Muon Magnetic Moment-Mass Conundrum and the Scale of New Physics

Vishnu Padmanabhan Kovilakam

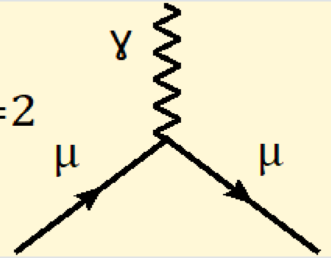
Oklahoma State University

In collaboration with K.S. Babu, and Sudip Jana

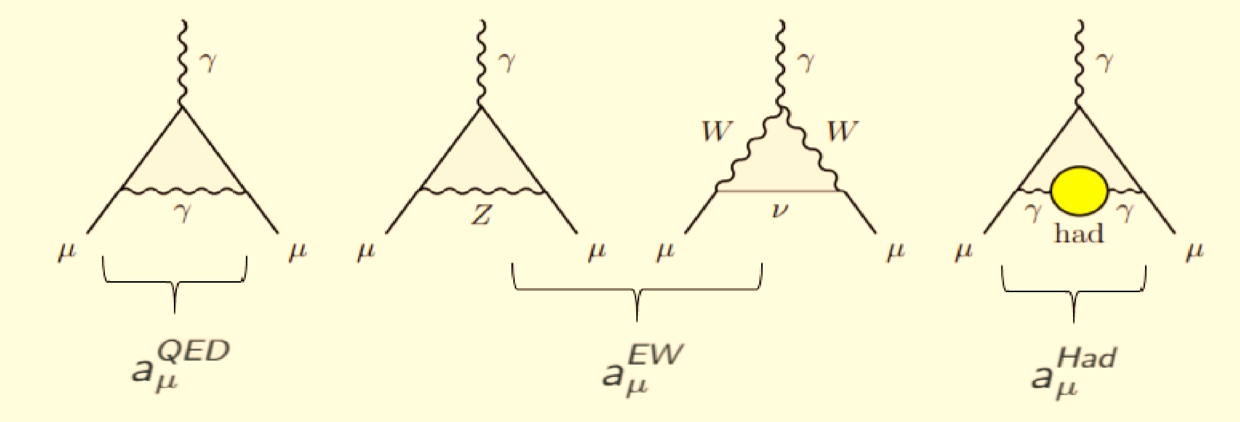
Oklahoma State University

Muon Magnetic Moment: Overview

- Magnetic moment of Leptons: $\vec{\mu}_B = g_\mu \frac{e}{2m_\mu}\vec{S}$
- Lande' g- factor: $g_\mu = 2$



- Due to Quantum corrections, $(g-2)_\mu \neq 0$.
- Anomalous Magnetic Moment: $a_\mu = \frac{(g-2)_\mu}{2}$

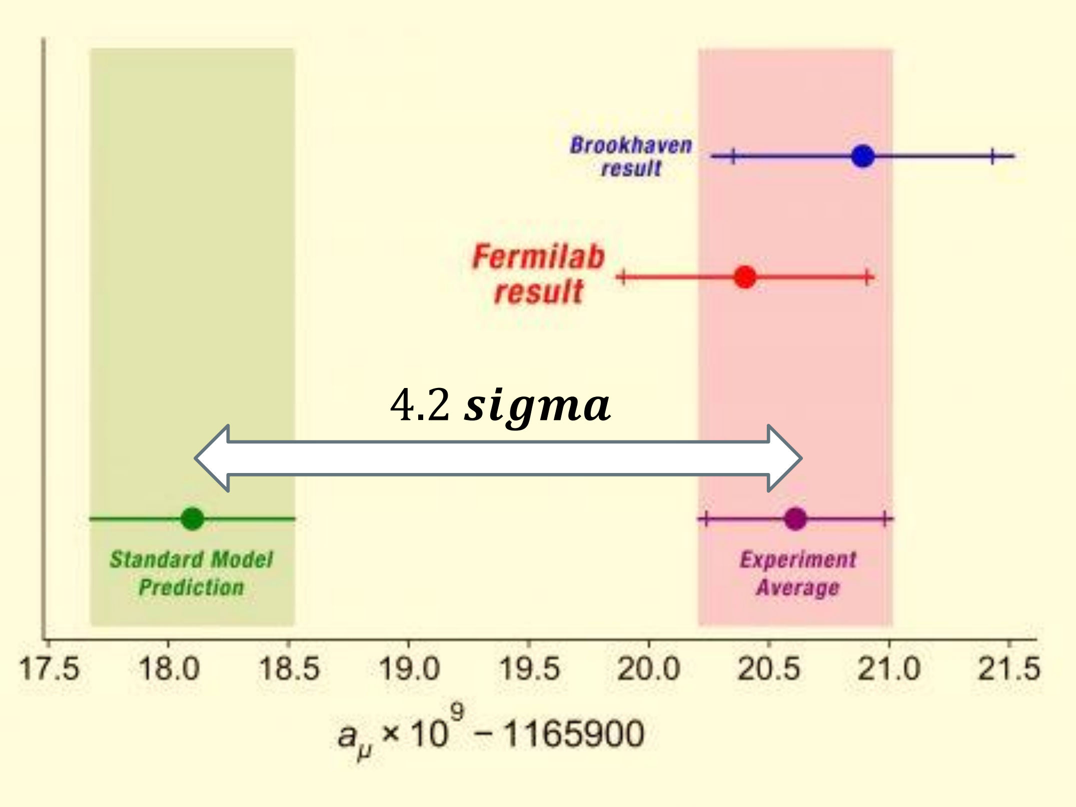


$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had}$$

Current Status of muon (g-2)

$$10^{11}\boldsymbol{a_\mu} = \begin{cases} 116591810(43)\ \textbf{SM} \\ 116592040(54)\ \textbf{Exp} \end{cases}$$

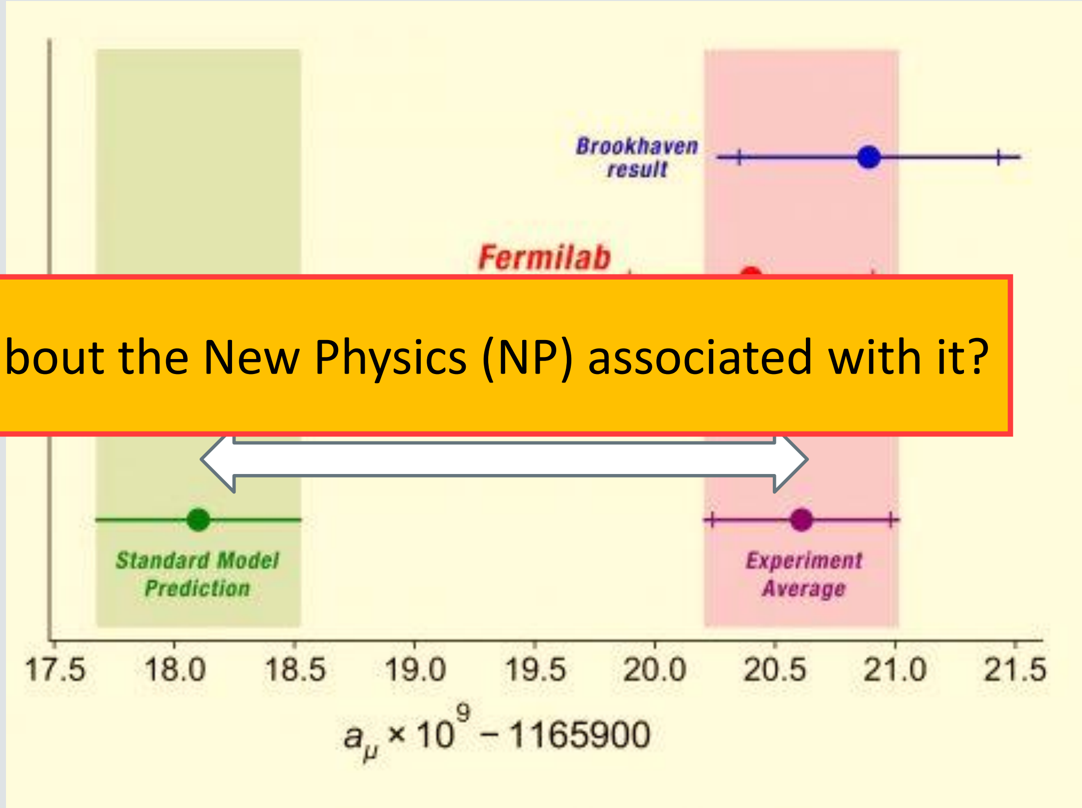
$$\Delta \boldsymbol{a_\mu} = \boldsymbol{a_\mu}^{\boldsymbol{exp}} - \boldsymbol{a_\mu}^{\boldsymbol{SM}} = 251(59) \times 10^{-11}$$



Fermilab Muon g-2 Collaboration, B. Abi *et al.* (2021)

Current Status of muon (g-2)

$$10^{11} a_\mu = \begin{cases} 116591810(43)\ \mathbf{SM} \\ 116592040(54)\ \mathbf{Exp} \end{cases}$$

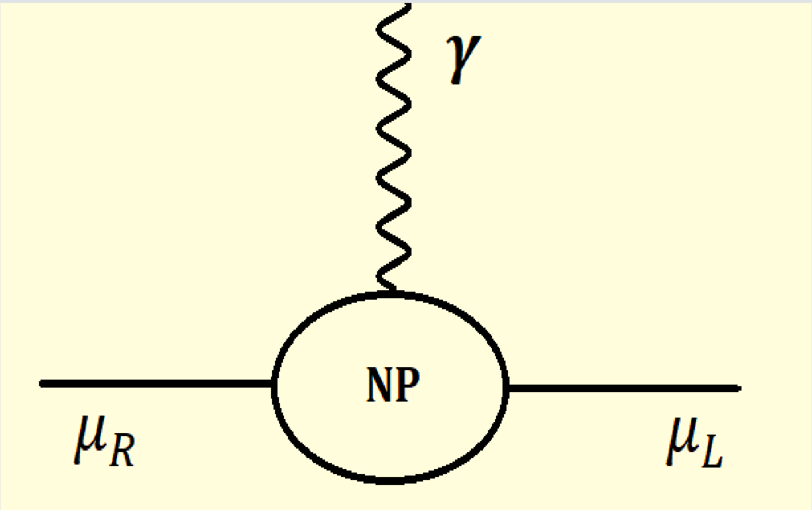


Fermilab Muon g-2 Collaboration, B. Abi *et al.* (2021)

If the muon (g-2) anomaly is real, what we can say about the New Physics (NP) associated with it?

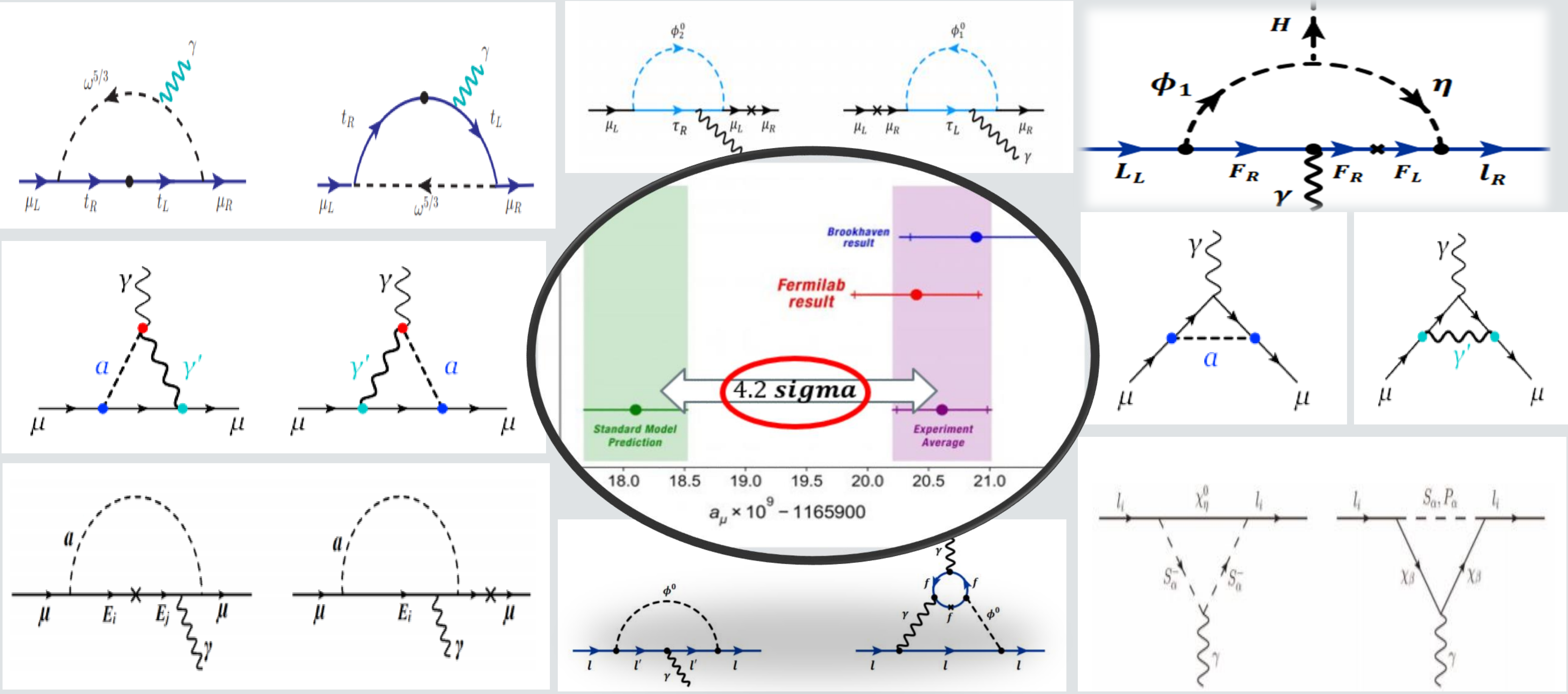
$$\Delta a_\mu = a_\mu{}^{exp} - a_\mu{}^{SM} = 251(59) \times 10^{-11}$$

