Non-perturbative Effects in a Simplified t-channel Dark Matter Model

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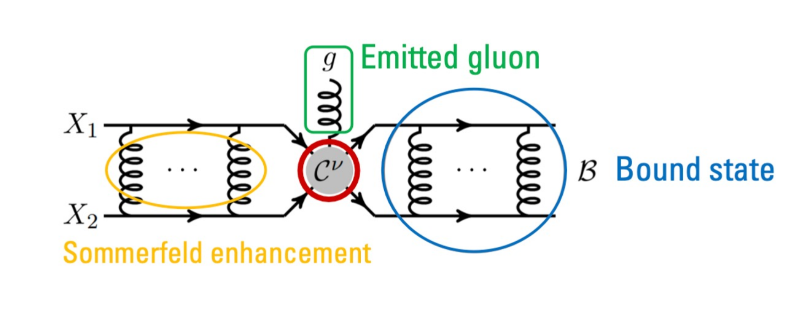

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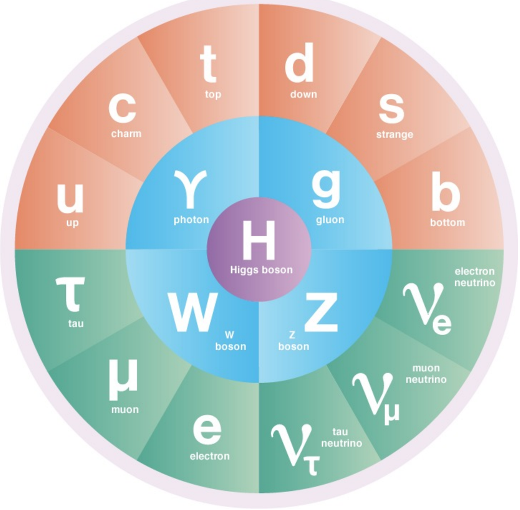
A Simplified t-channel Model

- SM + Dark Matter Particle (χ: $Majorana$) + mediator ($\tilde{X}_i$ $(\mathbf{3}, \mathbf{1}, +2/3)$)

$$\mathcal{L} \supset \sum_i (D^\mu \tilde{X}_i)^\dagger (D_\mu \tilde{X}_i) - m_X^2 \tilde{X}_i^\dagger \tilde{X}_i + g_{\rm DM} \tilde{X}_i^\dagger \overline{\chi} P_R q_i + h.c.$$



Mediator $\tilde{X}_i$

Dark Matter χ

Simple model with three parameters;

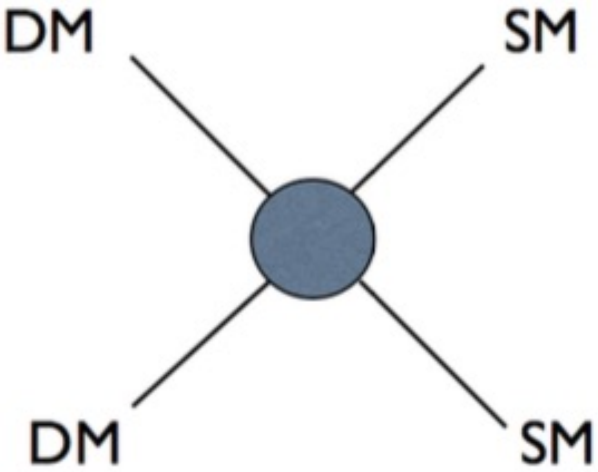
Model parameters: $\{m_\chi, \Delta = m_X - m_\chi, g_{\rm DM}\}$

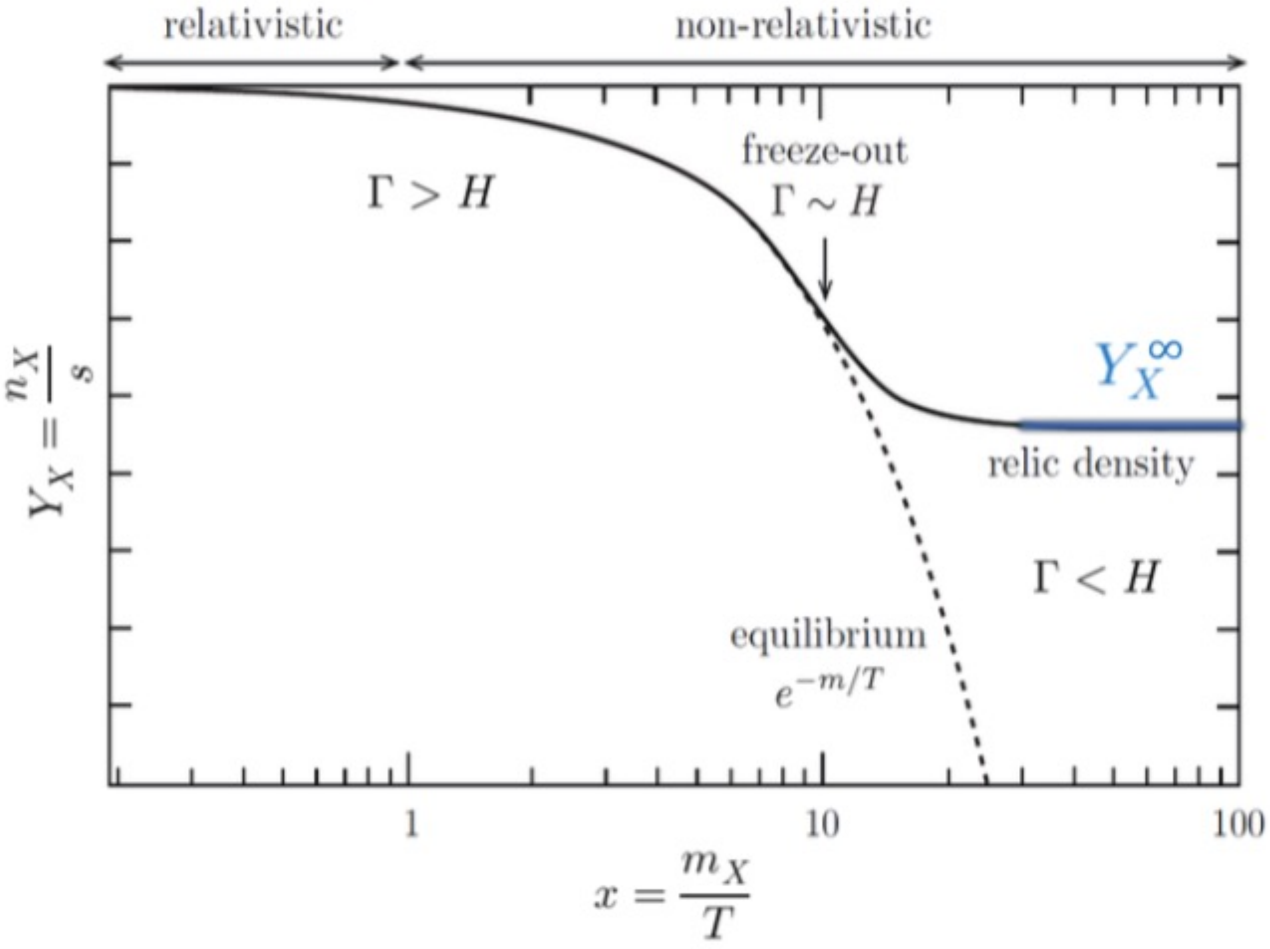
DiFranzo et. al. 2013, KM et. al 2019
Arina et. al. 2020 & 2021

