

# Constraining anomalous Higgs boson couplings to virtual photons.

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Based on work in [arxiv:2109.13363](https://arxiv.org/abs/2109.13363)



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# JHUGen Framework

See Talks: [H. Roskes at LHC EFT WG](#)  
[H. Roskes at Pheno 2020](#)  
[M. Xiao at ICHEP 2020](#)  
[U. Sarica at Higgs 2020](#)  
[A. Gritsan at LHC Higgs WG](#)  
[M. Schulze at LHC Higgs WG](#)

JHUGenerator <https://spin.pha.jhu.edu/>

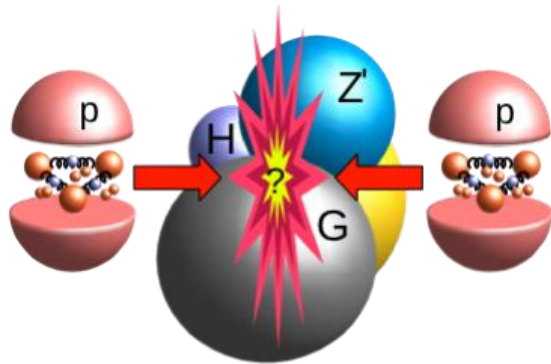
Simulate wide range of processes involving spin 0,1,2 particles with a general coupling model

JHUGen MELA – Matrix Element Likelihood Approach

Calculate observables to optimally isolate processes or operators  
Reweight generated samples from one hypothesis to another

JHUGenLexicon

Tool for translation between different EFT bases and the JHUGen amplitude basis convention



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*****  
*                               JHU Generator v7.5.1                               *  
*****  
*  
*   Spin and parity determination of single-produced resonances at hadron colliders   *  
*  
*   I. Anderson, S. Bolognesi, F. Caola, J. Davis, Y. Gao, A. V. Gritsan,          *  
*   L. S. Mandacaru Guerra, Z. Guo, C. B. Martin, T. Martini, K. Melnikov, R. Pan,  *  
*   R. Rontsch, J. Roskes, U. Sarica, M. Schulze, N. V. Tran, A. Whitbeck, M. Xiao, Y. Zhou *  
*   Phys.Rev. D81 (2010) 075022; arXiv:1001.3396 [hep-ph],                          *  
*   Phys.Rev. D86 (2012) 095031; arXiv:1208.4018 [hep-ph],                          *  
*   Phys.Rev. D89 (2014) 035007; arXiv:1309.4819 [hep-ph],                          *  
*   Phys.Rev. D94 (2016) 055023; arXiv:1606.03107 [hep-ph],                          *  
*   Phys.Rev. D102 (2020) 056022; arXiv:2002.09888 [hep-ph],                       *  
*   Phys.Rev. D102 (2021) 055045; arXiv:2104.04277 [hep-ph].                       *  
*   arXiv:2109.13363 [hep-ph].                                                     *  
*  
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# Anomalous Couplings and EFT

HVV couplings parameterized by tensor structures which allow for modelling of any EFT effects

$$A(HVV) = \frac{1}{v} \left\{ M_V^2 \left( g_1^{VV} + \frac{\kappa_1^{VV} q_1^2 + \kappa_2^{VV} q_2^2}{(\Lambda_1^{VV})^2} + \frac{\kappa_3^{VV} (q_1 + q_2)^2}{(\Lambda_Q^{VV})^2} + \frac{2q_1 \cdot q_2}{M_V^2} g_2^{VV} \right) (\varepsilon_1 \cdot \varepsilon_2) \right. \\ \left. - 2g_2^{VV} (\varepsilon_1 \cdot q_2)(\varepsilon_2 \cdot q_1) - 2g_4^{VV} \varepsilon_{\varepsilon_1 \varepsilon_2 q_1 q_2} \right\}.$$

Using JHUGenLexicon we can map these amplitude couplings to any other EFT basis we want:

Enforce SU(2) x U(1) to translate between Amplitude basis and EFT bases

$$\begin{aligned} \delta g_1^{ZZ} &= \frac{v^2}{\Lambda^2} \left( 2C_{H\Box} + \frac{6e^2}{s_w^2} C_{HWB} + \left( \frac{3c_w^2}{2s_w^2} - \frac{1}{2} \right) C_{HD} \right), & g_4^{ZZ} &= -2 \frac{v^2}{\Lambda^2} (s_w^2 C_{H\tilde{B}} + c_w^2 C_{H\tilde{W}} + s_w c_w C_{H\tilde{W}B}), \\ \kappa_1^{ZZ} &= \frac{v^2}{\Lambda^2} \left( -\frac{2e^2}{s_w^2} C_{HWB} + \left( 1 - \frac{1}{2s_w^2} \right) C_{HD} \right), & g_4^{Z\gamma} &= -2 \frac{v^2}{\Lambda^2} \left( s_w c_w (C_{H\tilde{W}} - C_{H\tilde{B}}) + \frac{1}{2} (s_w^2 - c_w^2) C_{H\tilde{W}B} \right), \\ g_2^{ZZ} &= -2 \frac{v^2}{\Lambda^2} (s_w^2 C_{HB} + c_w^2 C_{HW} + s_w c_w C_{HWB}), & g_4^{\gamma\gamma} &= -2 \frac{v^2}{\Lambda^2} (c_w^2 C_{H\tilde{B}} + s_w^2 C_{H\tilde{W}} - s_w c_w C_{H\tilde{W}B}), \\ g_2^{Z\gamma} &= -2 \frac{v^2}{\Lambda^2} \left( s_w c_w (C_{HW} - C_{HB}) + \frac{1}{2} (s_w^2 - c_w^2) C_{HWB} \right), & g_4^{gg} &= -2 \frac{v^2}{\Lambda^2} C_{HG}, \\ g_2^{\gamma\gamma} &= -2 \frac{v^2}{\Lambda^2} (c_w^2 C_{HB} + s_w^2 C_{HW} - s_w c_w C_{HWB}), \\ g_2^{gg} &= -2 \frac{v^2}{\Lambda^2} C_{HG}, \end{aligned}$$


Couplings of interest CP-Even  $Z\gamma, \gamma\gamma$   
and CP-Odd  $Z\gamma, \gamma\gamma$

# External Constraints

$M_W$  and Zff couplings could be affected by EFT operators

- EW precision measurements tightly constrain these shifts
- Allowed range much smaller than sensitivity of Higgs measurements

Can constrain both Zff and  $M_W$  to SM within our framework

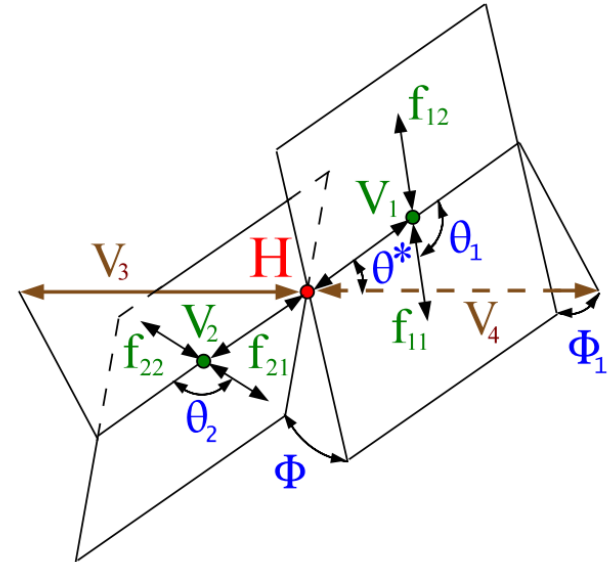
Leads to relations   $\delta g_1^{ZZ} = \frac{v^2}{\Lambda^2} \left( 2C_{H\Box} + \frac{6e^2}{s_w^2} C_{HWB} + \left( \frac{3c_w^2}{2s_w^2} - \frac{1}{2} \right) C_{HD} \right)$



	$\delta g_1^{ZZ} = \delta g_1^{WW}$	$\kappa_1^{ZZ}$	$g_2^{ZZ}$	$g_2^{Z\gamma}$	$g_2^{\gamma\gamma}$	$g_4^{ZZ}$	$g_4^{Z\gamma}$	$g_4^{\gamma\gamma}$	$\kappa_2^{Z\gamma}$	$\kappa_1^{WW}$	$g_2^{WW}$	$g_4^{WW}$
$c_{H\Box}$	0.1213	0	0	0	0	0	0	0	0	0	0	0
$c_{HD}$	0.2679	-0.0831	0	0	0	0	0	0	-0.1320	-0.1560	0	0
$c_{HW}$	0	0	-0.0929	-0.0513	-0.0283	0	0	0	0	0	-0.1212	0
$c_{HWB}$	0.1529	-0.0613	-0.0513	0.0323	0.0513	0	0	0	0.1763	0.0360	0	0
$c_{HB}$	0	0	-0.0283	0.0513	-0.0929	0	0	0	0	0	0	0
$c_{H\bar{W}}$	0	0	0	0	0	-0.0929	-0.0513	-0.0283	0	0	0	-0.1212
$c_{H\bar{W}B}$	0	0	0	0	0	-0.0513	0.0323	0.0513	0	0	0	0
$c_{H\bar{B}}$	0	0	0	0	0	-0.0283	0.0513	-0.0929	0	0	0	0

