sector.
- Different decay chains with different characteristic timescales can populate different regions of $g_v(v)$.





- By probing $g_v(v)$, we can glean information about how the DM was produced.

Particle Horizons

- The primary way in which the DM velocity distribution affects structure is through **free-streaming**.

- Rapidly-moving particles can stream out of overdense regions, thereby suppressing structure on distance scales below the corresponding **particle horizon**:

Speed extrapolated to present time

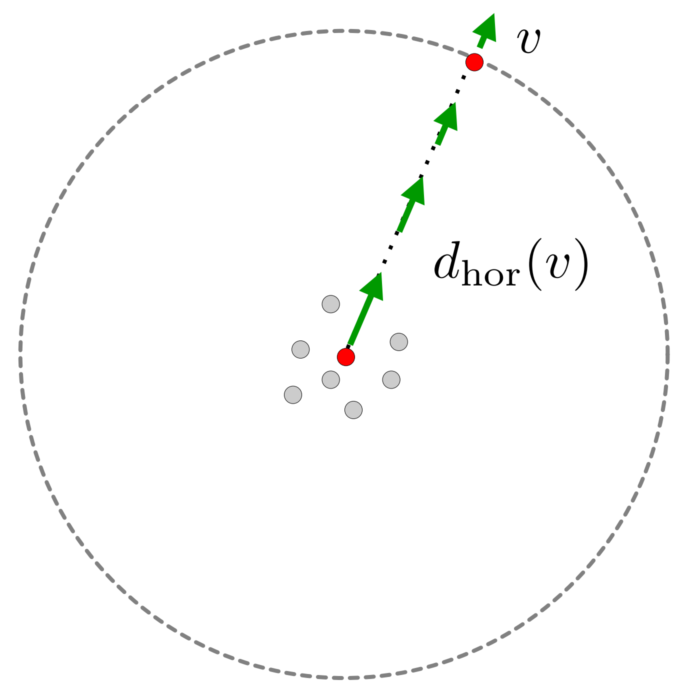
$$d_{\rm hor}(v) \equiv \int_{t_{\rm prod}}^{t_{\rm now}} \frac{dt}{a(t)} v(t) \; = \int_{a_{\rm prod}}^{1} \frac{da}{Ha^2} \frac{\gamma v}{\sqrt{\gamma^2 v^2 + a^2}}$$

Time at production

- The corresponding wavenumber above which structure is suppressed for a population of particles with exactly the same speed v (in the cosmological background frame) is

$$k_{\rm hor}(v) \equiv \xi \left[\int_{a_{\rm prod}}^{1} \frac{da}{Ha^2} \frac{\gamma v}{\sqrt{\gamma^2 v^2 + a^2}} \right]^{-1}$$

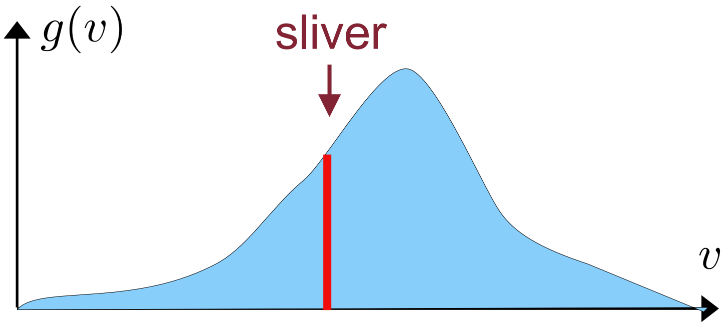
$\mathcal{O}(1)$ factor



The Free-Streaming Scale

- In general, the DM velocity distribution contains a ***spectrum*** of velocities contains a spectrum of $k_{\rm hor}(v)$, with each "sliver" of the spectrum suppressing structure below a different distance scale.

- The aggregate effect of free-streaming across the distribution is often estimated by evaluating $k_{\rm hor}(\langle v\rangle)$ for the ***average*** particle speed:



$$k_{\rm FSH}(v) \equiv \xi \left[\int_{a_{\rm prod}}^{1} \frac{da}{Ha^2} \frac{\gamma\langle v\rangle}{\sqrt{\gamma^2\langle v\rangle^2 + a^2}}\right]^{-1}$$

- This approximation does a good job of characterizing the effect of free-streaming when $g_v(v)$ consists of a single, relatively narrow peak.