$$\dot{n}(t) + 3H(t)n(t) = -\langle \sigma_{\chi\chi} v \rangle \left(n(t)^2 - n_{\rm eq}(t)^2 \right)$$

$$\frac{dY_X}{dx} = -\frac{s\langle \sigma v \rangle}{Hx}[Y_X^2 - (Y_X^{eq})^2]$$

- In the early universe, dark matter was in thermal equilibrium with SM



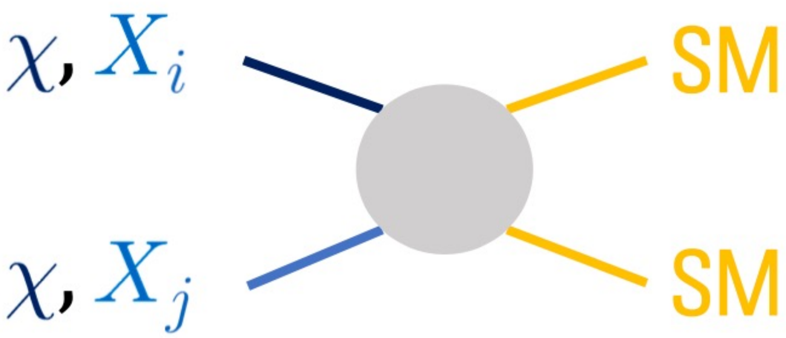


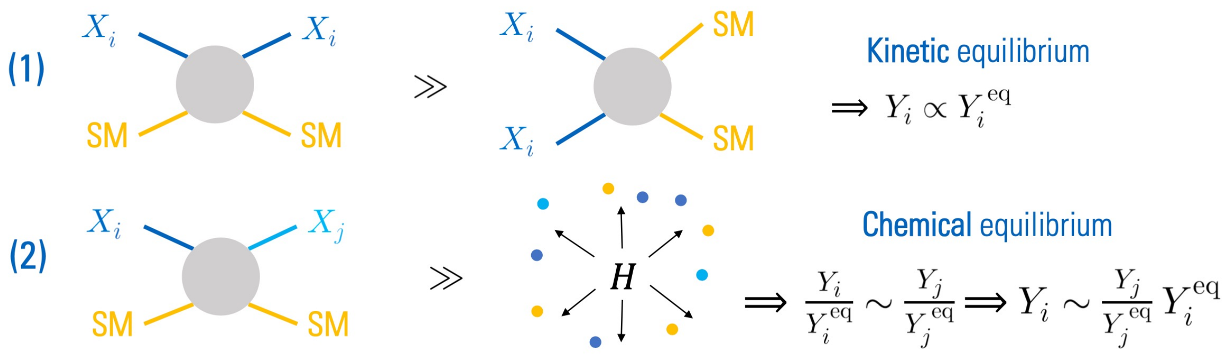
Co-annihilation

[Griest&Seckel (1991)], [Edsjö&Gondolo (1997)], …

- If $m_{X_i} \gg m_\chi$ then these are Boltzmann suppressed.
- If $m_{X_i} \sim m_\chi$ we would need a system of n Boltzmann equations