Above: A numerical example of the relationship between the  $C_{HX} = 1$  and mass eigenstate amplitude basis used in our analysis

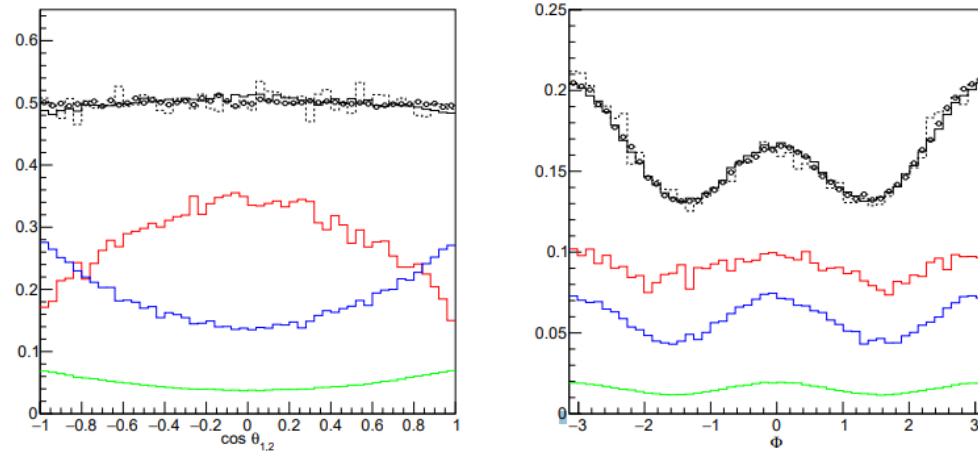
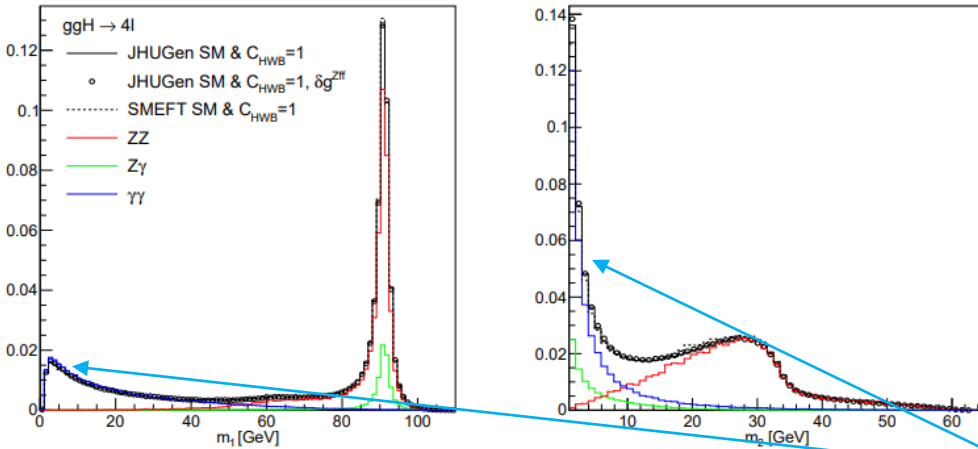
# Photons in $H \rightarrow 4l$



Anomalous photon couplings introduces contributions at low  $q^2$

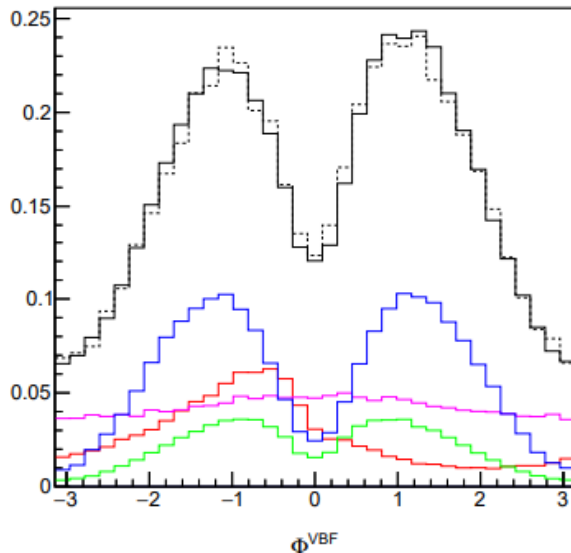
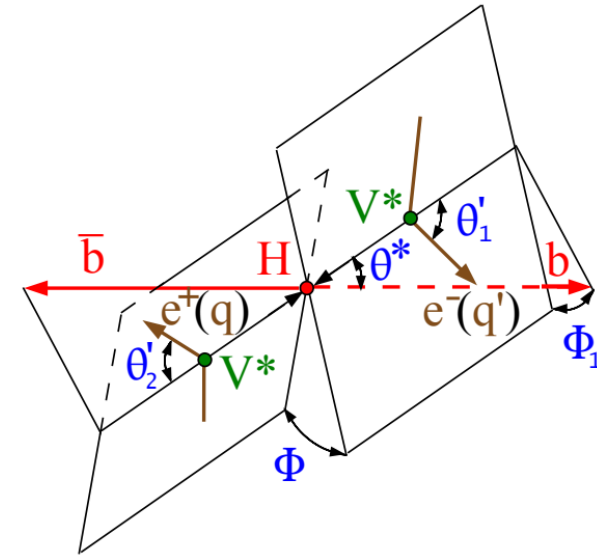
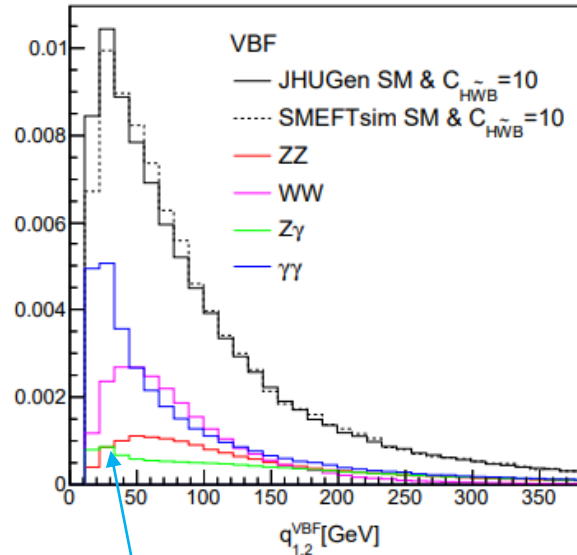
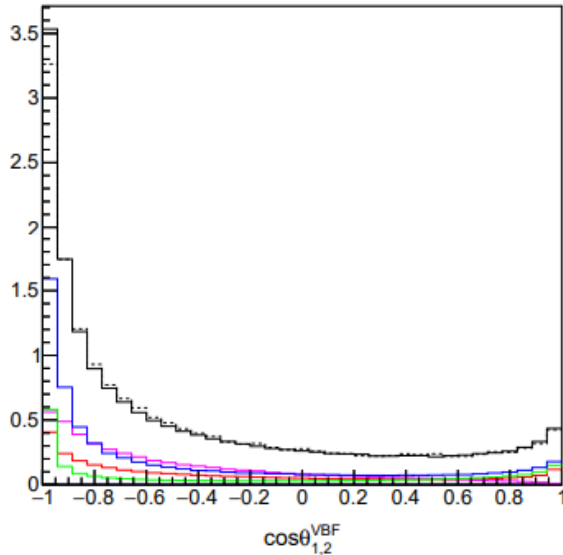
Example: Simulate SM &  $c_{HWB} = 10$

JHUGen (Amplitude basis) kinematic distributions equivalent to SMEFTSim (Warsaw Basis) with the proper rotation using JHUGenLexicon



$$c_{HWB} = -0.0513 g_2^{ZZ} - 0.0323 g_2^{Z\gamma} - 0.0513 g_2^{\gamma\gamma}$$

