Resolving discrepancies in anomalous magnetic moment in the Zee model

Phenomenology 2022 Symposium

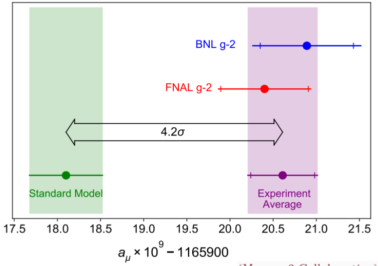
Ritu Dcruz[1]
In collaboration with Rahool K. Barman[1], Anil Thapa[2]

[1] Oklahoma State University

[2] University of Virginia

[JHEP03(2022)183]

Anomalous Magnetic Moment

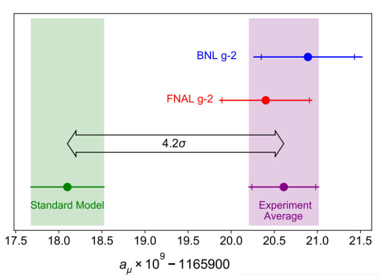


(Muon g-2 Collaboration)

- Discrepancy in $a_\ell = (g_\ell - 2)/2$
- $\Delta a_\mu = (251 \pm 59) \times 10^{-11}$ @ 4.2σ
- $\Delta a_e = (-8.8 \pm 3.6) \times 10^{-13}$ @ 2.4σ
- Opposite signs makes simultaneous explanation difficult

Motivation

Anomalous Magnetic Moment



(Muon g-2 Collaboration)

- Discrepancy in $a_\ell = (g_\ell - 2)/2$
- $\Delta a_\mu = (251 \pm 59) \times 10^{-11}$ @ 4.2σ
- $\Delta a_e = (-8.8 \pm 3.6) \times 10^{-13}$ @ 2.4σ
- Opposite signs makes simultaneous explanation difficult

Zee Model

(Zee, 1980)

- Tiny neutrino mass realized at one loop
- Extra doublet scalar and singly-charged scalar
- New doublet can give corrections to a_ℓ
- Charged scalars can induce potentially large diagonal NSI

(Babu, Dev, Jana, and Thapa, 2020)

Model Description

- THDM (Lee, 1973)+ $\eta^{\pm}$
- Higgs basis:

(Davidson and Haber, 2005)

$$\mathrm{H}_1 = \begin{pmatrix} \mathrm{G}^+ \\ \frac{1}{\sqrt{2}}\left(\mathrm{v} + \mathrm{H}_1^0 + i\mathrm{G}^0\right) \end{pmatrix}, \quad \mathrm{H}_2 = \begin{pmatrix} \mathrm{H}_2^+ \\ \frac{1}{\sqrt{2}}\left(\mathrm{H}_2^0 + i\mathrm{A}\right) \end{pmatrix}$$

- Physical fields:

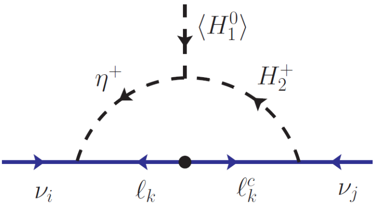
$$\begin{aligned} &\mathrm{A} && \\ &\mathrm{h},\ \mathrm{H} && \sin 2\widetilde{\alpha} = 2\lambda_6 \mathrm{v}^2/(\mathrm{m}_\mathrm{H}^2 - \mathrm{m}_\mathrm{h}^2) \\ &\mathrm{h}^+,\ \mathrm{H}^+ && \sin 2\varphi = -\sqrt{2}\mathrm{v}\mu/(\mathrm{m}_{\mathrm{H}^+}^2 - \mathrm{m}_{\mathrm{h}^+}^2) \end{aligned}$$

- Alignment limit: $\widetilde{\alpha} \to 0$

(Gunion and Haber, 2003)

- $-\mathcal{L}_\mathrm{Y} \supset \mathrm{f}_{\mathrm{ij}} \mathrm{L}_\mathrm{i}^\alpha \mathrm{L}_\mathrm{j}^\beta \epsilon_{\alpha\beta} \eta^+ + \mathrm{Y}_{\mathrm{ij}} \widetilde{\mathrm{H}}_2^\alpha \mathrm{L}_\mathrm{i}^\beta \ell_\mathrm{j}^c \epsilon_{\alpha\beta} +$ h.c.
- μ: coefficient of $\mathrm{H}_1^\alpha \mathrm{H}_2^\beta \epsilon_{\alpha\beta} \eta^-$; breaks the lepton number by two units

