

# Finding Evidence of Inflation and Galactic Magnetic Fields with CMB Surveys

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- Scenarios ‘1’ and ‘2’ generate *primordial magnetic fields* (PMFs) – nG scale PMFs at Mpc scales are adiabatically compressed to  $\mu\text{G}$  scale fields in galaxies.
- PMFs are an attractive scenario to explain the uniform distribution of magnetic fields in voids.

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- Scale invariant (or nearly) PMF power spectrum.

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- In the Standard Model, these are *not* **first-order**; some beyond-SM extensions can make them so.
- No evidence for any of these models so far.

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- More importantly, it will be a compelling evidence of inflation!!!
- If  $B_{\text{SI}}$  is constrained below 0.1 nG, inflation isn't the primary source of galactic fields.

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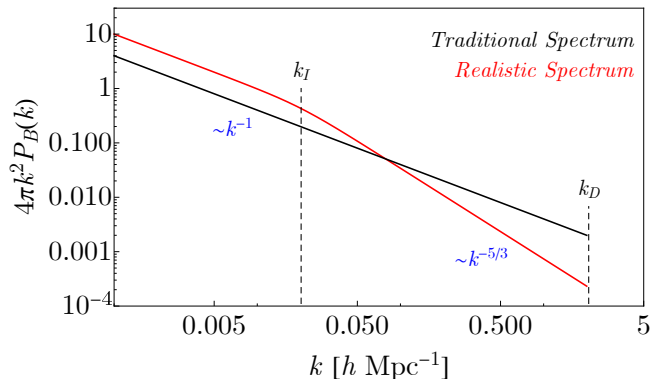
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- This scales as  $B^2$ .
- Birefringence can thus provide a tighter bound on the PMF strength from future surveys.

# Realistic PMF Spectrum

- PMF constitute a Gaussian random field in three dimensions – characterized by the power spectrum  $P_B(k)$ .

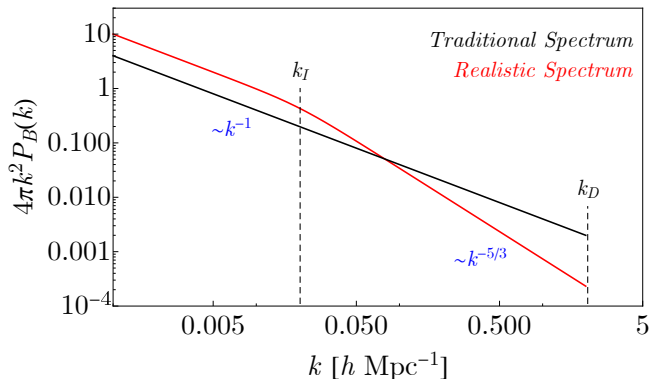
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- We use this to theoretically calculate the anisotropic birefringence.

# Birefringence Forecasts

- From the rotation angle  $\alpha(\hat{\mathbf{n}})$ , we get a power spectrum,  
 $\langle \alpha(\hat{\mathbf{n}})\alpha(\hat{\mathbf{n}}') \rangle \equiv \sum_l (2l + 1) C_l^{\alpha\alpha} P_l(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}') / 4\pi$ .

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# Birefringence Forecasts (Contd.)

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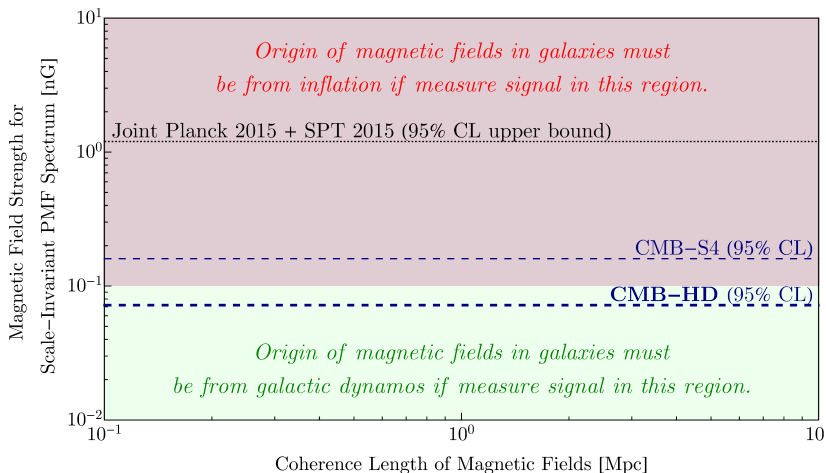
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- CMB-HD will improve the bound on  $A_\alpha$  by four orders of magnitude – giving tightest constraints on PMFs.

