

# The Migdal Effect in $H_2^+$

Ian Harris

University of Illinois at Urbana-Champaign

Yoni Kahn, Carlos Blanco, Jesus Perez Rios, Benjamin Lillard

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# Outline

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## ① Introduction

## ② Migdal Effect

Explanation of ME

Direct Detection in Different Systems

## ③ The Hydrogen Molecular Ion

Two Distinct Effects

Preliminary Detection Thresholds



# Daily Modulation

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- Great way to distinguish signal from background
- Sidereal vs. Earth day
- Standard Halo Model (SHM) ansatz for DM velocity distribution<sup>2</sup>
  - Gives time dependence over a sidereal day

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<sup>2</sup>Evans, O'Hare, McCabe; arXiv:1810.11468v2

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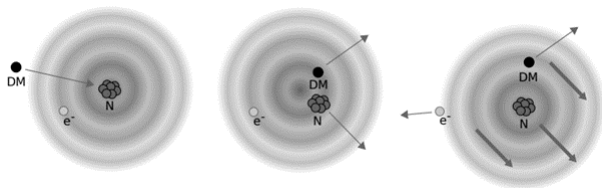
## ③ The Hydrogen Molecular Ion

Two Distinct Effects

Preliminary Detection Thresholds

# Migdal Effect

- Particle-nucleus scattering
  - Momentum transfer displaces nucleus, induces electronic transition probability



**Figure:** Migdal effect for a single atom. In this case, the electron is emitted.<sup>3</sup>

- Electron can be emitted or transition to a higher energy state
  - Need to determine initial, final wavefunctions

<sup>3</sup>Dolan, Kahlhoefer, McCabe; arXiv:1711.09906v2

# Semiconductor Migdal Effect

- Nucleus is not a free particle, so qualitatively different in bound systems

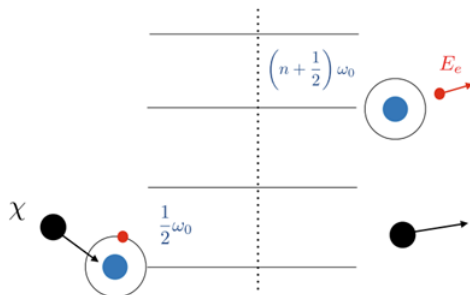


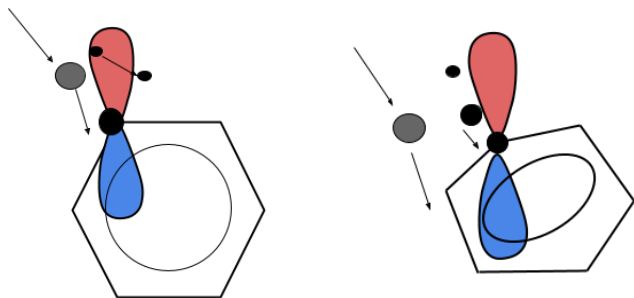
Figure: Migdal Effect in a lattice, approximated by a harmonic potential<sup>4</sup>

<sup>4</sup>Kahn, Krnjaic, Mandava; arXiv:2011.09477v1

## Electron Scattering vs. Migdal Effect

- DM can couple to electrons and/or nuclei
- Electron, Migdal scattering have identical final state: recoiling atom, excited electron
- Migdal rate can exceed electron scattering rate<sup>5</sup>:

$$\frac{dR_M/d\vec{q}}{dR_e/d\vec{q}} > Z^2 \left( \frac{m_e}{m_N} \right)^2 (\vec{q}r_a)^2 \quad (1)$$



<sup>5</sup>Baxter, Kahn, Krnjaic; arXiv:1908.00012v3



## Electron Scattering: Organic Molecules and Anisotropy

- Organic crystals can show directional dependence in scattering rate
  - Improves detection ability
- Linear Combination of Atomic Orbitals (LCAO) method for electronic states<sup>6</sup>

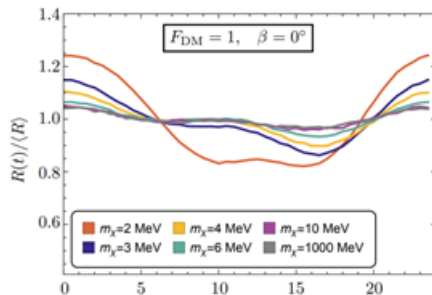


Figure: Daily modulation of electron scattering in trans-stilbene crystals<sup>7</sup>

- Goal: Exploit anisotropy for the Migdal Effect

<sup>6</sup>Blanco, Kahn, Lillard, McDermott; arXiv:2103.08601v2

<sup>7</sup>Blanco, Kahn, Lillard, McDermott; arXiv:2103.08601v2

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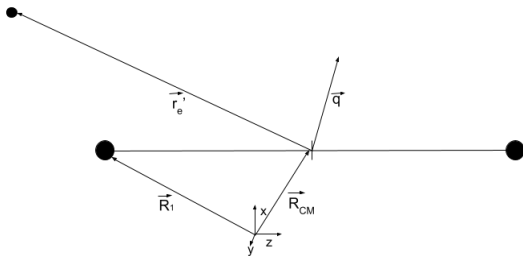
③ The Hydrogen Molecular Ion