# Photons in VBF Higgs Production

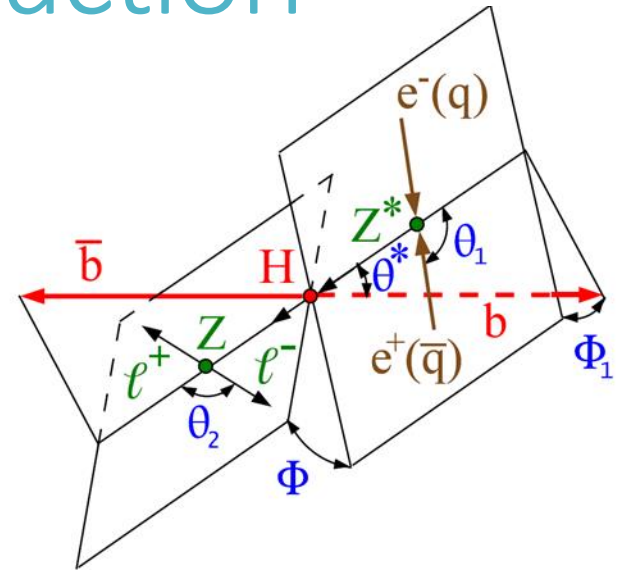
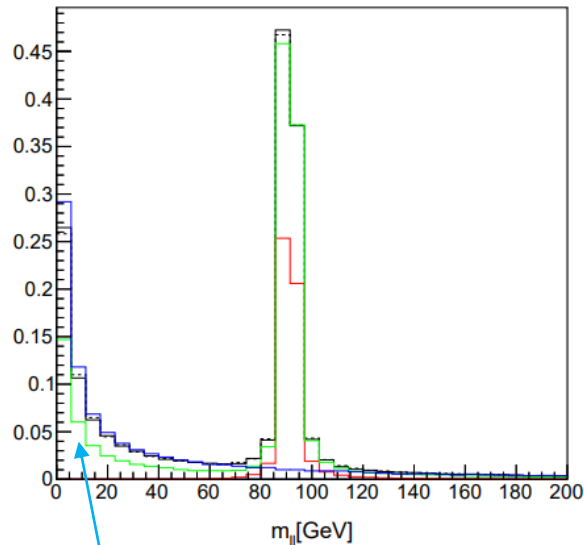
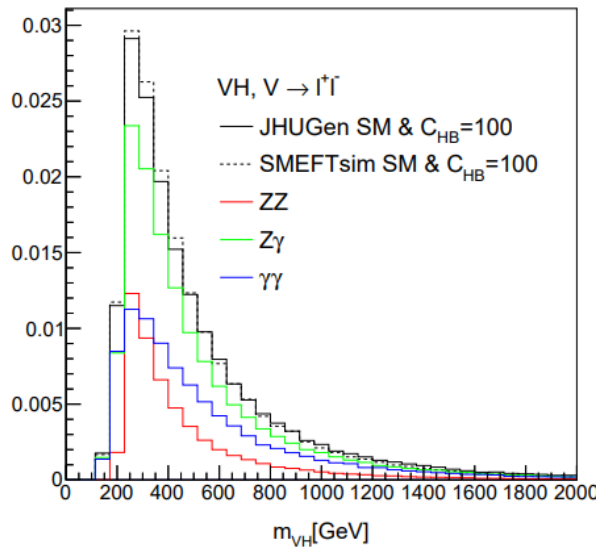


Anomalous photon couplings introduce enhanced  $\gamma\gamma$  and  $Z\gamma$  fusion contributions at low  $q^2$

Example: Simulate VBF with SM Couplings &  $C_{H\widetilde{W}B} = 10$

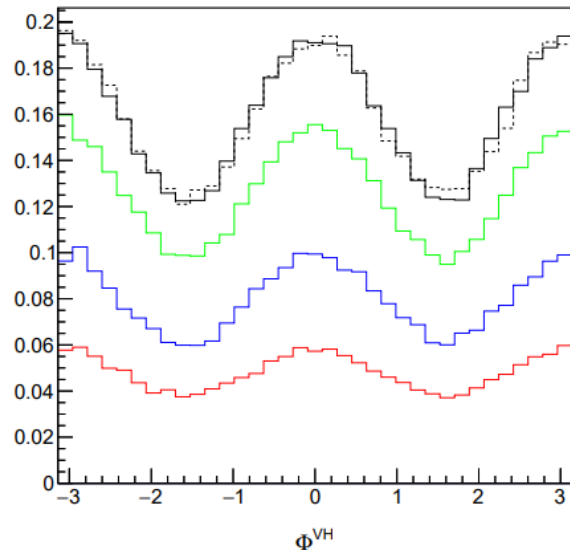
$$C_{H\widetilde{W}B} = -0.0513 g_4^{ZZ} + 0.0323 g_4^{Z\gamma} + 0.0513 g_4^{\gamma\gamma}$$

# Photons in $Z/\gamma^* H$ Production



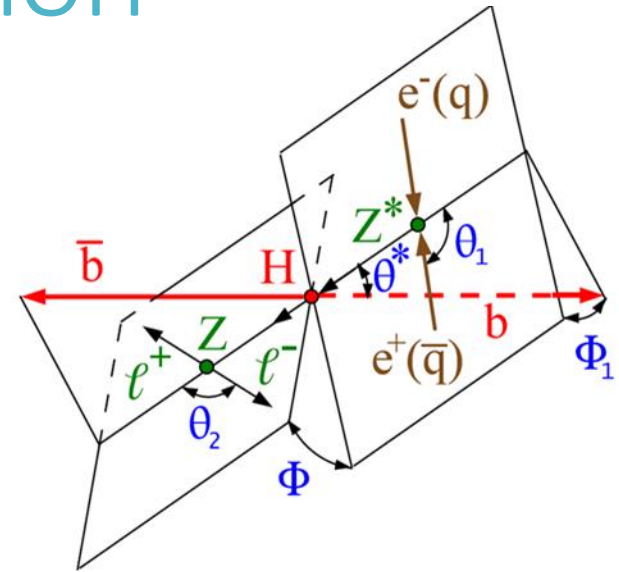
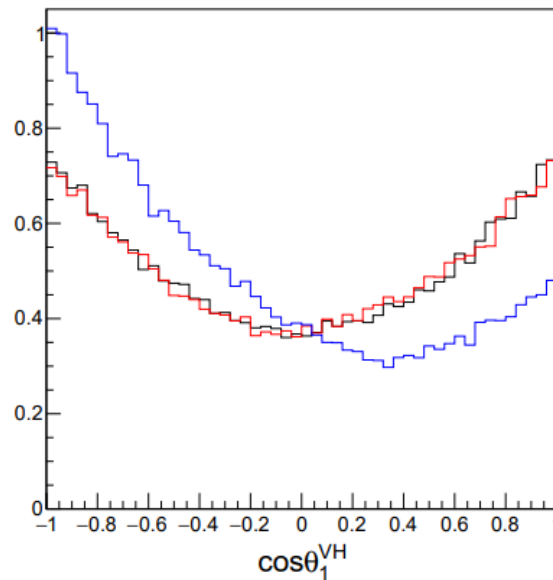
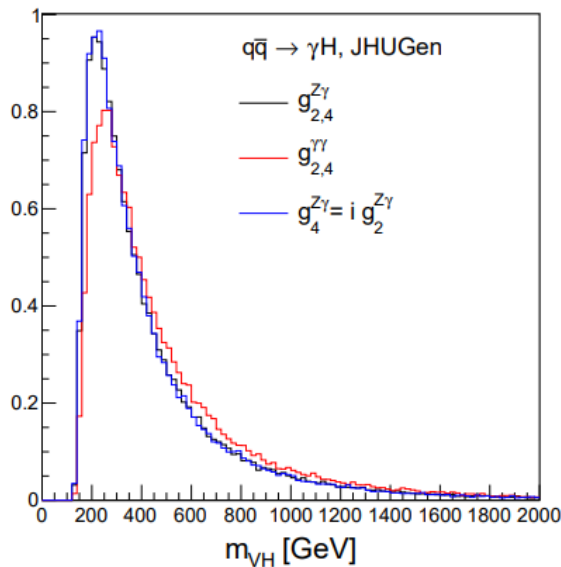
Anomalous photon couplings introduce  $\gamma^* H$  contributions at low  $q^2$

Example: Simulate  $Z/\gamma^* H$  with SM Couplings &  $C_{HB} = 100$



$$C_{HB} = -0.0283 g_2^{ZZ} + 0.0513 g_2^{Z\gamma} - 0.0929 g_2^{\gamma\gamma}$$

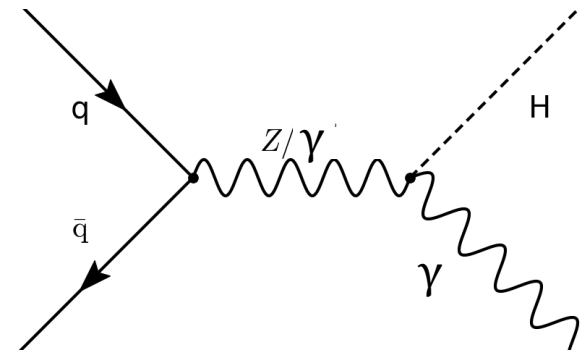
# Photons in $\gamma H$ Production



Anomalous photon couplings greatly increase rate of  $\gamma H$  production

Signal is a Higgs with high pT photon

Unable to distinguish between real valued CP-Even and CP-Odd couplings





# EFT Analysis

Measure Higgs cross sections as a function of anomalous couplings

$$\frac{d\sigma(i \rightarrow H \rightarrow f)}{d\vec{\Omega}} \propto \frac{\left( \sum \alpha_{jk}^{(i)} a_j a_k \right) \left( \sum \alpha_{lm}^{(f)} a_l a_m \right)}{\Gamma_{\text{tot}}}$$