NP Interpretation of Muon (g-2) anomaly



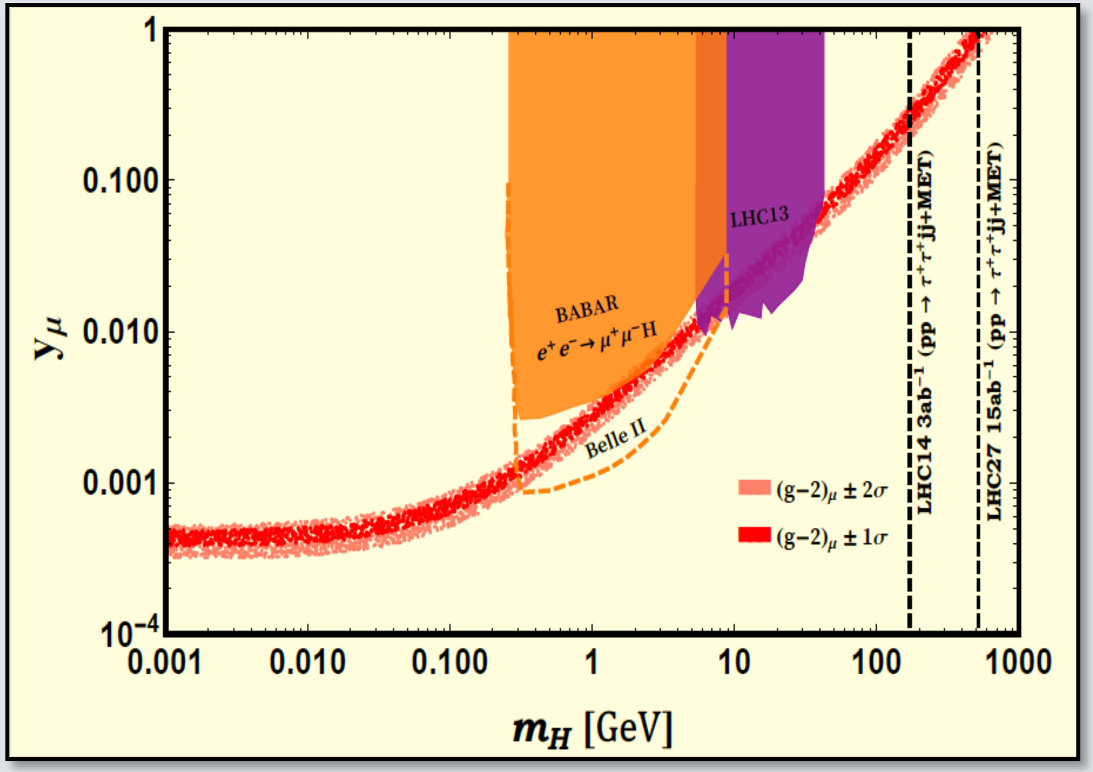
Many possible explanations in different contexts are available in the literature.

Possible Explanations in different contexts..



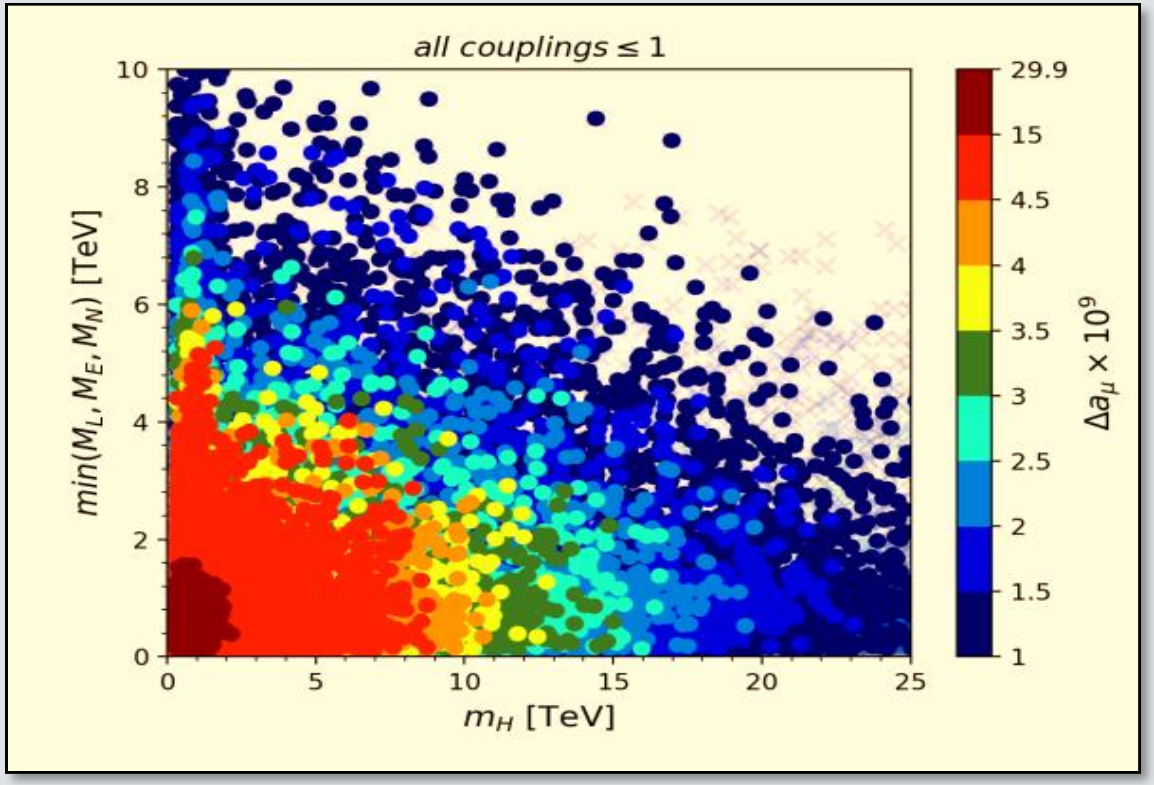
$(g\text{-}2)_\mu$ Anomaly and Scale of NP

Below a GeV Scale



S. Jana, VPK, S. Saad (2020)

Above a TeV Scale

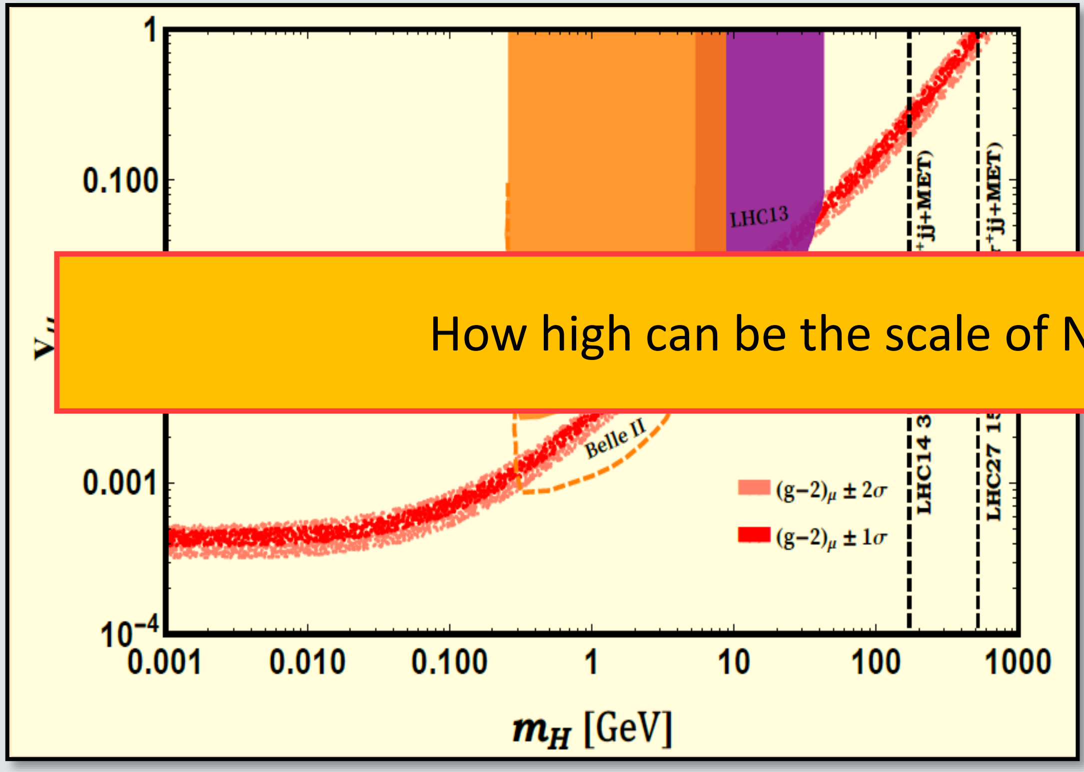


R. Dermisek, K. Hermanek, N. McGinnis (2021)

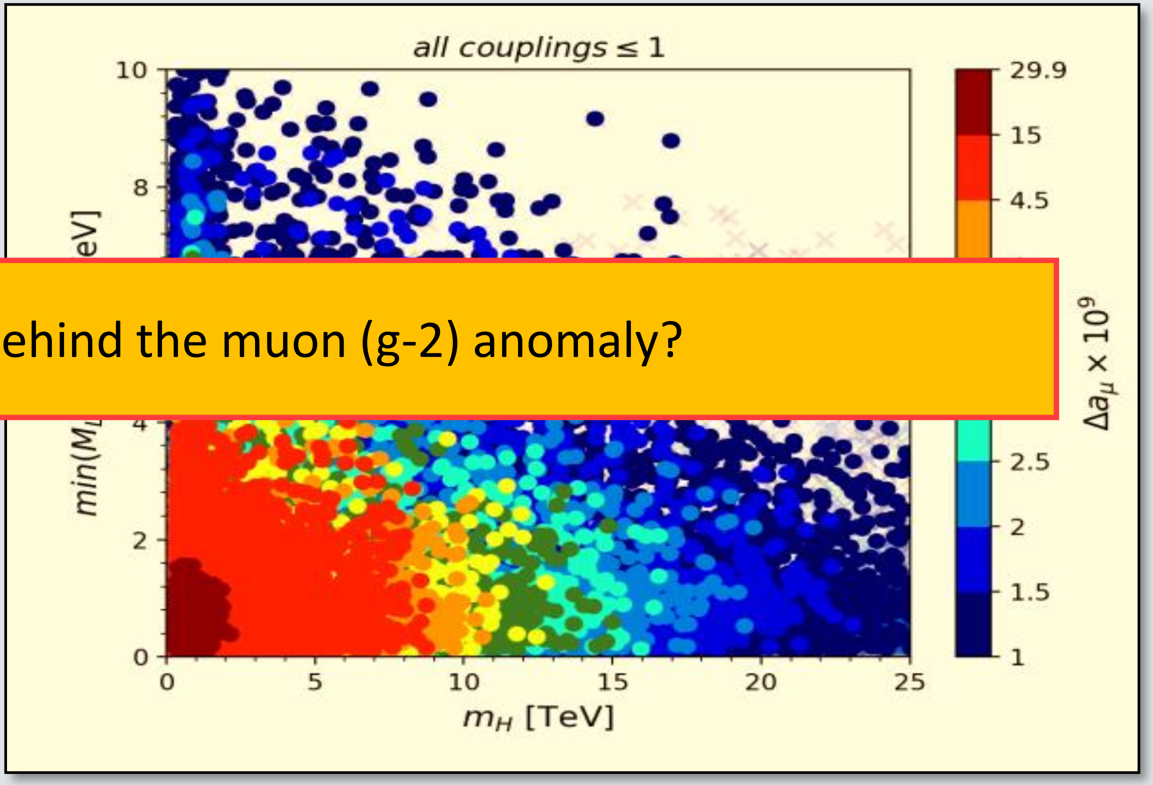
$(g\text{-}2)_\mu$ Anomaly and Scale of NP

Below a GeV Scale

Above a TeV Scale



S. Jana, VPK, S. Saad (2020)



R. Dermisek, K. Hermanek, N. McGinnis (2021)

How high can be the scale of NP behind the muon (g-2) anomaly?