Co-annihilations





(3) Eventually, all the X_i will decay into the LSP DM

dark sector = { $\chi, X_2, X_3, \ldots, X_n$ }

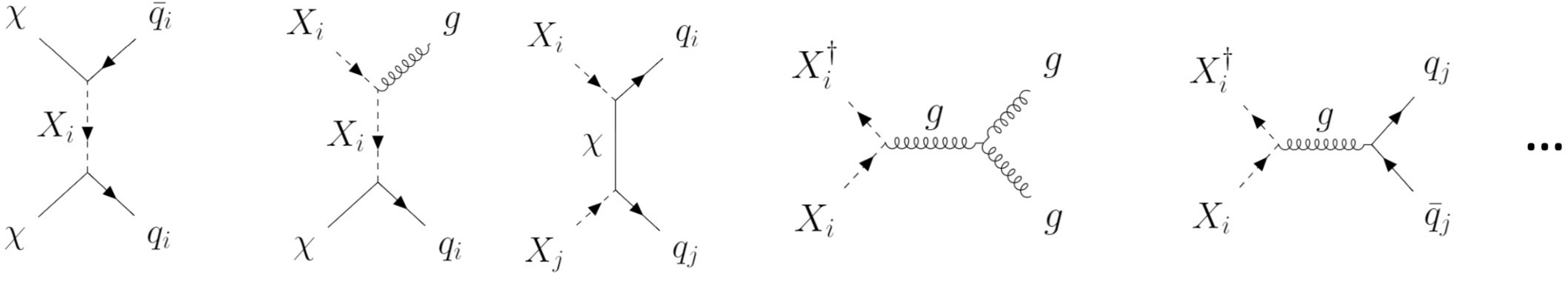
$$\tilde{Y} = Y_\chi + \sum Y_{X_i}$$

Effective Boltzmann equation

$$\frac{d\tilde{Y}}{dx} = -\frac{s}{H\,x}\langle\sigma_{\rm eff} v_{\rm rel}\rangle\left(\tilde{Y} - \tilde{Y}^{\rm eq}\right)$$

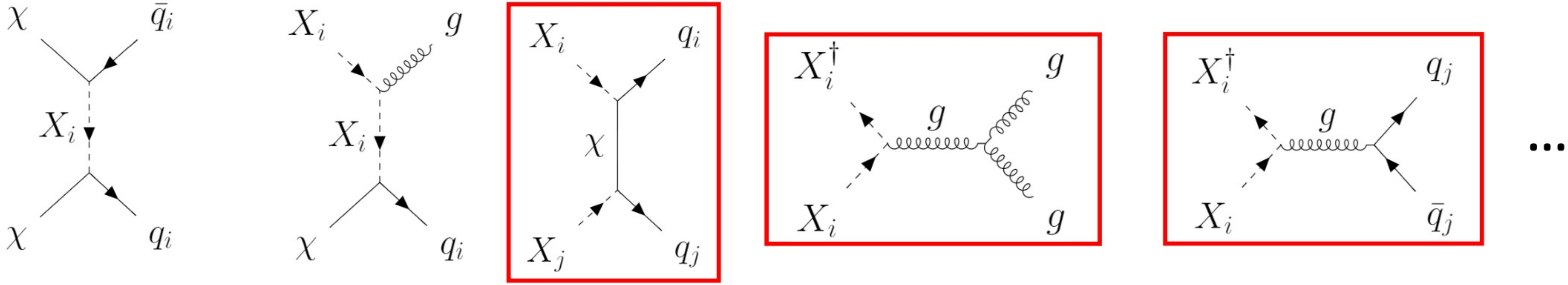
$$\langle\sigma_{\rm eff} v_{\rm rel}\rangle = \sum_{ij}\langle\sigma_{ij} v_{ij}\rangle\frac{Y_i^{\rm eq} Y_j^{\rm eq}}{\tilde{Y}^{\rm eq^2}}$$

Co-annihilation



g_{DM}^4 $\quad$ $g_{\mathrm{DM}}^2 g_s^2 e^{-\Delta/T}$ $\quad$ $g_{\mathrm{DM}}^4 e^{-2\Delta/T}$ $\quad$ $(\alpha g_{\mathrm{DM}}^2 + \beta g_s^2)^2\, e^{-2\Delta/T}$ $\quad$ $g_{\mathrm{t}}^4 e^{-2\Delta/T}$

Relevant when mass splitting Δm small



g_{DM}^4 $\quad$ $g_{\mathrm{DM}}^2 g_s^2 e^{-\Delta/T}$ $\quad$ $g_{\mathrm{DM}}^4 e^{-2\Delta/T}$ $\quad$ $(\alpha g_{\mathrm{DM}}^2 + \beta g_s^2)^2 e^{-2\Delta/T}$ $\quad$ $g_s^4 e^{-2\Delta/T}$

Relevant when mass splitting Δm small

! **Incoming colored states experience gluonic long-range force!**

Recall: $X \sim \mathbf{3}$ and $X^\dagger \sim \overline{\mathbf{3}}$ of $\mathrm{SU}(3)$

Sommerfeld Effect

A. Sommerfeld Ann. Phys. 403 (1931) 257

Slowly-moving massive particles experience the presence of the **NR potential** between them:

- Wavefunctions are distorted already at large distances (long-range effect)
- Probability of finding particle at interaction vertex is modified (non-perturbative)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_l}{dr}\right) = \left[2\mu\left[V(r) - E\right] + \frac{l(l+1)}{r^2}\right]R_l \qquad V_{\mathrm{Coulomb}}(r) = \pm\frac{\alpha}{r}$$