Maximum likelihood calculated for EFT hypothesis to match measured cross section/kinematic distributions

$\alpha$  are functions of kinematic observables  $\vec{\Omega}$   
factorized into both production and decay components

$\Gamma_{\text{tot}}$  purely dependent on anomalous couplings

# Higgs Width Calculation

$$\sigma(i \rightarrow H \rightarrow f) \propto \frac{\left(\sum \alpha_{jk}^{(i)} a_j a_k\right) \left(\sum \alpha_{lm}^{(f)} a_l a_m\right)}{\Gamma_{\text{tot}}}$$

Total width directly dependent on anomalous couplings

$$\Gamma_{\text{known}} = \Gamma_{\text{tot}}^{\text{SM}} \times \sum_f \left( \frac{\Gamma_f^{\text{SM}}}{\Gamma_{\text{tot}}^{\text{SM}}} \times \frac{\Gamma_f}{\Gamma_f^{\text{SM}}} \right) = \sum_f \Gamma_f^{\text{SM}} R_f \quad (\text{Function of anomalous couplings})$$

$$f = b\bar{b}, c\bar{c}, W^+W^-, gg, \tau^+\tau^-, ZZ/Z\gamma^*/\gamma^*\gamma^*$$

Full analytic formulas are calculated for each  $R_f$  including photon couplings

$R_f$  dependent on  $q^2$  cutoff

Values shown for  $q^2 > (2m_f)^2$

$$\begin{aligned} R_{ZZ/Z\gamma^*/\gamma^*\gamma^*} = & \left(\frac{g_1^{ZZ}}{2}\right)^2 + 0.17(\kappa_1^{ZZ})^2 + 0.09(g_2^{ZZ})^2 + 0.04(g_4^{ZZ})^2 + 0.10(\kappa_2^{Z\gamma})^2 \\ & + 79.95(g_2^{Z\gamma})^2 + 75.23(g_4^{Z\gamma})^2 + 29.00(g_2^{\gamma\gamma})^2 + 29.47(g_4^{\gamma\gamma})^2 \\ & + 0.81\frac{g_1^{ZZ}}{2}\kappa_1^{ZZ} + 0.50\frac{g_1^{ZZ}}{2}g_2^{ZZ} + 0 \times \frac{g_1^{ZZ}}{2}g_4^{ZZ} - 0.19\frac{g_1^{ZZ}}{2}\kappa_2^{Z\gamma} \\ & - 1.56\frac{g_1^{ZZ}}{2}g_2^{Z\gamma} + 0 \times \frac{g_1^{ZZ}}{2}g_4^{Z\gamma} + 0.06\frac{g_1^{ZZ}}{2}g_2^{\gamma\gamma} + 0 \times \frac{g_1^{ZZ}}{2}g_4^{\gamma\gamma} \\ & + 0.21\kappa_1^{ZZ}g_2^{ZZ} + 0 \times \kappa_1^{ZZ}g_4^{ZZ} - 0.07\kappa_1^{ZZ}\kappa_2^{Z\gamma} - 0.64\kappa_1^{ZZ}g_2^{Z\gamma} \\ & + 0 \times \kappa_1^{ZZ}g_4^{Z\gamma} + 0.00\kappa_1^{ZZ}g_2^{\gamma\gamma} + 0 \times \kappa_1^{ZZ}g_4^{\gamma\gamma} + 0 \times g_2^{ZZ}g_4^{ZZ} \\ & - 0.05g_2^{ZZ}\kappa_2^{Z\gamma} - 0.51g_2^{ZZ}g_2^{Z\gamma} + 0 \times g_2^{ZZ}g_4^{Z\gamma} - 0.02g_2^{ZZ}g_2^{\gamma\gamma} \\ & + 0 \times g_2^{ZZ}g_4^{\gamma\gamma} + 0 \times g_4^{ZZ}\kappa_2^{Z\gamma} + 0 \times g_4^{ZZ}g_2^{Z\gamma} + 0.36g_4^{ZZ}g_4^{Z\gamma} \\ & + 0 \times g_4^{ZZ}g_2^{\gamma\gamma} - 0.57g_4^{ZZ}g_4^{\gamma\gamma} + 1.80\kappa_2^{Z\gamma}g_2^{Z\gamma} + 0 \times \kappa_2^{Z\gamma}g_4^{Z\gamma} \\ & - 0.05\kappa_2^{Z\gamma}g_2^{\gamma\gamma} + 0 \times \kappa_2^{Z\gamma}g_4^{\gamma\gamma} + 0 \times g_2^{Z\gamma}g_4^{Z\gamma} - 1.84g_2^{Z\gamma}g_2^{\gamma\gamma} \\ & + 0 \times g_2^{Z\gamma}g_4^{\gamma\gamma} + 0 \times g_4^{Z\gamma}g_2^{\gamma\gamma} - 2.09g_4^{Z\gamma}g_4^{\gamma\gamma} + 0 \times g_2^{\gamma\gamma}g_4^{\gamma\gamma} \end{aligned}$$

# MELA Observables

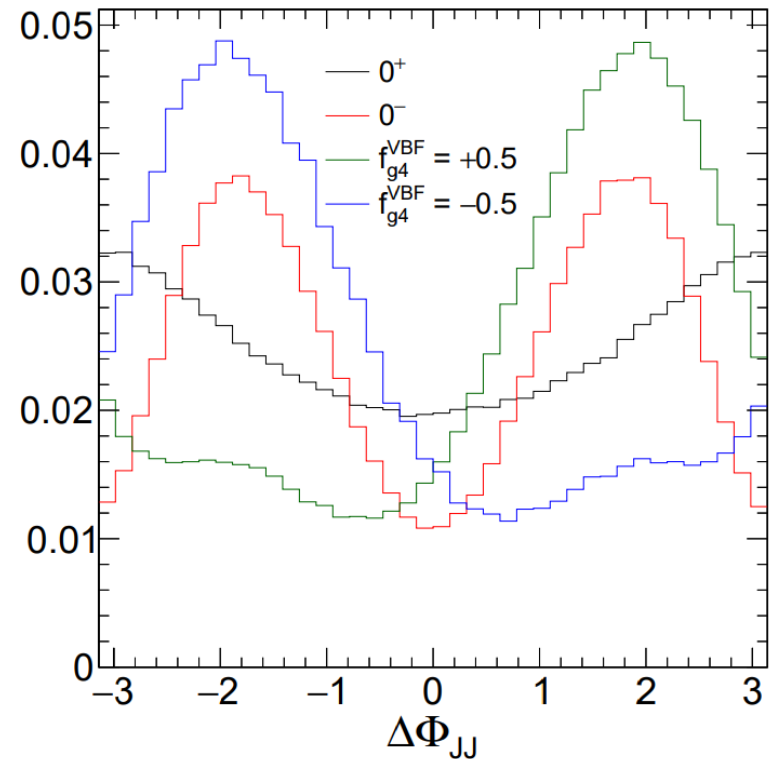
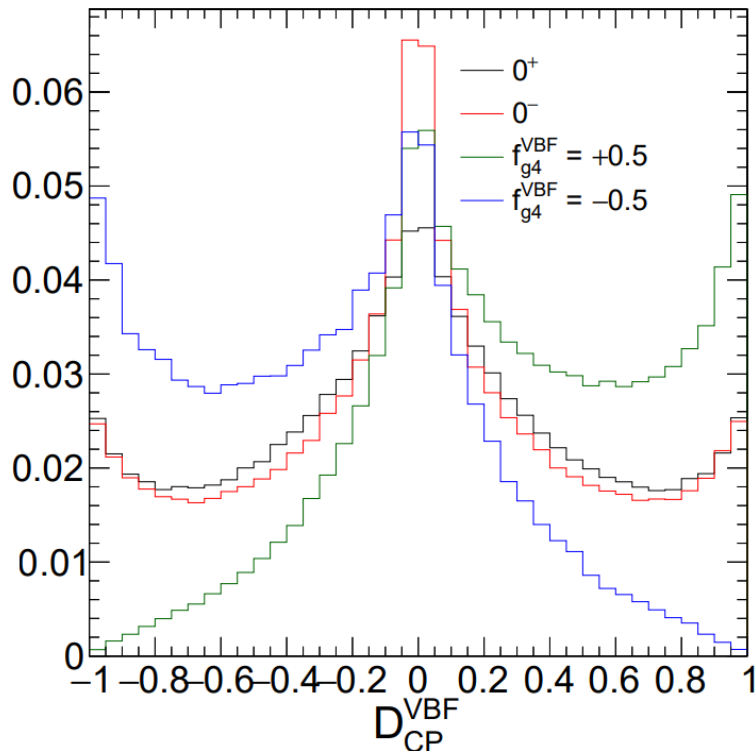
- Events have many kinematic observables
- We construct observables that utilize all kinematic information

$$\mathcal{D}_{\text{alt}}(\Omega) = \frac{\mathcal{P}_{\text{sig}}(\Omega)}{\mathcal{P}_{\text{sig}}(\Omega) + \mathcal{P}_{\text{alt}}(\Omega)}$$

$$\mathcal{D}_{\text{int}}(\Omega) = \frac{\mathcal{P}_{\text{int}}(\Omega)}{2\sqrt{\mathcal{P}_{\text{sig}}(\Omega) \times \mathcal{P}_{\text{alt}}(\Omega)}}$$

$$\frac{\left( \sum \alpha_{jk}^{(i)} a_j a_k \right) \left( \sum \alpha_{lm}^{(f)} a_l a_m \right)}{\Gamma_{\text{tot}}}$$