$(g\text{-}2)_\mu$ Anomaly and Scale of NP

From the EFT analysis NP scale can be $\mathcal{O}(100)$ TeV*.

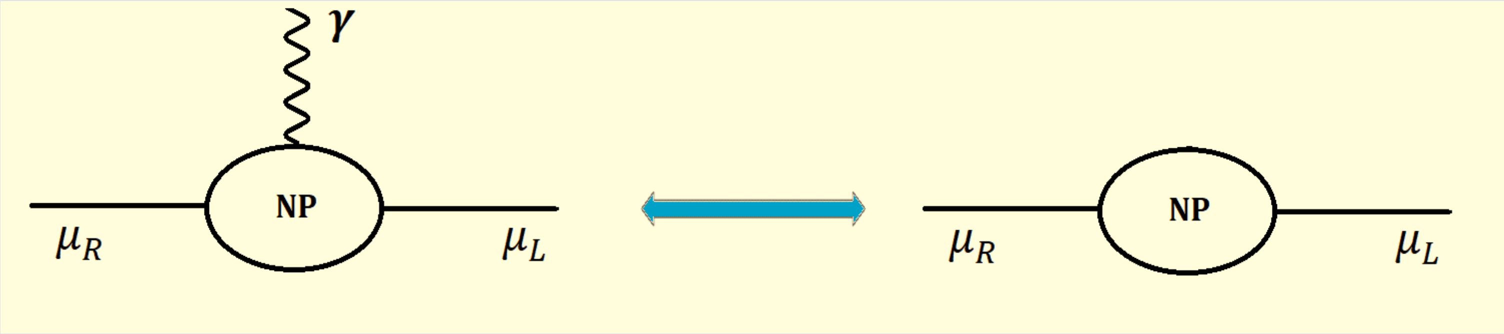
However, in renormalizable theories this is not the case.

*L. Allwicher, L. Di Luzio, M. Fedele, F. Mescia, M. Nardecchia(2021)

Muon Magnetic Moment- Mass Conundrum

Both Muon Magnetic Moment operator and Muon Mass operator are chirality flipping in nature.

In the absence of any additional symmetries (and without fine-tuning) one would expect that any NP contribution to Muon magnetic moment would generate a muon mass term.



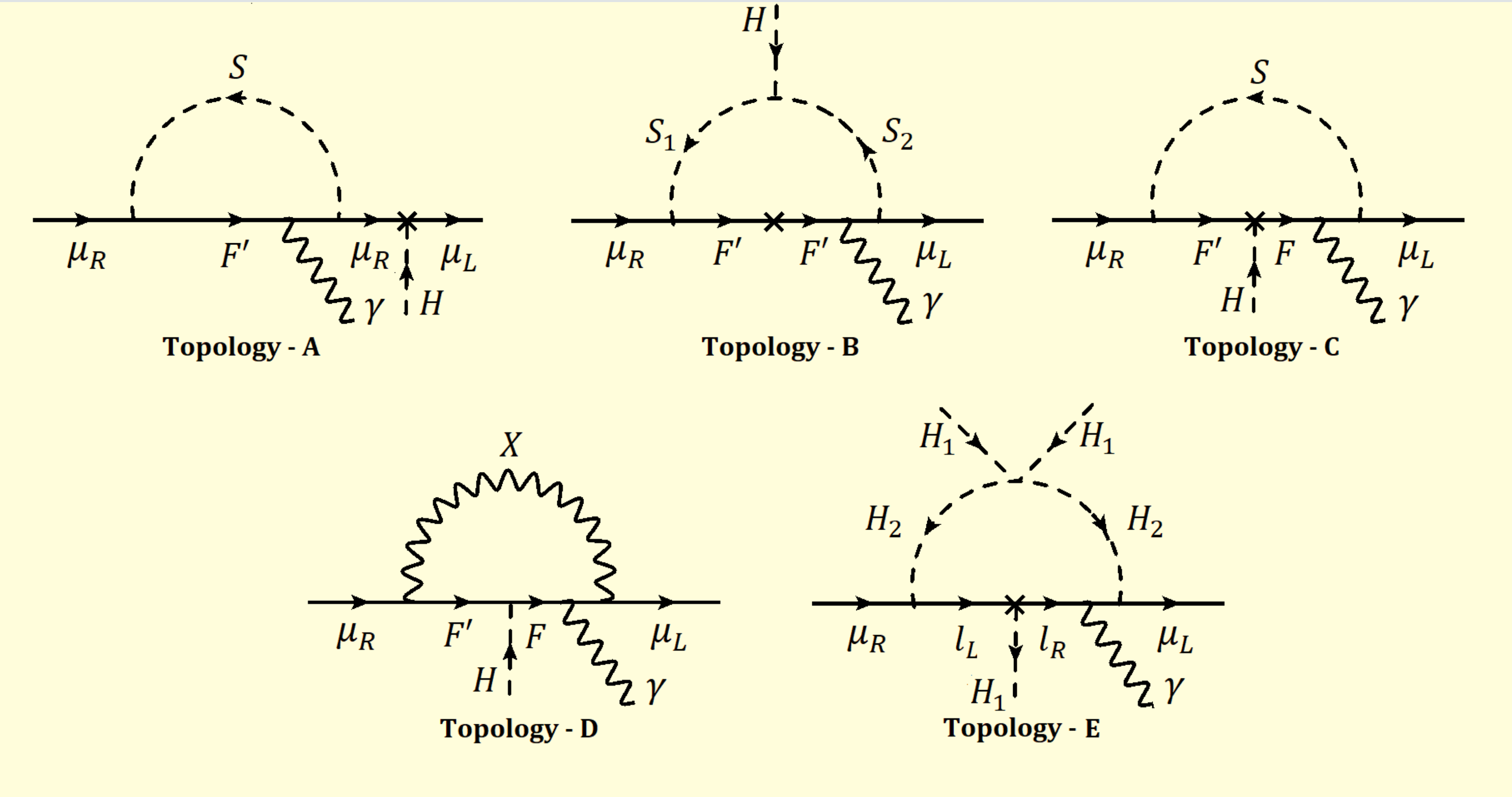
These muon mass corrections constraint the scale of NP.

Model-Exhaustive Analysis

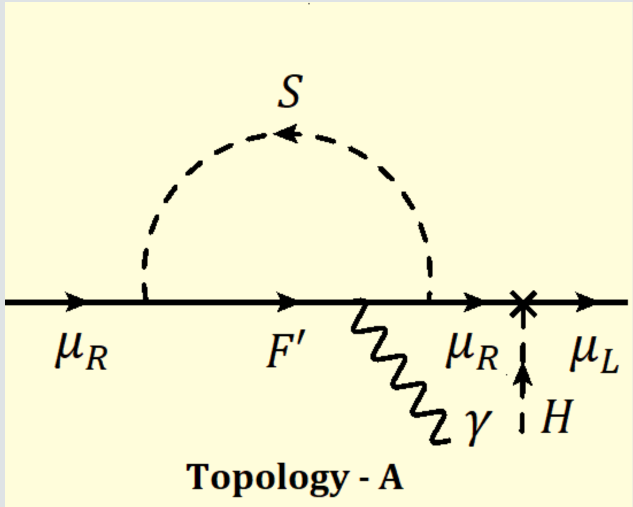
Assume NP contribution arises at one-loop level.

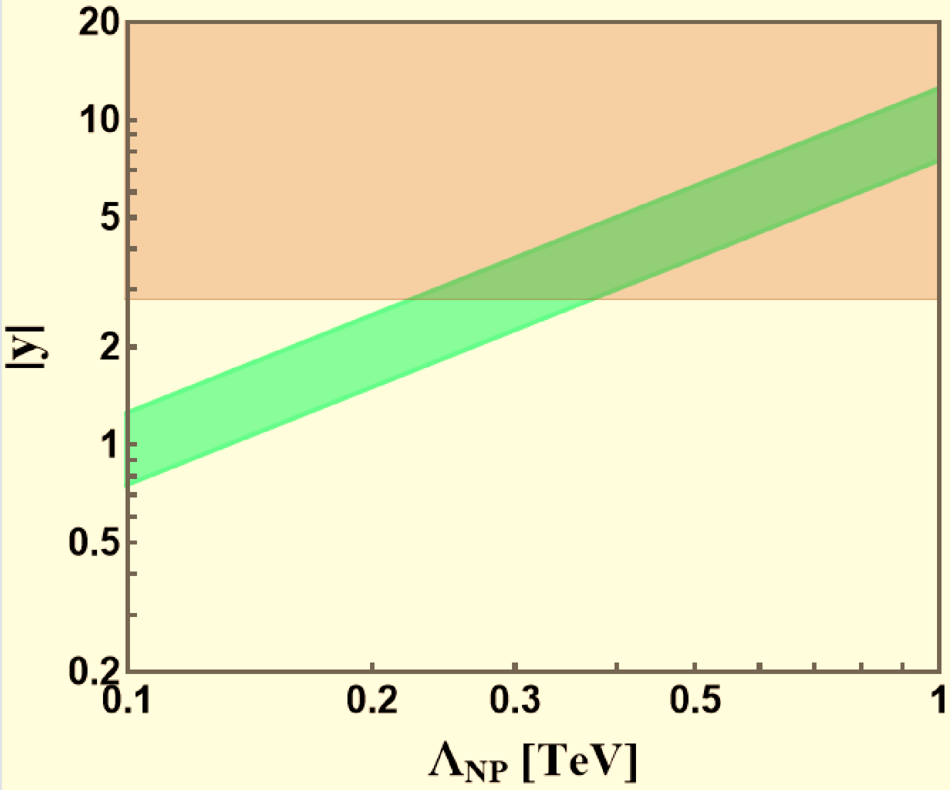
Then we consider all relevant topologies that could contribute to muon (g-2).