$$\mu = m_X/2$$

$$\sigma_\ell^S v_{\mathrm{rel}} = S_\ell\, \sigma_\ell^{\mathrm{pert.}}$$

$$\zeta = \alpha/v_{\mathrm{rel}}$$

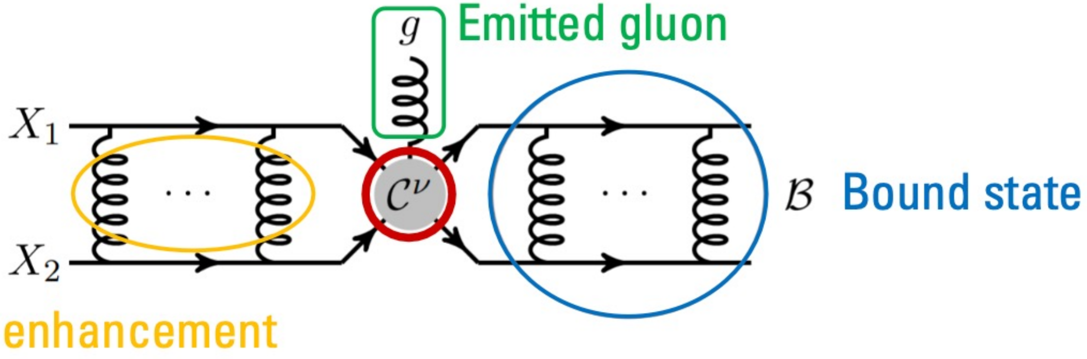
$$S_\ell = \left|\frac{(2\ell+1)!!}{|p|^\ell \ell!}\frac{\partial^\ell R_\ell(r)}{\partial r^\ell}\Big|_{r=0}\right|^2 \xrightarrow[\text{Coulomb}]{} S_\ell(\zeta) = \frac{2\pi\zeta}{1-e^{-2\pi\zeta}} \times \prod_{n=1}^{\ell}\left(1+\frac{\zeta^2}{n^2}\right)$$

Bound State Formation

[Harz and Petraki (2018)]
[Petraki et al. (2015)]

$$\mathbf{R_1} \otimes \mathbf{R_2} = \sum \hat{\mathbf{R}}$$

$C_2(\mathbf{R})$: quadratic Casimir of R



$$V_{\text{gluon}}^{[\hat{\mathbf{R}}]}(r) = -\frac{\alpha_g^{[\hat{\mathbf{R}}]}}{r} = -\frac{\alpha_s}{2r}\left[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\hat{\mathbf{R}})\right] \xrightarrow{\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}} V(r)_{\mathbf{3}\otimes\bar{\mathbf{3}}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & [\mathbf{1}] \\ +\frac{1}{6}\frac{\alpha_s}{r} & [\mathbf{8}] \end{cases}$$

$$(X + X^\dagger)_{[\mathbf{8}]} \to \left\{\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g\right\}_{[\mathbf{8}]}$$

$$\sigma_{\{100\}}^{[\mathbf{8}]\to[\mathbf{1}]} v_{\text{rel}} = \frac{2^7\, 17^2}{3^5} \frac{\pi \alpha_{s,[\mathbf{1}]}^{\text{BSF}} \alpha_{s,[\mathbf{1}]}^{B}}{m_X^2} S_{\text{BSF}}(\zeta_S, \zeta_B)$$

$$S_{\text{BSF}}(\zeta_S, \zeta_B) = \left(\frac{2\pi\zeta_S}{1 - e^{-2\pi\zeta_S}}\right)(1 + \zeta_S^2)\frac{\zeta_B^4 e^{-4\zeta_S \operatorname{arccot}(\zeta_B)}}{(1 + \zeta_B^2)^3}$$

Bound state decay and ionization

If X_1 and X_2 can (co-)annihilate into n lighter particles, then their bound states $\mathcal{B}(X_1, X_2)$ are unstable against decay into the same final-state particles.

Decay

$\mathcal{B}(XX^\dagger) \to gg$

→ New effective annihilation channel!

$$\Gamma_{\mathrm{dec},[\mathbf{R}]} = (\sigma^{s\text{-wave}}_{\mathrm{ann},[\mathbf{R}]} v_{\mathrm{rel}})|\psi^{[\mathbf{R}]}_{nlm}(0)|^2$$

$$|\psi^{[\mathbf{1}]}_{100}(0)|^2 = 8m_X^3 \mathfrak{S}\alpha^B_{s,[\mathbf{1}]}\mathfrak{T}^3/27\pi$$

$$\Gamma_{\mathrm{dec},[\mathbf{1}]} = \frac{32}{81} m_X (\alpha_s^{\mathrm{ann}})^2 (\alpha^B_{s,[\mathbf{1}]})^3$$

At $T \gg \mathcal{E}_{100} = \omega$, energetic gluons in thermal plasma can also dissociate/ionize B.S. into their constituents

Ionization

$\mathcal{B}(XX^\dagger) + g \to X + X^\dagger + g$

$$\Gamma_{\mathrm{ion}} = g_g \int_{\omega_{\min}}^{\infty} \frac{d\omega}{2\pi^2} \frac{\omega^2}{e^{\omega/T} - 1} \sigma_{\mathrm{ion}}$$

$$\sigma_{\mathrm{ion}} = \frac{g_X^2}{g_g g_{\mathcal{B}}} \left(\frac{\mu^2 v_{\mathrm{rel}}^2}{\omega^2}\right) \sigma_{\mathrm{BSF}}$$

BSF impact on Boltzmann equation

$$\langle \sigma_{\rm eff} v_{\rm rel} \rangle = \sum_{i,j=\{\chi, X\}} \langle \sigma_{ij} v_{ij} \rangle \frac{Y_i^{\rm eq} Y_j^{\rm eq}}{(\tilde{Y}^{\rm eq})^2} + \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} \frac{(Y_X^{\rm eq})^2}{(\tilde{Y}^{\rm eq})^2}$$

