Gravitational Waves from an Inflation Triggered First-Order Phase Transition

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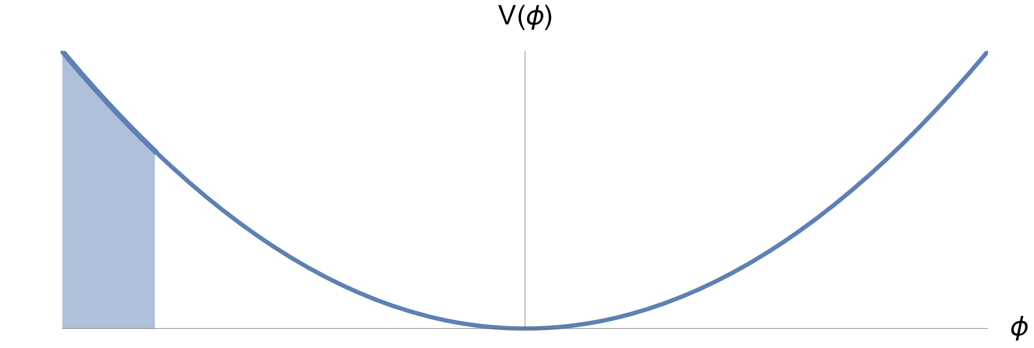
GW as a tool to probe the Early Universe

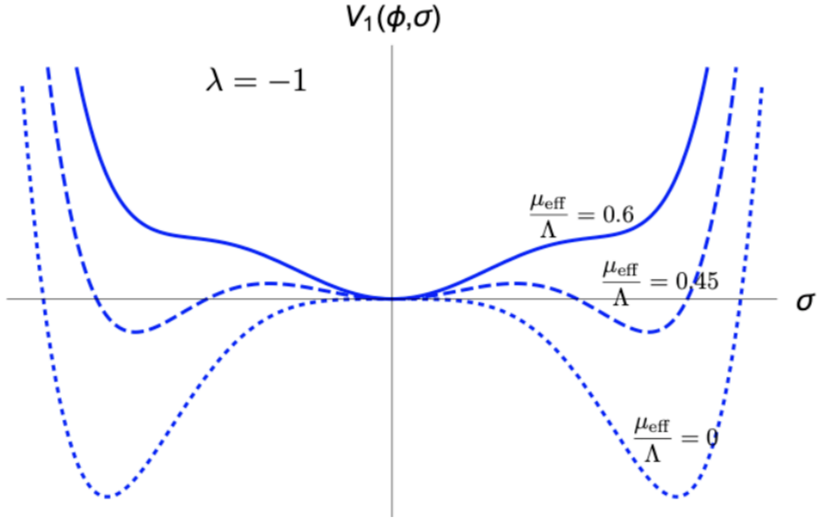
- Stochastic gravitational waves carry the information of its origin and the evolution history of the universe.
- It may be detected by the ground-based (LIGO, Virgo, etc.), space-based interferometers (LISA, BBO, etc.), radio telescope (SKA, etc.) or CMB in different frequency ranges.
- One interesting source is the strong first order phase transition during inflation.

Phase Transition induced by spectator field during inflation

- The inflaton field would roll over a wide range during the inflation era.
- If there is spectator sector weakly coupled to inflaton, the variation of inflaton value would lead to some significant phenomena in the spectator sector.



