Semisimple extensions of the SM

Joseph Tooby-Smith
Cornell University

Tue, 3 May 2022

Some Code

```
            for(int sol2=0;
sol2<=SM_alg_id_max;sol2++){if(SM_sub_embeddings[sol2][sol]) maximum=false;}
            if(maximum) latex_file<<"
$\\hyperref["<<sol<<"]{\\mathfrak{put}_{"<<sol<<"}}$ ";
        }
        latex_file<<endl;
        latex_file<<"\\subsection*{Minimal algebras}"<<endl;
        for(int sol=0; sol<=SM_alg_id_max;sol++){
            if(!Outer_automorphism_check[sol])continue;

            bool minimal=true;
            for(int sol2=0;
sol2<=SM_alg_id_max;sol2++){if(SM_sub_embeddings[sol][sol2]) minimal=false;}
            if(minimal) latex_file<<"
$\\hyperref["<<sol<<"]{\\mathfrak{put}_{"<<sol<<"}}$ ";
        }
        latex_file<<endl;
        latex_file<<endl;
        latex_file<<"\\subsection*{Outer automorphism classes}"<<endl;
        int outer_auto_count=0;
        for(int sol=0; sol<=SM_alg_id_max;sol++){
            if(!Outer_automorphism_check[sol])continue;

            //we check if this list has been done before
            bool done=false;
            for(int sol2=0; sol2<sol;sol2++)
if(Outer_automorphism[sol][sol2]!="False"){done=true;break;}
            if(done) continue;
            latex_file<<" ~\\\\~\\\\ "<<endl;
            outer_auto_count++;
            latex_file<<"Class "<<outer_auto_count<<":
$\\hyperref["<<sol<<"]{\\mathfrak{put}_{"<<sol<<"}}$";
            for(int sol2=sol+1; sol2<=SM_alg_id_max;sol2++){
                if( (Outer_automorphism[sol][sol2]=="False" &&
Outer_automorphism[sol2][sol]!="False")  ||(Outer_automorphism[sol][sol2]!="False"
&& Outer_automorphism[sol2][sol]=="False") ){cout<<"Something wrong with outer
automorphism check"<<endl;}
                if(Outer_automorphism[sol][sol2]!="False"){latex_file<<"
$\\hyperref["<<sol2<<"]{\\mathfrak{put}_{"<<sol2<<"}}$
("<<Outer_automorphism[sol][sol2]<<") ";}
            }
        }
        //closing the latex_file where we write data.
        latex_file.close();
        cout<<endl;
        cout<<"Program finished."<<endl;
```

The Cavendish (II)

The story begins

The Cavendish (II)