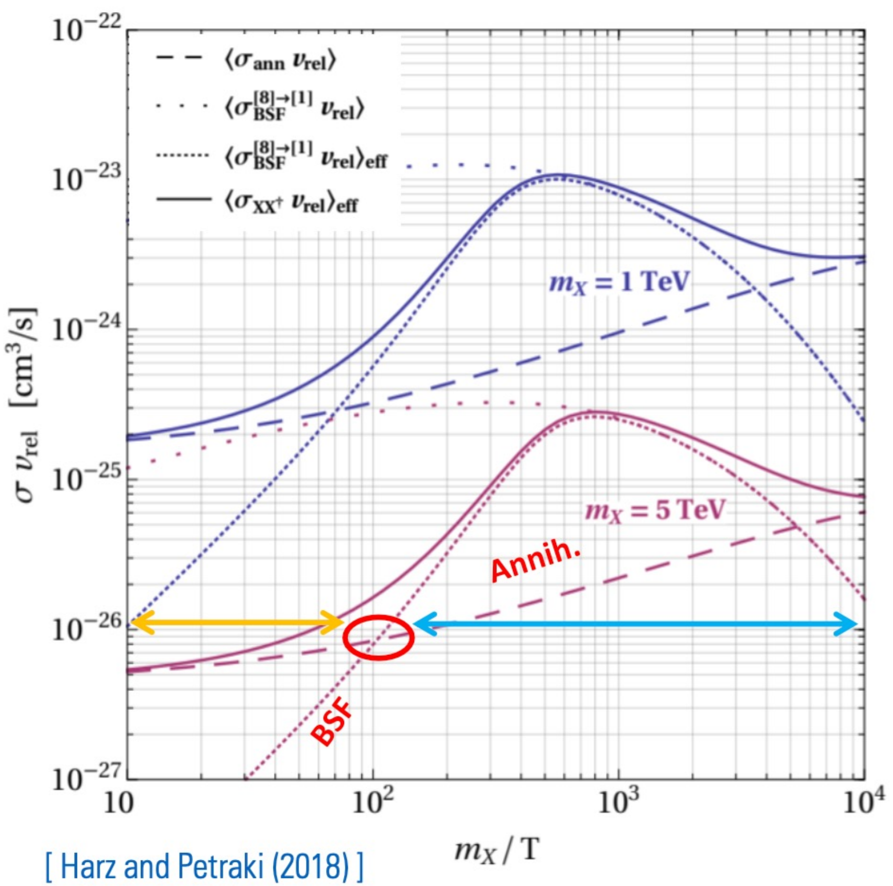
$$\mathcal{B}\mathrm{F}XX^\dagger\mathrm{G} \to g + g$$

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} \equiv \langle \sigma_{\rm BSF}^{[\mathbf{8}] \to [\mathbf{1}]} v_{\rm rel} \rangle \frac{\Gamma_{{\rm dec}[\mathbf{1}]}}{\Gamma_{{\rm dec}[\mathbf{1}]} + \Gamma_{{\rm ion},[\mathbf{1}]}}$$

$$\mathcal{B}_{\mathbb{W}} XX^\dagger{}_{\mathbb{X}} + g \to X + X^\dagger$$

Large T $\quad \Gamma_{\rm Ion} \gg \Gamma_{\rm Dec} \;\Rightarrow\; \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} \to 0$

Small T $\quad \Gamma_{\rm Ion} \ll \Gamma_{\rm Dec} \;\Rightarrow\; \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} \to \langle \sigma_{\rm BSF} v_{\rm rel} \rangle$



[Harz and Petraki (2018)]

Relic density computation

- Several tools for calculating DM relic density + experimental signatures (non-exhaustive list)
  - MadDM [Backovic et al. (2013)]
  - SuperIso Relic [Arbey&Mahmoudi (2009)]
  - DarkSUSY [Gondolo et al. (2004)]
  - MicrOMEGAs [Belanger et al. (2010)]

- BSF and Sommerfeld effects not included

  Exception: DarkSUSY includes Sommerfeld effect only for electroweak interactions.

MadDM

SuperIso Relic

Dark SUSY

MicrOMEGAs

We modified **MicrOMEGAs v.5.2.7** including:

1. **Sommerfeld** effect ($\mathbf{3} \otimes \bar{\mathbf{3}}$ and $\mathbf{3} \otimes \mathbf{3}$)
2. **BSF** (singlet ground state) for **colored** particles.

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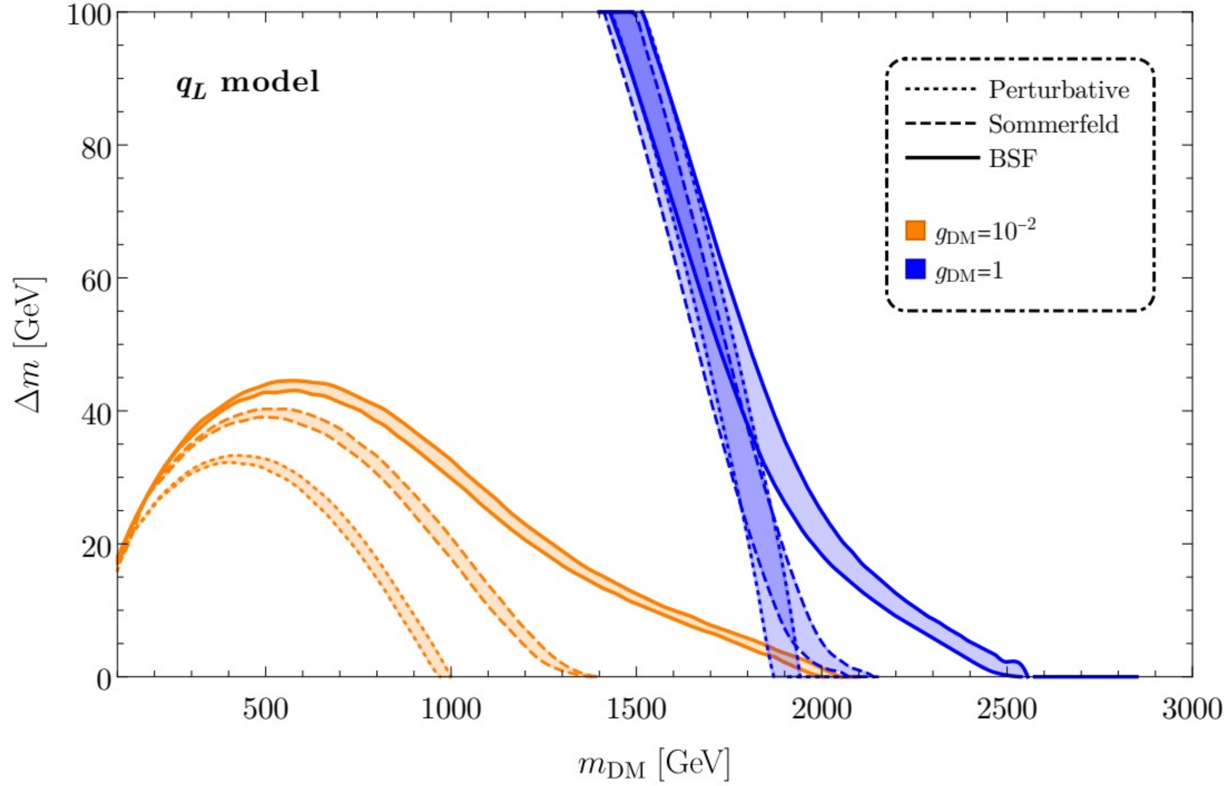