Neutrino mass



$$M_\nu = \kappa \left(f M_\ell Y + Y^T M_\ell f^T \right)$$

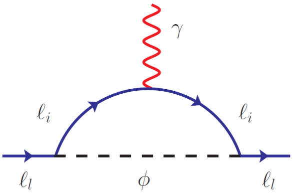
where, κ is the one-loop factor: $\kappa = \frac{1}{16\pi^2} \sin 2\varphi \log \left(\frac{m_{h^+}^2}{m_{H^+}^2} \right)$

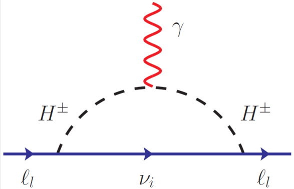
$$\left.\begin{array}{r} Y \ll 1 \text{ cannot resolve } \Delta a_\ell; \\ f \sim \mathcal{O}(1) \text{ cLFUV modifies } G_F \\ \text{cLFV restricts NSI from f to } \leq 10^{-8} \end{array}\right\} \boxed{Y \sim \mathcal{O}(1),\ f \ll 1}$$

(Herrero-García, Ohlsson, Riad and Wirén, 2017)

One-loop correction

(Leveille, 1978)

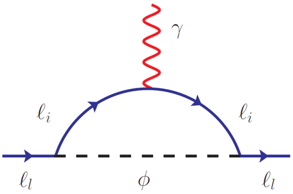


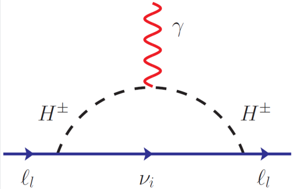


- $\Delta a_\ell^{(1)}(H^+) = \frac{-1}{96\pi^2}\frac{m_\ell^2}{m_{H^+}^2}|Y_{i\ell}|^2 \Rightarrow$ always -ve
- $\Delta a_\ell^{(1)}(\phi) = \frac{1}{16\pi^2}\frac{m_\ell^2}{m_\phi^2}\left(\frac{|Y_{\ell i}|^2+|Y_{i\ell}|^2}{6} \mp 2\frac{m_i}{m_\ell}\left(\frac{3}{4}+\log\left(\frac{m_i}{m_\phi}\right)\right)\Re(Y_{\ell i}Y_{i\ell})\right)$

One-loop correction

(Leveille, 1978)





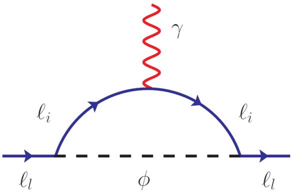
- $\Delta a_\ell^{(1)}(H^+) = \frac{-1}{96\pi^2}\frac{m_\ell^2}{m_{H^+}^2}|Y_{i\ell}|^2 \Rightarrow$ always -ve
- $\Delta a_\ell^{(1)}(\phi) = \frac{1}{16\pi^2}\frac{m_\ell^2}{m_\phi^2}\left(\frac{|Y_{\ell i}|^2+|Y_{i\ell}|^2}{6} \mp 2\frac{m_i}{m_\ell}\left(\frac{3}{4}+\log\left(\frac{m_i}{m_\phi}\right)\right)\Re(Y_{\ell i}Y_{i\ell})\right)$

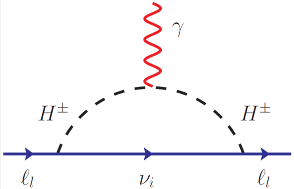
$$\Delta a_\mu \Rightarrow \begin{pmatrix} . & Y_{e\mu} & . \\ Y_{\mu e} & Y_{\mu\mu} & Y_{\mu\tau} \\ . & Y_{\tau\mu} & . \end{pmatrix} + \text{h.c.}, \qquad \Delta a_e \Rightarrow \begin{pmatrix} Y_{ee} & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & . & . \\ Y_{\tau e} & . & . \end{pmatrix} + \text{h.c.}.$$

AMM in Zee Model

One-loop correction

(Leveille, 1978)

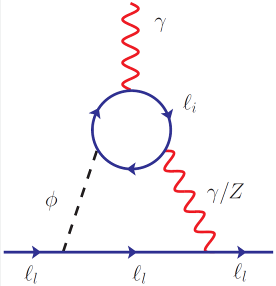




- $\Delta a_\ell^{(1)}(H^+) = \frac{-1}{96\pi^2}\frac{m_\ell^2}{m_{H^+}^2}|Y_{i\ell}|^2 \Rightarrow$ always -ve
- $\Delta a_\ell^{(1)}(\phi) = \frac{1}{16\pi^2}\frac{m_\ell^2}{m_\phi^2}\left(\frac{|Y_{\ell i}|^2+|Y_{i\ell}|^2}{6} \mp 2\frac{m_i}{m_\ell}\left(\frac{3}{4}+\log\left(\frac{m_i}{m_\phi}\right)\right)\Re(Y_{\ell i}Y_{i\ell})\right)$

$$\Delta a_\mu \Rightarrow \begin{pmatrix} . & Y_{e\mu} & . \\ Y_{\mu e} & Y_{\mu\mu} & Y_{\mu\tau} \\ . & Y_{\tau\mu} & . \end{pmatrix} + \text{h.c.}, \qquad \Delta a_e \Rightarrow \begin{pmatrix} Y_{ee} & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & . & . \\ Y_{\tau e} & . & . \end{pmatrix} + \text{h.c.}.$$