Spectator sector



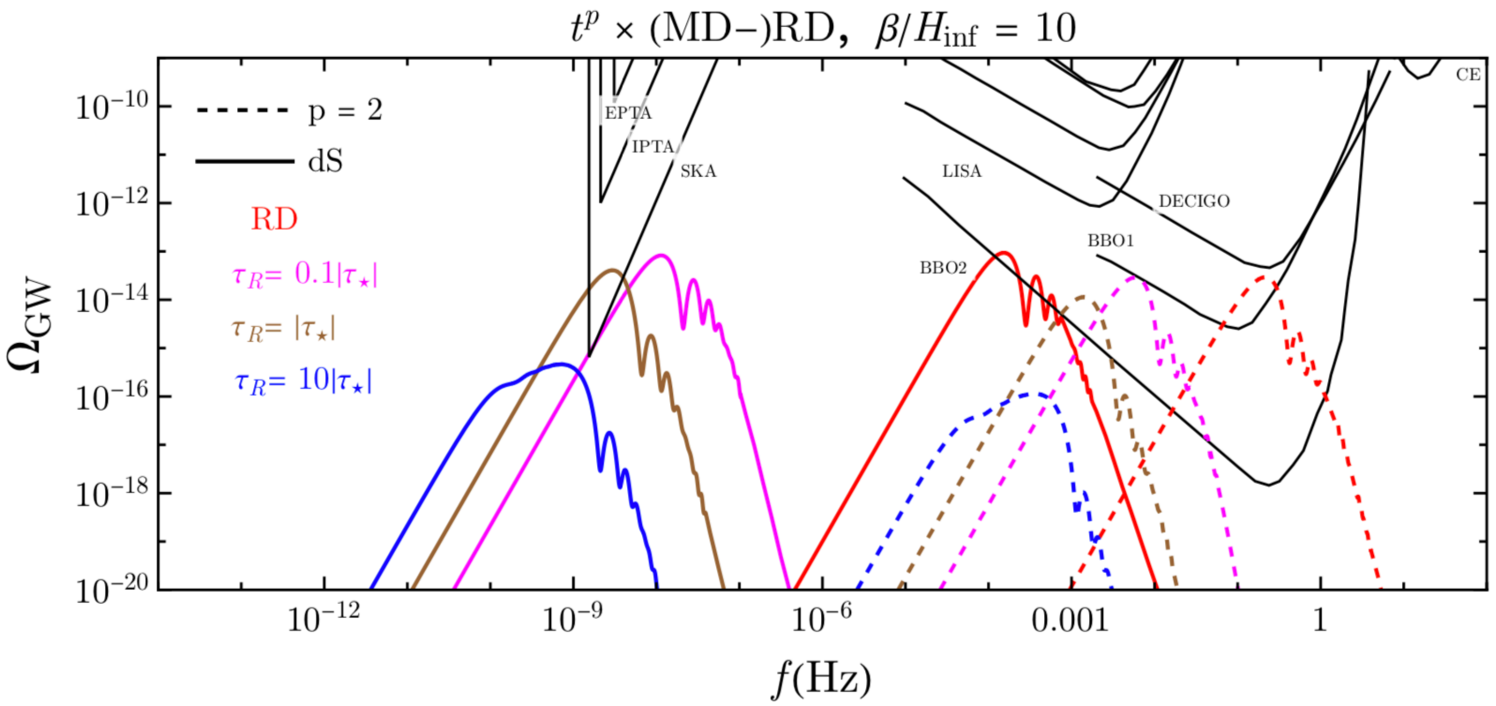
$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

- The mass or coupling in the spectator sector may be changed due to the large excursion of the inflaton.
- It may induce

Strong first order phase transition in spectator sector.

Strong First Order Phase Transition

- Bubble of true vacuum would be nucleated and expand if its size is larger than critical value.
- If the supercritical size bubbles nucleation rate $\beta \gg H$, the first order phase transition would complete
- Due to inflation, sound waves and magnetohydrodynamic turbulence can be ignored, we focus on bubble collision.