- Bands $\leftrightarrow \Omega_{\rm DM} h^2 = 0.120 \pm 0.005$
- Dramatic change in DM density with SE and SE+BSF for small $g_{\rm DM}$ when $\Delta m \ll m_{\rm DM}$.
- For $g_{\rm DM} \sim \mathcal{O}(1)$ still sizable effects
- Stronger effective annihilations
  $\Longrightarrow$ larger DM masses needed
  $\Longrightarrow$ larger mass splittings Δm

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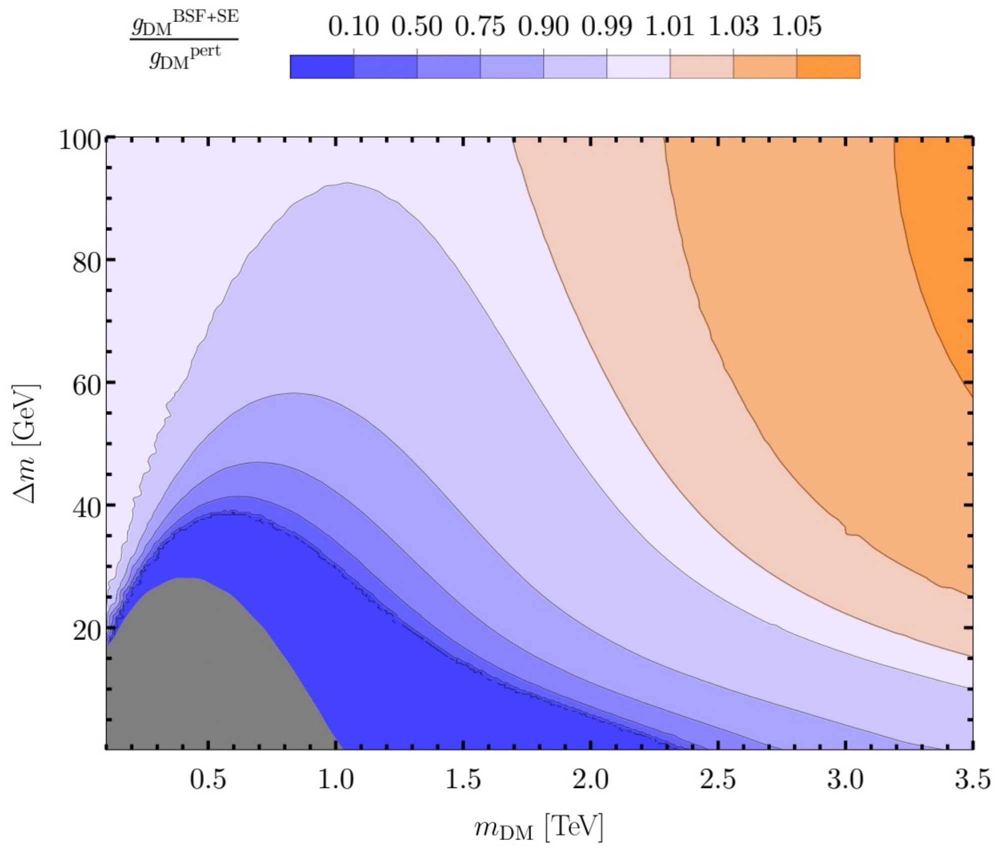
Cannot use k factor