Two-loop correction



(Ilisie, 2015)

- $\Delta a_\ell^{(2)} = \frac{\alpha_{\rm em}}{8\pi^3}\frac{m_\ell}{m_i}\frac{m_i^2}{m_\phi^2}\mathcal{F}\left(Y_{ii}Y_{\ell\ell}, \frac{m_i^2}{m_\phi^2}\right)$
- For couplings of same sign
  - $\phi = H \Rightarrow \mathcal{F} < 0$
  - $\phi = A \Rightarrow \mathcal{F} > 0$
- Chiral enhancement from τ lepton loop

$m_H < (m_A, m_{H^+}, m_{h^+})$ and $Y \sim \mathcal{O}(1)$

- One-loop correction
  - Real couplings
    - $Y_{ii}(i = e, \mu) \Rightarrow$ always +ve
    - Y_{ij} with relative negative sign $\Rightarrow$ constraints from LFV ✓
  - Complex couplings $\Rightarrow$ constrained by EDMs
  - $Y_{\mu\mu} \in \mathbb{R} \cap Y_{ee} \in \mathbb{I}$ ✓
- One- + two-loop corrections
  - $\Delta a_\ell^{(2)} \propto \frac{\alpha_{em}}{\pi} \frac{m_i}{m_\ell} \Delta a_\ell^{(1)}$
  - $m_H < m_A$ and $Y_{\tau\tau} \neq 0$ makes two-loop correction $(\propto m_\tau / m_e)$ -ve $\Rightarrow \Delta a_e$
  - Smaller $Y_{\tau\tau}$ can suppress $\Delta a_\mu^{(2)}$ $(\propto m_\tau / m_\mu)$; overall +ve correction

$m_H < (m_A, m_{H^+}, m_{h^+})$ and $Y \sim \mathcal{O}(1)$

- One-loop correction
  - Real couplings

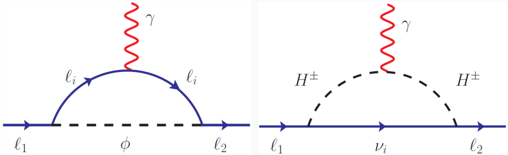
Real couplings Y_{ij} with relative negative sign

- One- + two-loop corrections
  - on ($\propto m_\tau / m_e$) -ve $\Rightarrow \Delta a_e$
  - erall +ve correction

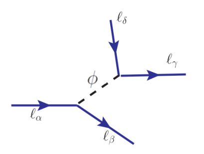
Y_{ii} with appropriate choice of $Y_{\tau\tau}$

Flavor Structures

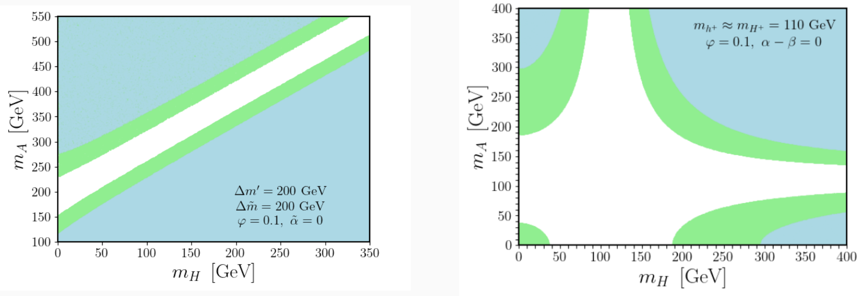
Low energy constraints



Radiative decay of charged leptons



Trilepton decay



T-parameter constraints on mass splitting

$$\text{TX-I:} \quad Y = \begin{pmatrix} Y_{ee} & 0 & 0 \\ 0 & Y_{\mu\mu} & \times \\ 0 & \times & Y_{\tau\tau} \end{pmatrix} \qquad \text{TX-II:} \quad Y = \begin{pmatrix} 0 & Y_{e\mu} & 0 \\ Y_{\mu e} & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

