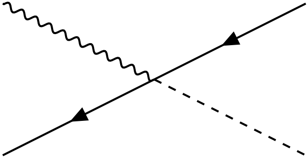
Generalized Partial Waves and Bottom-up EFT

Ming-Lei Xiao

Northwestern U & Argonne National Lab



May 10, 2022 @ **Phenomenology 2022 Symposium**

Overview of the Amplitude Basis Program

- ✓ Amplitude-Operator correspondence, on-shell basis for dim-6 SMEFT
  T.Ma, J.Shu, **M.-L.Xiao**, (19')
- ✓ Partial Waves and New Selection Rules
  M.Jiang, J.Shu, **M.-L.Xiao**, Y.-H.Zheng (20')
- ✓ Young Tensor basis – SMEFT, LEFT, $\nu\mathrm{SMEFT}$, GR
  H.-L.Li, Z.Ren, J.Shu, **M.-L.Xiao**, J.-H.Yu, Y.-H.Zheng (20',21')
- ✓ `Mathematica` package ABC4EFT [https://abc4eft.hepforge.org/]
  H.-L.Li, Z.Ren, **M.-L.Xiao**, J.-H.Yu, Y.-H.Zheng (22')
- ✓ Phase Space Integration of Partial Waves, Anomalous Dimension Matrix
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- ✓ Tree-level UV origin, Bottom-up EFT
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- ➠ Young Tensor basis – ChPT
  I.Low, J.Shu, **M.-L.Xiao**, Y.-H.Zheng (in progress)

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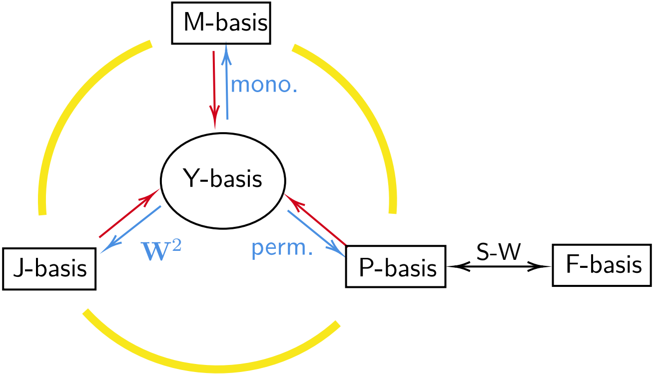

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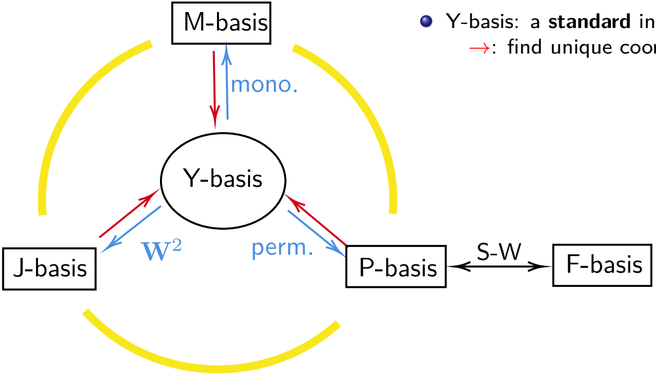

- Y-basis: a **standard** independent basis
  $\rightarrow$: find unique coordinate

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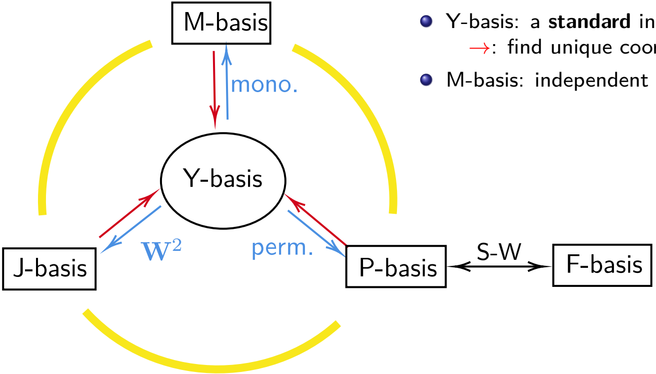