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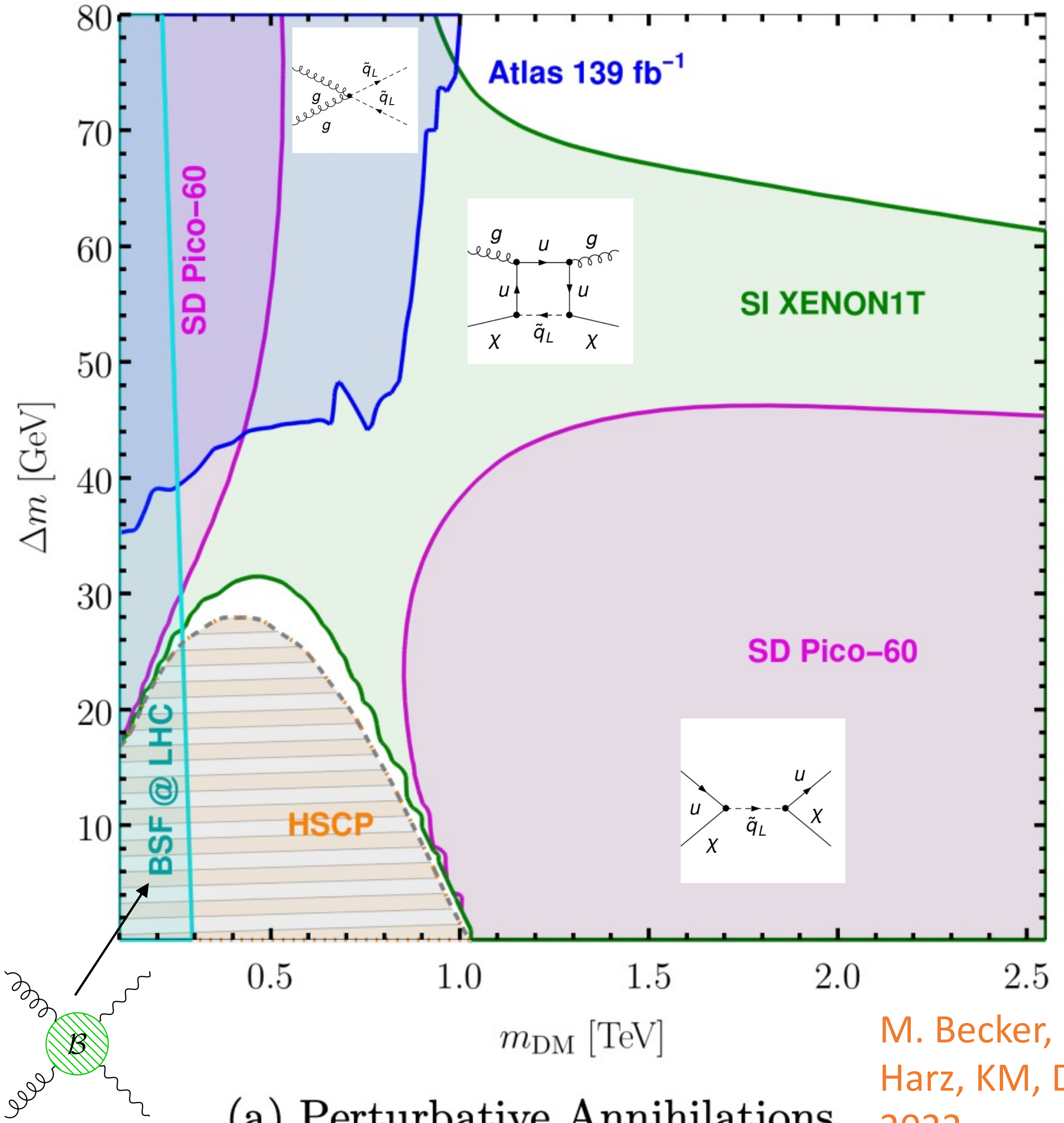

(a) Perturbative Annihilations

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- HSCP: Heavy stable charged particle searches (decays in or outside detector)
- BSF: Bound State Formation – Bound states form and decay to gauge bosons at LHC
- SI : Spin Independent Direct Detection
- SD: Spin Dependent Direct Detection

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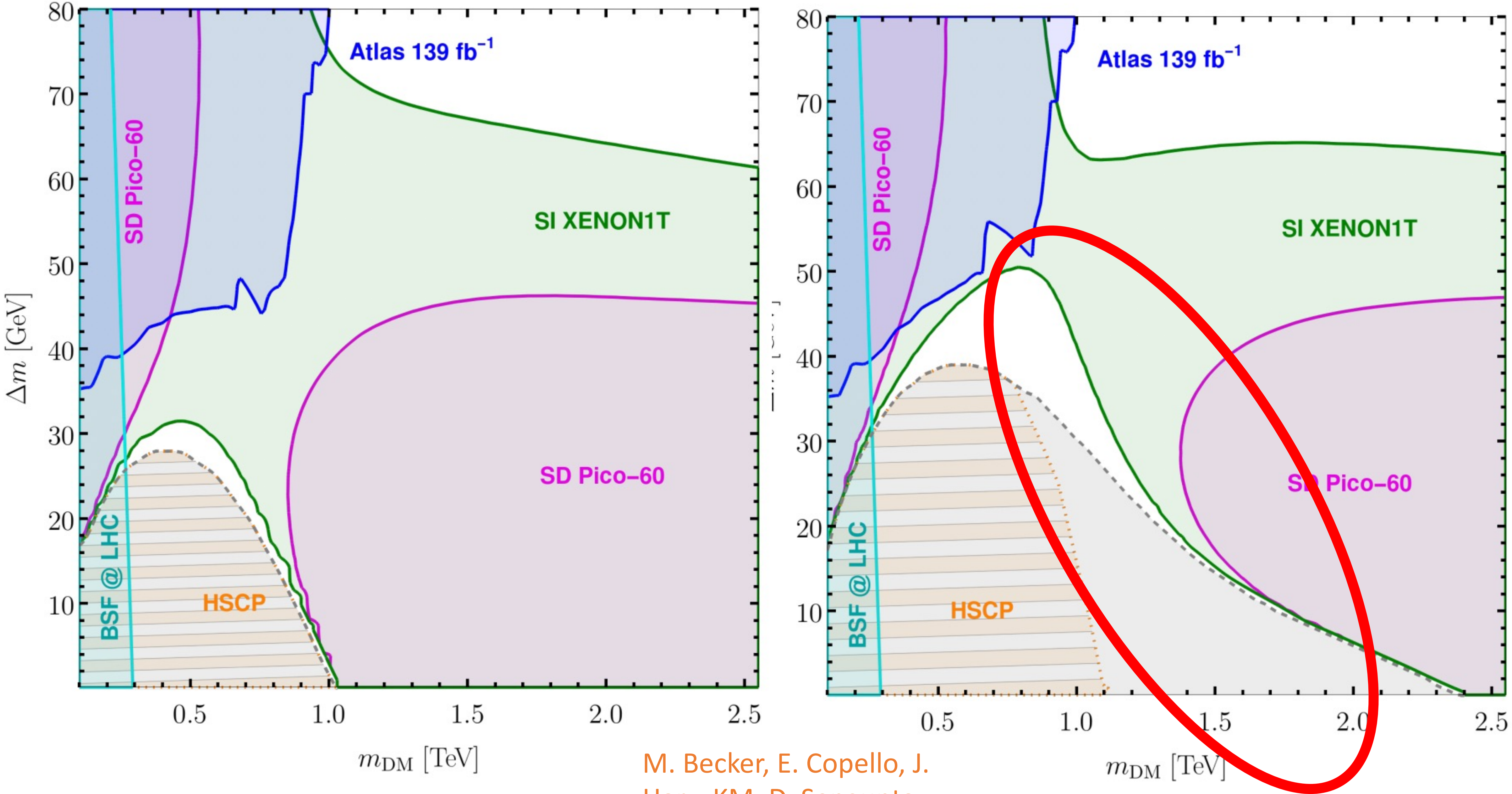