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# Forecasts on PMFs



# Subtracting Milky Way Birefringence

- MFs in our galaxy lead to CMB Birefringence of  $A_\alpha \sim 10^{-5} \text{ deg}^2$ , similar to  $\mathcal{O}(0.1 \text{ nG})$  PMFs.

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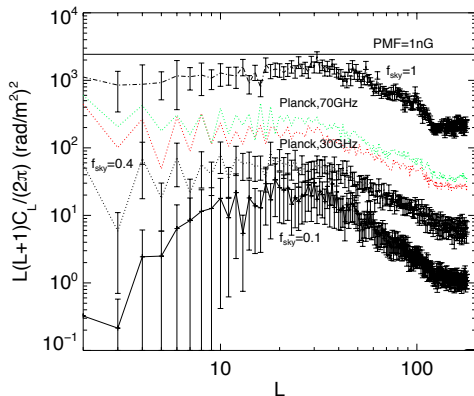
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- Independent MW-induced birefringence obtained from the  $\alpha(\hat{\mathbf{n}})$  of 40,000 extragalactic radio sources near the MW (NVSS Catalog).
- $\alpha(\hat{\mathbf{n}})$  is measured at multiple frequencies, giving a precise map of the MW birefringence.



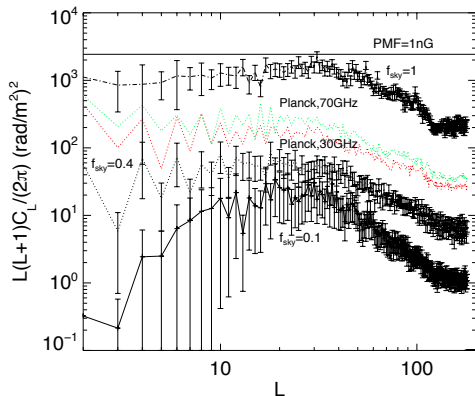
# Milky Way RM Spectra



(De et al., Phys. Rev. D 88.6.)

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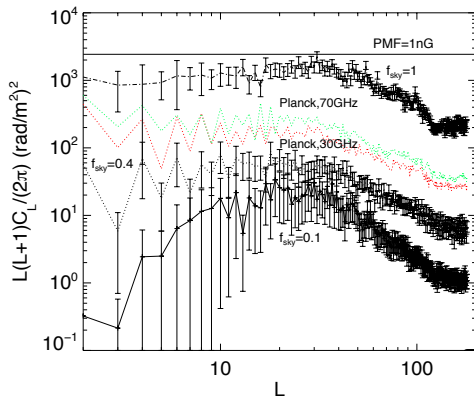
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- **The MW birefringence can thus be subtracted from the CMB measurement!!**

	SO	CMB-S4	CMB-HD
$\sigma(B_{SI})$ (nG)	0.47	0.08	0.036

- The current 95% CL upper bound on  $B_{\text{SI}}$  is 1.2 nG – comes from the Planck TT, EE, and TE, and SPT BB data.
- $A_\alpha$  measurements from CMB-S4 and CMB-HD will tighten it to 0.16 nG and 0.072 nG respectively.
- The CMB-HD bound is below the 0.1 nG threshold that distinguishes between purely inflationary and dynamo origins of galactic MFs.
- **Detection** of  $B_{\text{SI}} < 0.1$  nG will point to a dynamo origin of galactic MFs.
- **Detection** of  $B_{\text{SI}} > 0.1$  nG will be *a compelling evidence* for inflation!
- CMB-HD is capable of detecting inflationary PMFs at  $3\sigma$  significance.

*Thank You!*

# Supplementary Slides

# PMFs and the $H_0$ Tension

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- These inhomogeneties cause recombination to happen earlier, reducing the sound horizon and increasing  $H_0$ <sup>3</sup>.

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- These inhomogeneties cause recombination to happen earlier, reducing the sound horizon and increasing  $H_0$ <sup>3</sup>.
- A  $\sim 0.1$  nG PMFs before recombination is enough to resolve the Hubble tension.

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# Modeling Inflationary PMFs

- The comoving magnetic field  $\mathbf{B}$  is a Gaussian random field in three dimensions.
- Information about the energy of PMFs is encapsulated in the power spectrum  $P_B(k)$ ; magnetic helicity does not affect birefringence.
- Traditionally written as

$$P_B(k) = A_B k^{n_B}, \quad k \leq k_D \quad (1)$$

for some damping scale  $k_D$ ; For inflationary PMFs  $n_B = -3$

- We set  $k_D$  to the Silk damping scale  $2 \text{ Mpc}^{-1}$ ; PMFs on scales smaller than these have net rotation.