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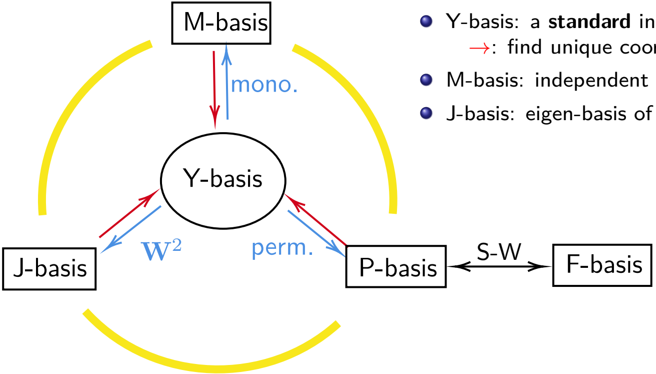

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- J-basis: eigen-basis of symmetry algebra.

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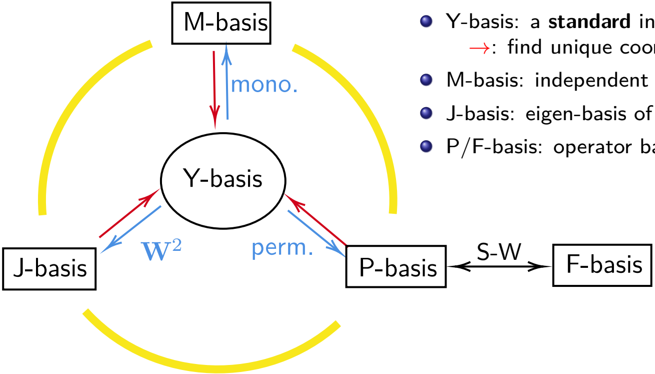

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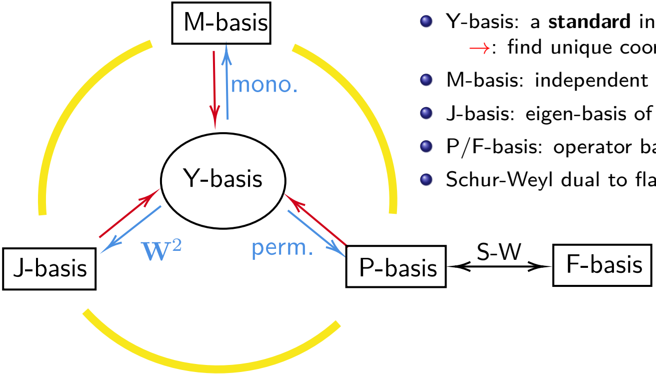

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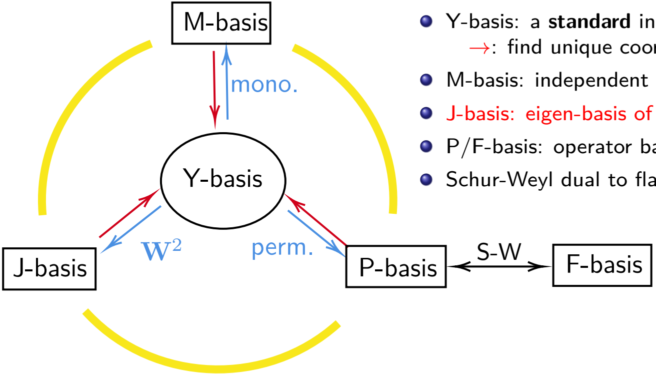

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Today's Focus

❒ Construction and application of generalized partial waves.

❒ J-basis, selection rules, algebraic phase space integration.

❒ Matching with UV resonances: Bottom-up EFT.

Generalization to $N \to M$ Scattering

The schematic definition of general partial wave expansion

$$\langle out|\mathbf{T}|in\rangle = \sum_{J,\sigma}\sum_{J',\sigma'}\int \mathrm{d}P\,\mathrm{d}P' \;\langle out|P,J,\sigma\rangle\langle P,J,\sigma|\mathbf{T}|P',J',\sigma'\rangle\langle P',J',\sigma'|in\rangle$$

$$\equiv \sum_{J} a_J \overline{B}^J(in \to out)\delta^{(4)}(p_{out}-p_{in})$$

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$$\overline{B}^J(in \to out) = \sum_\sigma \langle out|P, J, \sigma\rangle\langle P, J, \sigma|in\rangle$$