One-loop Topologies



Topology A





- Relevant Lagrangian

$$\mathcal{L}_\Lambda \supset -y\bar{\ell}_R S l_L + H.c.$$

- Highest possible mass of the lightest BSM state is:
  - PU constraints: 390 GeV

Topology B

- Class of models: Particle content transform under $SU(2)_L \times U(1)_Y$

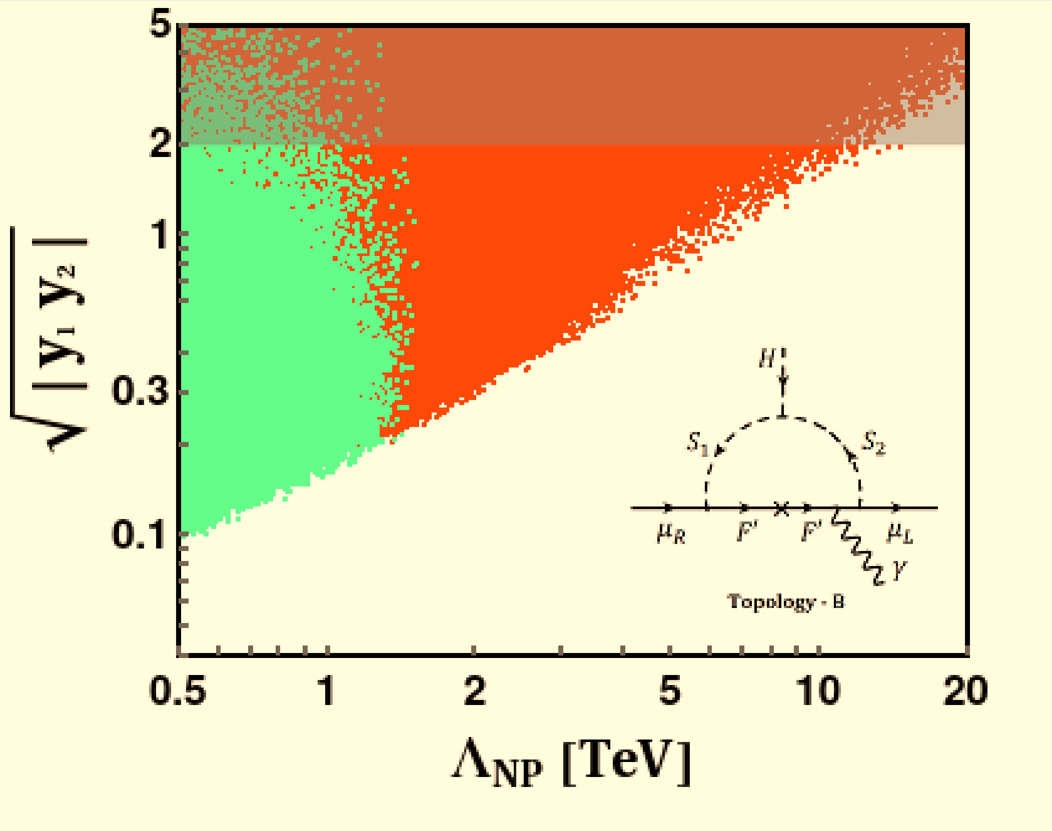
$$F'_{L,R} \sim (1,Q), \quad S_1 \sim (1, Q+1), \quad S_2 \sim (2, Q+\frac{1}{2}).$$

- Relevant Lagrangian

$$\mathcal{L}_B \supset -y_1 \bar{\ell}_R S_1^* F'_L - y_2 \bar{\ell}_L S_2^\dagger F'_R - m_{F'} \bar{F}'_L F'_R - \mu S_1^* H \epsilon S_2 + H.c.$$

- Correction to Muon mass:

$$\Delta m_\mu = \frac{y_1 y_2 \sin 2\alpha}{32\pi^2} (m_{F'}) \left[\frac{m_{H_2}^2}{m_{F'}^2 - m_{H_2}^2} \ln\left(\frac{m_{F'}^2}{m_{H_2}^2}\right) - \frac{m_{H_1}^2}{m_{F'}^2 - m_{H_1}^2} \ln\left(\frac{m_{F'}^2}{m_{H_1}^2}\right) \right].$$



Topology B

- Class of models: Particle content transform under $SU(2)_L \times U(1)_Y$

$$F'_{L,R} \sim (1, Q), \quad S_1 \sim (1, Q+1), \quad S_2 \sim (2, Q+\frac{1}{2}).$$

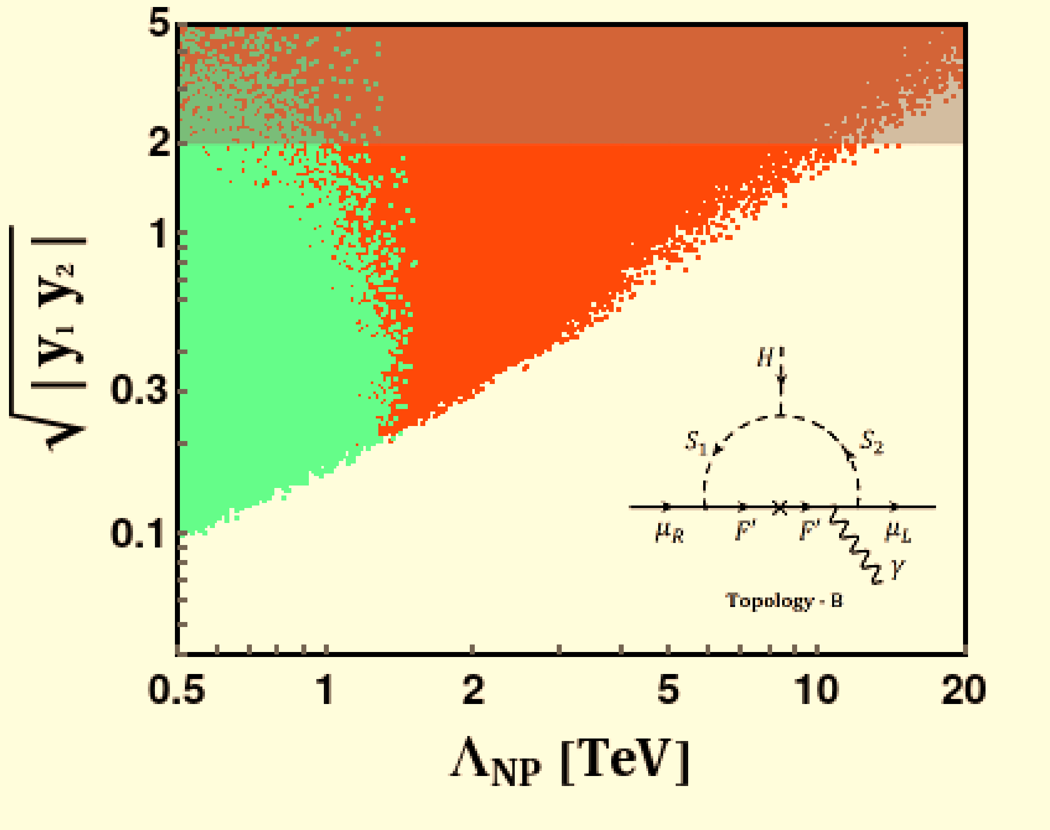
- Relevant Lagrangian

$$\mathcal{L}_B \supset -y_1 \bar{\ell}_R S_1^* F'_L - y_2 \bar{\ell}_L S_2^\dagger F'_R - m_{F'} \bar{F}'_L F'_R - \mu S_1^* H \epsilon S_2 + H.c.$$

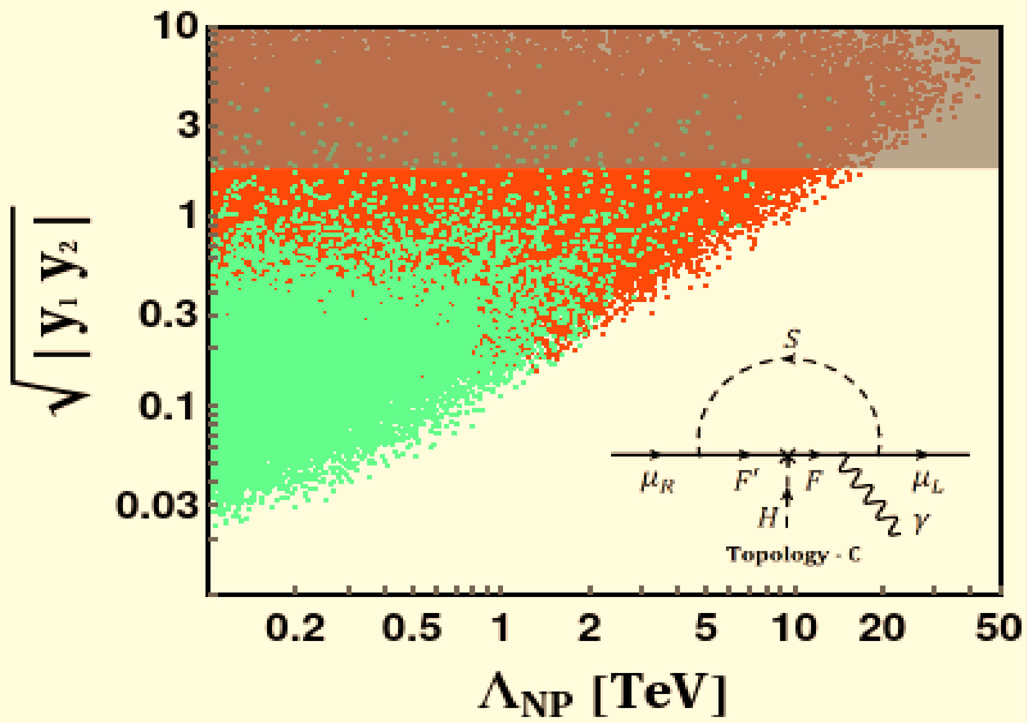
- Correction to Muon mass:

$$\Delta m_\mu = \frac{y_1 y_2 \sin 2\alpha}{32\pi^2} (m_{F'}) \left[\frac{m_{H_2}^2}{m_{F'}^2 - m_{H_2}^2} \ln\left(\frac{m_{F'}^2}{m_{H_2}^2}\right) - \frac{m_{H_1}^2}{m_{F'}^2 - m_{H_1}^2} \ln\left(\frac{m_{F'}^2}{m_{H_1}^2}\right) \right].$$

- Highest possible mass of the lightest BSM state is:
  - PU constraints: 12 TeV
  - PU Constraints + Δm_μ: 1.6 TeV



Topology C and D

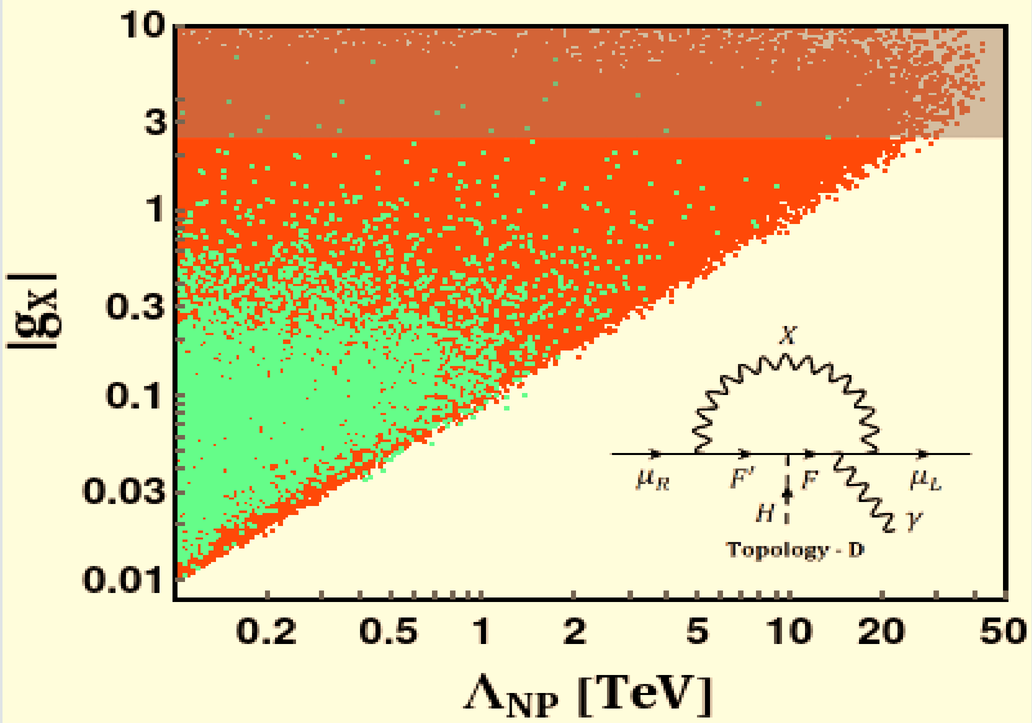


- Particle content transform under $SU(2)_L \times U(1)_Y$

$$F'_{L,R} \sim (1, Q), \quad F_{L,R} \sim (2, Q + \frac{1}{2}), \quad S \sim (1, Q+1).$$

- Relevant Lagrangian

$$\mathcal{L}_C \supset -y_1 \bar{\ell}_R S^* F'_L - y_2 \bar{\ell}_L S^* F_R - m_F \bar{F}_L F_R - m_{F'} \bar{F}'_L F'_R$$
$$- h_1 \bar{F}_R H F'_L - h_2 \bar{F}_L H F'_R + H.c.$$