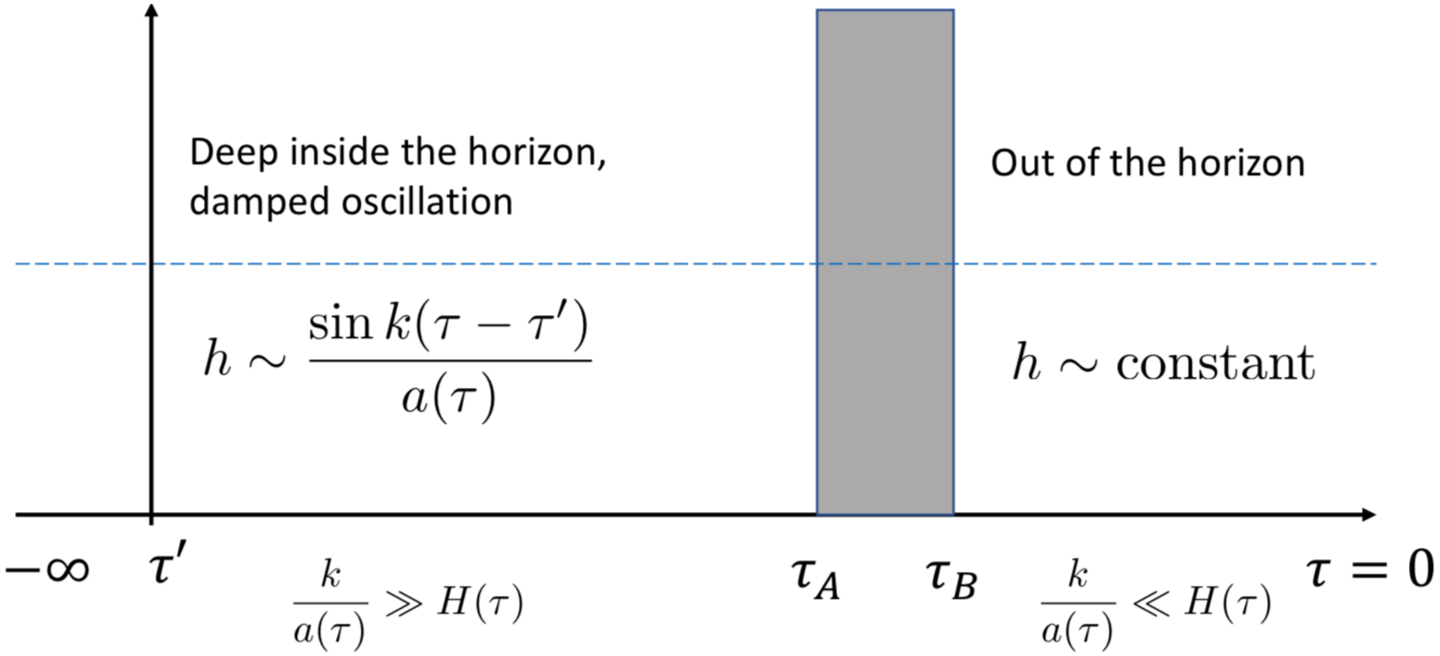


Multiple peaks and oscillatory pattern!

GW is not diluted away during inflation.

Solving the Green Function

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = \frac{16\pi G_N}{a(\tau_*)}T^{(0)}_{ij}\delta(\tau - \tau_*)$$

