S_3 3HDM and Flavor Constraints

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Introduction

- 3 fermion generations - 3 Higgs doublets
- S_3 group is a permutation group of a set of 3 elements
- The existing works focus on some specific situation of S_3 3HDM:
  - S_3 3HDM with extra Z_2 symmetry in lepton sector without scalar part considered. (J.Kubo, A. Mondragon, M. Mondragon, E Rodrigue)
  - Scalar potential part with CP conservation. (M. Gomez-Bock, M. Mondragon, A.Perez-Martinez)
- Scalar potential with soft breaking terms and flavor part of a general S_3-symmetric 3HDM
  - Flavor sector:
    - Masses, mixing matrix
    - FCNC
  - Scalar sector:
    - Stability/Unitarity
    - EDM

Yukawa Coupling Lagrangian

Flavor section in S_3 3HDM:

$$Q_L^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, \; u_R^i, \; d_R^i, \; L_L^i = \begin{pmatrix} \nu^i \\ \ell^i \end{pmatrix}_L, \; \ell_R^i, \; \nu_R^i, \; H_i$$

$$\begin{aligned}\mathcal{L}_{Y_d} = & - Y_1^d \overline{Q}_L^I H_3 d_R^I - Y_2^d \left(\overline{Q}_L^I \kappa_{IJ} H_1 d_R^J + \overline{Q}_L^I \eta_{IJ} H_2 d_R^J \right) \\ & - Y_3^d \overline{Q}_L^3 H_3 d_R^3 - Y_4^d \overline{Q}_L^3 H_I d_R^I - Y_5^d \overline{Q}_L^I H_I d_R^3 + h.c.\end{aligned}$$

$$\mathcal{L}_M = -M_1 \nu_R^{I,T} C \nu_R^I - M_3 \nu_R^{3,T} C \nu_R^3$$

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1 e^{i\theta_1}}{\sqrt{2}} \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2 e^{i\theta_2}}{\sqrt{2}} \end{pmatrix}, \quad \langle H_3 \rangle = \begin{pmatrix} 0 \\ \frac{v_3}{\sqrt{2}} \end{pmatrix}$$

With $\sqrt{v_1^2 + v_2^2 + v_3^2} = 246 \text{ GeV}$

Fermion Mass Matrices

$$M_{d,\ell} = \begin{pmatrix} \frac{Y_2^{d,\ell} v_2 e^{i\theta_2}}{\sqrt{2}} + \frac{Y_1^{d,\ell} v_3}{\sqrt{2}} & \frac{Y_2^{d,\ell} v_1 e^{i\theta_1}}{\sqrt{2}} & \frac{Y_5^{d,\ell} v_1 e^{i\theta_1}}{\sqrt{2}} \\ \frac{Y_2^{d,\ell} v_1 e^{i\theta_1}}{\sqrt{2}} & -\frac{Y_2^{d,\ell} v_2 e^{i\theta_2}}{\sqrt{2}} + \frac{Y_1^{d,\ell} v_3}{\sqrt{2}} & \frac{Y_5^{d,\ell} v_2 e^{i\theta_2}}{\sqrt{2}} \\ \frac{Y_4^{d,\ell} v_1 e^{i\theta_1}}{\sqrt{2}} & \frac{Y_4^{d,\ell} v_2 e^{i\theta_2}}{\sqrt{2}} & \frac{Y_3^{d,\ell} v_3}{\sqrt{2}} \end{pmatrix}$$

$$M_{u,\nu} = \begin{pmatrix} \frac{Y_2^{u,\nu} v_2 e^{-i\theta_2}}{\sqrt{2}} + \frac{Y_1^{u,\nu} v_3}{\sqrt{2}} & \frac{Y_2^{u,\nu} v_1 e^{-i\theta_1}}{\sqrt{2}} & \frac{Y_5^{u,\nu} v_1 e^{-i\theta_1}}{\sqrt{2}} \\ \frac{Y_2^{u,\nu} v_1 e^{-i\theta_1}}{\sqrt{2}} & -\frac{Y_2^{u,\nu} v_2 e^{-i\theta_2}}{\sqrt{2}} + \frac{Y_1^{u,\nu} v_3}{\sqrt{2}} & \frac{Y_5^{u,\nu} v_2 e^{-i\theta_2}}{\sqrt{2}} \\ \frac{Y_4^{u,\nu} v_1 e^{-i\theta_1}}{\sqrt{2}} & \frac{Y_4^{u,\nu} v_2 e^{-i\theta_2}}{\sqrt{2}} & \frac{Y_3^{u,\nu} v_3}{\sqrt{2}} \end{pmatrix}.$$