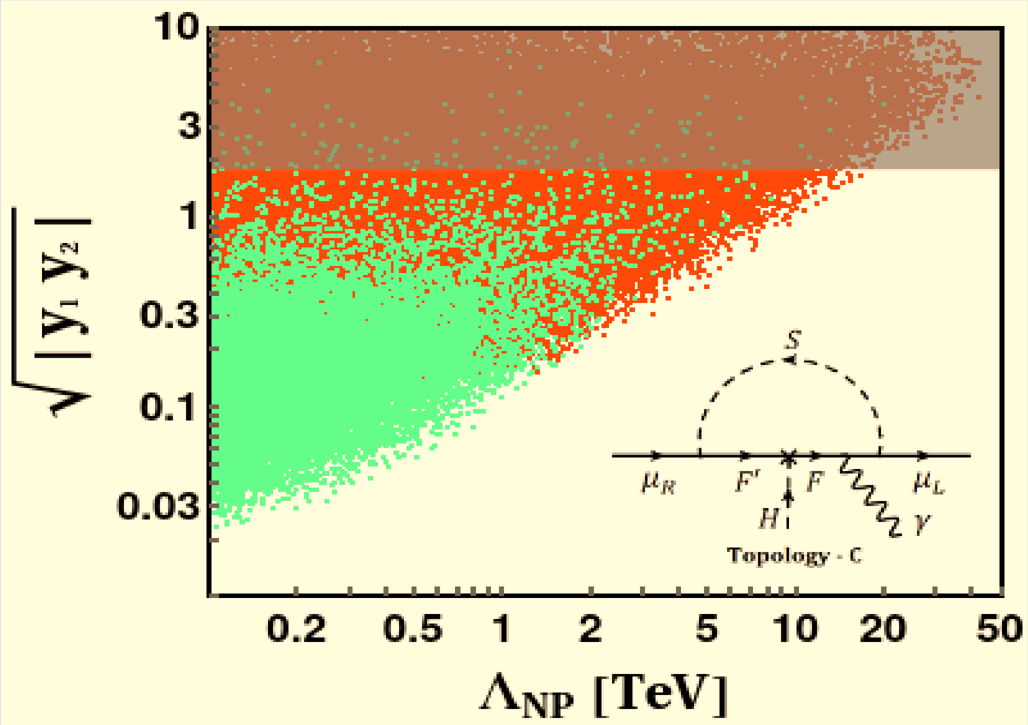
- Particle content transform under $SU(2)_L \times U(1)_Y$

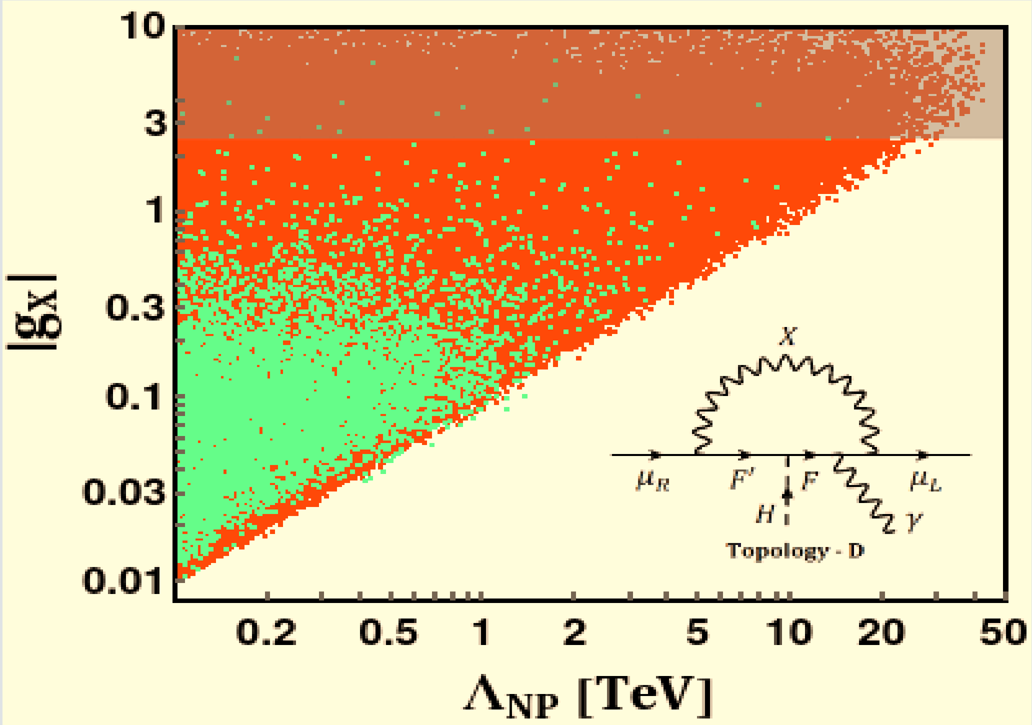
$$F'_{L,R} \sim (1, Q), \quad F_{L,R} \sim (2, Q + \frac{1}{2})$$

- Relevant Lagrangian

$$\mathcal{L}_D \supset g_X \left(\bar{\ell}_R \gamma^\rho F'_R + \bar{\ell}_L \gamma^\rho F_L \right) X_\rho - m_F \bar{F}_L F_R - m_{F'} \bar{F}'_L F'_R$$
$$- h_1 \bar{F}_R H F'_L - h_2 \bar{F}_L H F'_R + H.c.$$

Topology C and D

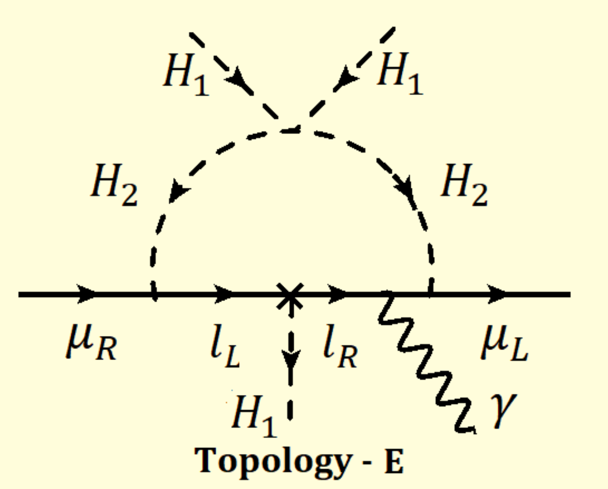


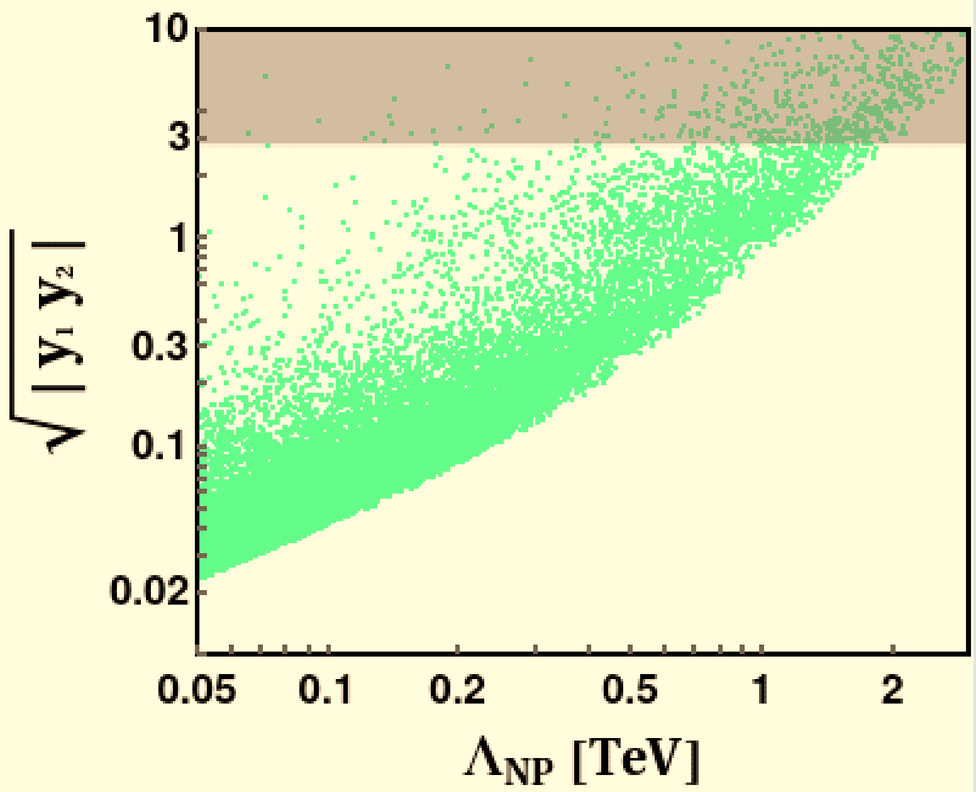


- Highest possible mass of the lightest BSM state is:
  - PU constraints: 19 TeV
  - PU Constraints + Δm_μ: 8 TeV

- Highest possible mass of the lightest BSM state is:
  - PU constraints: 25 TeV
  - PU Constraints + Δm_μ: 8.8 TeV

Topology E





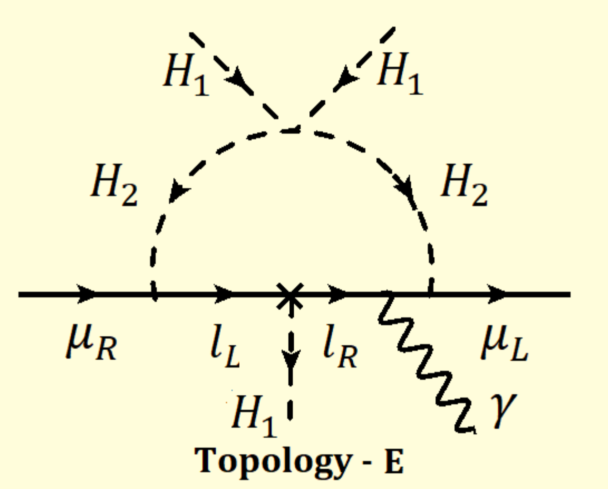
- Two-Higgs-doublet model:

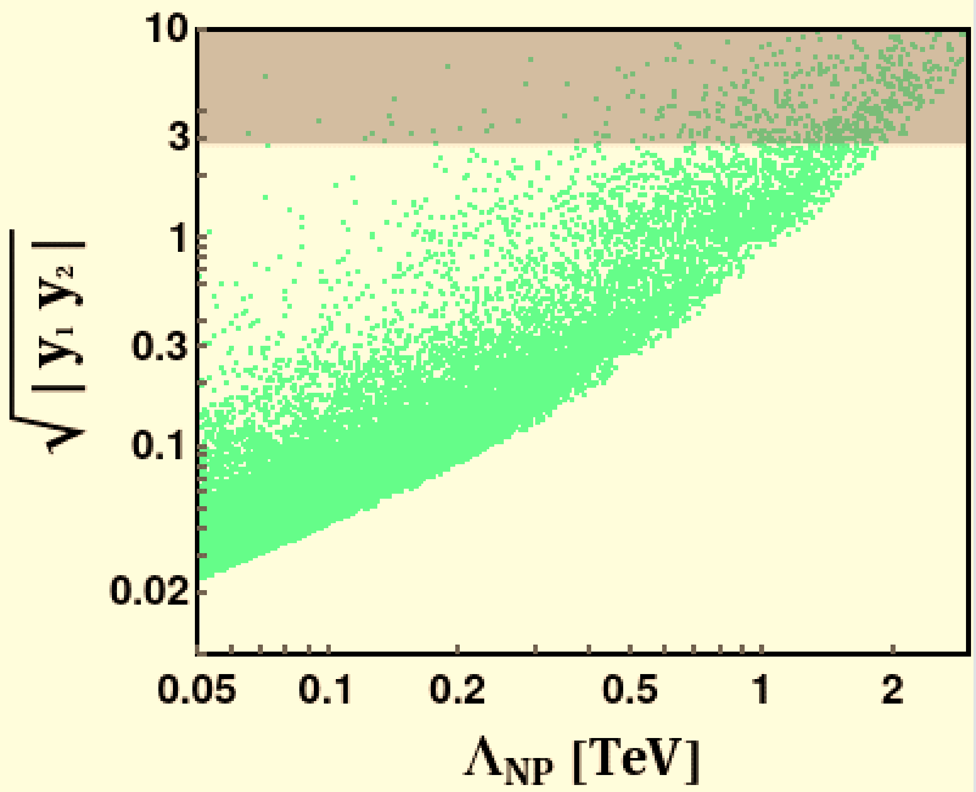
$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi_1^0 + iG^0) \end{pmatrix} \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\phi_2^0 + iA) \end{pmatrix}$$

- Relevant Lagrangian

$$-\mathcal{L}_E \supset Y \bar{\ell}_L H_1 \ell_R + \widetilde{Y} \bar{\ell}_L H_2 \ell_R + h.c.$$