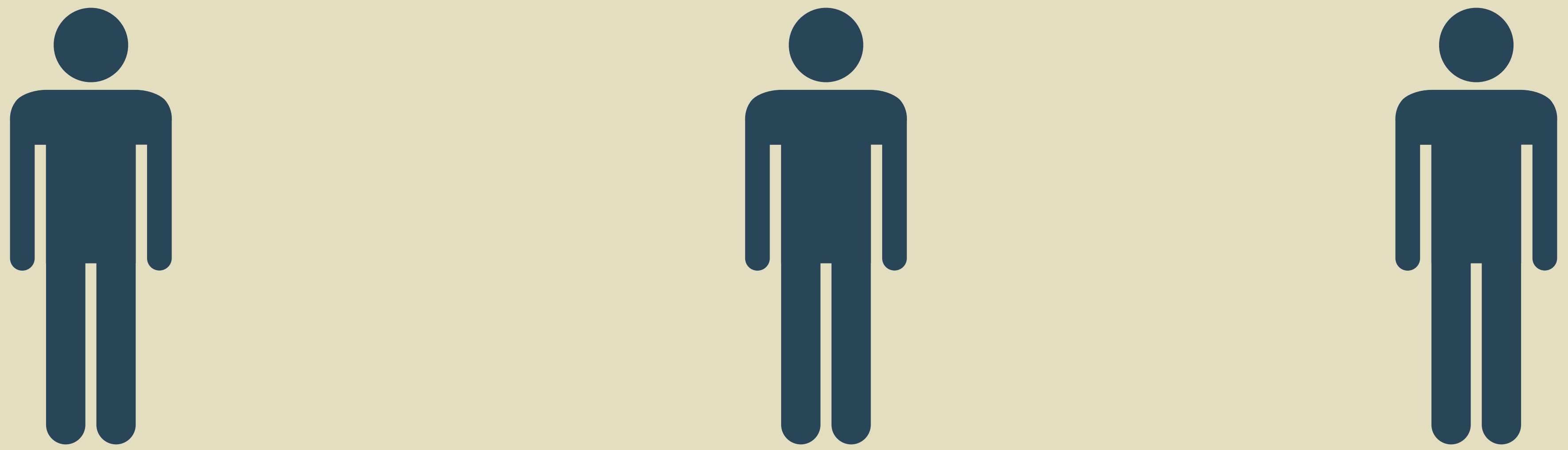
The SUSY working group

The story begins

The story begins

The Cavendish (II)

The ~~SUSY~~ working group
Pheno



Ben Allanach Joe Davighi Scott Melville

The u(1) extension

- Diagrams are not an accurate representation of the people.

Question: For the SM plus 3 RH neutrinos, what are the anomaly free sets of charge assignments under a $\mathfrak{u}(1)$-extension.

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y$$

$$\begin{array}{cccccc}
(2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\
(2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\
(2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0
\end{array}$$

Diophantine equations

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y$$

$$\begin{array}{cccccc} (2,3)_1 & (1,\bar{3})_{-4} & (1,\bar{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\ (2,3)_1 & (1,\bar{3})_{-4} & (1,\bar{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\ (2,3)_1 & (1,\bar{3})_{-4} & (1,\bar{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \end{array}$$

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y \oplus \mathfrak{u}(1)_X$$

$$\begin{array}{cccccc} (2,3)_{1,Q_1} & (1,\bar{3})_{-4,U_1} & (1,\bar{3})_{2,D_1} & (2,1)_{-3,L_1} & (1,1)_{6,E_1} & (1,1)_{0,N_1} \\ (2,3)_{1,Q_2} & (1,\bar{3})_{-4,U_2} & (1,\bar{3})_{2,D_2} & (2,1)_{-3,L_2} & (1,1)_{6,E_2} & (1,1)_{0,N_2} \\ (2,3)_{1,Q_3} & (1,\bar{3})_{-4,U_3} & (1,\bar{3})_{2,D_3} & (2,1)_{-3,L_3} & (1,1)_{6,E_3} & (1,1)_{0,N_3} \end{array}$$

$$\left\{ \text{Diophantine equations} \right\}$$

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y$$

$$\begin{array}{cccccc} (2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\ (2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \\ (2,3)_1 & (1,\overline{3})_{-4} & (1,\overline{3})_2 & (2,1)_{-3} & (1,1)_6 & (1,1)_0 \end{array}$$

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y \oplus \mathfrak{u}(1)_X$$

$$\begin{array}{cccccc} (2,3)_{1,Q_1} & (1,\overline{3})_{-4,U_1} & (1,\overline{3})_{2,D_1} & (2,1)_{-3,L_1} & (1,1)_{6,E_1} & (1,1)_{0,N_1} \\ (2,3)_{1,Q_2} & (1,\overline{3})_{-4,U_2} & (1,\overline{3})_{2,D_2} & (2,1)_{-3,L_2} & (1,1)_{6,E_2} & (1,1)_{0,N_2} \\ (2,3)_{1,Q_3} & (1,\overline{3})_{-4,U_3} & (1,\overline{3})_{2,D_3} & (2,1)_{-3,L_3} & (1,1)_{6,E_3} & (1,1)_{0,N_3} \end{array}$$

$$0 = \sum_{i=1}^{3} (6Q_i + 3U_i + 3D_i + 2L_i + E_i + N_i),\ 0 = \sum_{i=1}^{3} (3Q_i + L_i) 0 = \sum_{i=1}^{3} (2Q_i + U_i + D_i)$$

$$0 = \sum_{i=1}^{3} (Q_i + 8U_i + 2D_i + 3L_i + 6E_i), 0 = \sum_{i=1}^{3} (Q_i^2 - 2U_i^2 + D_i^2 - L_i^2 + E_i^2)$$