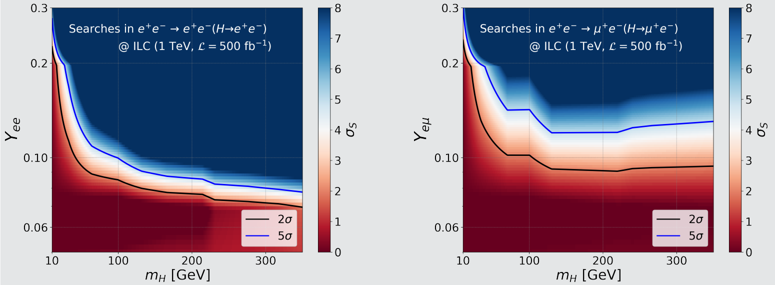
Δa_μ Δa_e NSI

Additional constraints

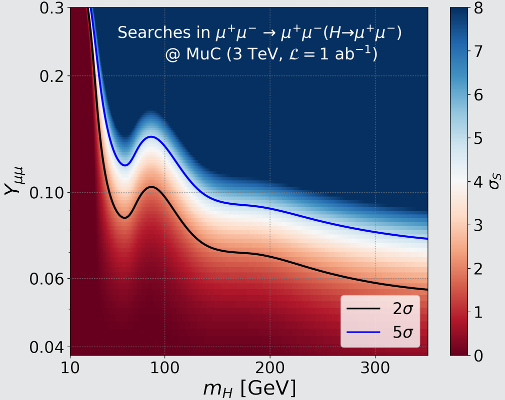
- Muonium-antimuonium oscillations: $e^-\mu^+ \to e^+\mu^- \Rightarrow$ constraints $(Y_{e\mu} Y_{\mu e})$
- Direct experimental constraints
  - Dark photon searches at KLOE and BABAR: $e^+e^- \to \gamma\,(A_d \to e^+e^-)$
  - Dark boson searches at BABAR: $e^+e^- \to \mu^+\mu^- H$
  - Z' searches at LHC: $pp \to \mu^+\mu^-(Z' \to \mu^+\mu^-)$
  - LEP searches: $e^+e^- \to \ell^+\ell^-$ and $e^+e^- \to \ell^+\ell^-(H \to \ell^+\ell^-)$ $(\ell = e, \mu)$

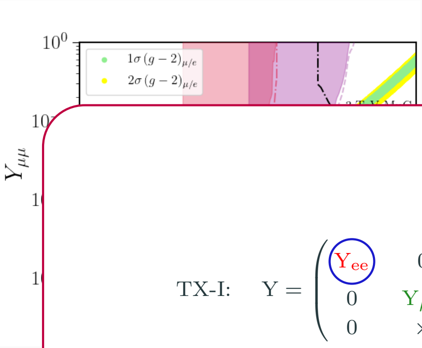
Probing Y_{ee}, $Y_{\mu\mu}$ and $Y_{e\mu}$ via $\ell^+\ell^- \to \ell^+\ell^- H \to \ell^+\ell^- \left(H \to \ell^+\ell^-\right)$

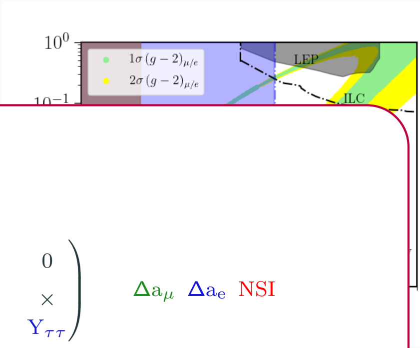
ILC



Muon Collider





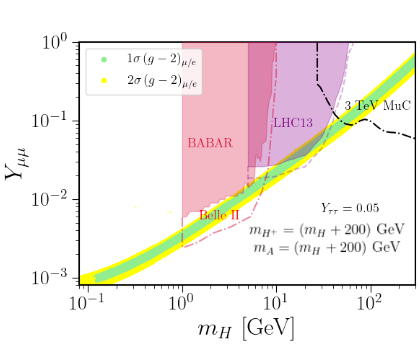


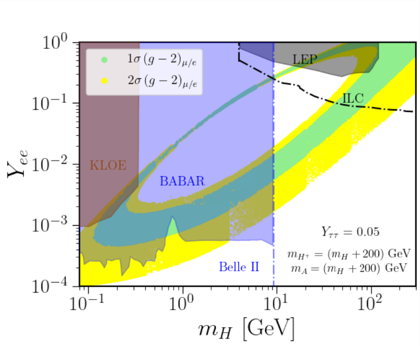
$$\text{TX-I:} \quad Y = \begin{pmatrix} Y_{ee} & 0 & 0 \\ 0 & Y_{\mu\mu} & \times \\ 0 & \times & Y_{\tau\tau} \end{pmatrix} \qquad \Delta a_\mu \quad \Delta a_e \quad \text{NSI}$$

