How to measure the W Mass: A Theory Perspective

Joshua Isaacson

Based on: arxiv:2205.02788

In Collaboration with: Yao Fu and C.-P. Yuan

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Standard Model: W Mass

Standard Model EW Fit

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\left(1+\Delta r\right)$$

$$\Delta r=\Delta\alpha-\frac{c_W^2}{s_W^2}\Delta\rho+\Delta r_{\text{rem}}\left(M_H\right)\ ,$$

where s_W^2 is the Weinberg angle, $\Delta\alpha$ is the correction to α from the light fermions, $\Delta\rho$ is the correction to the ρ parameter, and Δr_{rem} contains all corrections containing the Higgs mass.

Parameter	Fit Result
G_μ [GeV^{-2}]	1.1663787 $\times 10^{-5}$
$\alpha(0)^{-1}$	137.035999139
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	0.027627 $\pm$ 0.000096
M_Z [GeV]	91.1883 $\pm$ 0.0021
M_H [GeV]	125.21 $\pm$ 0.12
m_t [GeV]	172.75 $\pm$ 0.44
M_W [GeV]	80.3591 $\pm$ 0.0052

Table reproduced from: HEPFit Group (2112.07274).

Experimental Measurements

- CDF Run II results most precise
- 7σ tension with SM
- 3σ tension between CDF-II and ATLAS result
- Missing LHCb result: 80,354 $\pm$ 32 MeV
- For more details see Joey Huston's talk

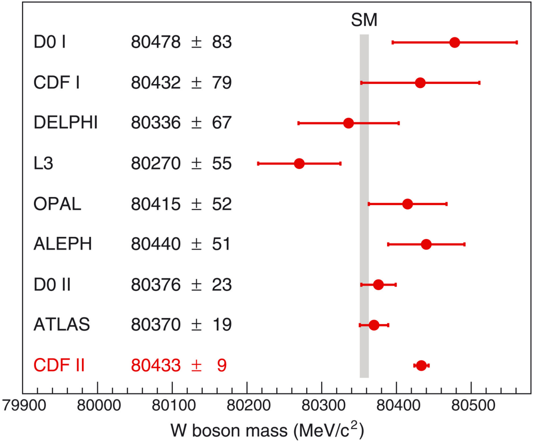
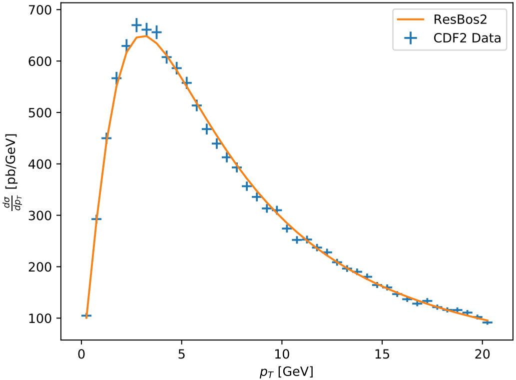


Figure reproduced from CDF-II measurement (Science 376, 170).

Theory Calculation

Breakdown of Fixed Order

- Perturbative series has terms proportional to $\alpha_s^n \log^m\left(\frac{p_T^2}{M_W^2}\right)$, $m \leq 2n$
- As $p_T^W \to 0$ the series no longer converges
- Need to include corrections to all orders by resumming the series



Analytic vs. Numeric Resummation

Analytic:

- Formal resummation (focus here on b-space CSS resummation)
- Pros:
  - High precision and accuracy
- Cons:
  - Inclusive only
  - Numerically expensive
- Used by CDF to obtain M_W

Numerical

- Parton Showers (Pythia, Sherpa, Herwig, Dire, Vincia)
- Pros:
  - Exclusive final states
  - Quick
- Cons:
  - Currently only LL with some subleading effects included
- Used by ATLAS to obtain M_W

Collins-Soper-Sterman Formalism

Resummation

$$\frac{d\sigma_{\text{res}}}{dQ^2 d^2\vec{q_T} dy d\Omega} = \sigma \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}} \tilde{W},$$

$$\tilde{W} = e^{-S(b)} \; C \otimes f(x_A, C_3/b) C \otimes f(x_B, C_3/b)$$

$$S(b) = \int_{\frac{C_1^2}{b^2}}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) A(\bar{\mu}) + B(\bar{\mu}) \right]$$

[Collins, Soper, Sterman, '85] [...]

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- Electroweak cross section

[Collins, Soper, Sterman, '85] [...]

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- Sudakov factor

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- Sudakov factor
- Collinear factors

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- Electroweak cross section
- Sudakov factor
- Collinear factors
- Perturbative Coefficients (A, B, C)

[Collins, Soper, Sterman, '85] [...]