$$0 = \sum_{i=1}^{3} (6Q_i^3 + 3U_i^3 + 3D_i^3 + 2L_i^3 + E_i^3 + N_i^3)$$

Diophantine equations

Ben Allanach Joe Davighi Scott Melville

Numerical solution to SM+u(1) ACCs

A history

- 1812.04602
- 1905.13729
- 1912.04804

A history

- 1812.04602
- 1905.13729
- 1912.04804

Ben Allanach   Joe Davighi   Scott Melville

Numerical solution to SM+u(1) ACCs

Davi Costa   Bogdan Dobrescu   Patrick Fox

Analytic solution to u(1) ACCs

$$\sum_i x_i = 0, \quad \sum_i x_i^3 = 0$$

A history

- 1812.04602
- 1905.13729
- 1912.04804

Ben Allanach Joe Davighi Scott Melville

Numerical solution to SM+u(1) ACCs

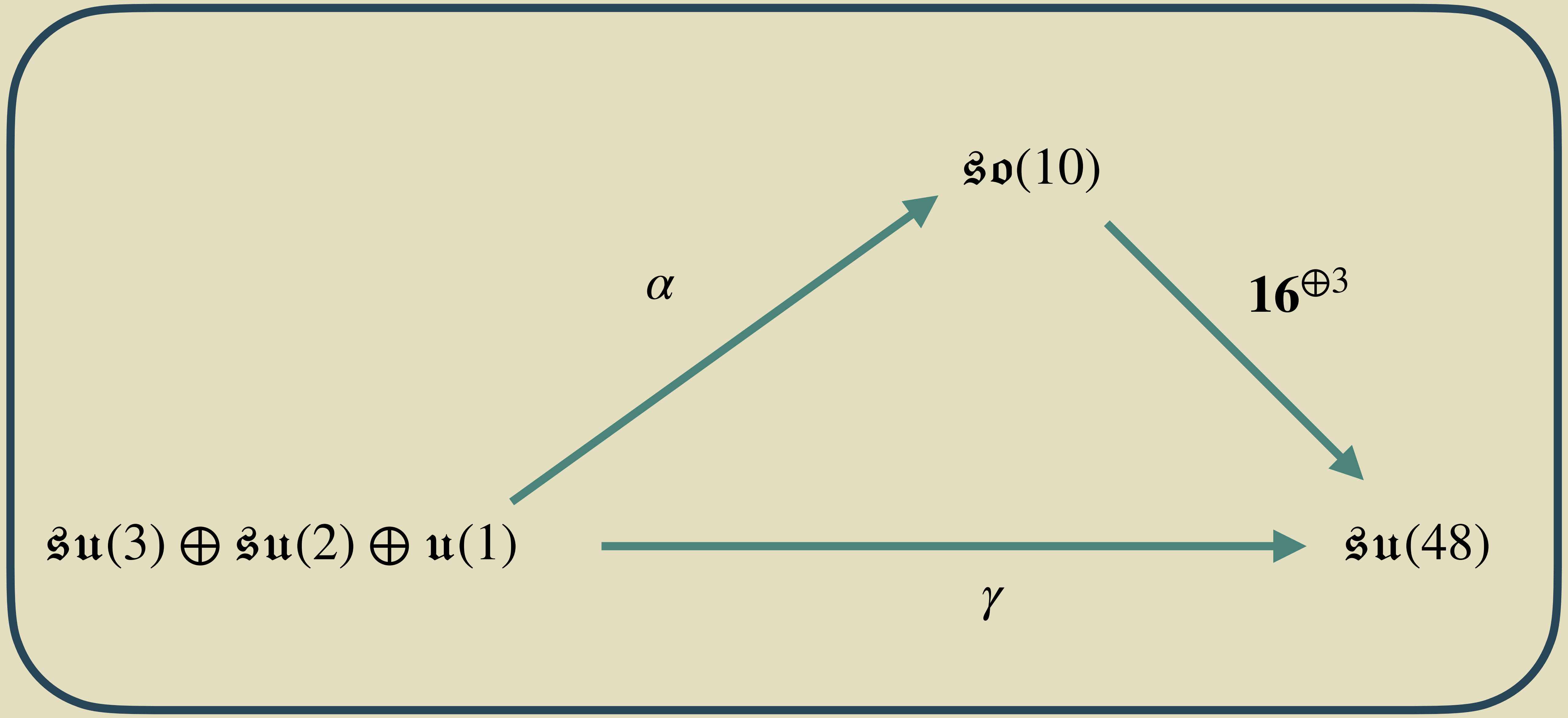
Davi Costa Bogdan Dobrescu Patrick Fox

Analytic solution to u(1) ACCs

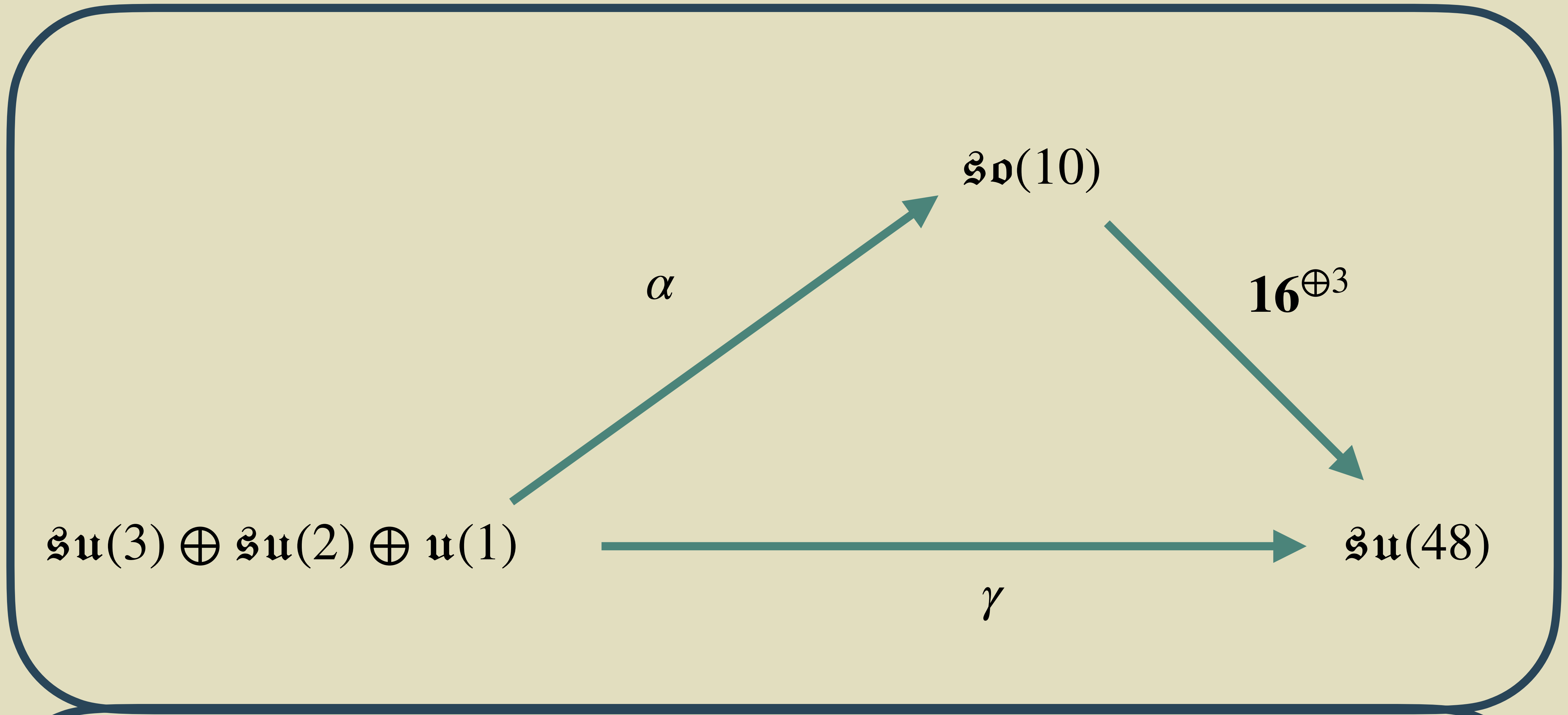
$$\sum_i x_i = 0, \quad \sum_i x_i^3 = 0$$