MELA calculates optimal observables from matrix elements to distinguish between various anomalous coupling hypotheses



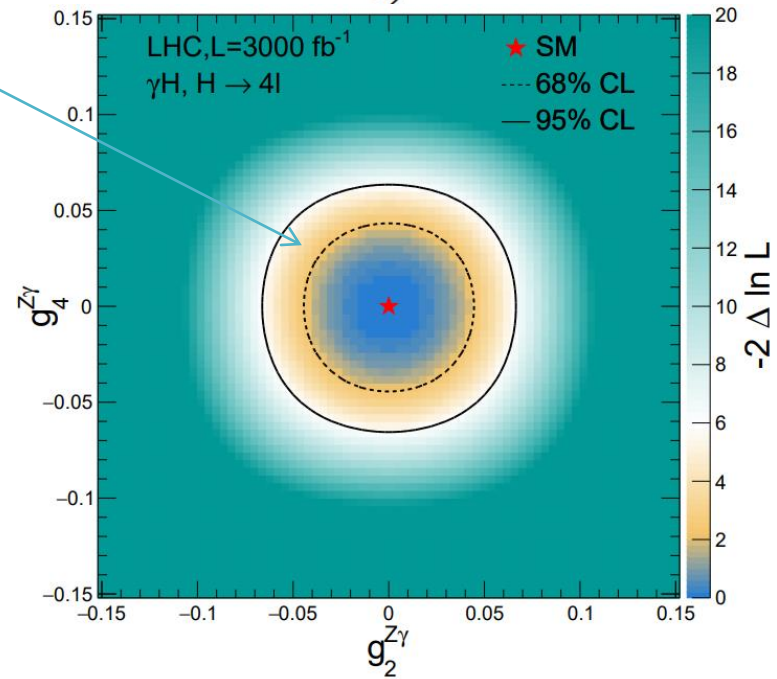
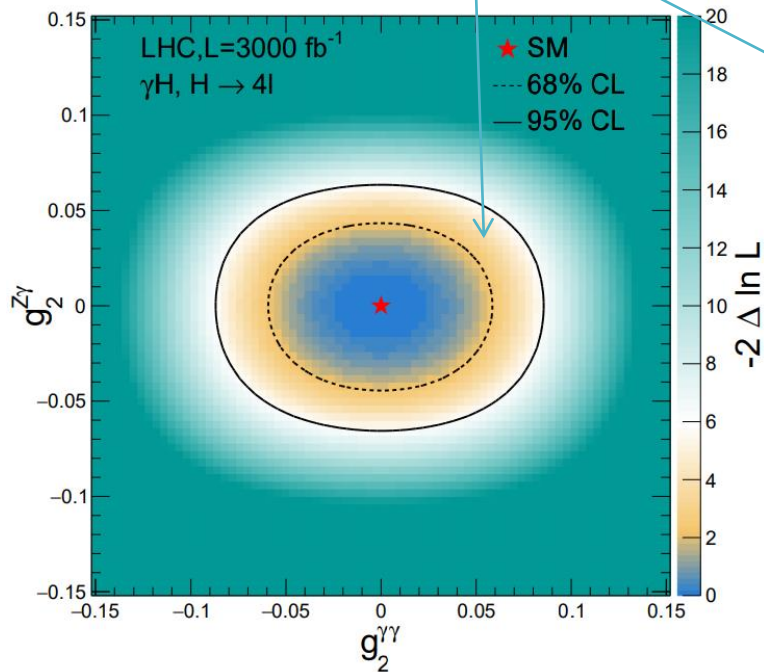
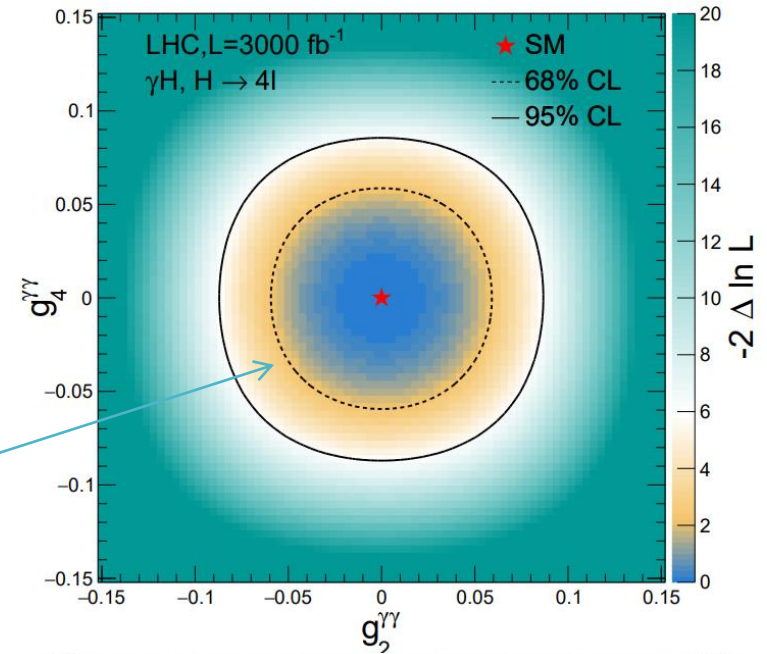
# $\gamma H$ Constraints

Only use production info from  $\gamma H$

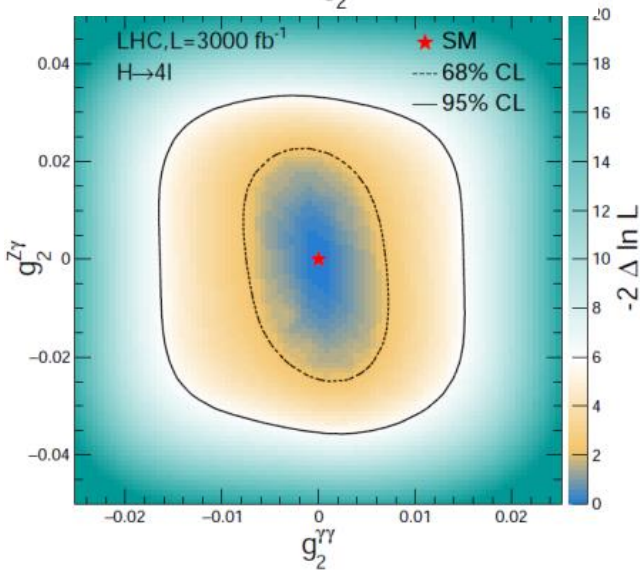
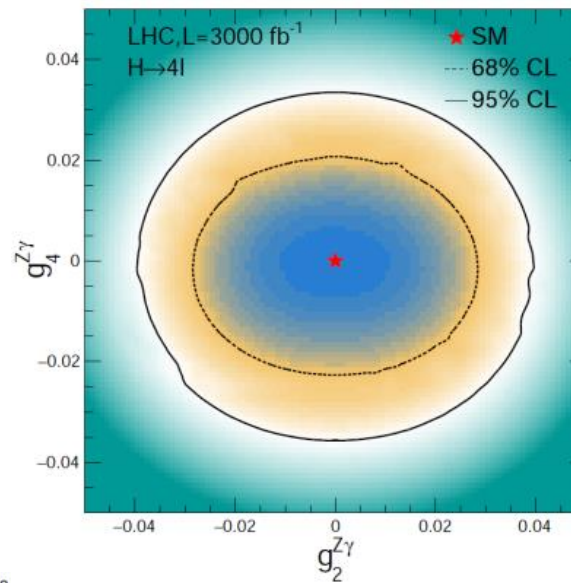
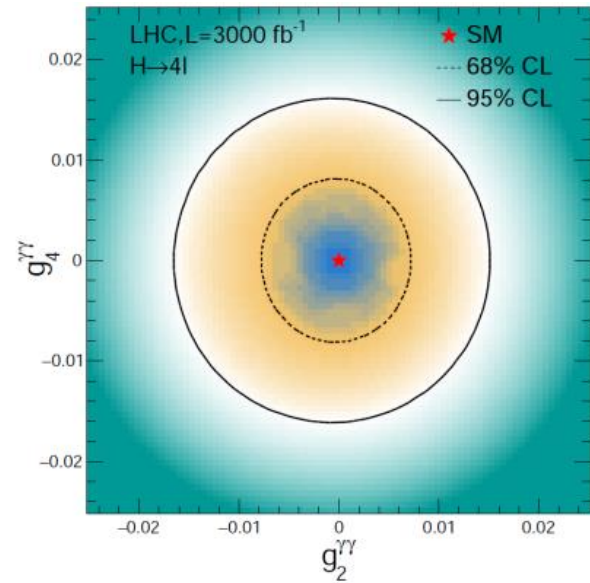
Unable to distinguish between

