Simulating 3+1 Dimension Schwinger Pair Production with Quantum Computers

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ArXiv: 2112.06863, in collaboration with Wei Xue

Phenomenology 2022 Symposium
May 10, 2022

Motivation

Why do we need quantum computers?

- There are problems too hard to solve analytically, e.g. quantum chromodynamics (QCD).
- There are limitations of classical computations, e.g. the sign problem in lattice QCD.
- Quantum computers *can* simulate the real time dynamics of such highly entangled systems, hence solve the sign problem.

Motivation

Why do we simulate the Schwinger effect?

- The Schwinger effect is a very well understood phenomenon of QED in which electron-positron pairs are spontaneously created in the presence of an electric field.
- It has never been directly observed due to the extremely strong electric-field strengths required. (The Schwinger limit: $E_c \simeq 10^{18} V/m$)
- It is a non-perturbative effect of vacuum decay. This makes this elusive effect of great interest for other theories, such as QCD and gravitational physics.

Theoretical Setup

Momentum factorization

- Suppose the electric field is along z-direction, we decompose the Fermion field in Fourier space transverse to the electric field direction

$$\psi(\mathbf{x}) = \int \frac{dp_x dp_y}{(2\pi)^2} e^{i(p_x x + p_y y)} \sum_s \psi_s(p_x, p_y, z)$$

- The Hamiltonian is factorized to be

$$H = \int d^3x \bar{\psi}(-i\vec{\gamma} \cdot \nabla + m)\psi = \int \frac{dp_x dp_y}{(2\pi)^2} \sum_s H_s(p_x, p_y, z)$$

where $H_s(m; p_x, p_y, z) \simeq H_{1+1}(m' = \sqrt{m^2 + p_x^2 + p_y^2}; z)$ with a unitary transformation.

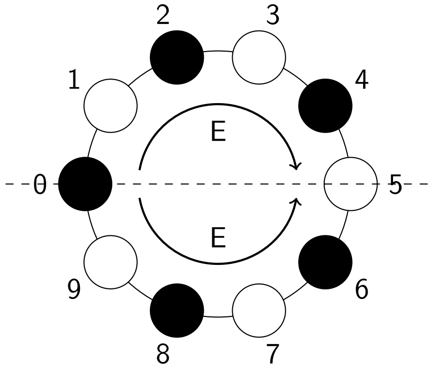
Theoretical Setup

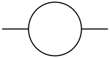
Spatial discretization

- Discretize the space along z-axis to N sites with spacing a
  $L_z = Na, z = na, (n = 0, 1, \ldots, N-1)$.
- Staggered Fermion (the Kogut-Susskind formulation): put the upper (lower) components of the spinor on the even (odd) lattice points.

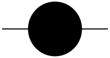
$$\phi(n)/\sqrt{a} \to \begin{cases} \psi_{upper}(x), & n \text{ even}, \\ \psi_{lower}(x), & n \text{ odd}. \end{cases}$$

- Periodic boundary condition: $\phi(n+N) = \phi(n)$



 $= |0\rangle$:

unoccupied electron (odd)
occupied positron (even)

 $= |1\rangle$:

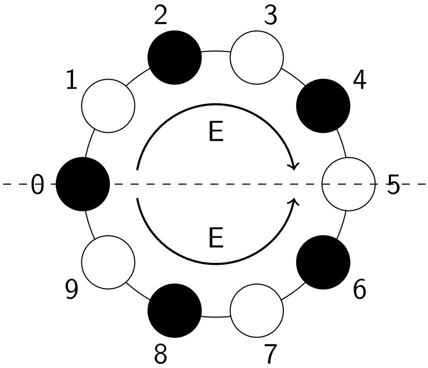
occupied electron (odd)
unoccupied positron (even)