# The Birefringence Spectrum

- From the rotation angle  $\alpha(\hat{\mathbf{n}})$ , we get a power spectrum,  $\langle \alpha(\hat{\mathbf{n}})\alpha(\hat{\mathbf{n}}') \rangle \equiv \sum_l (2l + 1) C_l^{\alpha\alpha} P_l(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}') / 4\pi$ .
- The amplitude of anisotropic birefringence is

$$A_\alpha \equiv \frac{l(l+1)C_l^{\alpha\alpha}}{2\pi} \propto \frac{B^2}{\nu_0^4} \quad (2)$$

for frequency  $\nu_0$  of observation.

- For a scale-invariant PMFs,  $A_\alpha$  is independent of  $l$  in the multipole region of interest.

# The Birefringence Spectrum (Contd.)

- However,  $A_\alpha$  is frequency dependent, and CMB surveys observe at two frequencies.
- Since  $A_\alpha \propto \nu_0^{-4}$ , we can construct an *effective frequency* for our theoretical prediction,

$$\frac{1}{\nu_{\text{eff}}^4} = \frac{1}{2} \left( \frac{1}{\nu_1^4} + \frac{1}{\nu_2^4} \right). \quad (3)$$

- Equivalent to taking an arithmetic mean of the measurements on the channels – assuming equal noise levels.
- For the channels of 90 and 150 GHz, we find  $\nu_{\text{eff}} = 103.8$  GHz.

# Birefringence Forecasts

Experiment	White noise	Beam	$f_{\text{sky}}$	Delensing Fraction
SO-SAT	$3 \mu\text{K}'$	$17'$	0.1	0.3
CMB-S4	$2 \mu\text{K}'$	$2'$	0.5	0.15
CMH-HD	$0.7 \mu\text{K}'$	$0.4'$	0.5	0.1

The error bars on  $A_\alpha$  are computed as:

$$\frac{1}{\sigma^2(A_\alpha)} = \sum_l f_{\text{sky}} \frac{2l+1}{2} \frac{(C_l^{\alpha\alpha, \text{fid}})^2}{(N_l^{\alpha\alpha})^2}, \quad (4)$$

where  $C_l^{\alpha\alpha, \text{fid}} = 2\pi/l(l+1)$  and  $N_l^{\alpha\alpha}$  is the reconstruction noise spectrum.

Multipole ranges of  $100 < l < 5000$  are used for this calculation.

# Subtracting Milky Way Birefringence (Contd.)

- At our effective frequency  $\nu_{\text{eff}} = 103.8$  GHz, we have

$$A_\alpha = 2.363 \times 10^{-7} \left( \frac{A_{\text{RM}}}{1 \text{ rad/m}^2} \right)^2 \text{ deg}^2, \quad (5)$$

where  $A_{\text{RM},l}^2 \equiv l(l+1)C_l^{\text{RM}}/2\pi \approx A_{\text{RM}}^2$ .

- $\sigma_{A_{\text{RM},l}^2}$  comes from both sample variance and measurement uncertainty.
- For the cleanest 40% of the sky, the galactic contribution is  $A_{\text{RM},l}^2 \approx 70 l^{-0.17} (\text{rad/m}^2)^2$ .
- The associated error is  $\sigma_{A_{\text{RM},l}^2} \approx 0.7 A_{\text{RM},l}^2$ .



# Subtracting Milky Way Birefringence (Contd.)

- $A_{\text{RM}}$  is related to  $B_{\text{SI}}$  as  $A_{\text{RM}} = 68 \text{ rad/m}^2 (B_{\text{SI}}/1 \text{ nG})$ .
- The Galactic  $A_{\text{RM}} \approx \sqrt{70} \text{ rad/m}^2 \approx 8 \text{ rad/m}^2$  gives  $B_{\text{SI,G}} \approx 0.12 \text{ nG}$  on Mpc scales.
- SNR for detecting MW-induced  $A_{\text{RM}}$  is

$$\left(\frac{S}{N}\right)^2 = \sum_l \frac{\left(A_{\text{RM},l}^2\right)^2}{\sigma_{A_{\text{RM},l}^2}^2} \approx 26^2. \quad (6)$$

- This is likely optimistic – we have ignored covariance between the  $\sigma_{A_{\text{RM},l}^2}$ .
- Let's be conservative and take  $\text{SNR} = 10$  – this gives  $\sigma_{A_{\text{RM},l}} \approx 0.4 \text{ rad/m}^2$ , and thus  $\sigma_{B_{\text{SI,G}}} \approx 0.006 \text{ nG}$ .