$f = f_{\mu\tau} \ldots \quad 0 \quad 1 \quad 0$, $\quad \ldots \times 10^{-4}$; $\quad 0 \quad 2.6 \times 10^{-5} \quad 0.01$

NSI $\varepsilon_{ee} = 4.2\%$ for $Y_{ee} = 0.31, \varphi = 0.1$, and $m_{H^+} = 285\,\text{GeV}$

Results: TX-I

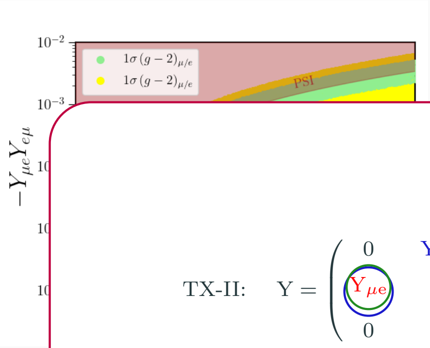


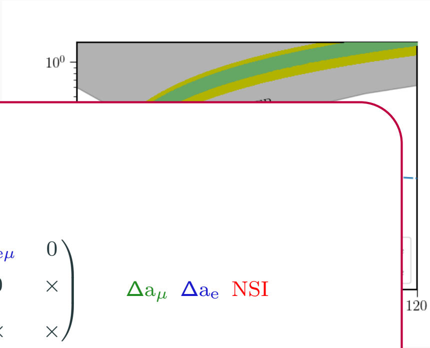


With $\mathrm{a_0} = \kappa\, \mathrm{f}_{\mu\tau} = 2.95 \times 10^{-7}$ and $\mathrm{m_H} = 85$ GeV,

$$\mathrm{f} = \mathrm{f}_{\mu\tau} \begin{pmatrix} 0 & 2.14 \times 10^{-3} & 1.18 \times 10^{-4} \\ & 0 & 1 \\ & & 0 \end{pmatrix}, \qquad \mathrm{Y} = \begin{pmatrix} 0.31 & 0 & 0 \\ 0 & 0.169 & -4.4 \times 10^{-4} \\ 0 & 2.6 \times 10^{-5} & 0.01 \end{pmatrix}$$

NSI $\varepsilon_{\mathrm{ee}} = 4.2\%$ for $\mathrm{Y_{ee}} = 0.31, \varphi = 0.1$, and $\mathrm{m_{H+}} = 285\,\mathrm{GeV}$





$$\text{TX-II:} \quad Y = \begin{pmatrix} 0 & Y_{e\mu} & 0 \\ Y_{\mu e} & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

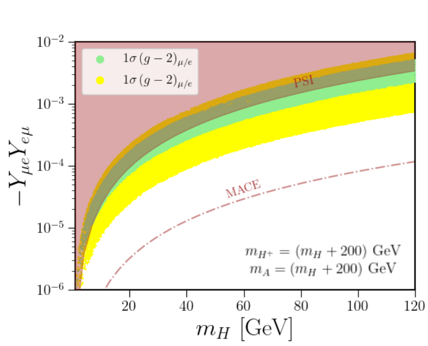
Δa_μ Δa_e NSI

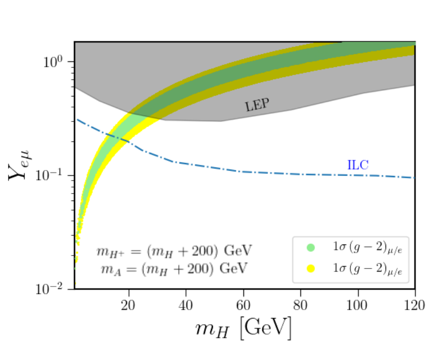
V

$$f = \ldots \quad 0 \quad \ldots \quad 1.18 \times 10^{-5} \quad 0 \quad 5.19 \times 10^{-6}$$

No observable NSI ($\epsilon_{\mu\mu} \propto |Y_{\mu e}|^2$)

Results: TX-II





With $a_0 = \kappa\, f_{\mu\tau} = 1.80 \times 10^{-6}$ and $m_H = 22$ GeV,

$$f = f_{\mu\tau}\begin{pmatrix} 0 & 0.119 & -0.198 \\ & 0 & 1.0 \\ & & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0.3 & 0 \\ -3.5\times 10^{-4} & 0 & 5.56\times 10^{-5} \\ 1.18\times 10^{-5} & 0 & 5.19\times 10^{-6} \end{pmatrix}$$