$|P, J, \sigma\rangle$ are Poincaré irreducible states ($\mathbf{W}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}P_\nu M_{\rho\sigma}$)

$$\mathbf{P}|P, J, \sigma\rangle = P|P, J, \sigma\rangle, \qquad \mathbf{W}^2|P, J, \sigma\rangle = -P^2 J(J+1)|P, J, \sigma\rangle.$$

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N-particle Poincaré Clebsch-Gordan Coefficient (CGC):

$$\langle P,J,\sigma|\Psi_1,\ldots,\Psi_N\rangle \equiv \mathcal{C}_N^{J,\sigma}\delta^{(4)}(P - \sum_i p_i)$$

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Example: Two-body CGC (unique solution)

$$\mathcal{C}_{(h_1,h_2)}^{J,\sigma} \sim [12]^{J+h_1+h_2}(\langle 1\chi\rangle^{J-h_1+h_2}\langle 2\chi\rangle^{J+h_1-h_2})^{\{I_1,\ldots,I_{2J}\}}$$

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Example: Three-body CGC (degenerate)

$$\mathcal{C}_{(0,0,0)}^{J=1,\sigma,1} \sim [12]\langle 1\chi^{\{I_1}\rangle\langle 2\chi^{I_2\}}\rangle$$
$$\mathcal{C}_{(0,0,0)}^{J=1,\sigma,2} \sim [23]\langle 2\chi^{\{I_1}\rangle\langle 3\chi^{I_2\}}\rangle$$
$$\cdots$$

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We can formally define the partial wave basis

$$\overline{B}^J = \sum_\sigma [\mathcal{C}^{J,\sigma}(out)]^* \mathcal{C}^{J,\sigma}(in) \equiv \mathcal{C}^J(\overline{out}) \cdot \mathcal{C}^J(in)$$

Angular Momentum from the Casimir $\mathbf{W}^2$

Notice the total angular momentum J was defined via $\mathbf{W}^2$

$$W^2\mathcal{C}_N^{J,\sigma} \equiv \int \mathrm{d}^4P \, \langle P, J, \sigma | \mathbf{W}^2 | \Psi_1, \ldots, \Psi_N \rangle = -\, sJ(J+1)\mathcal{C}_N^{J,\sigma}$$

$s = P^2 = \left(\sum_i^N p_i\right)^2$

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Poincaré Algebra for Functions of Spinor Variables

$$W^2 = \frac{1}{8}P^2(\mathrm{Tr}[M^2] + \mathrm{Tr}[\tilde{M}^2]) - \frac{1}{4}\mathrm{Tr}[P^{\intercal}MP\widetilde{M}]$$

$$M_{\alpha\beta} = i\sum_{i=1}^{N}\left(\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + \lambda_{i\beta}\frac{\partial}{\partial\lambda_i^{\alpha}}\right), \quad \widetilde{M}_{\dot\alpha\dot\beta} = i\sum_{i=1}^{N}\left(\tilde\lambda_{i\dot\alpha}\frac{\partial}{\partial\tilde\lambda_i^{\dot\beta}} + \tilde\lambda_{i\dot\beta}\frac{\partial}{\partial\tilde\lambda_i^{\dot\alpha}}\right)$$

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Define the action on amplitudes

$$W^2_{\mathcal{I}}\overline{B}^J(\mathcal{I}\to\mathcal{I}') \equiv \left(W^2\mathcal{C}^J_{\mathcal{I}}\right)\cdot\mathcal{C}^J_{\bar{\mathcal{I}}'} = -s_{\mathcal{I}}J(J+1)\overline{B}^J(\mathcal{I}\to\mathcal{I}')$$

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$$W^2_{\mathcal{I}}\mathcal{M}(\mathcal{I}\to\mathcal{I}') \equiv \sum_J a_J\left(W^2\mathcal{C}^J_{\mathcal{I}}\right)\cdot\mathcal{C}^J_{\bar{\mathcal{I}}'} = -s_{\mathcal{I}}\sum_J a_J J(J+1)\overline{B}^J(\mathcal{I}\to\mathcal{I}')$$

W^2 Construction of Partial Waves

```
In[3]:= W2[ab[1, 3] × ab[2, 4] + ab[1, 4] × ab[2, 3], {1, 2}] // Ampform // Simplify
Out[3]= -2 s12 (⟨14⟩ ⟨23⟩ + ⟨13⟩ ⟨24⟩)
```