- However, as we shall see, it often fails to do so in DM scenarios with more complicated $g_v(v)$ distributions.

- In this talk, I'll focus on an example $g_v(v)$ distribution consisting of two Gaussian peaks in $(\log v)$-space:

$$g_v(v) = \sum_{i=0}^{1} \frac{\mathcal{N}\Omega_i}{\sqrt{2\pi}\sigma_i\Omega_{\rm DM}} \exp\left\{-\frac{1}{2\sigma_i^2}\left[\log\left(\frac{v}{\langle v\rangle}_i\right) + \frac{1}{2}\sigma_i^2\right]^2\right\}$$

Matter Power Spectrum

- In the linear regime, density perturbations can be characterized in terms of the linear matter power spectrum.

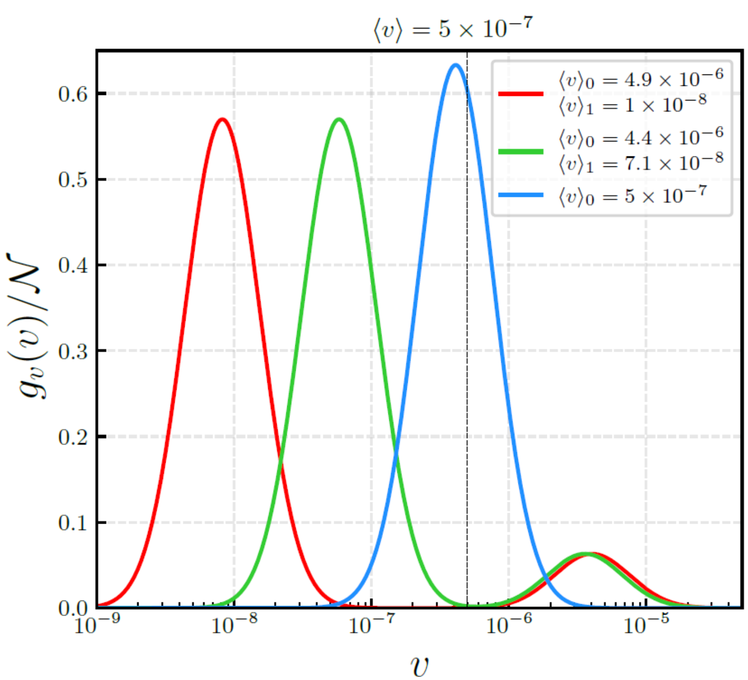
$$P(k,t) \equiv 4\pi \int dr\, r^2 \frac{\sin(kr)}{kr} \xi(r,t)$$

Two-point correlation function for the fractional overdensity

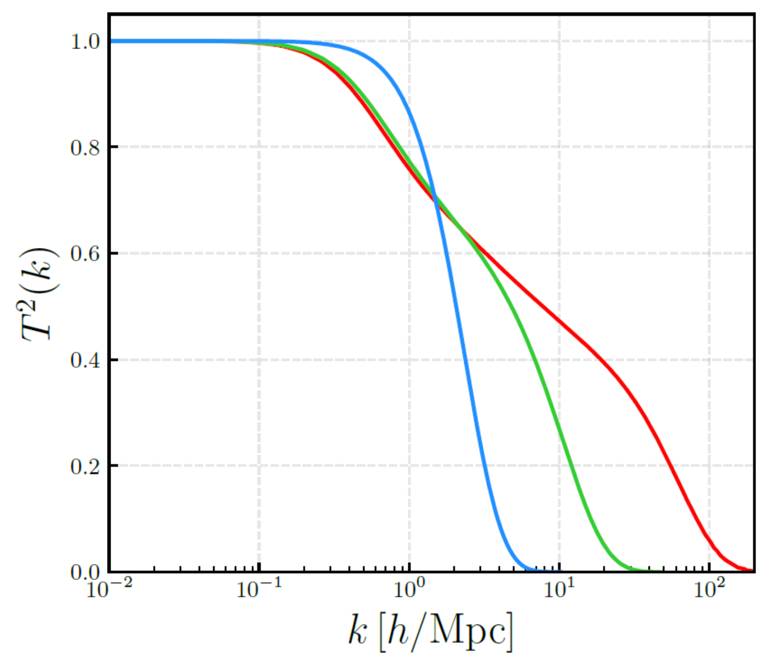
- Deviations from the pure CDM case can be parametrized in terms of the **transfer function**.