No observable NSI ($\epsilon_{\mu\mu} \propto |Y_{\mu e}|^2$)

- Proposed a novel scenario in the Zee model that resolves the anomalies in muon and electron AMMs, while being consistent with neutrino oscillation data
- Neutral scalar in the second Higgs doublet resolves the two anomalies, with the two-loop Barr-Zee diagram playing an essential role in accommodating the relative sign between the two
- Only two textures of flavor structure exist in the model that can explain all the observables, while being consistent with flavor and collider constraints
- The model features charged scalars which can potentially induce NSI ϵ_{ee} as large as 8%.
- Scalar mass ranges currently allowed: 10-300 GeV in TX-I and 1-30 GeV in TX-II
- Sizeable μEDM can be probed in the future
- Most of the parameter space can be probed by future experiments

Thank You

Expressions for AMM

One-loop

- $\Delta a_\ell^{(1)}(\phi) = \frac{m_\ell^2}{32\pi^2}\Big(\left\{|Y_{\ell i}|^2 + |Y_{i\ell}|^2\right\} F_\phi(x,1) \pm 2\frac{m_i}{m_\ell}\Re(Y_{\ell i}Y_{i\ell})F_\phi(x,0)\Big)$, where,

$$F_\phi(x,\epsilon) = \int_0^1 \frac{x^2 - \epsilon x^3}{m_\ell^2 x^2 + (m_i^2 - m_\ell^2)x + m_\phi^2(1-x)}\,dx.$$

  $+$ and $-$ correspond to H and A, respectively.

- $\Delta a_\ell^{(1)}(H^+) = \frac{m_\ell^2}{16\pi^2}|Y_{i\ell}|^2 \int_0^1 \frac{x^3 - x^2}{m_\ell^2 x^2 + (m_{H^+}^2 - m_\ell^2)x}\,dx.$

Two-loop

- $\Delta a_\ell^{(2)} = \frac{\alpha_{em}}{4\pi^3}\frac{m_\ell}{m_i}\frac{z}{2}\Big(-C^\phi_{S_\ell}C^\phi_{S_i}G(x,z,1) + C^\phi_{P_\ell}C^\phi_{P_i}G(x,z,0)\Big)$, where,

$$G(x,z,\epsilon) = \int_0^1 \frac{1 - 2\,\epsilon\, x(1-x)}{x(1-x) - z}\log\frac{x(1-x)}{z}\,dx,$$

  with $z = m_i^2/m_\phi^2$, and the coefficients

$$C^H_{S_i} = C^A_{P_i} = \frac{1}{\sqrt{2}}\mathrm{Re}(Y_{ii}), \quad -C^H_{P_f} = C^A_{S_f} = \frac{i}{\sqrt{2}}\mathrm{Im}(Y_{ii}).$$

Electric Dipole Moment

- $d_\ell(\phi) = \frac{\mp q_i m_i}{16\pi^2 m_\phi^2} \frac{\mathrm{Im}(Y_{i\ell}^* Y_{\ell i}^*)}{2} I\left(m_i^2/m_\phi^2, m_\ell^2/m_\phi^2\right),$
with $+(-)$ corresponding to A (H), and

$$I(r,s) = \int_0^1 \frac{x^2}{1 - x + rx - sx(1-x)}\,dx.$$

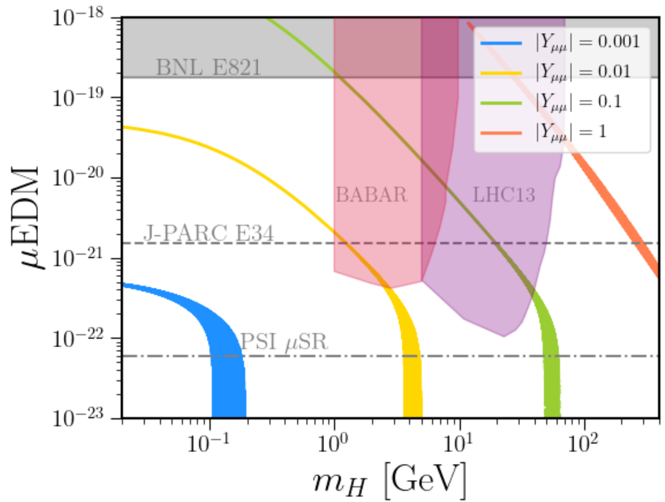
(Ecker, Grimus and Neufeld, 1983)

- $|d_e| \leq 1.1 \times 10^{-29}$ e-cm

(AMCE Collaboration, 2018)

- Δa_μ cannot be satisfied for $\theta \in [\frac{\pi}{4}, \frac{3\pi}{4}]$ since the dominant chirally enhanced term is $\propto \cos 2\theta$ which is ≤ 0 regardless of the value of $Y_{\mu\mu}$ and m_H.