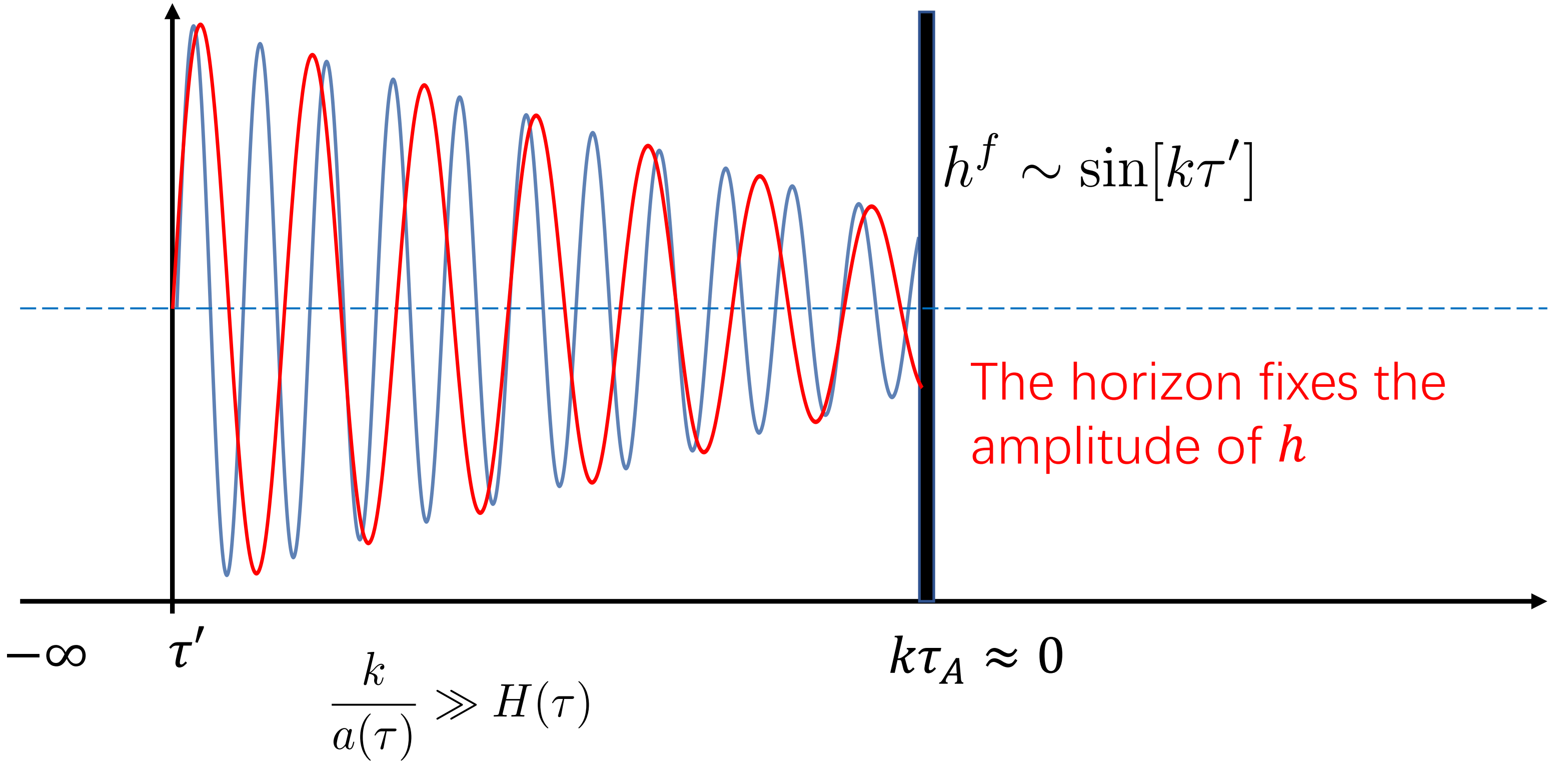


Horizon crossing

$$\frac{k}{a} \sim H$$

- Almost all modes would become super-horizon at the end of inflation hence constant amplitude.

Inflation: Oscillatory Pattern



- After horizon crossing, the amplitude is frozen and different modes exit the horizon at different time.
- The amplitude is oscillating with k at the horizon.

Finite Duration of Phase Transition

- The phase transition happens during finite time interval.
- For $\beta/H_{\rm inf} \gg 1$, we take the time integration with the source.
- Need detailed simulation for $\beta/H_{\rm inf} \sim O(1)$.

Convoluted with Green Function!

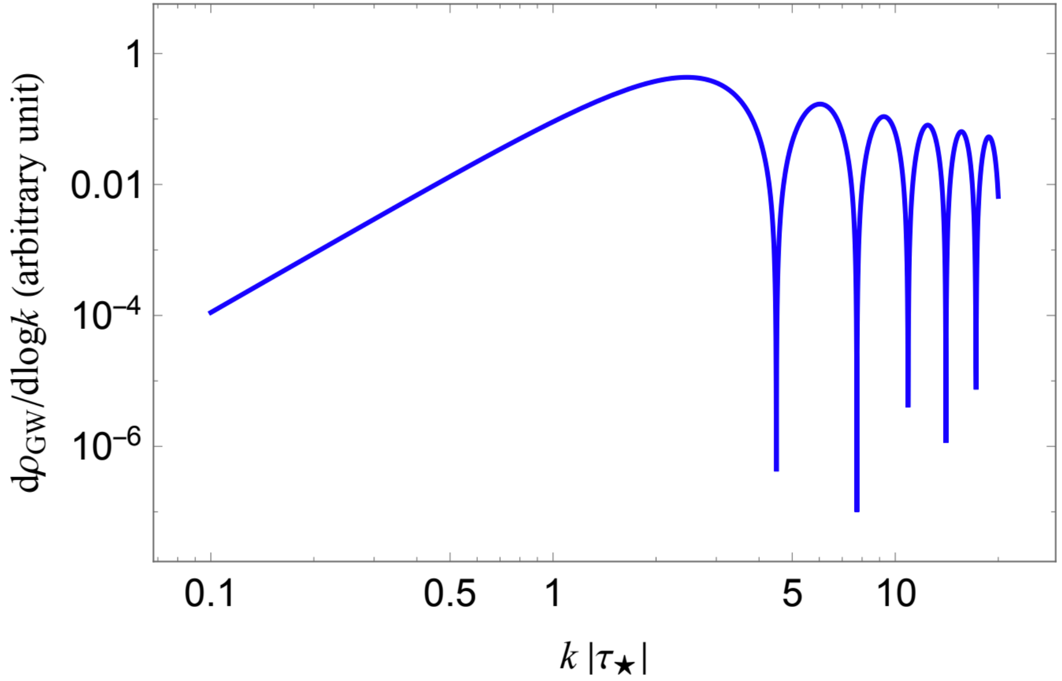
De Sitter Inflation

$$\tilde{h}_{ij}(\eta,\mathbf{k}) = \frac{16\pi G_N}{k}\int d\eta' \frac{\tilde{\mathcal{G}}(\eta,\eta')}{a(\eta)} a^3(\eta')\tilde{\sigma}^{\rm TT}_{ij}(\eta',\mathbf{k})$$

$$a(\tau) = -\frac{1}{H_{\rm inf}\tau}$$

$$\left.\frac{\mathcal{G}(\eta,\eta')}{a(\eta)}\right|_{\eta\to 0} = \frac{H_{\rm inf}}{k}\left(\frac{\sin\eta'}{\eta'} - \cos\eta'\right)$$

$$\tilde{h}_{\mathbf{k}}(\tau) \approx -\frac{16\pi G_N H_{\rm inf}\tilde{T}^{(0)}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau_\star}\right)\cos k(\tau-\tau_\star) + \left(1 + \frac{1}{k^2(\tau\tau_\star)}\right)\sin k(\tau-\tau_\star)\right]$$