$$M_\nu^{\mathrm{Majorana}} = M_\nu \tilde{M}^{-1} (M_\nu)^T$$

where $\tilde{M} = \mathrm{diag}(M_1, M_1, M_3)$

Quark Mass Matrix Parametrization

$$U^{\dagger}_{d(u,\ell)L} M_{d(u,\ell)} U_{d(u,\ell)R} = \mathrm{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)})$$

$$M_d = \begin{pmatrix} \epsilon & x m_1 & x m_2 \\ x m_1 & -2m_1 + \epsilon & m_2 \\ x m_2^{'} & m_2^{'} & m_3 \end{pmatrix}$$

Where

$$\epsilon = m_0 + m_1 \quad x = \frac{v_1 e^{i\theta_1}}{v_2 e^{i\theta_2}} \quad m_1 = \frac{Y_2^d v_2 e^{i\theta_2}}{\sqrt{2}} \quad m_2 = \frac{Y_5^d v_1 e^{i\theta_1}}{\sqrt{2}}$$

$$m_3 = \frac{Y_3^d v_3 e^{i\theta_3}}{\sqrt{2}} \quad m_2^{'} = \frac{Y_4^d v_2 e^{i\theta_2}}{\sqrt{2}} \quad m_0 = \frac{Y_1^d v_3 e^{i\theta_3}}{\sqrt{2}}$$

Quark Masses/CKM Matrix Fit Result

m(GeV)	m_u	m_c	m_t
Exp	0.0011±0.000375	0.532±0.002	150.7±0.5
Fit	0.000873	0.531	150.8
m(GeV)	m_d	m_s	m_b
Exp	0.0025±0.000325	0.047±0.008	2.43±0.025
Fit	0.00248	0.0443	2.446

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.974 & 0.226 & 0.00387 \\ 0.226 & 0.9732 & 0.0397 \\ 0.00808 & 0.03908 & 0.9992 \end{pmatrix}$$

$$|V_{\mathrm{CKM}}^{\mathrm{exp}}| = \begin{pmatrix} 0.9737 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.041 \pm 0.0014 \\ 0.008 \pm 0.0003 & 0.0388 \pm 0.0011 & 0.9992 \pm 0.0001 \end{pmatrix}$$

$$J = \mathrm{Im}\left[(V_{\mathrm{CKM}})_{11}(V_{\mathrm{CKM}})_{22}(V_{\mathrm{CKM}}^*)_{12}(V_{\mathrm{CKM}}^*)_{21}\right] = 3.05 \times 10^{-5}$$

$$J^{\mathrm{exp}} = 3 \times 10^{-5} \pm 1.2 \times 10^{-6}$$

Lepton Masses/PMNS Matrix Fit Result

m(**GeV**)	e	μ	τ
Exp	0.000511±0.0000051	0.1056±0.001056	1.777±0.0177
Fit	0.000516	0.1062	1.7799
m(**eV**)	ν_e	ν_μ	ν_τ
Majarona	5.352×10^{-3}	0.0102	0.04969

Δm^2(**GeV**)	Δm^2_{21}	Δm^2_{31}
Exp	$7.42\pm0.21\times10^{-23}$	$2.517\pm0.028\times10^{-21}$
Fit	7.61810^{-23}	2.44110^{-21}

$$|V_{\rm PMNS}| = \begin{pmatrix} 0.821 & 0.552 & 0.147 \\ 0.424 & 0.546 & 0.722 \\ 0.382 & 0.630 & 0.675 \end{pmatrix}$$

$$|V_{\rm PMNS}^{\rm exp}| = \begin{pmatrix} 0.823\pm0.022 & 0.546\pm0.033 & 0.1495\pm0.0065 \\ 0.37\pm0.137 & 0.5775\pm0.1165 & 0.7045\pm0.0735 \\ 0.3935\pm0.1325 & 0.586\pm0.115 & 0.686\pm0.075 \end{pmatrix}$$