$$T^2(k) = \frac{P(k)}{P_{\mathrm{CDM}}(k)}$$

$g_v(v)$ Distributions



Transfer Functions (Calculated Using CLASS)



Matter Power Spectrum

- At a quantitative level, the distinctiveness of the power spectra to which our double-peak $g_v(v)$ distributions give rise can be assessed as follows.

- Compare to warm dark matter (WDM) scenarios, wherein $T^2(k)$ depends effectively on the DM mass m_{WDM} alone.

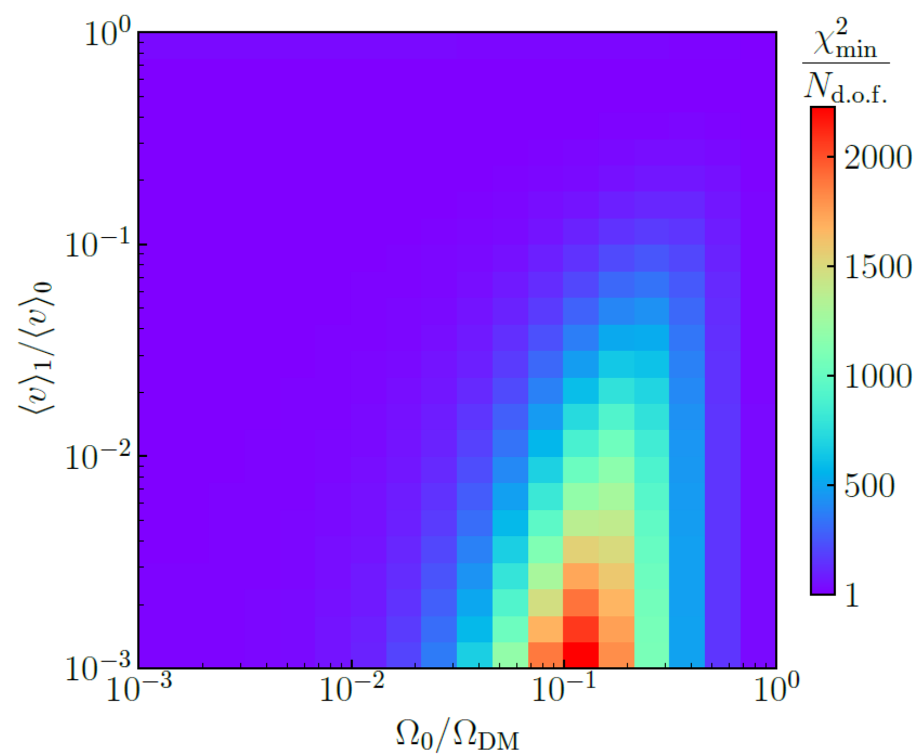
$$T^2_{\text{WDM}}(k) \approx \left[1 + (\alpha k)^{2\nu}\right]^{-10/\nu}, \quad \text{where} \quad \alpha \propto \left(\frac{m_{\text{WDM}}}{\text{keV}}\right)^{-1.11}$$

- Survey a broad range of m_{WDM}. For each, evaluate the goodness-of-fit statistic

$$\chi^2(m_{\text{WDM}}) = \sum_j \frac{[T^2(k_j) - T^2_{\text{WDM}}(k_j)]^2}{\sigma^2_{T^2}(k_j)}$$