EDM (ctd...)



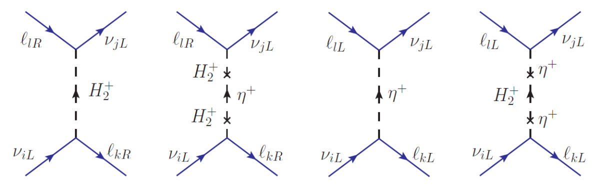
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
μEDM
BNL E821
BABAR
LHC13
J-PARC E34
PSI μSR
$|Y_{\mu\mu}| = 0.001$
$|Y_{\mu\mu}| = 0.01$
$|Y_{\mu\mu}| = 0.1$
$|Y_{\mu\mu}| = 1$
10^{-1}
10^{0}
10^{1}
10^{2}
m_H [GeV]

Non Standard Neutrino Interactions

(Wolfenstein, 1978)

Neutrino interactions different from SM weak interactions characterized by

$$-\mathcal{L}_{\mathrm{NSI}}^{\mathrm{eff}} = \varepsilon_{\alpha\beta}^{\mathrm{fP}} 2\sqrt{2}\mathrm{G_F} \left(\bar{\nu}_\alpha \gamma_\rho \mathrm{L} \nu_\beta\right) \left(\bar{\mathrm{f}}\gamma^\rho \mathrm{P f}\right)$$



$$\varepsilon_{\mathrm{ij}} \equiv \varepsilon_{\mathrm{ij}}^{\left(\mathrm{h}^+\right)} + \varepsilon_{\mathrm{ij}}^{\left(\mathrm{H}^+\right)} = \frac{1}{4\sqrt{2}\mathrm{G_F}} \mathrm{Y_{ie}} \mathrm{Y_{je}^*} \left(\frac{\sin^2\varphi}{\mathrm{m}_{\mathrm{h}^+}^2} + \frac{\cos^2\varphi}{\mathrm{m}_{\mathrm{H}^+}^2} \right),$$

- Off-diagonal NSI, ϵ_{ij} subject to strong constraints from LFV
- Diagonal NSI, ϵ_{ii} potentially large contribution from $(\mathrm{Y_{ee}}, \mathrm{Y_{\mu e}}, \mathrm{Y_{\tau e}})$

(Babu, Dev, Jana, and Thapa, 2020)

Constraints from Radiative Decay

Process	Exp. Bound	Constraints
$\mu \to e\gamma$	$BR < 4.2 \times 10^{-13}$	$\lvert Y_{\mu f} Y_{ef} \rvert^2 + \lvert Y_{f\mu} Y_{fe} \rvert^2 < 1.89 \times 10^{-9} \left(\frac{m_\phi}{100 GeV}\right)^4$
		$\left(\lvert Y_{ef} Y_{f\mu} \rvert^2 + \lvert Y_{\mu f} Y_{fe} \rvert^2\right) \mathcal{C} < 5.84 \times 10^{-13} \left(\frac{m_\phi}{100\ GeV}\right)^4 \left(\frac{1\ GeV}{m_f}\right)^2$
$\tau \to e\gamma$	$BR < 3.3 \times 10^{-8}$	$\lvert Y_{\tau f} Y_{ef} \rvert^2 + \lvert Y_{f\tau} Y_{fe} \rvert^2 < 8.31 \times 10^{-4} \left(\frac{m_\phi}{100 GeV}\right)^4$
		$\left(\lvert Y_{ef} Y_{f\tau} \rvert^2 + \lvert Y_{\tau f} Y_{fe} \rvert^2\right) \mathcal{C} < 7.29 \times 10^{-5} \left(\frac{m_\phi}{100\ GeV}\right)^4 \left(\frac{1\ GeV}{m_f}\right)^2$
$\tau \to \mu\gamma$	$BR < 4.4 \times 10^{-8}$	$\lvert Y_{\tau f} Y_{\mu f} \rvert^2 + \lvert Y_{f\tau} Y_{f\mu} \rvert^2 < 1.11 \times 10^{-3} \left(\frac{m_\phi}{100 GeV}\right)^4$
		$\left(\lvert Y_{\mu f} Y_{f\tau} \rvert^2 + \lvert Y_{\tau f} Y_{f\mu} \rvert^2\right) \mathcal{C} < 9.72 \times 10^{-5} \left(\frac{m_\phi}{100\ GeV}\right)^4 \left(\frac{1\ GeV}{m_f}\right)^2$