Higgs Potential

Scalar Potential with S_3 symmetry:

$$
\begin{aligned}
V =&\mu_0^2(h_1^\dagger h_1 + h_2^\dagger h_2) + \mu_1^2(h_s^\dagger h_s) + \lambda_1(h_1^\dagger h_1 + h_2^\dagger h_2)^2 \\
&+ \lambda_2(h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3[(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&+ [\lambda_4 e^{ic_4}[(h_s^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_s^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2)] + h.c.] \\
&+ \lambda_5[(h_s^\dagger h_s)(h_1^\dagger h_1 + h_2^\dagger h_2)] + \lambda_6[(h_s^\dagger h_1)(h_1^\dagger h_s) + (h_s^\dagger h_2)(h_2^\dagger h_s)] \\
&+ [\lambda_7 e^{ic_7}[(h_s^\dagger h_1)^2 + (h_s^\dagger h_2)^2] + h.c.] + \lambda_8(h_s^\dagger h_s)^2
\end{aligned}
$$

The soft-breaking terms:

$$
\begin{aligned}
V_2' =&\mu_2^2(h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}(\mu_3^2 e^{ib_3} h_1^\dagger h_2 + h.c.) + \frac{1}{2}(\mu_4^2 e^{ib_4} h_1^\dagger h_s + h.c.) \\
&+ \frac{1}{2}(\mu_5^2 e^{ib_5} h_2^\dagger h_s + h.c.)
\end{aligned}
$$

Higgs Basis Transformation

Rotating into Higgs basis:

$$R_H = \begin{pmatrix} \frac{e^{-i\theta_1} v_1}{v} & \frac{e^{-i\theta_2} v_2}{v} & \frac{v_3}{v} \\ 0 & \frac{e^{-i\theta_2} v_3}{v_{23}} & -\frac{v_2}{v_{23}} \\ -\frac{e^{-i\theta_1} v_{23}}{v} & \frac{e^{-i\theta_2} v_{12}}{v v_{23}} & \frac{v_1 v_3}{v v_{23}} \end{pmatrix}$$

$$v_{23} = \sqrt{v_2^2 + v_3^2} \quad v_{12} = \sqrt{v_1^2 + v_2^2} \quad v = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Yukawa Coupling Matrices After All Transformations

$$U_k^L(ab)B_H(ij)X_j^k(bc)O(in)U_k^R(cd) = X_n^k(ad)$$

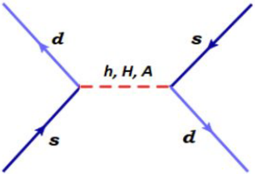
where $i, j, n = 1, ..., 6; \quad k = u, d, e; \quad a, b, c, d = 1, 2, 3$

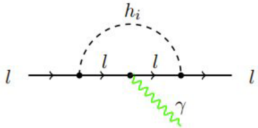
$$U_k^L(ab)R_H(ij)S_j^k(bc)P(in)U_k^R(cd) = S_n^k(ad)$$

where $i, j, n = 1, 2, 3; \quad k = u, d, e; \quad a, b, c, d = 1, 2, 3$

FCNC,EDM BFB,Unitarity

- FCNC and EDM Constraints





- Unitarity (Dipankar Das, Ujjal Kumar Dey)
- BFB, necessary conditions:

$$\lambda_1 > 0, \quad \lambda_1 - \lambda_2 > 0, \quad \lambda_1 + \lambda_3 > 0, \quad \lambda_8 > 0$$

For the best fitting point, BFB is further checked numerically.

Higgs Masses Result

Higgs	h_2	A_2	$h_2^{\pm}$
m(GeV)	7114.614	7124.454	7123.214
Higgs	h_3	A_3	$h_3^{\pm}$
m(GeV)	348487.001	348487.021	348487.105

Conclusion

- General S_3 3HDM with CPV, soft-breaking terms
- Global Fitting:
  - Fermions masses (improvement of neutrino mass fit is in process), CKM/PMNS matrix
  - FCNC constraints
  - EDM
  - BFB/Unitarity
- Lowest heavy Higgs masses found to be 7114.614 GeV
  - Still space for us to improve to get lighter mass result
- Future studies:
  - Fitting against both Normal/Inverse Hierarchy
  - Collider Phenomenology
    - Heavy Majorana Neutrino
    - Heavy Higgs