The system H is invariant under the parity transformation:

$$P : \phi(m) \to (-1)^m \phi(N - m)$$

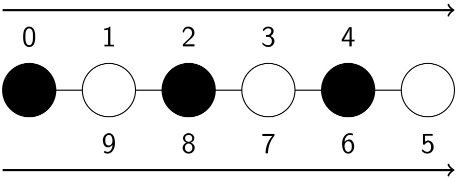
Theoretical Setup

Reduction by Parity



The system H is invariant under the parity transformation:

$$P : \phi(m) \to (-1)^m \phi(N-m)$$



Define the parity even and odd fields

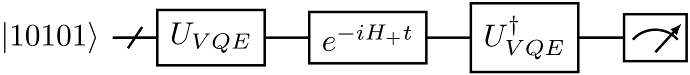
$$\phi_\pm(m) = \frac{\phi(m) \pm (-1)^m \phi(N-m)}{\sqrt{2}}$$

The Hamiltonian is further divided into two parts

$$H = H_+ + H_-$$

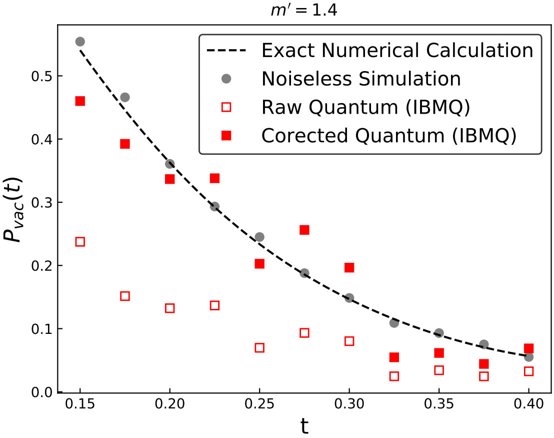
Description of Algorithms

1. Prepare the ground state of the Hamiltonian with both the electric field and the nearest-neighbor lattice-site interactions turned off, which is $|10101\rangle$ for H_+ and $|01010\rangle$ for H_-.
2. Adiabatic turn on the nearest-neighbor lattice site interactions or using the Variational Quantum Eigensolver (VQE) method to find the ground state of the free Hamiltonian.
3. Evolve in time, via Suzuki-Trotter formulae, according to the full Hamiltonian. It is during this time evolution that pair production may occur.
4. Adiabatically turn off the nearest-neighbor lattice site interactions or apply the inverse VQE method.
5. Measure the persistence probability of the ground state.



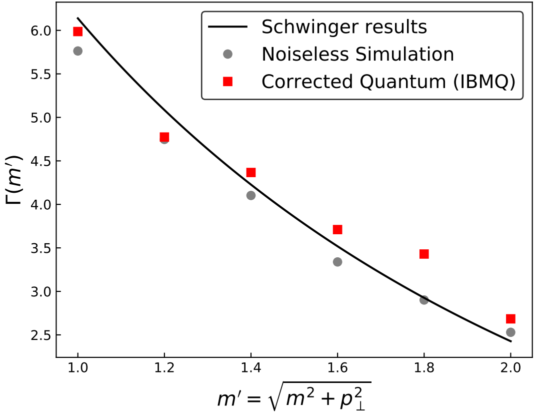
Schwinger pair production results (1+1D)

For a particular choice of the parameters (natural units):
$m = 1, a = 0.45, eE = 20$, we get $\Gamma = 4.37$, while QED predicts $\Gamma = 4.22$.



Schwinger pair production results (3+1D)

- Integrating over the transverse momentum gives the pair production rate in (3+1)D: $\Gamma_{3+1}(m) = 2\int \frac{d^2 p_\perp}{(2\pi)^2}\Gamma_{1+1}(m' = \sqrt{m^2 + p_\perp^2})$
- Theoretical predication of QED gives $\Gamma_{3+1} = 0.58$, while corrected quantum computer result gives $\Gamma_{3+1} = 0.60$.