CP-Odd and CP-Even with only  $\gamma H$

Constraints of Order  $\sim 0.05$



# Constraints: Production + Decay



Using information from VBF, VH production +  $4l$  decay

Tighter constraints on all couplings, Order  $\sim 0.01-0.02$

$\gamma\gamma$  couplings better than  $Z\gamma$

# Constraints from Decays with On-Shell photons

Measurements of the rate of  $H \rightarrow \gamma\gamma$  and  $H \rightarrow Z\gamma$  can be used to constrain anomalous  $H\gamma\gamma$  and  $HZ\gamma$  couplings

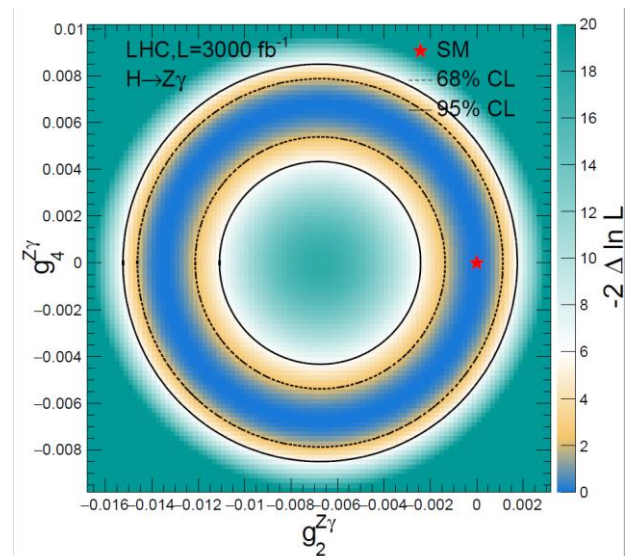
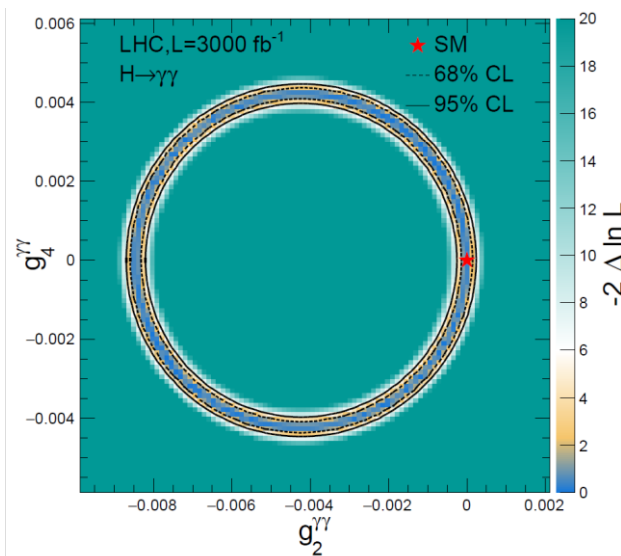
Expected constraints taken from Higgs Working Group 2 Report [arxiv:1902.00134](https://arxiv.org/abs/1902.00134)

$$R_{\gamma\gamma} \simeq \frac{1}{\left(g_2^{\gamma\gamma, \text{SM}}\right)^2} \left[ \left(g_2^{\gamma\gamma, \text{SM}} + g_2^{\gamma\gamma}\right)^2 + \left(g_4^{\gamma\gamma}\right)^2 \right]$$

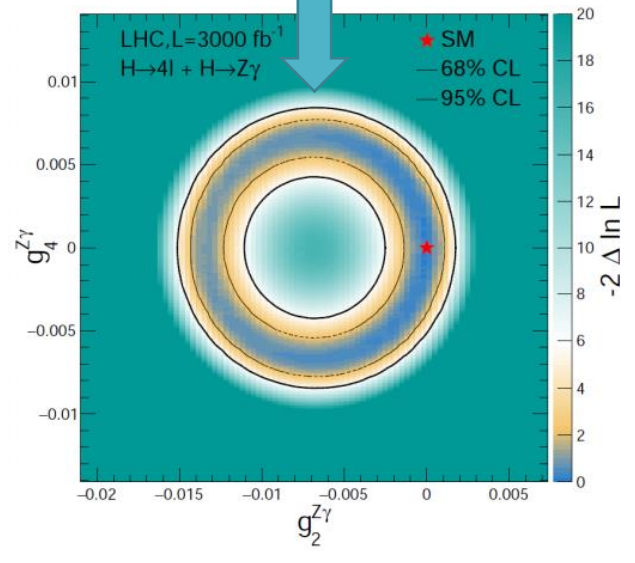
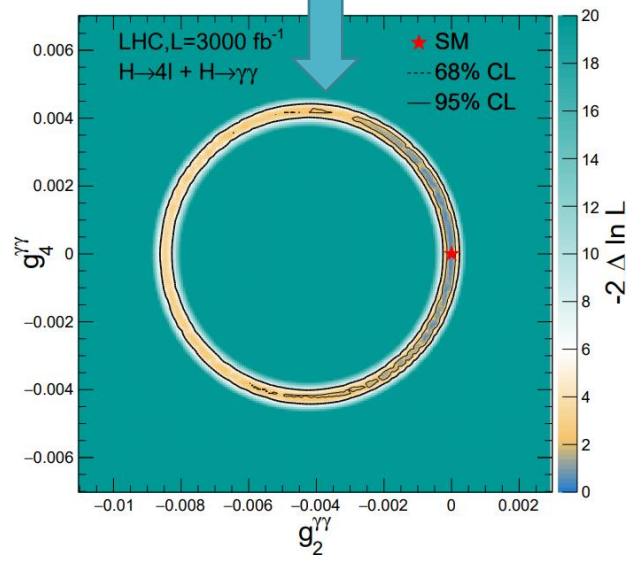
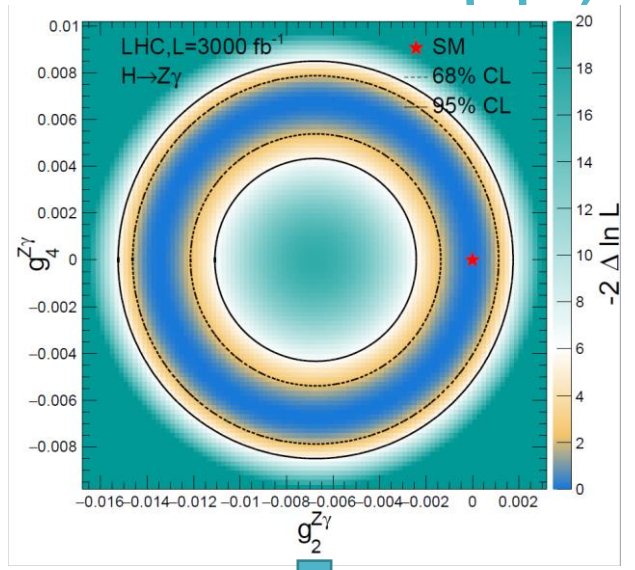
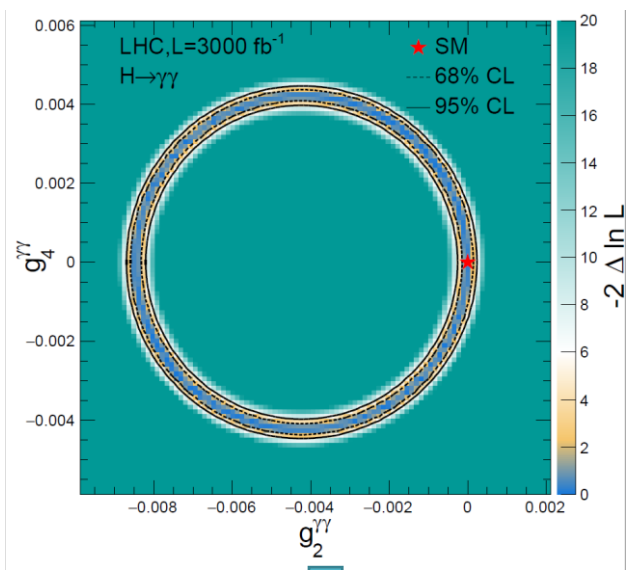
$$R_{Z\gamma} \simeq \frac{1}{\left(g_2^{Z\gamma, \text{SM}}\right)^2} \left[ \left(g_2^{Z\gamma, \text{SM}} + g_2^{Z\gamma}\right)^2 + \left(g_4^{Z\gamma}\right)^2 \right]$$

$$R_{\gamma\gamma} \simeq 1.00 \pm 0.05$$

$$R_{Z\gamma} \simeq 1.00 \pm 0.24$$



# Full Combination: $H \rightarrow 4l$ and $H \rightarrow \gamma\gamma, Z\gamma$



# Conclusion

To increase sensitivity of Higgs coupling to virtual photons, need to push  $q^2$  of analysis as low as possible