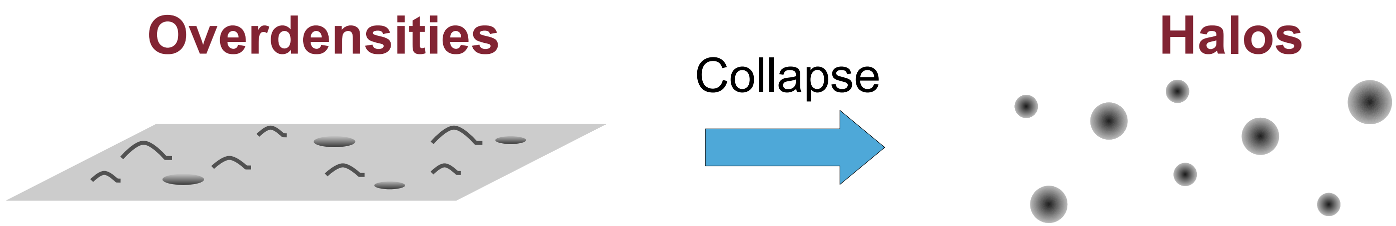
Chosen so that $\chi^2(m_{\text{WDM}}) = 1$ for a single Gaussian with the same width σ ≈ 063 as a WDM distribution.

- Take the ***minimum*** from among these $\chi^2(m_{\text{WDM}})$ values as our ultimate measure of the distinctiveness of $T^2(k)$.



Halo-Mass Function

- In the ***non-linear regime***, things become more complicated.



- Analytic approach to modeling the number density of halos per unit halo mass (the ***halo-mass function***) provided by the Press-Schechter formalism. [Press, Schechter: ApJ 1974; Bond, Cole, Efstathiou, Kaiser, ApJ 1991]

- In this approach, $dn/d(\log M)$ depends on $P(k)$ through the spatially-averaged variance of the fractional overdensity:

$$\sigma^2(t,R) \equiv \int_{-\infty}^{\infty} d\log k \; W^2(k,R) \frac{k^3 P(k,t)}{2\pi^2}$$

Window function

- In particular, the halo-mass function takes the form:

$$\frac{dn}{d\log M} = \frac{\overline{\rho}}{2M}\eta(M)\frac{d\log \nu(M)}{d\log M}$$

where $\nu(M) \equiv \dfrac{\delta_c^2}{\sigma^2(t_{\rm now}, R(M))}$

Critical overdensity $\delta_c \approx 1.686$

R as a function of halo mass M

"Mass function": depends on M only through $\nu(M)$

Halo-Mass Function

- Concrete functional forms for $W(k,R)$ and $\eta(M)$ can be posited on the basis of numerical simulations.
  - Window function (k-space top-hat): $$W(k,R) = \Theta(1 - kR)$$

    This form of $W(k,R)$ makes the relationship between R and M somewhat ambiguous: $$M \equiv \frac{4\pi}{3}\overline{\rho}(c_W R)^3$$

    Constant (fit to simulation data)
  - Mass function: $$\eta(M) = \sqrt{\frac{2\nu(M)}{\pi}} A \left[1 + \nu^{-\alpha}(M)\right] e^{-\nu(M)/2}$$

[Sheth, Tormen: astro-ph/9901; Sheth, Mo, Tormen: astro-ph/9907024]

- Concrete functional forms for $W(k,R)$ and $\eta(M)$ can be posited on the basis of numerical simulations.

- Nevertheless, this combination of $W(k,R)$ and $\eta(M)$ have been shown to accord well with the results of N-body simulations of mixed WDM/CDM models. [Parimbelli, Scelfo, Giri, Schneider, Archidiacono, Camera, Viel: 2106.04588]
- Modifying the functional forms for $W(k,R)$ and $\eta(M)$ would affect our numerical results, but not our overall takeaway.

Cluster-Number Counts

- Cluster-number counts provide an observational handle on the DM velocity distribution.
- Simply the number of galaxy clusters observed within a given region of the sky.
- Directly related to the halo-mass function:

$$N_\mathrm{C} = \int_0^{z_\mathrm{max}} z \frac{dV}{dz} \int_{\log M_\mathrm{th}(z)}^{\infty} d\log M \frac{dn}{d\log M}$$

Mass threshold (z-dependent)

- Comoving volume element: $$\frac{dV}{dz} = 4\pi\Delta\Omega \frac{c\chi^2(z)}{H(z)}$$