$-2 s_{12} (\langle 14\rangle \langle 23\rangle + \langle 13\rangle \langle 24\rangle)$

```
In[4]:= W2[ab[1, 3]^2 ab[2, 4]^2 + 4 ab[1, 3] × ab[2, 4] × ab[1, 4] × ab[2, 3] + ab[1, 4]^2 ab[2, 3]^2, {1, 2}] // Ampform //
        Simplify
```

Out[4]= $-6 s_{12} \left(\langle 14\rangle^2 \langle 23\rangle^2 + 4 \langle 13\rangle \langle 14\rangle \langle 23\rangle \langle 24\rangle + \langle 13\rangle^2 \langle 24\rangle^2\right)$

In[5]:= W2Diagonalize$\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, 2, \{1, 2\}\right]$

Out[5]= ⟨| basis → {ab[1, 2] ab[3, 4]^2 sb[3, 4], −ab[1, 3] ab[2, 4]^2 sb[2, 4], ab[1, 3] × ab[2, 4] × ab[3, 4] × sb[3, 4]},

j → {2, 1, 0}, transfer → $\begin{pmatrix} -1 & -6 & 6 \\ -1 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}$,

j-basis → $\{-6 s_{24} \langle 13\rangle \langle 24\rangle - 6 s_{34} \langle 13\rangle \langle 24\rangle + s_{34} \langle 12\rangle \langle 34\rangle, -2 s_{34} \langle 13\rangle \langle 24\rangle + s_{34} \langle 12\rangle \langle 34\rangle, -s_{34} \langle 12\rangle \langle 34\rangle\}$ |⟩

```
In[6]:= PWExpand[ab[1, 3] × ab[2, 4] × s[1, 4], 4, {1, 2}]
```

Out[6]= ⟨| j → {2, 1, 0}, j-basis →

{6 ab[1, 3] ab[2, 4]^2 sb[2, 4] + 6 ab[1, 3] × ab[2, 4] × ab[3, 4] × sb[3, 4] − ab[1, 2] ab[3, 4]^2 sb[3, 4],

2 ab[1, 3] × ab[2, 4] × ab[3, 4] × sb[3, 4] − ab[1, 2] ab[3, 4]^2 sb[3, 4],

ab[1, 2] ab[3, 4]^2 sb[3, 4]}, coeff → $\left\{\frac{1}{6}, 0, \frac{1}{6}\right\}$ |⟩

Inner Products and Phase Space Integrations

Important property of partial wave amplitudes – orthonormality:

$$\int \mathrm{d}\Omega_1\, D^J_{\Delta'\Delta}(\Omega_1) D^{J'}_{\Delta\Delta''}(\Omega - \Omega_1) = \frac{4\pi}{2J+1}\delta^{JJ'} D^J_{\Delta'\Delta''}(\Omega).$$

This is exactly a two-body phase space (PS) integration $\mathrm{dLIPS}_2 = \frac{1}{8}\mathrm{d}\Omega$.

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$$\begin{aligned}\left\langle \mathcal{C}_n^{J,\sigma,a},\, \mathcal{C}_n^{J',\sigma',a'} \right\rangle &\equiv \int \mathrm{dLIPS}_n\, \mathcal{C}_n^{J,\sigma,a} \left(\mathcal{C}_n^{J',\sigma',a'}\right)^* \\ &= g_n^{aa'}(J)\delta^{JJ'}\delta^{\sigma\sigma'}\end{aligned}$$

[2111.08019]

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CGC can be ortho-normalized by $g(J) = \frac{\pi}{2(2J+1)}$!

$$\int \mathrm{dLIPS}_{x,y}\, \overline{B}^J_{\mathrm{L}}(1,2,\bar{x},\bar{y})\, \overline{B}^{J'}_{\mathrm{R}}(x,y,3,4) = \mathcal{C}^J(1,2) \cdot \left\langle \mathcal{C}^J(x,y),\, \mathcal{C}^{J'}(x,y) \right\rangle \cdot \mathcal{C}^{J'}(3,4) = g(J) \delta^{JJ'}\, \overline{B}^J(1,2,3,4).$$