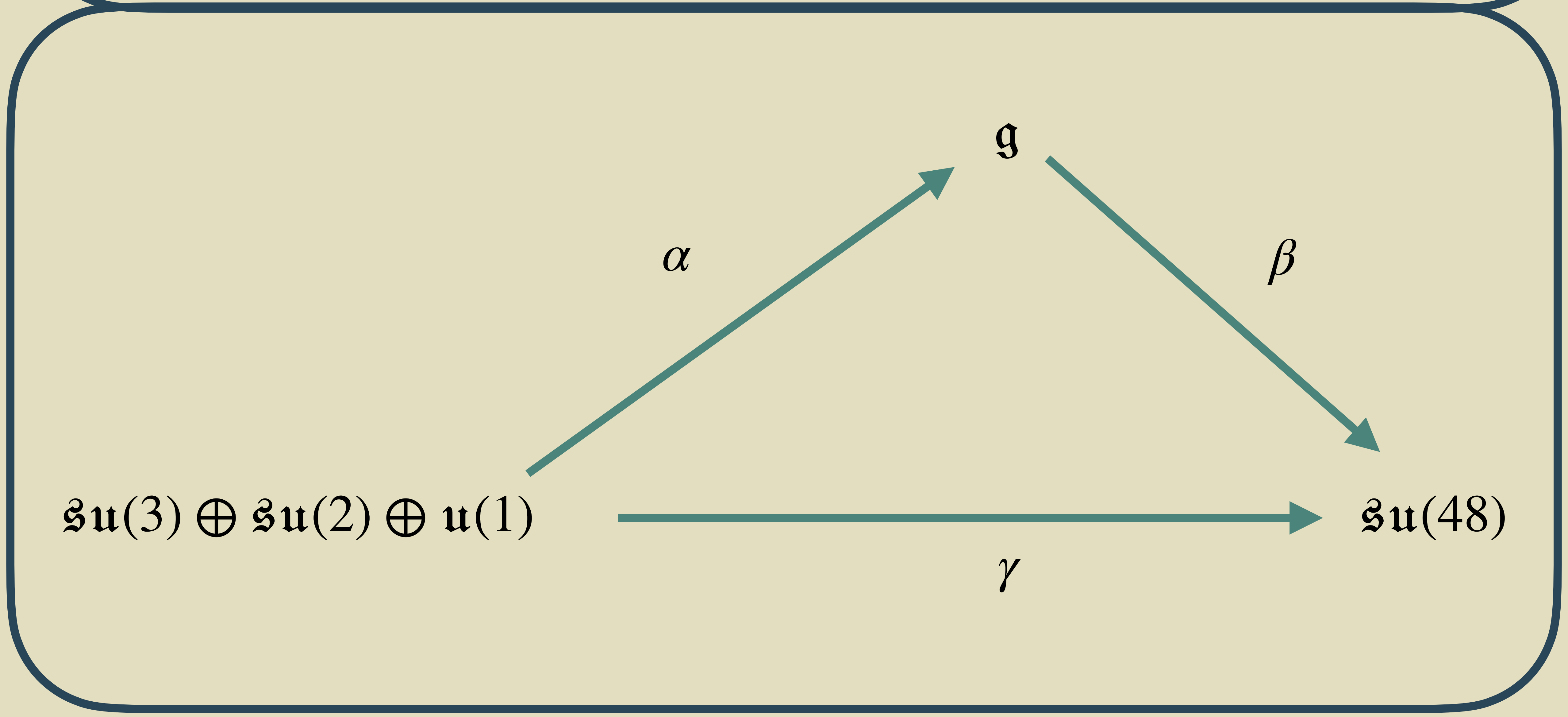
Ben Allanach Ben Gripaios JTS

Analytic solution to SM+u(1) ACCs



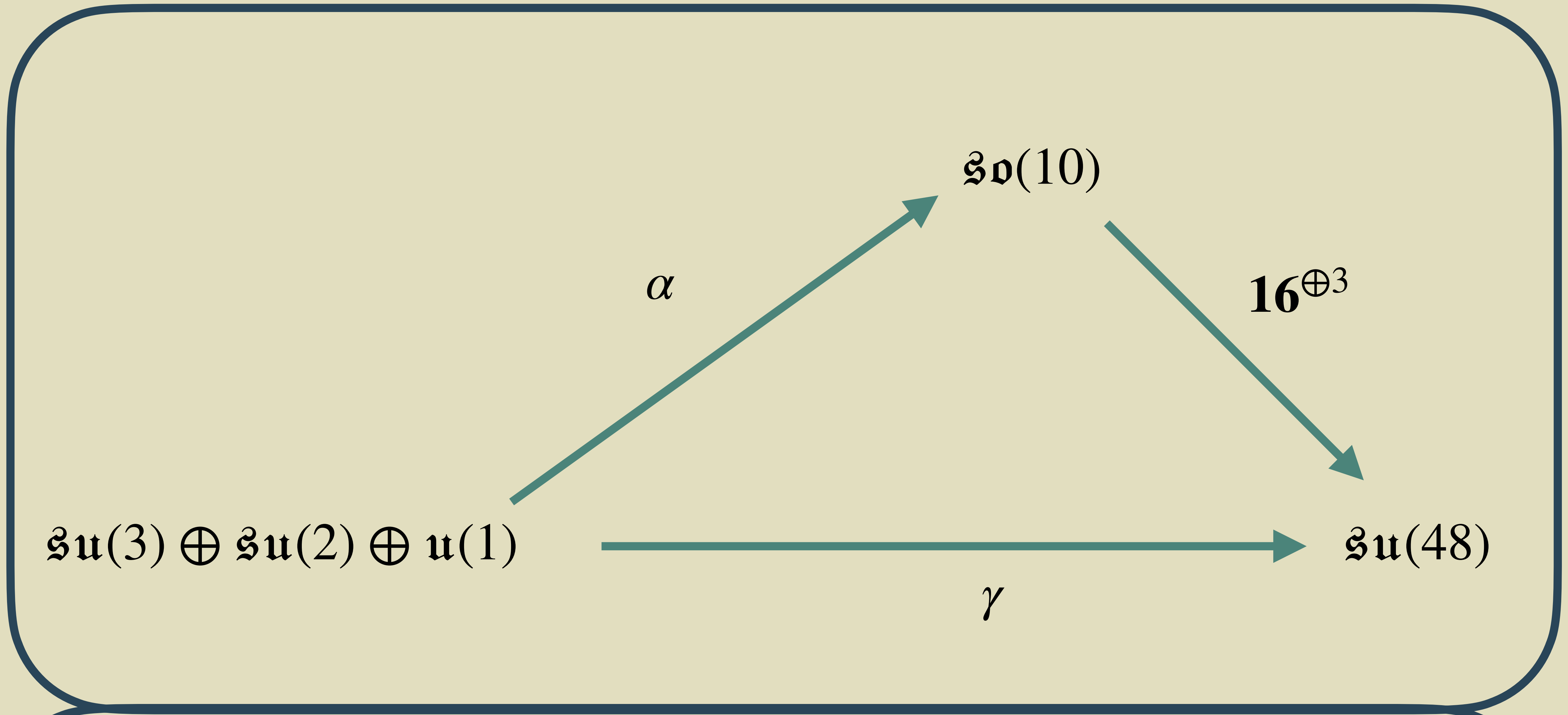
A precise statement