Calculated  $\Gamma_H$  as a function of anomalous couplings

Measuring  $\gamma H$  cross section can provide decent constraints on virtual photon couplings

Using full information from VBF, VH production and  $H \rightarrow 4l$  decay provides the tightest constraints from HVV measurements

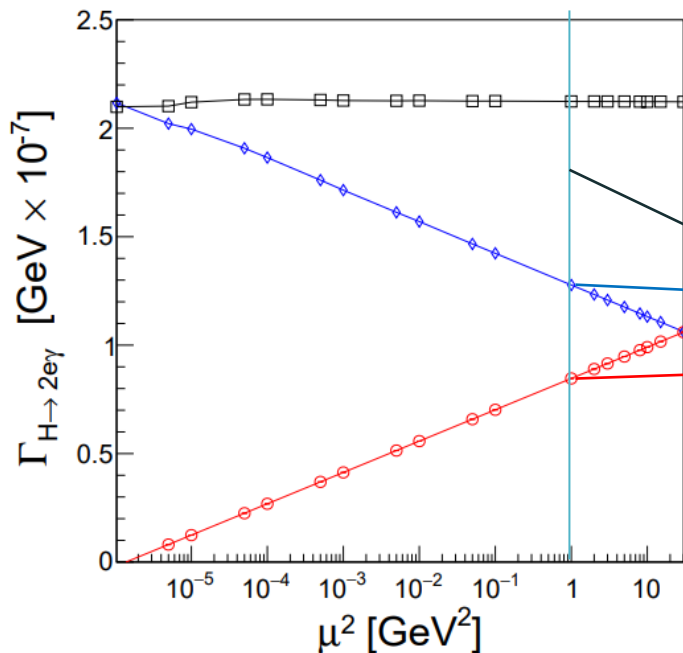


# Total Width Calculation

Higgs decay to fermions through virtual photons have sharp peaks for small  $q^2 = (p_{f+} + p_{f-})^2$

Developed procedure to handle singularities at low  $q^2$

For details on derivation see [arxiv:2109.13363](https://arxiv.org/abs/2109.13363)



Approximate  $\Gamma_{H \rightarrow 2e\gamma}$  width by measuring partial widths at given  $q^2$  cut.

$$\Gamma_{H \rightarrow 2l\gamma} = \Gamma_{H \rightarrow 2l\gamma} \Big|_{q_{2l}^2 \geq \mu^2} + SF \Gamma_{H \rightarrow \gamma\gamma}$$

Lepton scale factor SF is a function of  $q^2$

Similar forms could be calculated for decays to quarks

# Analytic Decay Rates

$$\begin{aligned}
 R_{\gamma\gamma} = & 1.60932 \left( \frac{g_1^{WW}}{2} \right)^2 - 0.69064 \left( \frac{g_1^{WW}}{2} \right) \kappa_t + 0.00912 \left( \frac{g_1^{WW}}{2} \right) \kappa_b - 0.49725 \left( \frac{g_1^{WW}}{2} \right) (N_c Q^2 \kappa_Q) \\
 & + 0.07404 \kappa_t^2 + 0.00002 \kappa_b^2 - 0.00186 \kappa_t \kappa_b \\
 & + 0.03841 (N_c Q^2 \kappa_Q)^2 + 0.10666 \kappa_t (N_c Q^2 \kappa_Q) - 0.00136 \kappa_b (N_c Q^2 \kappa_Q) \\
 & + 0.20533 \tilde{\kappa}_t^2 + 0.00006 \tilde{\kappa}_b^2 - 0.00300 \tilde{\kappa}_t \tilde{\kappa}_b \\
 & + 0.10252 (N_c Q^2 \tilde{\kappa}_Q)^2 + 0.29018 \tilde{\kappa}_t (N_c Q^2 \tilde{\kappa}_Q) - 0.00202 \tilde{\kappa}_b (N_c Q^2 \tilde{\kappa}_Q) .
 \end{aligned}$$

$$g_2^{\gamma\gamma, Q} = -\frac{\alpha}{3\pi} N_c Q^2 \kappa_Q, \quad g_4^{\gamma\gamma, Q} = -\frac{\alpha}{2\pi} N_c Q^2 \tilde{\kappa}_Q.$$

Effective  $H\gamma\gamma$  couplings mimic heavy quark effects

$R_{\gamma\gamma}$  can be cast in terms of anomalous photon couplings

- Fix HWW couplings to SM value
- Fix Hff couplings to SM value

$$R_{\gamma\gamma} \simeq \frac{1}{(g_2^{\gamma\gamma, \text{SM}})^2} \left[ \left( g_2^{\gamma\gamma, \text{SM}} + g_2^{\gamma\gamma} \right)^2 + (g_4^{\gamma\gamma})^2 \right]$$

# Higgs production with photon ( $\gamma H$ )

To date there has been no search for  $\gamma H(125)$  production at LHC or LEP

$\gamma H$  production has extremely low SM cross-section

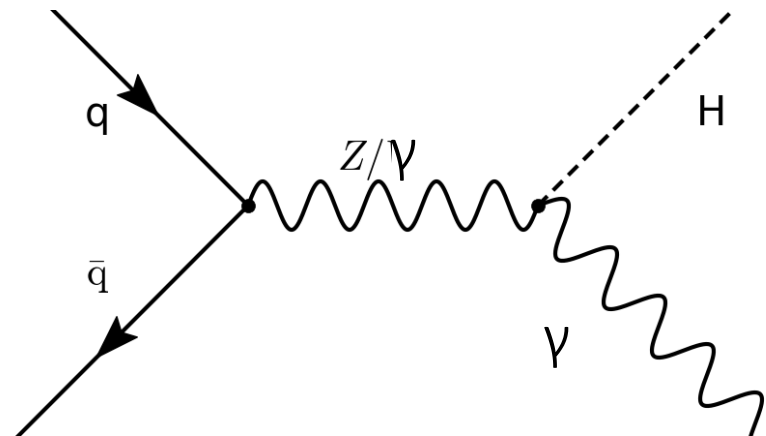
-  $\sim .1$  events expected with entire LHC and HL-LHC dataset

Anomalous  $HZ\gamma$  and  $H\gamma\gamma$  couplings dramatically enhance expected  $\gamma H$  yield above SM for relatively small coupling values

$$\frac{\sigma(q\bar{q} \rightarrow \gamma H)}{\sigma_{\text{ref}}^{\gamma H}} = \left(g_2^{Z\gamma}\right)^2 + \left(g_4^{Z\gamma}\right)^2 + 0.553 \left(g_2^{\gamma\gamma}\right)^2 + 0.553 \left(g_4^{\gamma\gamma}\right)^2 - 0.578 g_2^{Z\gamma} g_2^{\gamma\gamma} - 0.578 g_4^{Z\gamma} g_4^{\gamma\gamma}$$