Oscillatory pattern:
oscillating with k

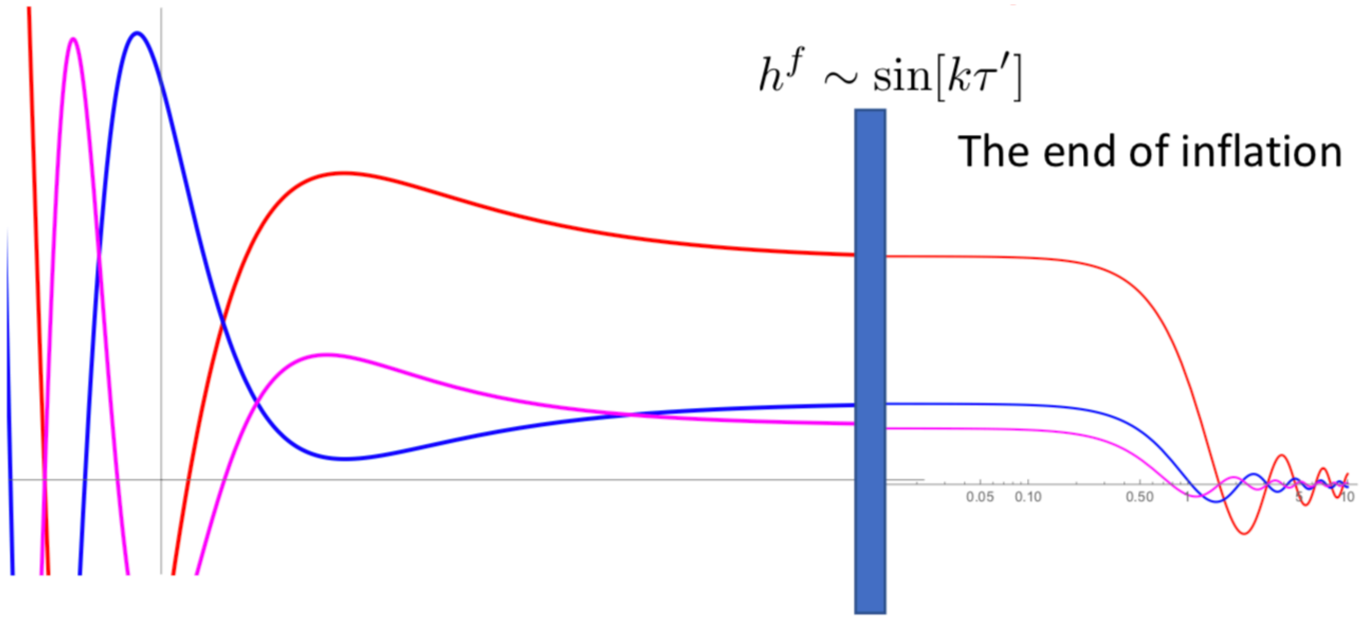
Post Inflationary Evolution

$$\tilde{h}_{ij}(\tau, \mathbf{k}) = \tilde{h}^{f}_{ij}(\mathbf{k})\mathcal{E}(k\tau)$$

After entering deep RD $\mathcal{E}(\eta) = \tilde{\mathcal{E}}^{i}_{0} a^{-1} \sin(\eta + \phi)$