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$$\int \mathrm{dLIPS}_{x,y} \, \overline{B}_{\mathrm{L}}^{J,a}(N \to \bar{x},\bar{y}) \, \overline{B}_{\mathrm{R}}^{J',b}(x,y \to M) = \mathcal{C}_N^{J,a} \cdot \left\langle \mathcal{C}^J(x,y),\, \mathcal{C}^{J'}(x,y) \right\rangle \cdot \mathcal{C}_M^{J',b} = g(J) \delta^{JJ'} \, \overline{B}^{J,ab}(N \to M).$$

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$$\int \mathrm{dLIPS}_n\, \overline{B}_{\mathrm{L}}^{J,ac}(N\to n)\, \overline{B}_{\mathrm{R}}^{J',db}(n\to M) = \mathcal{C}_N^{J,a}\cdot\left\langle \mathcal{C}_n^{J,c},\, \mathcal{C}_n^{J',d}\right\rangle\cdot\mathcal{C}_M^{J',b} = g(J)\delta^{JJ'}\,\overline{B}^{J,ab}(N\to M).$$

J-basis Operators

Definition

J-basis operators correspond to product of gauge and Poincaré partial waves

$$\mathcal{O}^{J,\mathbf{R}}_{\mathcal{I}\to\mathcal{I}'} \sim \mathcal{T}(\mathbf{R})\overline{B}^J(\mathcal{I}\to\mathcal{I}') \quad \begin{cases} W_{\mathcal{I}}^2\overline{B}^J = -s_{\mathcal{I}}J(J+1)\overline{B}^J \\ \mathbb{C}_{\mathcal{I}}\mathcal{T}(\mathbf{R}) = C(\mathbf{R})\mathcal{T}(\mathbf{R}) \end{cases}$$

When acting on the multi-particle state we have the conservation law

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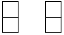
```
In[3]:= LoadModel["SMEFT.m"];

In[4]:= GetJBasisForType[SMEFT, "ec" "L" "Q" "uc" "D"^2, {{1, 3}, {2, 4}}]

Out[4]= ⟨|basis → {ϵ^ij (ec_p L_ri) ((D_μ Q_saj) (D^μ uc_t^a)), ϵ^ij (ec_p Q_saj) ((D_μ L_ri) (D^μ uc_t^a)),
      i ϵ^ij (ec_p σ_μν L_ri) ((D^μ Q_saj) (D^ν uc_t^a))}, groups → {SU3c, SU2w, Spin}, j-basis →
     {<|{L2, uc4} → {{0, 1}, {1}, 2}, {ec1, Q3} → {{1, 0}, {1}, 2}|> → {{-6, 2, -6}},
      <|{L2, uc4} → {{0, 1}, {1}, 1}, {ec1, Q3} → {{1, 0}, {1}, 1}|> → {{2, 2, -2}},
      <|{L2, uc4} → {{0, 1}, {1}, 0}, {ec1, Q3} → {{1, 0}, {1}, 0}|> → {{0, -2, 0}}}|⟩
```

Relation with “Real” Operators

Example of H^4D^2 operators in SMEFT, for channel $H_1^\dagger, H_2 \to H_3^\dagger, H_4$:

$\mathrm{Sym}_{H,H^\dagger}$	P-basis	$\mathcal{K}^{\mathrm{pj}}$	J-basis	$\mathbf{R}$	J
	$Q_{\varphi\Box}$	$3 \quad -1 \quad -1 \quad -1$	$(H_1^\dagger H_2)D^2(H_3^\dagger H_4)$	$\mathbf{1}$	0
	$Q_{\varphi D}$	$0 \quad 1 \quad -1 \quad 0$	$(H_1^\dagger \tau^I H_2)D^2(H_3^\dagger \tau^I H_4)$	$\mathbf{3}$	
	$Q'_{\varphi\Box}$	$5 \quad 1 \quad 1 \quad 1$	$(H_1^\dagger i\overleftrightarrow{D}_\mu H_2)(H_3^\dagger i\overleftrightarrow{D}^\mu H_4)$	$\mathbf{1}$	1
	$Q'_{\varphi D}$	$2 \quad 1 \quad 1 \quad 0$	$(H_1^\dagger i\tau^I \overleftrightarrow{D}_\mu H_2)(H_3^\dagger i\tau^I \overleftrightarrow{D}^\mu H_4)$	$\mathbf{3}$	