$$\sigma_{\text{ref}}^{\gamma H} = 1.33 \times 10^4 \text{ fb.}$$

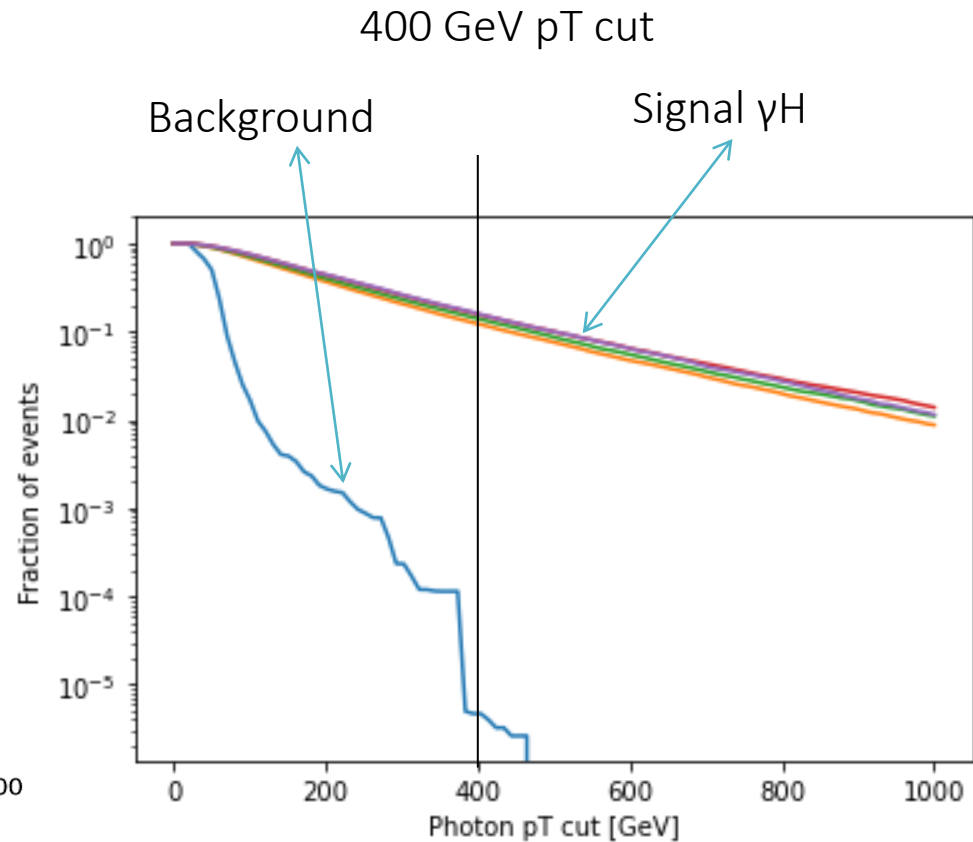
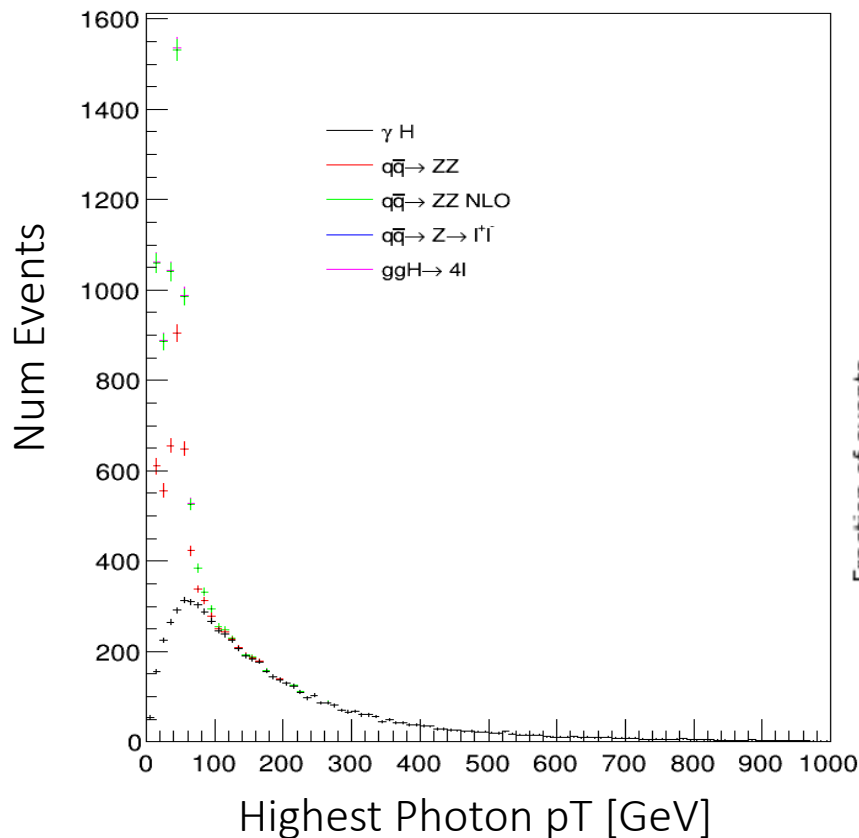
For  $g_2^{Z\gamma} = 1$ , expect  $\sim 2 \times 10^5$  events



# $\gamma H$ Background

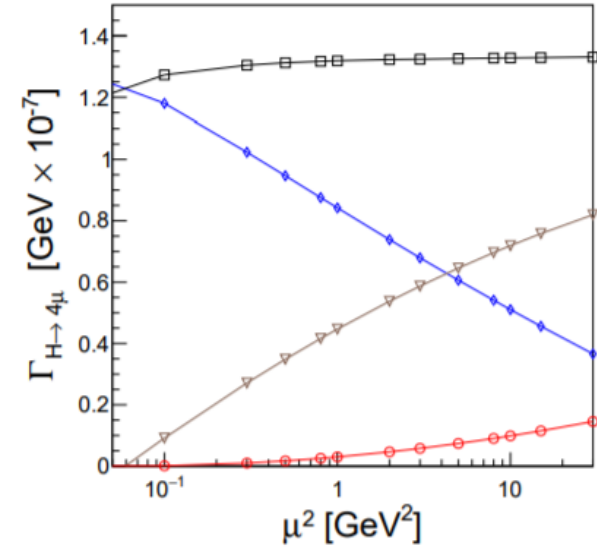
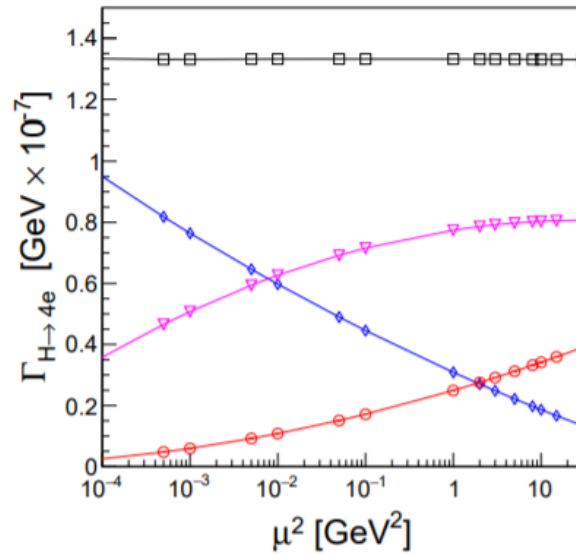
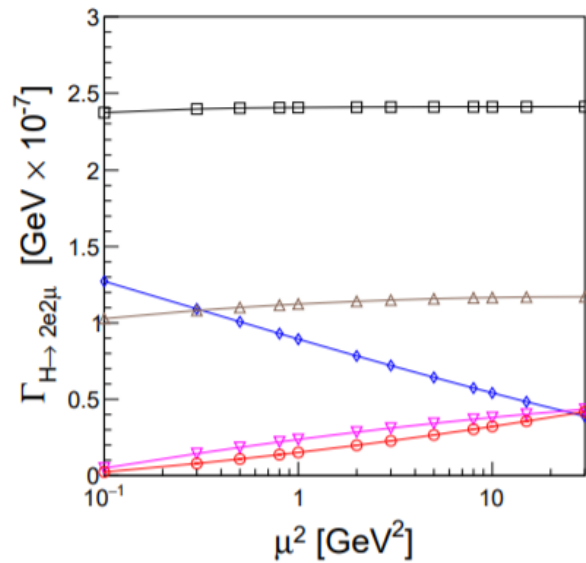
Main Sources of background are  $ggH$  and  $qqZZ$

$qq\bar{q}\bar{q}\rightarrow ZZ/Z\gamma^*\rightarrow 4l$  background is generated with POWHEG + Pythia hadronization.



# Total Width Calculation

$$\begin{aligned}
 \Gamma_{H \rightarrow 2\ell 2\ell'} &= \Gamma_{H \rightarrow 2\ell 2\ell'} \Big|_{q_{2\ell}^2 \geq \mu^2, q_{2\ell'}^2 \geq \mu^2} \\
 &+ \frac{\alpha}{2\pi} \left[ \frac{2}{3} \log \left( \frac{\mu_\ell^2}{m_\ell^2} \right) - \frac{10}{9} \right] \Gamma_{H \rightarrow 2\ell' \gamma} \Big|_{q_{2\ell'}^2 \geq \mu^2} + \frac{\alpha}{2\pi} \left[ \frac{2}{3} \log \left( \frac{\mu_{\ell'}^2}{m_{\ell'}^2} \right) - \frac{10}{9} \right] \Gamma_{H \rightarrow 2\ell \gamma} \Big|_{q_{2\ell}^2 \geq \mu^2} \\
 &+ \left( \frac{\alpha}{2\pi} \right)^2 \left[ \frac{2}{3} \log \left( \frac{\mu_\ell^2}{m_\ell^2} \right) - \frac{10}{9} \right] \left[ \frac{2}{3} \log \left( \frac{\mu_{\ell'}^2}{m_{\ell'}^2} \right) - \frac{10}{9} \right] 2\Gamma_{H \rightarrow \gamma\gamma} + \mathcal{O}(\mu^2/M_H^2),
 \end{aligned}$$



# Total Width Calculation

Similarly, we can use this procedure for decays to quarks

$$\Gamma_{H \rightarrow 2j\gamma} = \Gamma_{H \rightarrow 2j\gamma} \Big|_{q_{2q}^2 \geq \mu^2} + \left[ \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) + \frac{\alpha}{\pi} \frac{11}{9} \log\left(\frac{\mu^2}{M_Z^2}\right) \right] 2\Gamma_{H \rightarrow \gamma\gamma} + \mathcal{O}(\mu^2/M_Z^2)$$

Hadron form factor unknown/ hard to derive (resonances)