Topology E

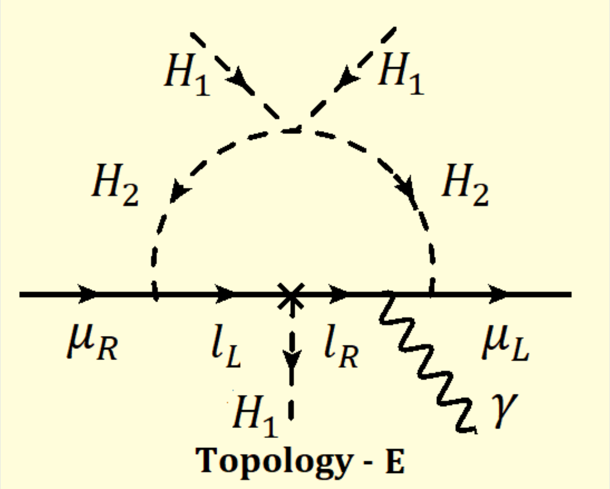


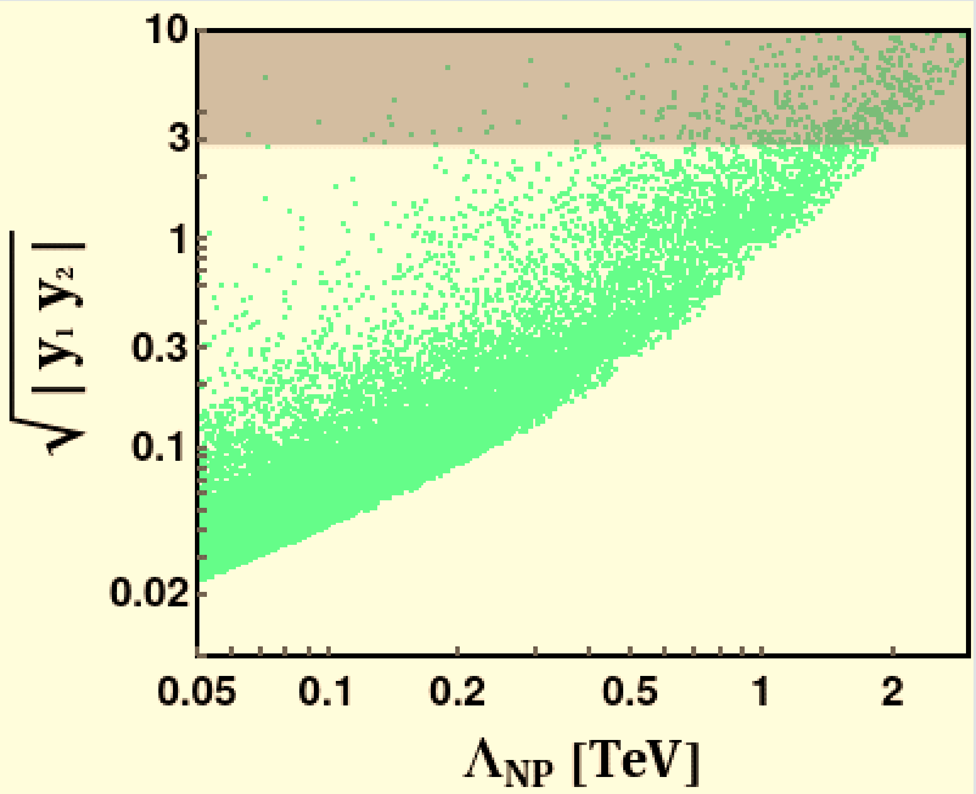


- ❑ Correction to Muon mass:

$$\Delta m_\mu = \frac{y_1 y_2}{32\pi^2}(m_\tau)\left[\frac{m_H^2}{m_\tau^2 - m_H^2}\ln\left(\frac{m_\tau^2}{m_H^2}\right) - \frac{m_A^2}{m_\tau^2 - m_A^2}\ln\left(\frac{m_\tau^2}{m_A^2}\right)\right].$$

Topology E





- Correction to Muon mass:

$$\Delta m_\mu = \frac{y_1 y_2}{32\pi^2}(m_\tau)\left[\frac{m_H^2}{m_\tau^2 - m_H^2}\ln\left(\frac{m_\tau^2}{m_H^2}\right) - \frac{m_A^2}{m_\tau^2 - m_A^2}\ln\left(\frac{m_\tau^2}{m_A^2}\right)\right].$$

- Highest possible mass of the lightest BSM state is:

  - PU Constraints + Δm_μ: 1.9 TeV

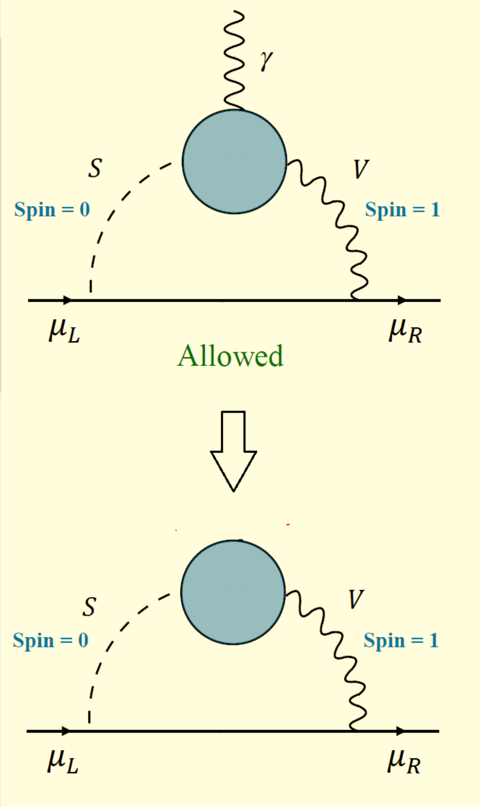
One-loop Topologies: Summary

	NP Scale (TeV)	
	PU	PU + Δm_μ
Topology: A	**0.4**	**0.4**
Topology: B	**12**	**1.6**
Topology: C	**19**	**8**
Topology: D	**25**	**8.8**
Topology: E	**1.9**	**1.9**

Decoupling Muon Magnetic Moment from its Mass: Spin Symmetry Mechanism*

In renormalizable gauge theories there are no direct couplings of the type ***γVS*** where ***S*** is a scalar field, and ***V*** is a gauge boson field.

However, such a coupling could be generated via loops. At the two-loop level, this vertex could contribute to muon (g-2).



*Originally proposed in the context of neutrino magnetic moments by *Barr, Freire, and Zee* (1990)

Decoupling Muon Magnetic Moment from its Mass: Spin Symmetry Mechanism*

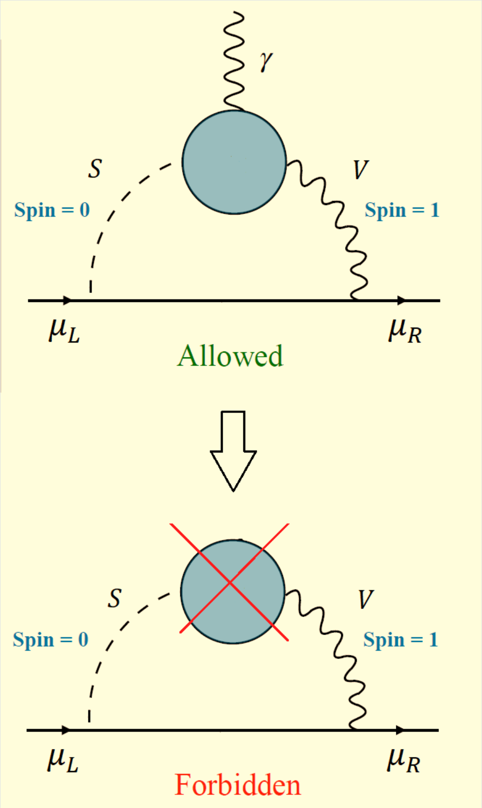
In renormalizable gauge theories there are no direct couplings of the type ***γVS*** where ***S*** is a scalar field, and ***V*** is a gauge boson field.

However, such a coupling could be generated via loops. At the two-loop level, this vertex could contribute to muon (g-2).

As for its contribution to Δm_μ, it is well known that for transversely polarized vector bosons, the transition from spin 1 to spin 0 cannot occur.

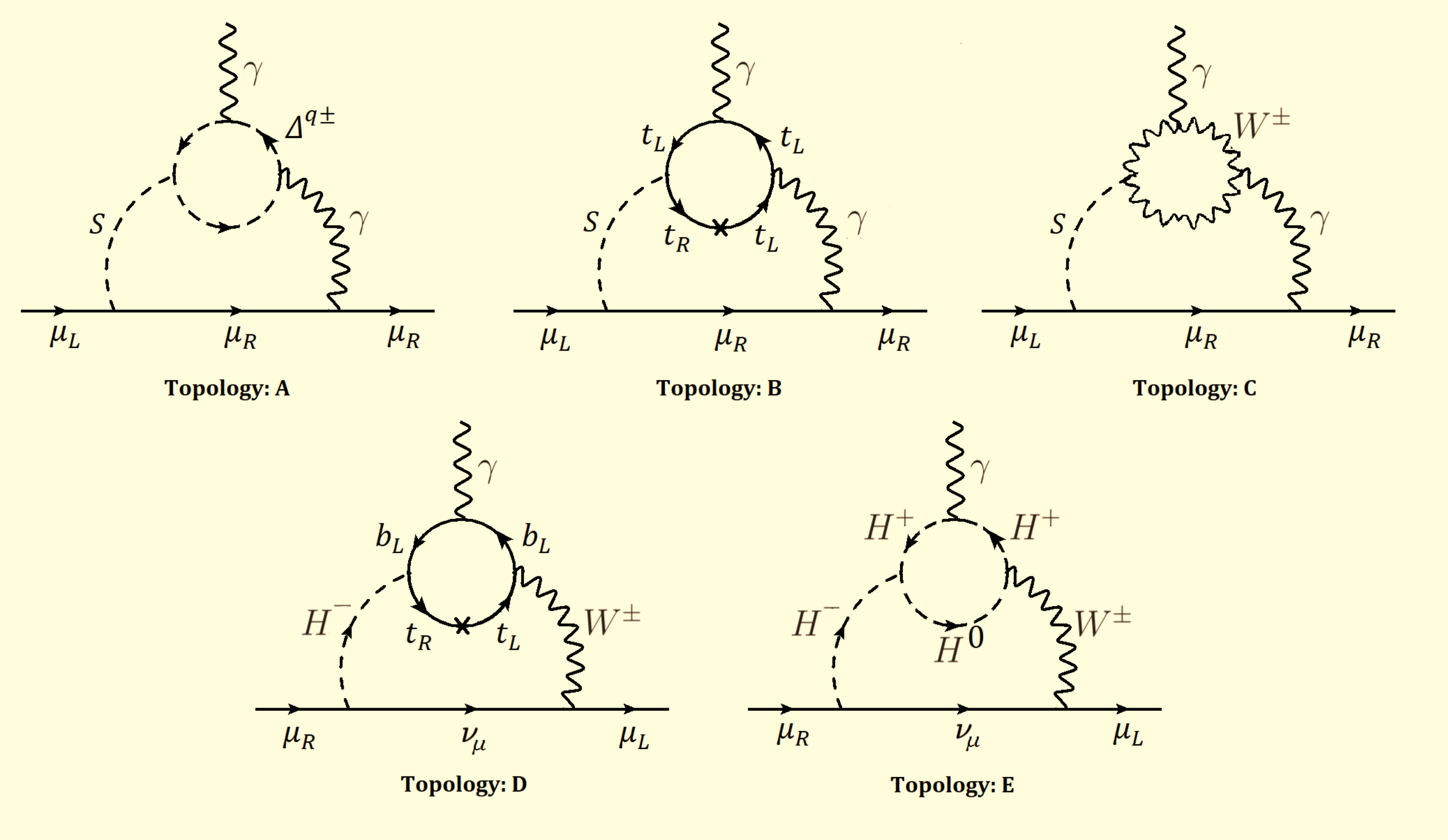
Only the longitudinal mode, the Goldstone mode, would contribute to such transitions.

This implies that in the two-loop diagram utilizing the $\gamma W^+ S^-$ for generating Δa_μ , if the photon line is removed, only the longitudinal mode of $W^\pm$ bosons will contribute, leading to a suppression factor of m_l^2/m_W^2 in the muon mass.