Instantaneous Reheating

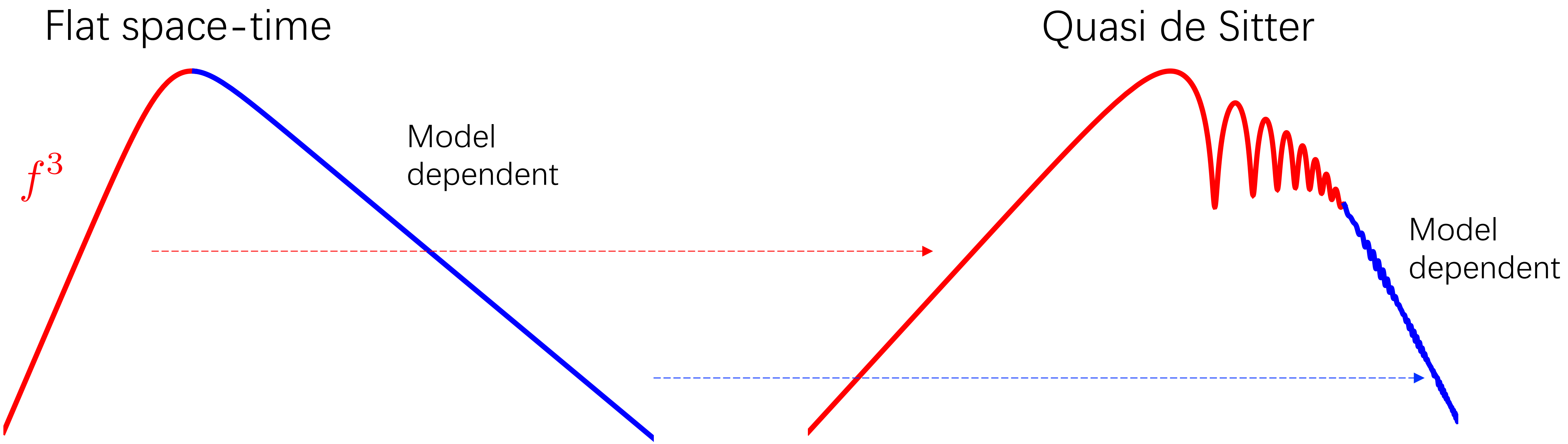
RD: $$\mathcal{E}(\eta) = \frac{\sin\eta}{\eta}$$



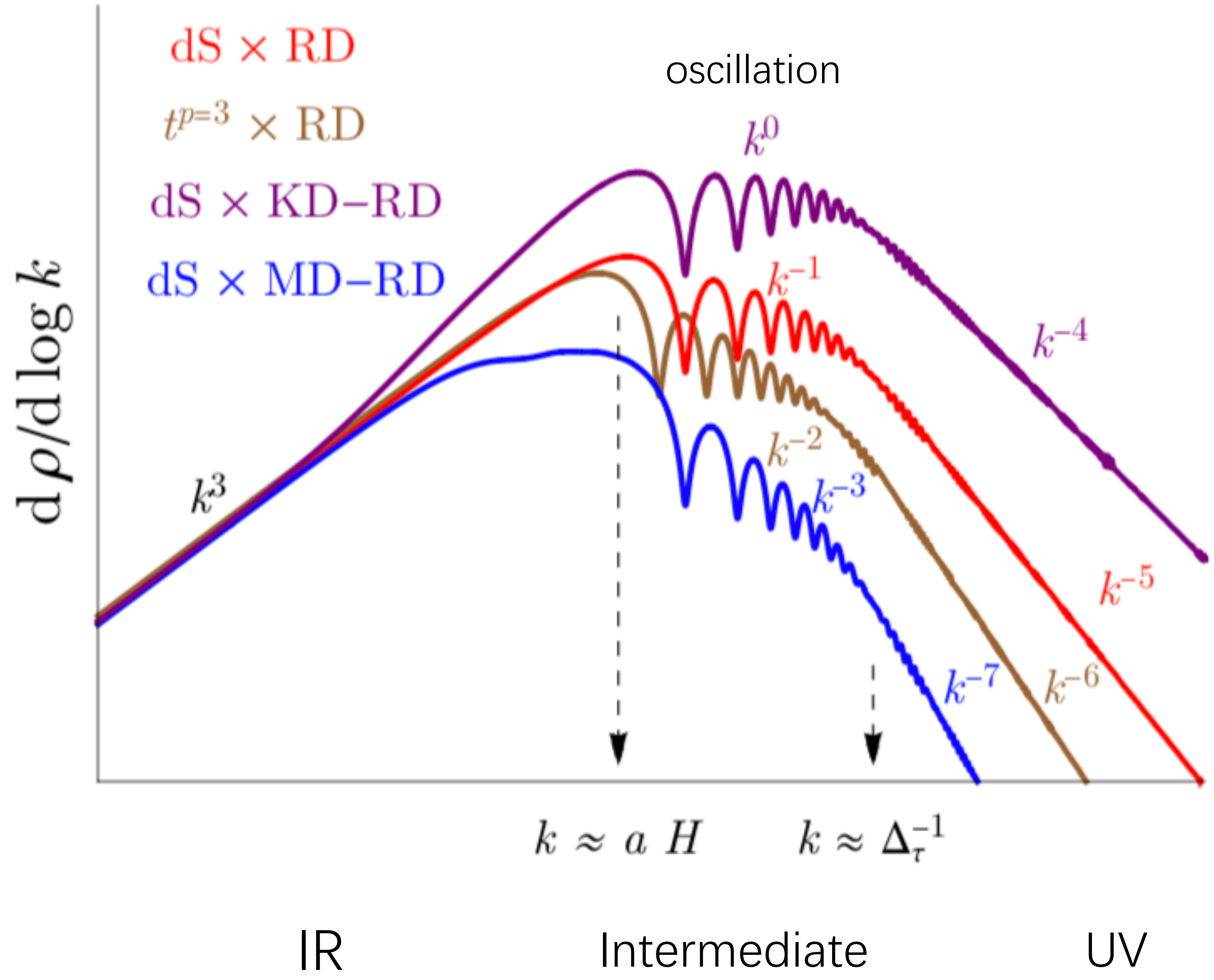
Other cases:
Slow reheating, primordial black hole dominated intermediate stage