(a) Perturbative Annihilations

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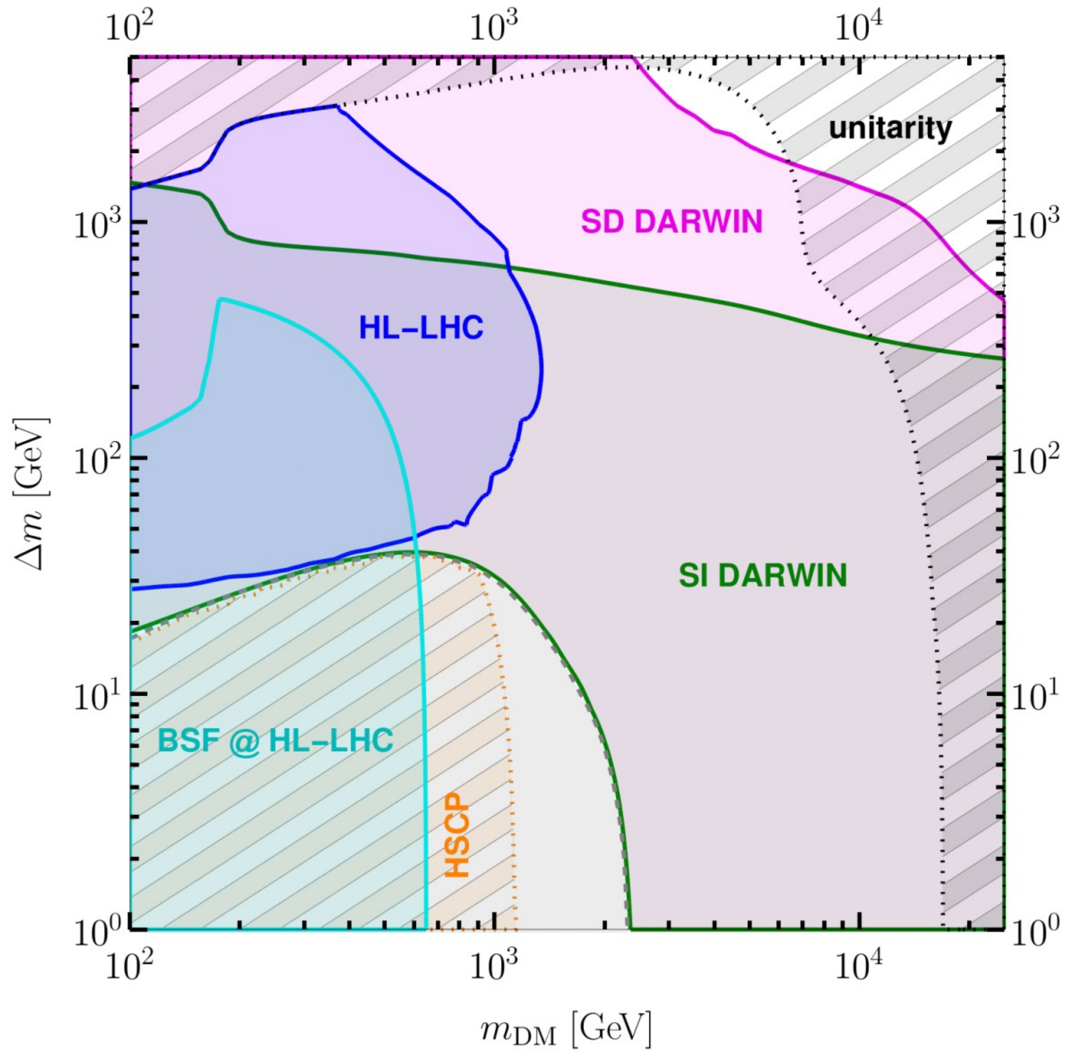
(c) Sommerfeld+BSF

Future Projection

- HSCP: Heavy stable charged particle searches (decays in or outside detector)
- BSF: Bound State Formation – Bound states form and decay to gauge bosons at LHC
- SI : Spin Independent Direct Detection
- SD: Spin Dependent Direct Detection

FCC 100TeV ?



(c) Sommerfeld+BSF

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Summary

- Looked at the impact of Sommerfeld enhancement and Bound State Formation on the relic density.
  - Strong effect on the Simplified t-channel parameter space – and should be taken seriously.
  - Next steps
    - Formalism allows to be generalized to many models
    - Plans to release tool for community use.