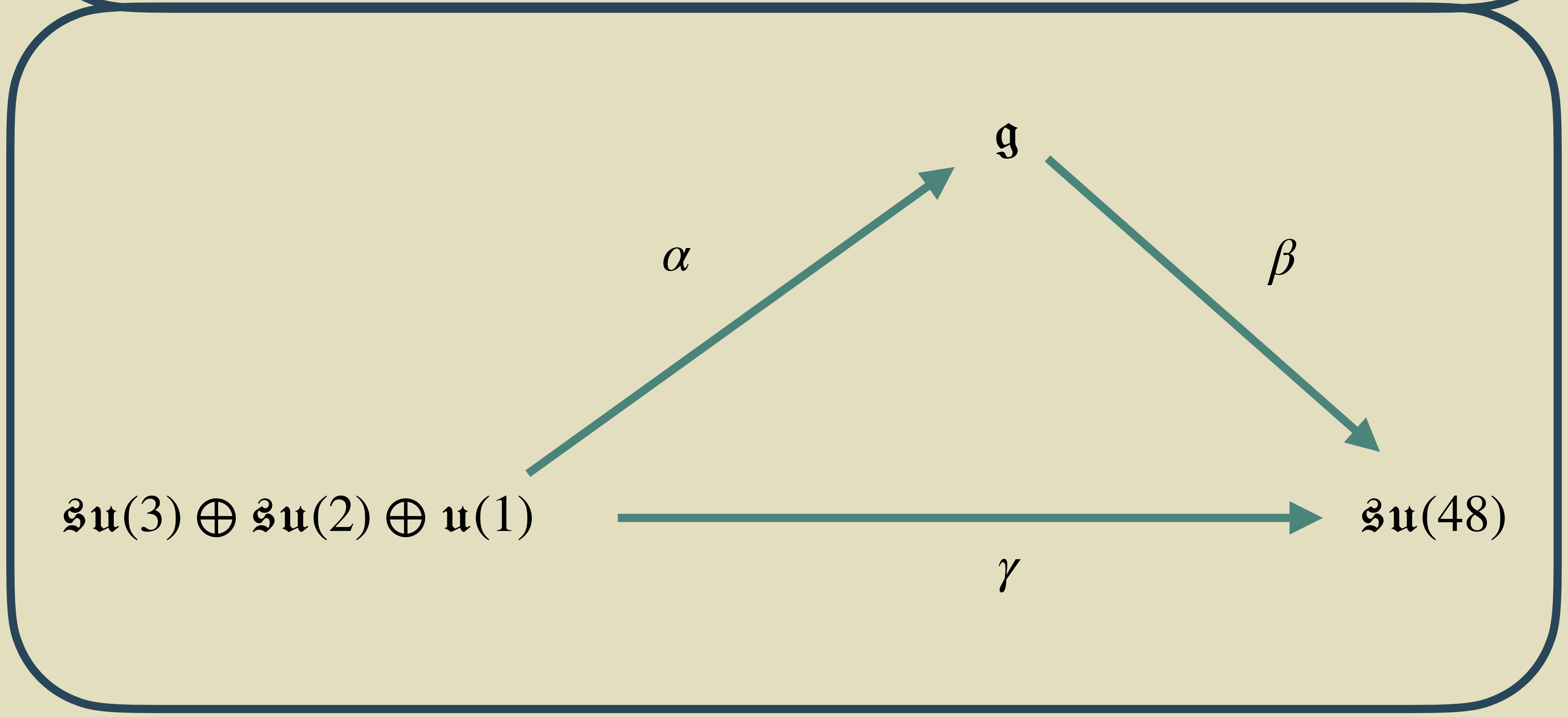




A precise statement

A precise statement

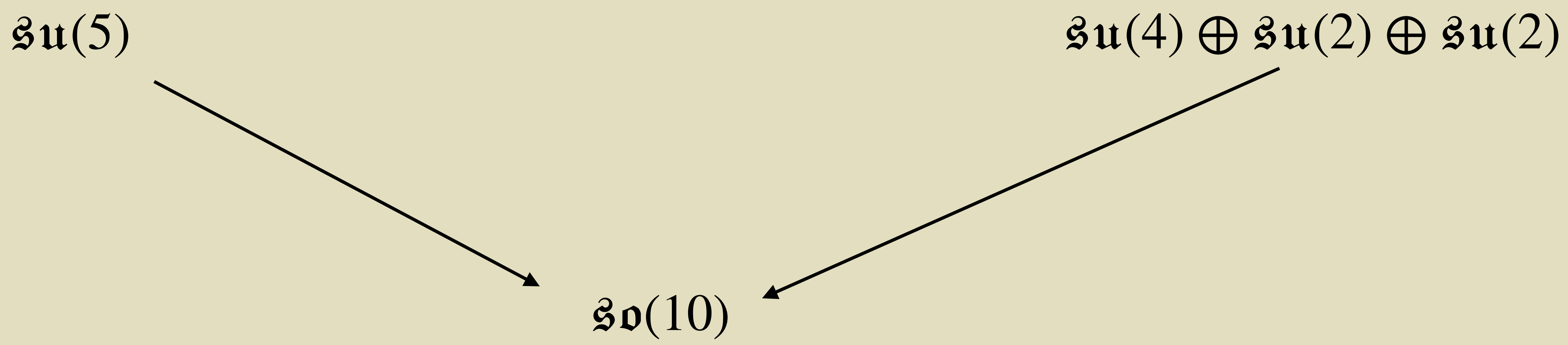




$\mathfrak{g}$ is semi-simple
e.g.
$\mathfrak{su}(4) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$

The representation β is free of *local* anomalies

One family:



The results!

The results!