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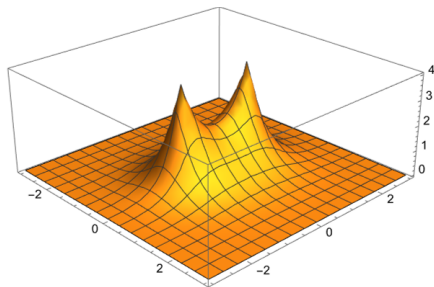
Preliminary Detection Thresholds

# $H_2^+$ Wavefunctions

- $H_2^+$  is a toy model; goal is to generalize results to realistic detector molecules
- Electronic wavefunctions numerically determined using the "shooting method"<sup>8</sup>



(a) Coordinate diagram for  $H_2^+$



(b) Ground state of  $H_2^+$

<sup>8</sup>Grivet, The Hydrogen Molecular Ion Revisited

# Center of Mass Recoil

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- General probability amplitude  $\langle f | \exp [i\vec{q} \cdot \vec{R}] | i \rangle$  becomes

$$M(q) \simeq 2 \langle \phi_{e'[\rho_0]} | \exp \left[ i \frac{m_e}{2m_N + m_e} \vec{q} \cdot \vec{r}_e \right] | \phi_{e[\rho_0]} \rangle \quad (2)$$

- Nuclear coordinate contains relative electron coordinate

## Center of Mass Results

- Directional dependence yields time dependence in a crystal

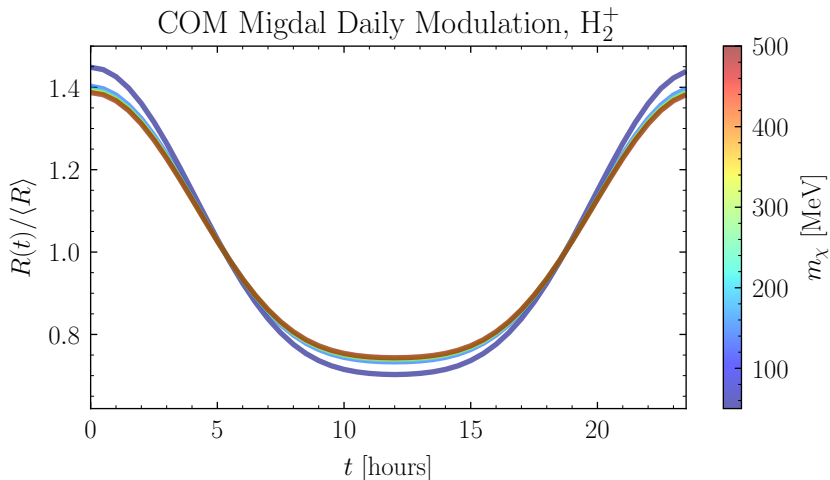


Figure: Daily modulation for center of mass recoil calculations. Across an order of magnitude in mass,  $\mathcal{O}(1)$  modulation is expected.

# Non-Adiabatic Coupling

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- Factorize 3-body Schrodinger equation:<sup>9</sup>

$$\frac{1}{m_e} \nabla^2 \Psi + \frac{1}{m_N} \sum_k \nabla_k^2 \Psi + 2(E - V) \Psi = 0 \quad (3)$$

$$\Psi(\vec{r}) = \psi(\vec{r}) \chi(\vec{r}) \quad (4)$$

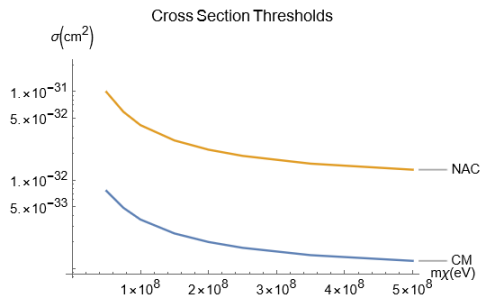
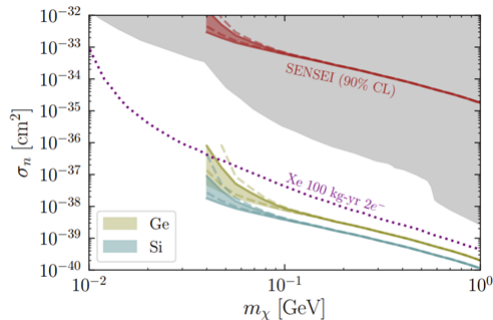
- Terms beyond Born-Oppenheimer treated as perturbations
- If we sum over all states, transition probability is isotropic!

$$P(\text{any electronic transition})_{\text{NAC}} = \frac{q^2}{M_{\text{total}}^2 \omega^2} \int d^3 r'_e \left( \frac{\partial \psi_0(\vec{r}'_e)}{\partial \rho} \right)_{\rho=\rho_0}^2 \quad (5)$$

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<sup>9</sup>Lovesey, Bowman, Johnson; Z. Phys. B - Condensed Matter 47, 137-147 (1982) 

# Thresholds for $H_2^+$ and Semiconductors

(a)  $H_2^+$  (preliminary)

(b) Semiconductor

Figure:  $H_2^+$  vs. semiconductor<sup>10</sup> detection thresholds<sup>10</sup>Knapen, Kozaczk, Lin; arXiv:2011.09496v1

# Summary

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- The Migdal Effect, alongside electronic scattering, can be used to study sub-GeV dark matter
- Daily modulation in anisotropic systems helps to distinguish signal from background
- The two distinct physical effects that I studied in  $H_2^+$  could eventually be studied in real detector molecules