More complicated post-inflationary evolution.

Spectrum distortion



Cai, Pi, Sasaki, 1909.13728



dS × RD
$t^{p=3}$ × RD
dS × KD–RD
dS × MD–RD
oscillation
k^0
k^{-1}
k^{-4}
k^{-2}
k^{-3}
k^3
k^{-5}
k^{-7}
k^{-6}
$\mathrm{d}\,\rho/\mathrm{d}\log k$
$k \approx a\,H$
$k \approx \Delta_\tau^{-1}$
IR
Intermediate
UV

Observed GW signals today

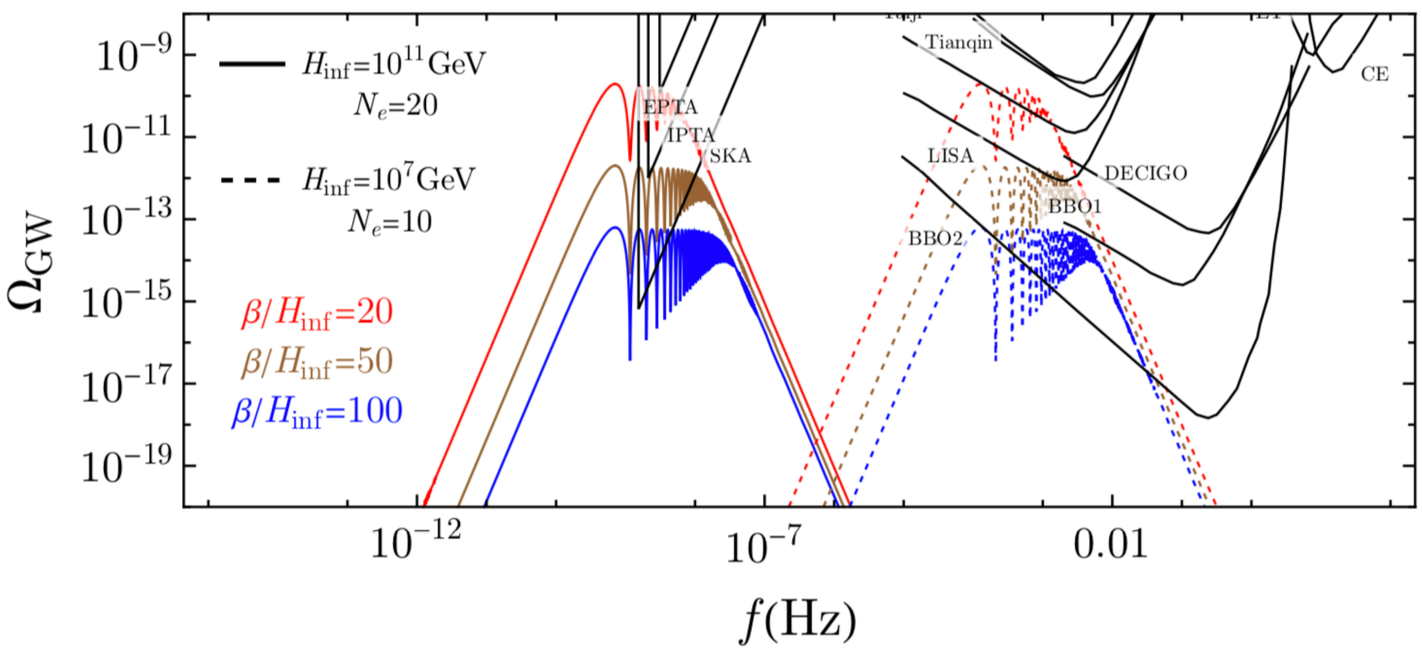
- Peak frequency today

$$f_{\text{today}} = f_\star \times \frac{a(\tau_\star)}{a_r}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_r^2}{8\pi G_N}\right)\right]^{1/4}}$$