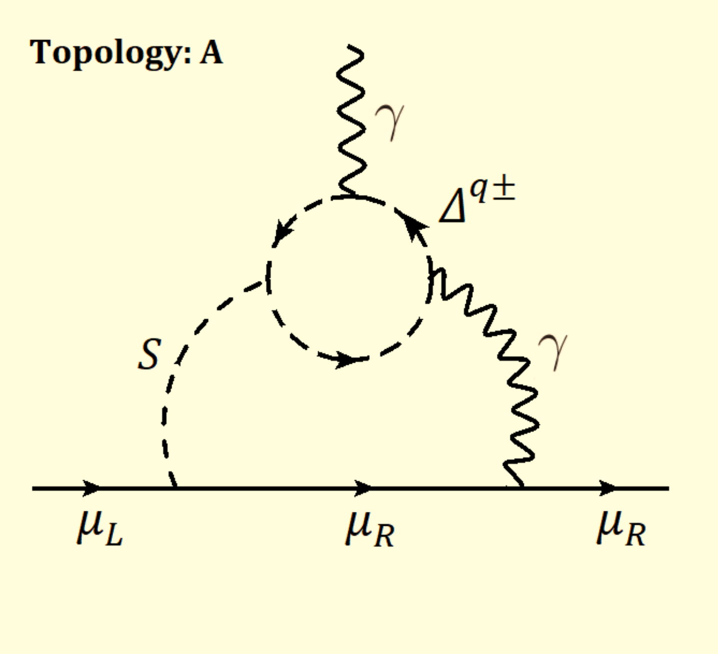


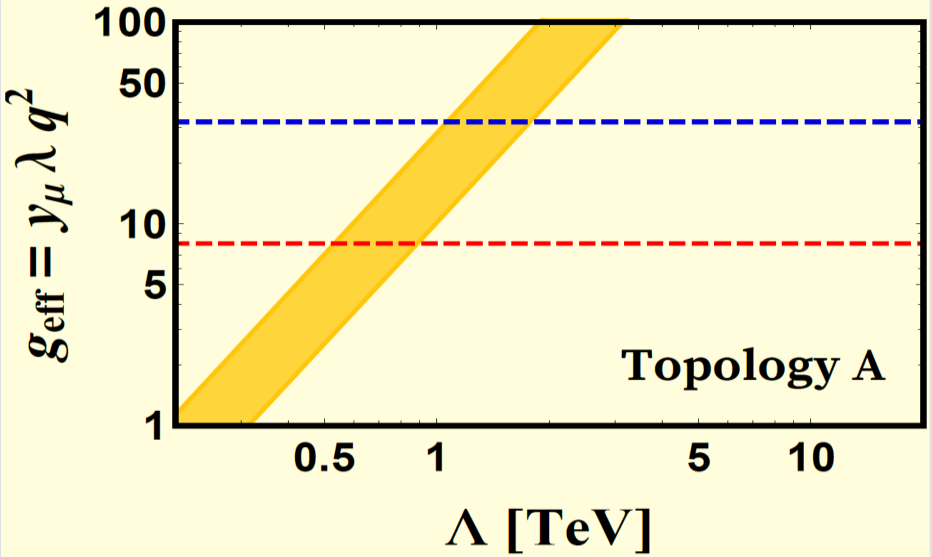
*Originally proposed in the context of neutrino magnetic moments by *Barr, Freire, and Zee* (1990)

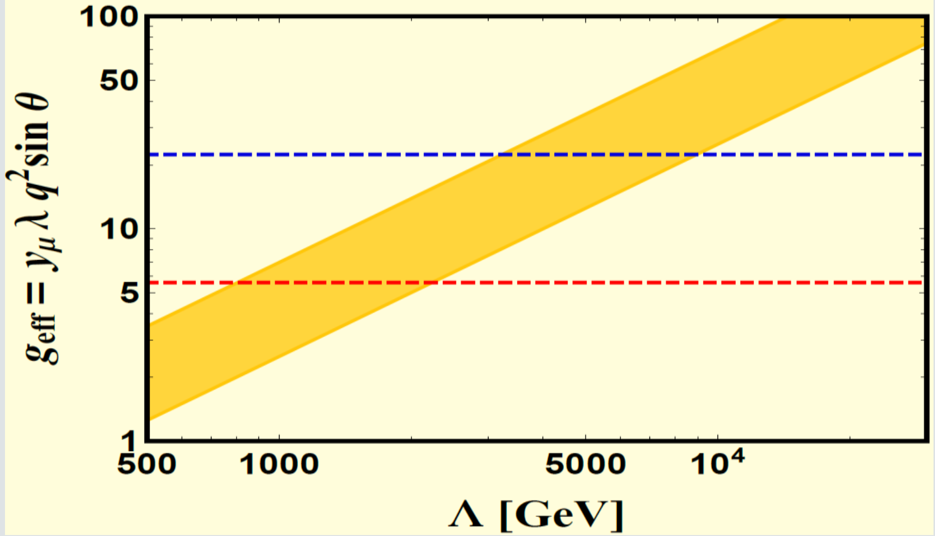
Spin Symmetry Mechanism: Two-loop topologies



Topology A

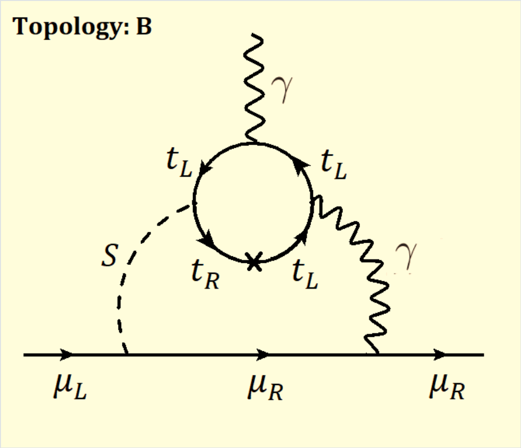


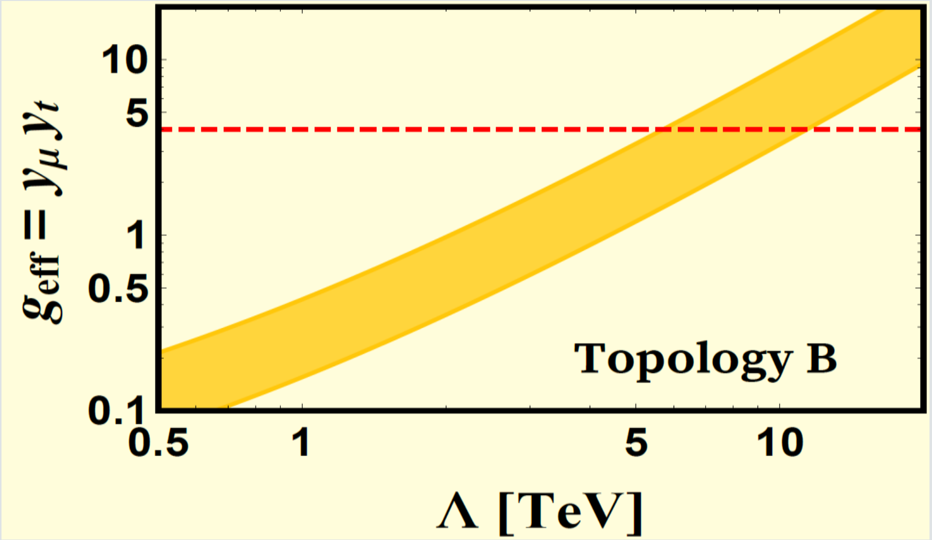




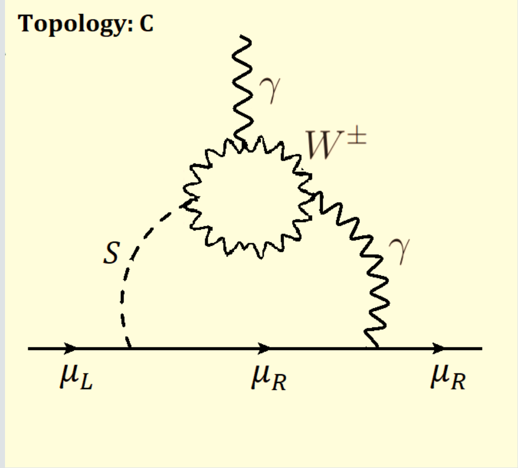
- Highest possible mass of the lightest BSM state is:
  - PU Constraints : 0.9 TeV
    : 2.2 TeV

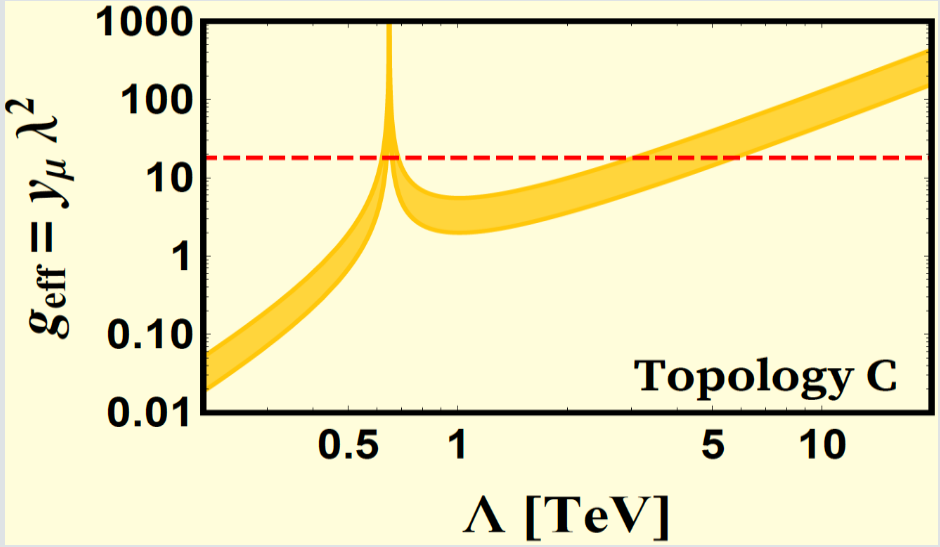
Topology B and C





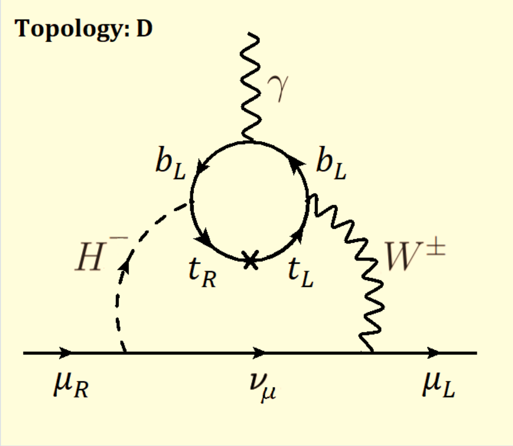
- Highest possible mass of the lightest BSM state is:
  - PU Constraints : 10 TeV

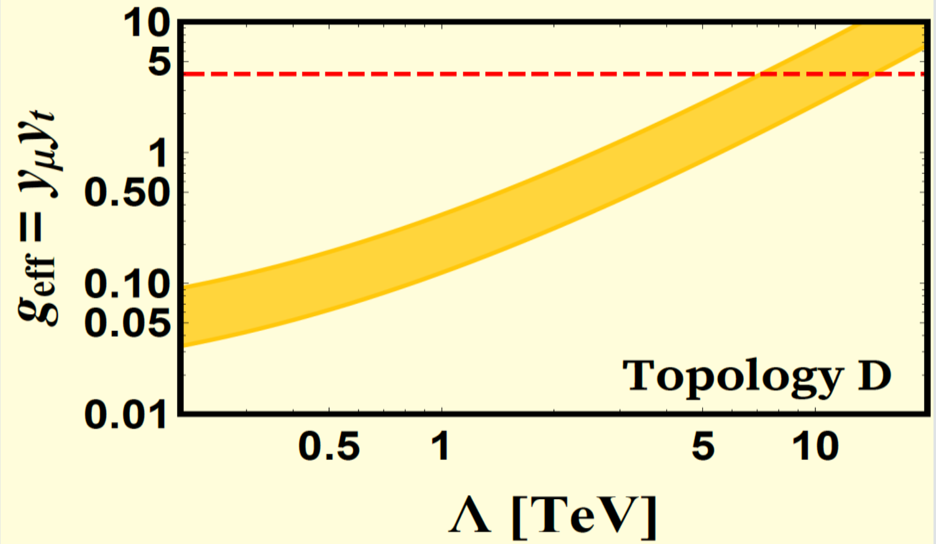




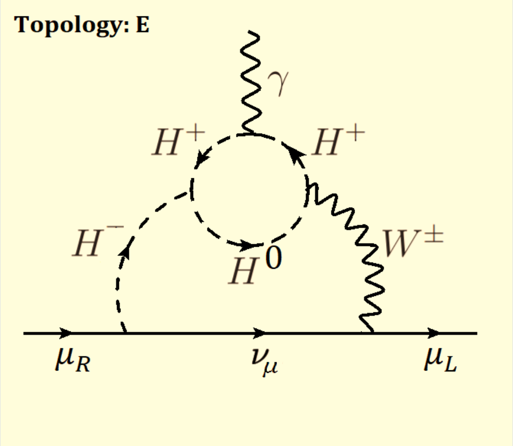
- Highest possible mass of the lightest BSM state is:
  - PU Constraints : 5.5 TeV