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$\text{Sym}_{H,H^\dagger}$	P-basis	$\mathcal{K}^{\text{pj}}$	J-basis	$\mathbf{R}$	J
$\yng(2)\ \ \yng(2)$	$Q_{\varphi\Box}$	$3 \quad -1 \quad -1 \quad -1$	$(H_1^\dagger H_2)D^2(H_3^\dagger H_4)$	$\mathbf{1}$	0
	$Q_{\varphi D}$	$0 \quad 1 \quad -1 \quad 0$	$(H_1^\dagger \tau^I H_2)D^2(H_3^\dagger \tau^I H_4)$	$\mathbf{3}$	
$\yng(1,1)\ \ \yng(1,1)$	$Q'_{\varphi\Box}$	$5 \quad 1 \quad 1 \quad 1$	$(H_1^\dagger i\overleftrightarrow{D}_\mu H_2)(H_3^\dagger i\overleftrightarrow{D}^\mu H_4)$	$\mathbf{1}$	1
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Relation of Operators	$Q_{\varphi\Box} = 3\mathcal{O}^{0,\mathbf{1}} - \mathcal{O}^{0,\mathbf{3}} - \mathcal{O}^{1,\mathbf{1}} - \mathcal{O}^{1,\mathbf{3}}$ $Q_{\varphi D} = \mathcal{O}^{0,\mathbf{3}} - \mathcal{O}^{1,\mathbf{1}}$
Relation of Coefficients	$C^{0,\mathbf{1}} = 3C_{\varphi\Box}, \quad C^{0,\mathbf{3}} = -C_{\varphi\Box} + C_{\varphi D},$ $C^{1,\mathbf{1}} = -C_{\varphi\Box} - C_{\varphi D}, \quad C^{1,\mathbf{3}} = -C_{\varphi\Box}.$

Vanishing Loops

N.Craig, M.Jiang, Y.-Y.Li, D.Sutherland [2001.00017]

$$\mathcal{A}^{1-\text{loop}} = \sum_i b_i I_2^i + c_i I_3^i + d I_4 + R$$

Non-Abelian

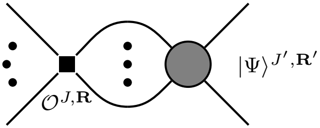
		(4,0)				(4,2)	
		$V^+V^+V^-V^-$	$V^+V^-\psi^+\bar\psi^-$	$V^+V^-\phi\phi$	$V^+\psi^-\bar\psi^-\phi$	$V^+V^+V^+V^-$	$V^+V^+\psi^+\bar\psi^-$
(4,0)	$\psi^2\bar\psi^2$	×	0	×	0*	×	R
	$\phi^4 D^2$	×	×	0	×	×	×
	$\phi^2\psi\bar\psi D$	×	0	0	0	×	R
(4,2)	$F\psi^2\phi$	×	R	R	R	×	0
	$F^2\phi^2$	R	0	R	R	0*	0*
	ψ^4	×	0	×	0	×	0
(4,−2)	$\bar F\bar\psi^2\phi$	×	R	R	R	×	0
	$\bar F^2\phi^2$	R	0	R	R	0	0
	$\bar\psi^4$	×	0	×	R	×	0

Abelian

	(4,0)				(4,2)	
	$V^+V^+V^-V^-$	$V^+V^-\psi^+\bar\psi^-$	$V^+V^-\phi\phi$	$V^+\psi^-\bar\psi^-\phi$	$V^+V^+V^+V^-$	$V^+V^+\psi^+\bar\psi^-$
$\psi^2\bar\psi^2$	×	0	×	0*	×	0
$\phi^4 D^2$	×	×	0	×	×	×
$\phi^2\psi\bar\psi D$	×	0	0	0	×	0
$F\psi^2\phi$	×	R	R	R	×	0
$F^2\phi^2$	R	0	R	R	0	0
ψ^4	×	0	×	0	×	0
$\bar F\bar\psi^2\phi$	×	R	R	R	×	0
$\bar F^2\phi^2$	R	0	R	R	0	0
$\bar\psi^4$	×	0	×	R	×	0