One family:

$$\mathfrak{su}(5) \rightarrow \mathfrak{so}(10) \leftarrow \mathfrak{su}(4) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$

Two families:

45 algebras, for example:

$\mathfrak{su}(4) \oplus \mathfrak{sp}(4) \oplus \mathfrak{sp}(4)$
$(\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{4})$

$\mathfrak{so}(10) \oplus \mathfrak{su}(2)$
$(\mathbf{16}, \mathbf{2})$

$\mathfrak{su}(5) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$
$(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1})^{\oplus 2} \oplus (\mathbf{10}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{2})$

$\mathfrak{su}(8) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$
$(\bar{\mathbf{8}}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{8}, \mathbf{1}, \mathbf{2})$

Three family: 340 algebras in total
24 maximal

1) $\mathfrak{so}(10) \oplus \mathfrak{su}(2)$
$(\mathbf{16}, \mathbf{3})$

2) $\mathfrak{so}(10)^{\oplus 3}$
$(\mathbf{16}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{16}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{16})$

3) $\mathfrak{so}(10)^{\oplus 2} \oplus \mathfrak{su}(2)$
$(\mathbf{16}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{16}, \mathbf{2})$

4) $\mathfrak{su}(4) \oplus \mathfrak{sp}(6)^{\oplus 2}$
$(\bar{\mathbf{4}}, \mathbf{6}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{6})$

5) $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{sp}(6)$
$(\bar{\mathbf{4}}, \mathbf{6}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{6})$

6) $\mathfrak{su}(12) \oplus \mathfrak{su}(2)^{\oplus 2}$
$(\overline{\mathbf{12}}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{12}, \mathbf{1}, \mathbf{2})$

7) $\mathfrak{su}(4) \oplus \mathfrak{sp}(4)^{\oplus 2} \oplus \mathfrak{so}(10)$
$(\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{4}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{16})$

8) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{5}}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})$

9) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{5}}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})$

10) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})$

11) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})$

12) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})$

13) $\mathfrak{su}(5) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})$

14) $\mathfrak{su}(5)^{\oplus 2} \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)$
$(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

15) $\mathfrak{su}(5)^{\oplus 3} \oplus \mathfrak{su}(2)$
$(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10}, \mathbf{1}, \mathbf{1})$
$\oplus (\mathbf{1}, \mathbf{1}, \bar{\mathbf{5}}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})$

16) $\mathfrak{su}(8) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$
$(\mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{8}}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{8}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

17) $\mathfrak{su}(4) \oplus \mathfrak{sp}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$
$(\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})$

18) $\mathfrak{su}(4) \oplus \mathfrak{sp}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 2}$
$(\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})$

19) $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{4}}, \mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

20) $\mathfrak{su}(4) \oplus \mathfrak{sp}(6) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{4}}, \mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

21) $\mathfrak{su}(4)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\bar{\mathbf{4}}, \mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

22) $\mathfrak{su}(5) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$

23) $\mathfrak{su}(5)^{\oplus 2} \oplus \mathfrak{su}(2)^{\oplus 3}$
$(\mathbf{1}, \bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})$

24) $\mathfrak{su}(4) \oplus \mathfrak{so}(10) \oplus \mathfrak{su}(2)^{\oplus 4}$
$(\mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})$

The third family solutions

The paper

Floccinaucinihilipilification

B C Allanach[*]
DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom

Ben Gripaios[†] and Joseph Tooby-Smith[‡]
Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge, CB3 0HE, United Kingdom

We show how one may classify all semisimple algebras containing the $\mathfrak{su}(3)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$ symmetry of the Standard Model and acting on some given matter sector, enabling theories beyond the Standard Model with unification (partial or total) of symmetries (gauged or global) to be catalogued. With just a single generation of Standard Model fermions plus a singlet neutrino, the only gauged symmetries correspond to the well-known algebras $\mathfrak{su}(5), \mathfrak{so}(10)$, and $\mathfrak{su}(4)\oplus\mathfrak{su}(2)\oplus\mathfrak{su}(2)$, but with two or more generations a limited number of exotic symmetries mixing flavor, color, and electroweak symmetries become possible. We provide a complete catalogue in the case of 3 generations or fewer and describe how the method can be generalized to include additional matter.

Thank you!
Questions + criticism welcome!

Joseph Tooby-Smith
Cornell University

Tue, 3 May 2022