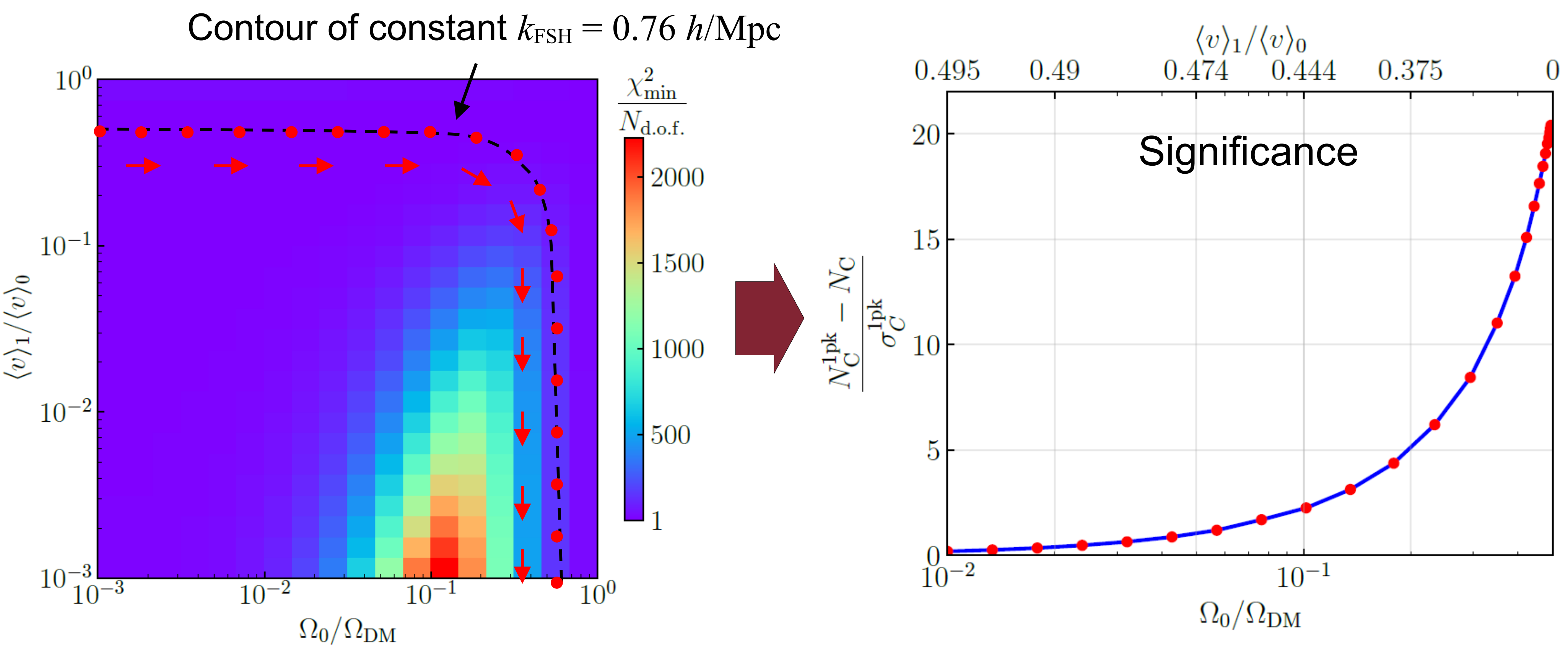
Comoving distance

Hubble parameter

- We model our analysis in a way which allow us to compare our results with results from the Euclid survey. [Sartoris et al.: 1505.02165]