Conclusions

- We developed and implemented a novel quantum algorithm of simulating the 3+1D Schwinger effect on a 5-qubit quantum computer.
- We introduced several techniques to simplify and parallelize the quantum simulation, such as background field method, dimension reduction, parity symmetry.
- Hardware errors are relieved by constraining to the charge-conserving subspace.
- With slight modifications, the algorithm can be applied to other particle generation processes such as reheating after inflation, black hole superradiance, etc.

Thank you!

Backup slides

Trotter–Suzuki Formulae

Express the Hamiltonian as a sum of easy to simulate Hamiltonians, then approximate the total evolution as a sequence of these simpler evolutions:

$$e^{-i\sum_{j=1}^{m} H_j t} = \left(\prod_{j=1}^{m} e^{-iH_j t/n_t}\right)^{n_t} + O(m^2 t^2/n_t)$$

For our system, $H_+ = H_{+1} + H_{+2} + H_{+3}$, where

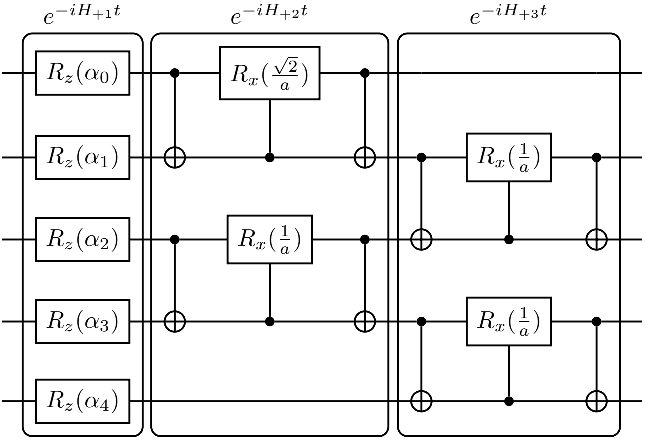
$$H_{+1} = \sum_{n=0}^{4} [(-1)^n m + eEan] \frac{\sigma_3(n)}{2}$$

$$H_{+2} = \frac{1}{\sqrt{2}a}[\sigma^+(0)\sigma^-(1) + \sigma^+(1)\sigma^-(0)] + \frac{1}{2a}[\sigma^+(2)\sigma^-(3) + \sigma^+(3)\sigma^-(2)]$$

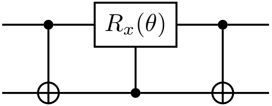
$$H_{+3} = \frac{1}{2a} \sum_{n=1,3} [\sigma^+(n)\sigma^-(n+1) + \sigma^+(n+1)\sigma^-(n)] \ ,$$

Demonstration of circuits

Real time evolution

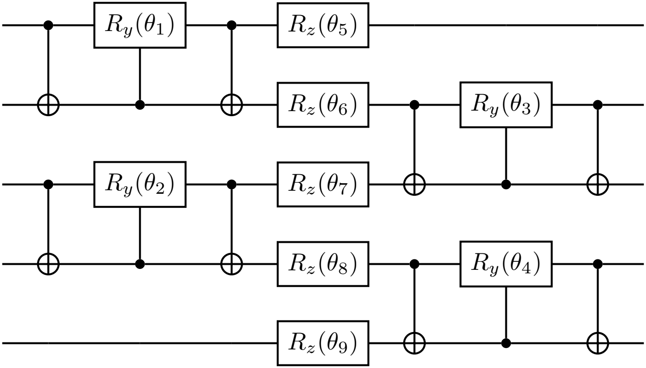


$$e^{-i\frac{\theta}{2}[\sigma^+(n)\sigma^-(n+1)+\sigma^+(n+1)\sigma^-(n)]} =$$



Demonstration of circuits

The VQE ansatz



Error Analysis

- Discretization error
- Truncation error
- Trotterizarion error
- Diabatic error
- Statistical error
- Hardware noise