Vanishing Loops

All-Loop Selection Rule



$\sim \delta^{JJ'}\delta^{\mathbf{RR}'}$

Vanishing Loops

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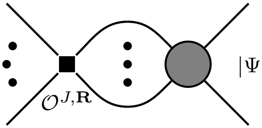

$$|\Psi\rangle^{J',\mathbf{R}'} \sim \delta^{JJ'}\delta^{\mathbf{R}\mathbf{R}'}$$

A $J' \geq |\Delta h|$

$$\mathcal{A}^{1-\text{loop}}(\psi^+, \psi^- \to V^+, V^-) = \sum_{J\geq 2} a_J \overline{B}^J$$

Vanishing Loops

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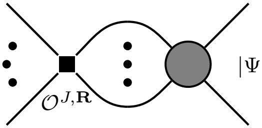

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B Two identical particles $\Rightarrow$ J' is even

Abelian: $\mathcal{A}^{1-\text{loop}}(\psi^+,\psi^-\to V^+,V^+)=\sum_{n\in\mathbb{Z}} a_{2n}\overline{B}^{2n}$

non-Abelian: $\mathcal{A}^{1-\text{loop}}(\psi^+,\psi^-\to V^{+a},V^{+b})=\sum_{n\in\mathbb{Z}} a_{2n}\delta^{ab}\overline{B}^{2n}+a_{2n+1}f^{abc}T^c\overline{B}^{2n+1}$

Calculate Anomalous Dimension Matrix

It is possible to explicitly compute ADM after partial wave expansion

$$\mathcal{A}^{1-\text{loop}} = \sum_i b_i I_2^i + \sum_j c_j I_3^j + dI_4 + R = \sum_i \frac{b_i}{\epsilon} + O(\epsilon^0)$$

The UV divergence only come from bubble cuts $b_i \sim \int \text{dLIPS}_2 \, \mathcal{A}_{i\text{L}} \times \mathcal{A}_{i\text{R}}$

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$$b_{N\to M} = \int \text{dLIPS}_2\, \overline{B}_{\text{L}}^{J,a}(N\to 2)\, \overline{B}_{\text{R}}^{J,b}(2\to M) = g(J)\overline{B}^{J,ab}(N\to M)$$

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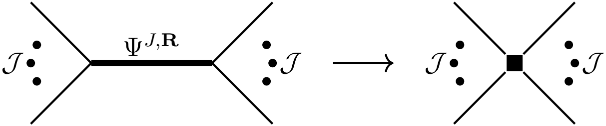
Selection rule for ADM: $\gamma = 0$ when $\{J\} \cap \{J\} = \emptyset$.

J-basis from Resonance

- Top-down: $\mathcal{L}_{\rm UV} \supset \Psi^{J,\mathbf{R}}_{\rm heavy} \cdot \mathcal{J}_{\rm light} \xrightarrow{\rm CDE} \mathcal{J}_{\rm light} \cdot \mathcal{J}_{\rm light}$



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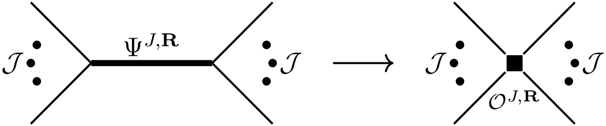

- Bottom-up: $\mathcal{O}^{J,\mathbf{R}} \longrightarrow \Psi^{J,\mathbf{R}}_{\text{heavy}}$ exhaust possible resonances without models!

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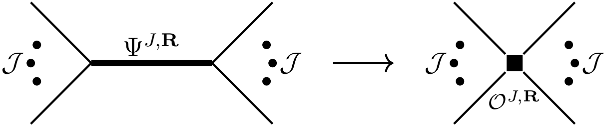

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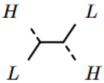
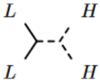
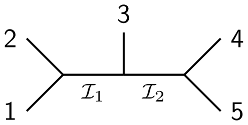
Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$	Model
	$\mathcal{B}^{J=1/2,\mathbf{R}=1}_{\{13\}} = \mathcal{B}^p_1 + \mathcal{B}^p_2.$	$\{\frac{1}{2}, 1, 0\}$	Type I
	$\mathcal{B}^{J=1/2,\mathbf{R}=3}_{\{13\}} = -\mathcal{B}^p_1 + 3\mathcal{B}^p_2,$	$\{\frac{1}{2}, 3, 0\}$	Type III
	$\mathcal{B}^{J=0,\mathbf{R}=3}_{\{12\}} = -2\mathcal{B}^p_1,$	$\{0, 3, -1\}$	Type II
	$\mathcal{B}^{J=0,\mathbf{R}=1}_{\{12\}} = 2\mathcal{B}^p_2.$	$\{0, 1, -1\}$	N/A