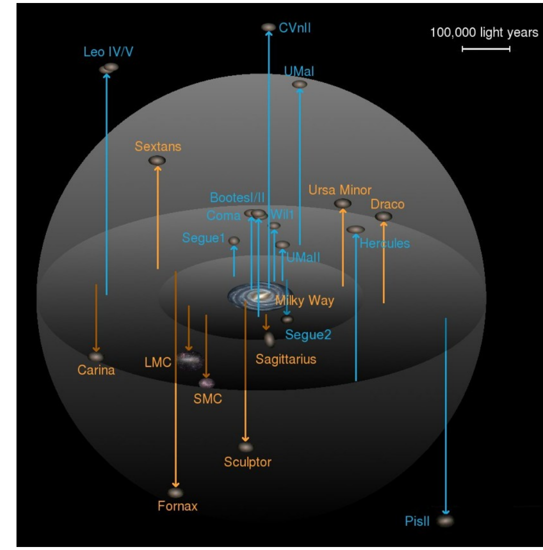
Cluster-Number Counts

- We compare N_C to the cluster-number count $N_C{}^{1\text{pk}}$ obtained from a $g(v)$ distribution consisting of a single Gaussian with the same nominal k_{FSH}.
- We assess the statistical significance of this difference based on the Poisson uncertainty $\sigma_C{}^{1\text{pk}}$ of the single-peak distribution.



- **<u>The Upshot</u>**: $g(v)$ distributions with the same kFSH but different detailed shapes can lead to drastically different cluster-number counts.

Satellite Counts



- Another observable handle on $g(v)$ comes from the **numbers of satellites** observed within the halos of larger galaxies – and, in particular, the Milky Way.
- Predictions for satellite counts follow from the **conditional mass function**.

$$\frac{dN(M, z|M_0, z_0)}{dM} = -\frac{M_0}{M}\sigma^2(M)\zeta(M, z|M_0, z_0)\frac{d\sigma^2(M)}{dM}$$

Differential number of halos per unit mass M present at redshift z which, on average, get incorporated into a single host halo of mass M_0 by redshift $z_0 < z$.

- For spherical collapse, the probability $\zeta(M,z|M_0,z_0)$ for a particle within a halo of mass M at z to be incorporated into a halo of mass M_0 by z_0 is

Universal growth factor

$$\zeta(M, t|M_0, t_0) = \frac{\delta_c/D(z) - \delta_c/D(z_0)}{(2\pi)^{1/2}[\sigma^2(M) - \sigma^2(M_0)]^{3/2}} \exp\left(-\frac{[\delta_c/D(z) - \delta_c/D(z_0)]^2}{2[\sigma^2(M) - \sigma^2(M_0)]}\right)$$

- The differential satellite number per unit subhalo mass is:

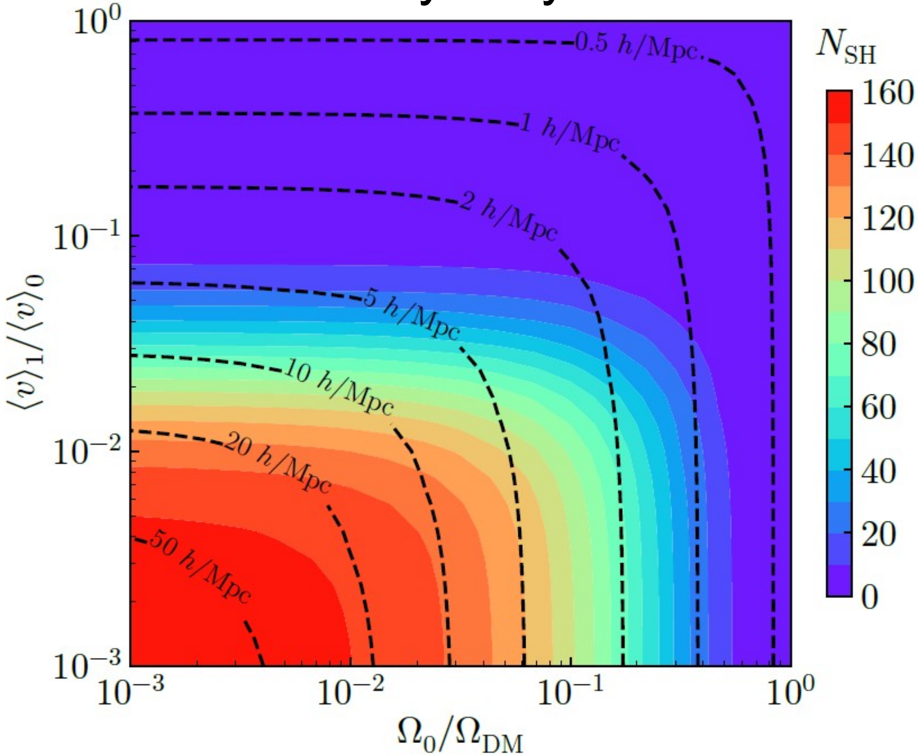
$$\frac{dN_{\rm SH}}{dM} = \frac{1}{\mathcal{N}_{\rm SH}}\int_0^\infty \frac{dN(M, t|M_0, t_0)}{dM}\frac{d\delta(z)}{dz}dz$$