Process	Exp. Bound	Constraints
$\mu \to e\gamma$	$BR < 4.2 \times 10^{-13}$	$\lvert Y_{f\mu} Y_{fe} \rvert^2 < 1.89 \times 10^{-9} \left(\frac{m_{H^-}}{100 GeV}\right)^4$
$\tau \to e\gamma$	$BR < 3.3 \times 10^{-8}$	$\lvert Y_{f\tau} Y_{fe} \rvert^2 < 8.31 \times 10^{-4} \left(\frac{m_{H^-}}{100 GeV}\right)^4$
$\tau \to \mu\gamma$	$BR < 4.4 \times 10^{-8}$	$\lvert Y_{f\tau} Y_{f\mu} \rvert^2 < 1.11 \times 10^{-3} \left(\frac{m_{H^-}}{100 GeV}\right)^4$

Constraints from Trilepton Decay

Process	Exp. Bound	Constraints
$\mu^- \to e^- e^+ e^-$	$\mathrm{BR} < 1.0 \times 10^{-12}$	$\lvert Y_{ee}\rvert^2(\lvert Y_{e\mu}\rvert^2 + \lvert Y_{\mu e}\rvert^2) < 1.16 \times 10^{-12} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to e^- e^+ e^-$	$\mathrm{BR} < 2.7 \times 10^{-8}$	$\lvert Y_{ee}\rvert^2(Y_{e\tau}\rvert^2 + \lvert Y_{\tau e}\rvert^2) < 1.76 \times 10^{-7} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to \mu^- e^+ e^-$	$\mathrm{BR} < 1.8 \times 10^{-8}$	$\lvert Y_{ee}\rvert^2(Y_{\mu\tau}\rvert^2 + \lvert Y_{\tau\mu}\rvert^2) < 8.78 \times 10^{-8} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to \mu^- \mu^+ \mu^-$	$\mathrm{BR} < 2.1 \times 10^{-8}$	$\lvert Y_{\mu\mu}\rvert^2(\lvert Y_{\mu\tau}\rvert^2 + \lvert Y_{\tau\mu}\rvert^2) < 1.37 \times 10^{-7} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to e^- \mu^+ \mu^-$	$\mathrm{BR} < 2.7 \times 10^{-8}$	$\lvert Y_{\mu\mu}\rvert^2(\lvert Y_{e\tau}\rvert^2 + \lvert Y_{\tau e}\rvert^2) < 1.32 \times 10^{-7} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to e^- \mu^+ e^-$	$\mathrm{BR} < 1.5 \times 10^{-8}$	$\lvert Y_{e\mu}\rvert^2(\lvert Y_{e\tau}\rvert^2 + 2\lvert Y_{\tau e}\rvert^2) + (\mu \leftrightarrow e) < 2.93 \times 10^{-7} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$
$\tau^- \to \mu^- e^+ \mu^-$	$\mathrm{BR} < 1.7 \times 10^{-8}$	$\lvert Y_{e\mu}\rvert^2(\lvert Y_{\tau\mu}\rvert^2 + 2\lvert Y_{\mu\tau}\rvert^2) + (\mu \leftrightarrow e) < 3.32 \times 10^{-7} \left(\frac{m_\phi}{100\ \mathrm{GeV}}\right)^4$

Muonium-Antimuonium Oscillations

- $$P(M \to \bar{M}) = \frac{64\alpha_{em}^6 m_{red}^6 \tau_\mu^2}{\pi^2} G_{M\bar{M}}^2 \simeq 1.95 \times 10^5 \; G_{M\bar{M}}^2$$

  where, $m_{red} = m_e m_\mu/(m_e + m_\mu)$

  (Cvetič, Dib, Kim and Kim, 2005)

- Measured by the PSI Collaboration, with $P(M \leftrightarrow \bar{M}) < 8.3 \times 10^{-11}$ at 95% C.L.

  (Willmann et al., 1999)

- Effective Hamiltonian

  $$\mathcal{H}_{eff} = \frac{G_{M\bar{M}}}{\sqrt{2}} \; [\bar{\mu}\gamma_\mu(1+\gamma_5)e][\bar{\mu}\gamma^\mu(1-\gamma_5)e] \,.$$

- $G_{M\bar{M}} \leq 1.77 \times 10^{-3} \Rightarrow Y_{e\mu}Y_{\mu e} \leq 2.37 \times 10^{-7}(m_H/\mathrm{GeV})^2$.

- Expected sensitivity from MACE experiment:
  $\mathcal{O}(10^{-13}) \Rightarrow Y_{e\mu}Y_{\mu e} \leq 8.11 \times 10^{-9}(m_H/\mathrm{GeV})^2$.