Multi-Partite Partial Waves

1. Find all tree topologies $\mathcal{P} = \{\mathcal{I}_i\}$.



$$\mathcal{I}_1 = \{1,2\} \simeq \{3,4,5\}$$
$$\mathcal{I}_2 = \{1,2,3\} \simeq \{4,5\}$$

$$[W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_2}] = 0\ , \quad [\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_2}] = 0$$

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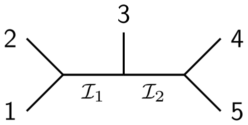

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$$[W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_2}] = 0 \ , \quad [\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_2}] = 0$$

2. Find the simultaneous eigen-functions for all channels $\mathcal{I}_i$:

$$W^2_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = -s_{\mathcal{I}_i} J_i(J_i+1) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} ,$$
$$\mathbb{C}_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = C(\mathbf{R}_i) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$$

Multi-partite partial waves describe cascade decay processes in experiments.

Multi-Partite Partial Waves

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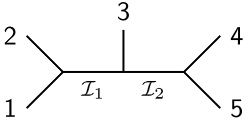

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Multi-partite partial waves describe cascade decay processes in experiments.

3. Obtain the corresponding operators $\mathcal{O}_{\mathcal{P}}^{\{J_i\},\{\mathbf{R}_i\}} \sim \mathcal{B}_{\mathcal{P}}^{\{J_i\},\{\mathbf{R}_i\}}$
They are generated after integrating out heavy particles $\Psi_i^{J_i,\mathbf{R}_i}$

Bottom-up Approach to Tree-Level UV Origins

	Top-down		Bottom-up
	UV model	Exhaustive Search	
Scan over Topologies	✘	✔	✔
UV Lagrangian	✔	✔	✘
Matching	✔	✔	✘
Complete UV	✘	✔	✔

- Complete list of Tree-level UV origins with arbitrary spins and gauge irreps.
- Automatic Code Implementation, applicable to generic EFT.
- Including non-renormalizable UV couplings (can be excluded if needed).
- Classification: tree v.s. loop origins.

Summary

- We generalized the definition of Partial Waves
  - Lorentz covariance
  - Arbitrary particle numbers
  - Multi-partite

  Orthonormal conditions are extended, applied to phase space integration.
- Casimir method of getting Poincaré/Guage partial waves
- Construction of J-basis operators, selection rules.
- Associate to tree-level UV origins: EFT inverse problem!
- Outlook: Improve theoretical bounds – unitarity, positivity ...

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Thank you for your attention!

Backup Slide

- Amplitude-Operator Correspondence

$$\mathcal{O} \sim \mathcal{B} \equiv \int \mathrm{d}^4x \, \langle 0|\mathcal{O}(x)|\Psi_i(p_i)\rangle$$

- $SU(N)$ Casimir $\mathbb{C}_2 \equiv \mathbf{T}^A\mathbf{T}^A$ has eigenvalue $C_2(\mathbf{R})$

$$\mathbf{T}^A \mathcal{T}_b^{aB} = (T^A)_c^a \mathcal{T}_b^{cB} + (T^A)_b^c \mathcal{T}_c^{aB} + i f^{ABC} \mathcal{T}_b^{aC}$$

e.x. $\mathcal{T}(\mathbf{3})_{ik}^{jl} \sim (\tau^I)_i^j (\tau^I)_k^l$.

- Higher spin UV couplings

$$\mathcal{L}_{\mathrm{UV}} \supset \Psi^{J\geq 1} \cdot \mathcal{J} \quad \begin{cases} \mathcal{J} \text{ is conserved with spin } J & \checkmark \\ \mathcal{J} \propto p \text{ with spin } \leq J & \stackrel{\mathrm{EOM}}{\Rightarrow} \mathcal{J}\cdot\mathcal{J} \equiv \mathcal{O}^{\leq J} \end{cases}$$

e.x. $V_\mu \partial^\mu \Phi \Rightarrow \Phi \Box \Phi$.