Normalization factor

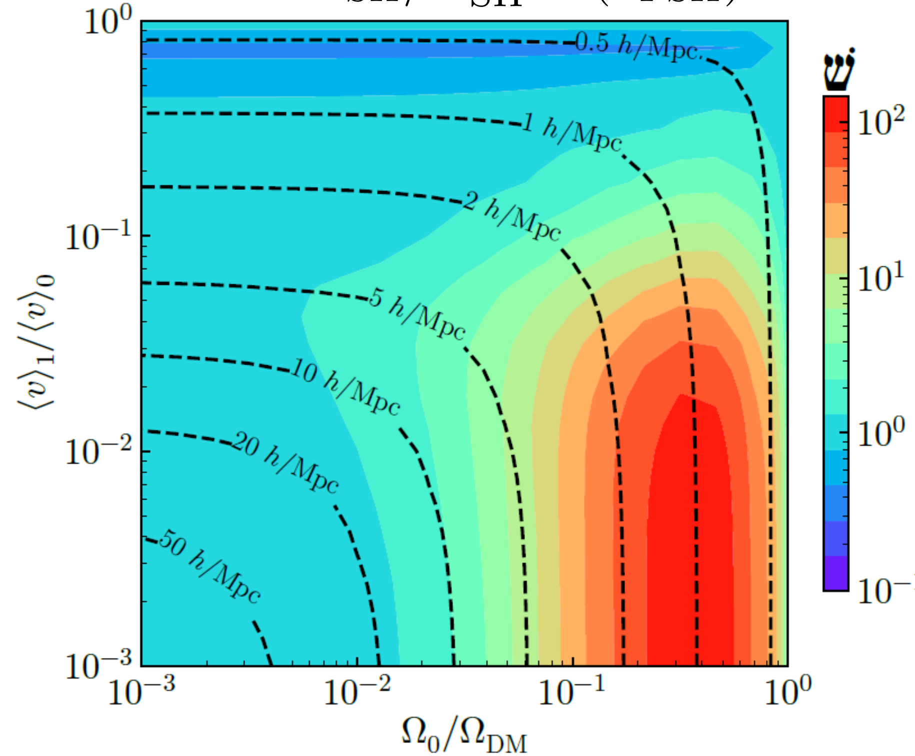
Satellite Counts

- We can integrate dN_{SH}/dM above some observability threshold $M_{\min}$ to obtain a characteristic number of subhalos N_{SH} for a given M_0. For the Milky Way, we take $M_{\min} = 10^8\, M_\odot$.
- We compare to the result $N_{\mathrm{SH}}^{\mathrm{WDM}}(k_{\mathrm{FSH}})$ for a WDM model with the same nominal free-streaming scale.

N_{SH} for Milky-Way Sized M_0



$שׁ \equiv N_{\mathrm{SH}}/N_{\mathrm{SH}}^{\mathrm{WDM}}(k_{\mathrm{FSH}})$



- **<u>The Upshot</u>**: $g_v(v)$ distributions with the same k_{FSH} but different detailed shapes can lead to drastically different Milky-Way satellite counts.

Summary

- Non-thermal DM scenarios can give rise to a wide variety of primordial DM velocity distributions.
- The ***detailed shape*** of these distributions *beyond* the nominal free-streaming scale can have observable consequences for structure formation.
- We have also demonstrated that the detailed shape of $g_v(v)$ can affect other astrophysical observables such as **cluster-number** and ***satellite counts***.
- The results I have shown are based on analytic modeling techniques and can (and should!) be refined through use of N-body and hydrodynamic simulations.
- Indeed, one of the primary purposes of this work is to motivate such numerical studies and identify the kinds of $g_v(v)$ profiles for which they are most needed.