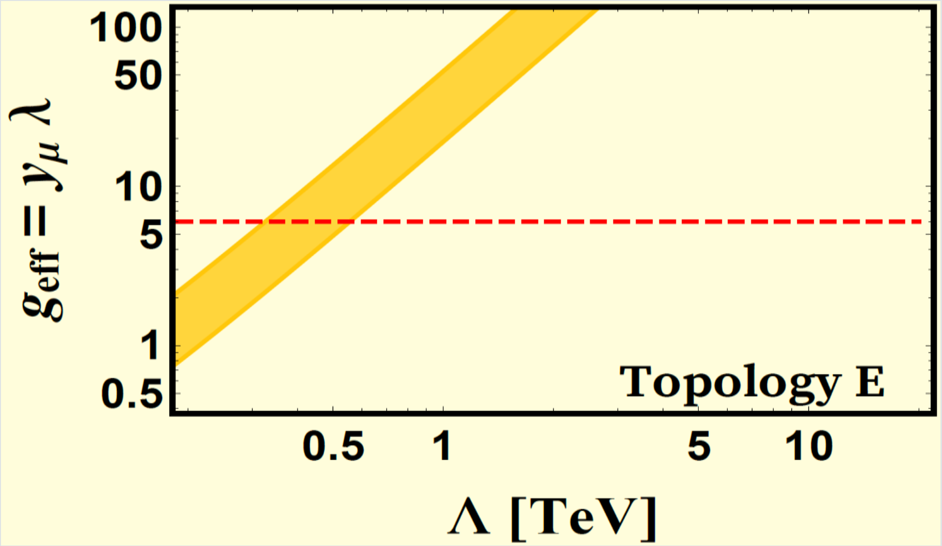
Topology D and E





- Highest possible mass of the lightest BSM state is:
  - PU Constraints : 11 TeV





- Highest possible mass of the lightest BSM state is:
  - PU Constraints : 0.5 TeV

Decoupling Muon Magnetic Moment from its Mass: Voloshin –type Symmetry

While the muon mass operator and magnetic operator both are chirality flipping, there is one important difference in their Lorentz structures.

The mass operator, being a Lorentz scalar, is symmetric, while the magnetic moment, being a Lorentz tensor operator is antisymmetric in the two fermion fields.

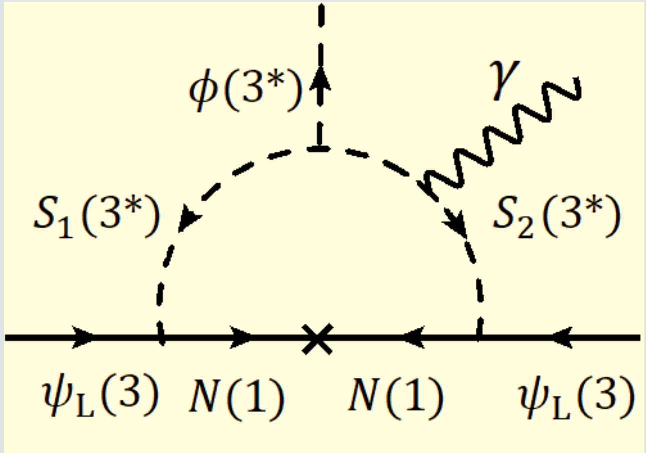
Under a symmetry which interchange $\boldsymbol{\mu}_L \longrightarrow (\boldsymbol{\mu}_R)^c$ and $\boldsymbol{\mu}_R \longrightarrow -(\boldsymbol{\mu}_L)^c$

muon mass is forbidden, while the muon magnetic moment operator is invariant.

However, for incorporating this idea into an UV-complete theory, $(\boldsymbol{\mu}_R)^c$ needs to be in the same multiplet with $\boldsymbol{\mu}_L$.

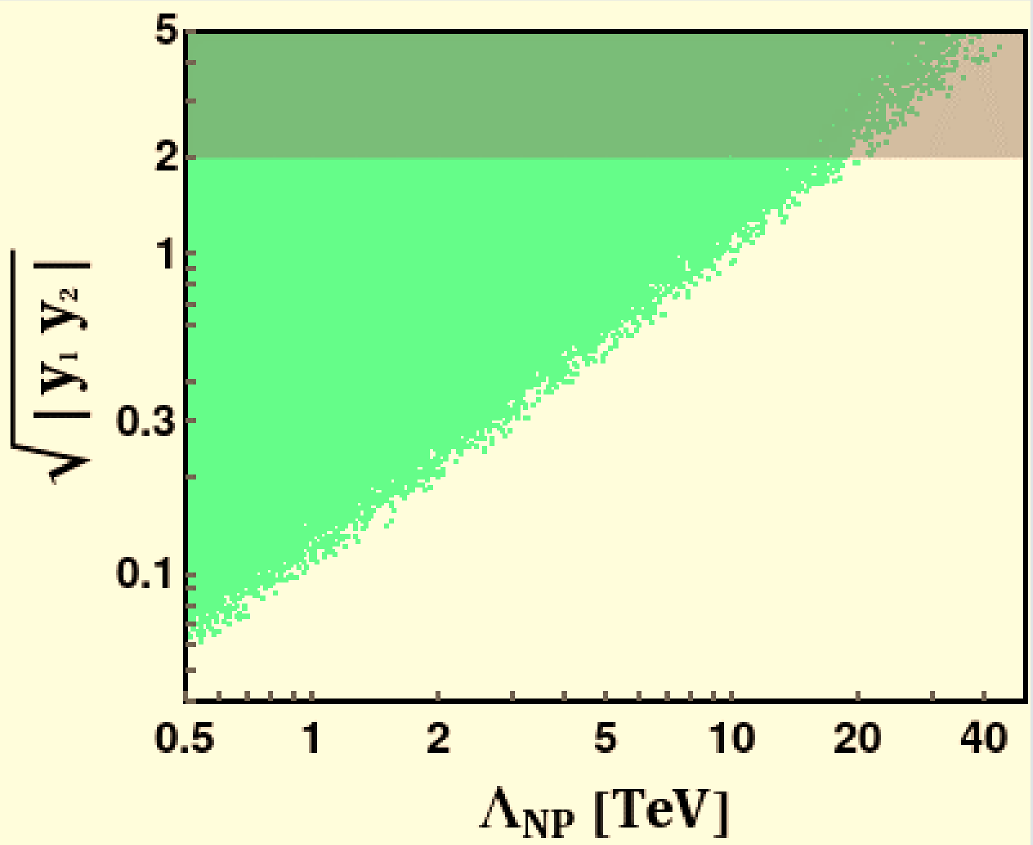
The simplest possible implementation is to enlarge the EW gauge symmetry to $SU(3)_L \times U(1)_X$.

Decoupling Muon Magnetic Moment from its Mass: Voloshin –type Symmetry

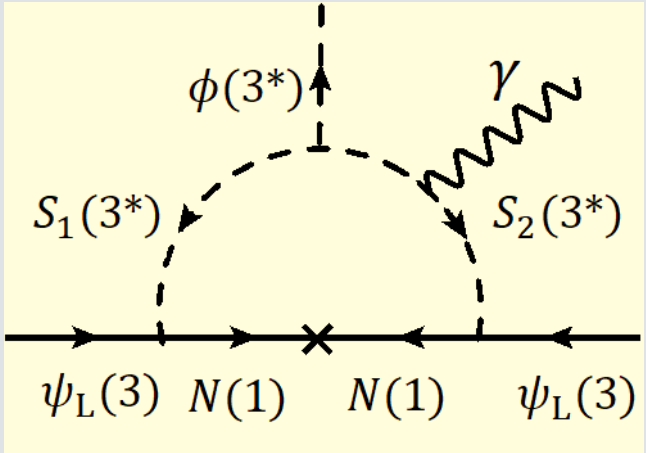


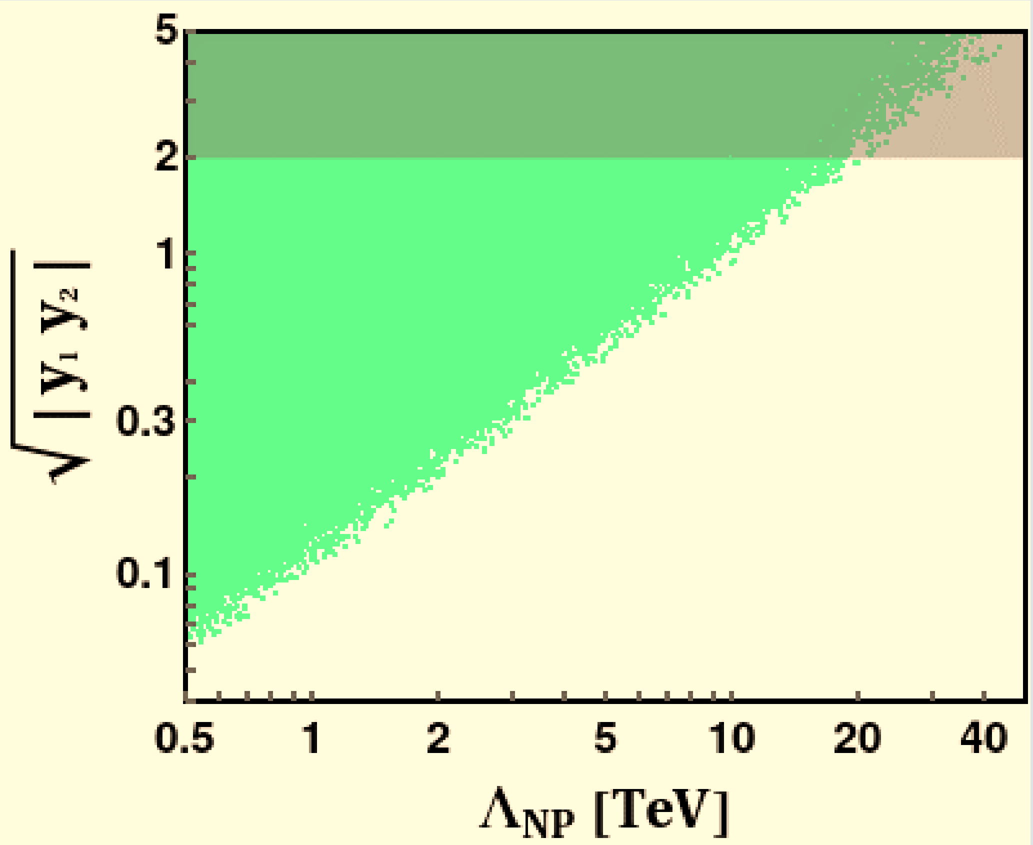
$$\psi_L = \begin{pmatrix} \nu \\ \mu \\ \mu^c \end{pmatrix} \qquad N$$

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^+ \\ \phi^- \end{pmatrix} \qquad S_{1,2} = \begin{pmatrix} S^0_{1,2} \\ S^+_{1,2} \\ S^-_{1,2} \end{pmatrix}$$



Decoupling Muon Magnetic Moment from its Mass: Voloshin –type Symmetry





- ❑ Highest possible mass of the lightest BSM state is:
  - ❑ PU constraints: 20 TeV

Conclusions

- We studied the constraints imposed by the muon mass corrections on the scale of NP interpretation of muon (g-2) anomaly.

- In the absence of any additional symmetries (and without severe fine-tuning) the NP scale can be as large as 9 TeV.

- We investigated a spin symmetry mechanism that can contribute to muon magnetic moment while keeping the muon mass correction to be small.

- We have proposed a simplified model based on $SU(3)_L \times U(1)_X$ gauge symmetry that can contribute to muon magnetic moment without inducing large correction to the muon mass. In this setup the NP scale can be as large as 20 TeV.

Thank You !