In dS-$t^{\tilde{p}}$-RD

$$\tilde{f}_{\text{today}}^{\text{peak}} = 1.1\times 10^{11}\text{Hz}\times\left(\frac{H_{\text{end}}}{m_{\text{pl}}}\right)^{1/2}\left(\frac{a_r}{a_{\text{end}}}\right)^{-\frac{1}{2\tilde{\alpha}-1}-\frac{1}{2}}\frac{a(\tau_\star)}{a_{\text{end}}}$$

KD intermediate stage, dS $\times t^{\tilde{p}=1/3}$-RD



Conclusion

- The power spectrum we get at the end of inflation shows oscillatory feature.
- The GWs might be observed on the space or ground based GW observatories.
- The observed GWs today produced by strong first order phase transition would carry the information of

(a) the details of the phase transition

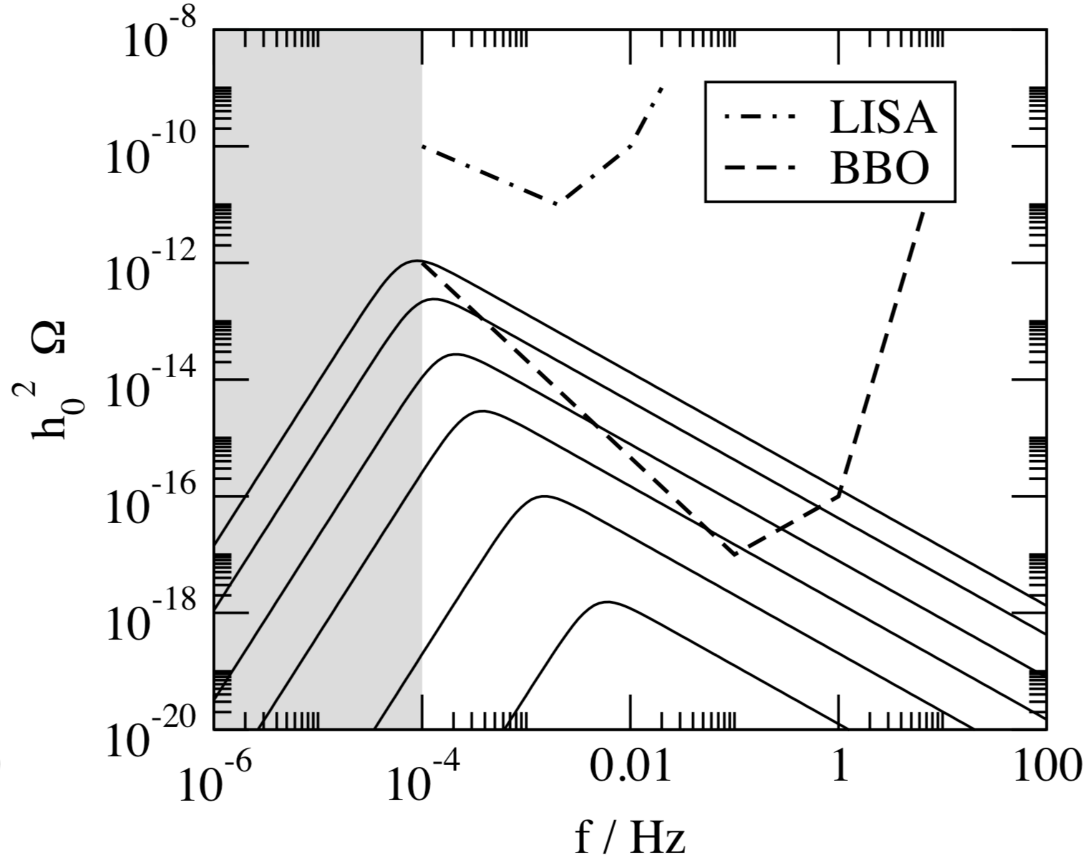
(b) the inflation

(c) the post-inflationary evolution history

Thank you!

Backup

Simulation result under the envelope approximation done in flat space



$$\beta \equiv -\frac{dS_4}{dt}\bigg|_{t_*}$$

determine the size and time interval of the phase transition

GW Power spectrum

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac}d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Flat space spectrum

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$$\frac{a(\tau_\star)}{a_1} \to e^{-N_e}$$

N_e: e-folds before the end of inflation