Order Definitions

Order	Boundary Condition	Anomalous Dimension γ_i (non-cusp)	Γ_{cusp}, β	Fixed Order Matching
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL' (+ NLO)	α_s	1-loop	2-loop	α_s
NNLL (+ NLO)	α_s	2-loop	3-loop	α_s
NNLL' (+ NNLO)	α_s^2	2-loop	3-loop	α_s^2
N^3LL (+ NNLO)	α_s^2	3-loop	4-loop	α_s^2
N^3LL' (+ N^3LO)	α_s^3	3-loop	4-loop	α_s^3
N^4LL (+ N^3LO)	α_s^3	4-loop	5-loop	α_s^3

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- ■ Accuracy used by CDF

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- Accuracy used by CDF
- Current accuracy available in ResBos code

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- ■ Accuracy used by CDF
- ■ Current accuracy available in ResBos code
- ■ All terms known to this accuracy

Non-Perturbative Fit

$$S(b) = \int_{\frac{C_1^2}{b^2}}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \right]$$

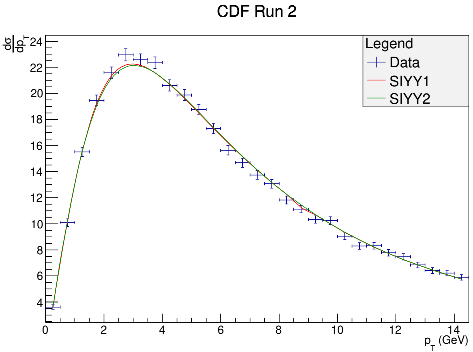
- Lower limit goes to zero as b goes to infinity
- Requires evaluation of $\alpha_s(C_1/b)$ which is non-perturbative
- Need to introduce a non-perturbative cutoff (b^*-prescription):

$$b^* = \frac{b}{\sqrt{1 + \frac{b^2}{b_{\text{max}}^2}}}$$

BLNY Form

$$S_{NP}(b) = -b^2 \left(g_1 + g_2 \log\left(\frac{Q}{2Q_0}\right) + g_1 g_3 \log(100 x_1 x_2) \right)$$

- g_1 and g_3 extracted from global fit
- g_2 tuned to reproduce CDF-II p_T^Z
- M_W vs. M_Z captured in Q dependence
- No flavor dependence included
- No consideration of uncertainty from changing form, but expected to be small



NOTE: SIYY2 is the same functional form as BLNY, but with $b_{\text{max}} = 1.5\ \text{GeV}^{-1}$

Flavor Dependence

- Study on flavor dependence for $\sqrt{s} = 7$ TeV LHC
- $S_{NP}(b) = -b^2(g_a + g_{evo} \log(Q^2/Q_0^2)$, where g_a is the flavor dependent piece
- Found shift could be up to 10 MeV
- Additional studies are required to validate
- Unclear what the global shift would be

Set	u_v	d_v	u_s	d_s	others
1	0.34	0.26	0.46	0.59	0.32
2	0.34	0.46	0.56	0.32	0.51
3	0.55	0.34	0.33	0.55	0.30
4	0.53	0.49	0.37	0.22	0.52
5	0.42	0.38	0.29	0.57	0.27

Set	ΔM_W^+		ΔM_W^-	
	M_T	p_T^ℓ	M_T	p_T^ℓ
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3

Table reproduced from: Phys. Letters B 788 (2019) 542-545

Results

Methodology

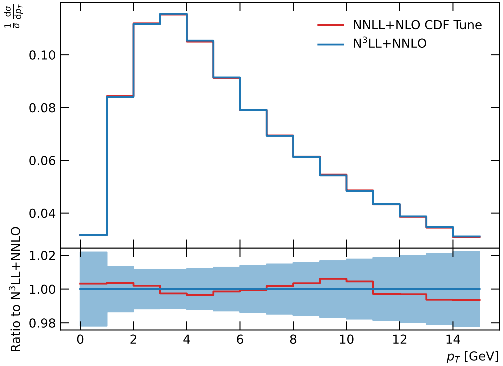
Our Procedure:

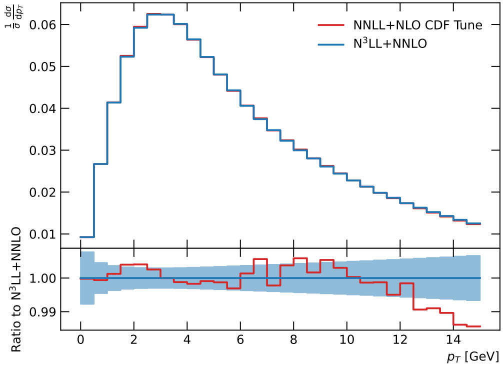
- Generate pseudodata using N^{3}LL+NNLO prediction
- Tune NNLL+NLO prediction to reproduce $p_T(Z)$ data
- Validate tuned result against $p_T(W)$ data
- Use tuned result to generate mass templates
- Extract W mass from template fit for each observable
- Calculate the mass shift from the input value for pseudodata

Details:

- Pseudodata $M_W = 80,358$ MeV
- Cuts:
  - $p_T(Z) < 15$ GeV, $p_T(W) < 15$ GeV
  - $30 < p_T(\ell) < 55 GeV$, $30 < p_T(\nu) < 55$ GeV
  - $|\eta(\ell)| < 1$
  - $66 < M_{\ell\ell} < 116$ GeV (Z events), $60 < m_T < 100$ GeV (W events)
- Number of Events:
  - 1,811,700 $W \to e\nu$
  - 66,180 $Z \to ee$
  - 2,424,486 $W \to \mu\nu$
  - 238,534 $Z \to \mu\mu$

Tuning to Pseudodata

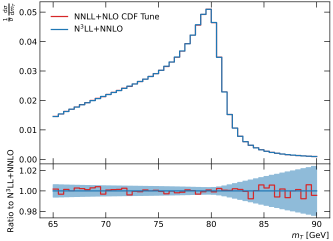




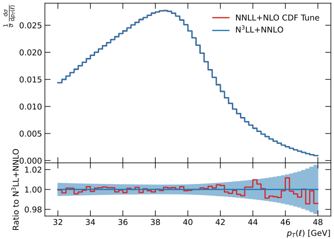
Tuned result:

- Fit to $p_T(Z) < 15$ GeV
- $g_2 = 0.662$ GeV^2
- $\alpha_S(M_Z) = 0.120$
- Tuned PDF set: `CT18NNLO_as_120`

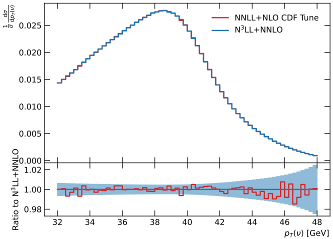
Results



Best Fit: $M_W = 80,386$ MeV



Best Fit: $M_W = 80,388$ MeV



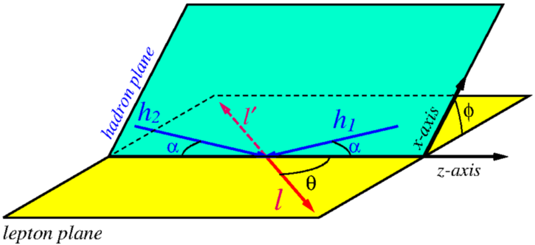
Best Fit: $M_W = 80,389$ MeV

	Mass Shift [MeV]	
Observable	ResBos2	+Detector Effect+FSR
m_T	1.5 ± 0.5	0.2 ± 1.8 ± 1.0
$p_T(\ell)$	3.1 ± 2.1	4.3 ± 2.7 ± 1.3
$p_T(\nu)$	4.5 ± 2.1	3.0 ± 3.4 ± 2.2

Conclusions

- CDF used ResBos code at NNLL+NLO accuracy
- ResBos v2 is able to go to N^3LL+NNLO accuracy
- ResBos2 corrected major criticism of incorrect angular functions in the ResBos code
- Mimic CDF analysis using pseudoexperiments at N^3LL+NNLO accuracy
- Find shift to be consistent with 0 MeV and up to 10 MeV in worse case

Backup



$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi$$
$$+\frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\Big\}.$$

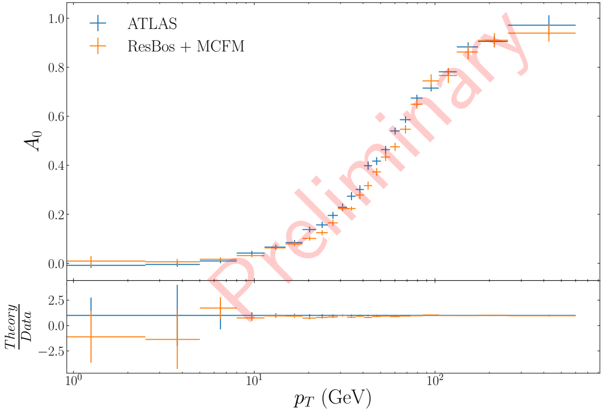
$$\langle P(\cos\theta,\phi)\rangle = \frac{\int P(\cos\theta,\phi)d\sigma(\cos\theta,\phi)d\cos\theta d\phi}{\int d\sigma(\cos\theta,\phi)d\cos\theta d\phi}.$$

$$\langle\frac{1}{2}(1-3\cos^2\theta)\rangle = \frac{3}{20}(A_0-\frac{2}{3});\quad \langle\sin 2\theta\cos\phi\rangle = \frac{1}{5}A_1;\quad \langle\sin^2\theta\cos 2\phi\rangle = \frac{1}{10}A_2;$$
$$\langle\sin\theta\cos\phi\rangle = \frac{1}{4}A_3;\quad \langle\cos\theta\rangle = \frac{1}{4}A_4;\quad \langle\sin^2\theta\sin 2\phi\rangle = \frac{1}{5}A_5;$$
$$\langle\sin 2\theta\sin\phi\rangle = \frac{1}{5}A_6;\quad \langle\sin\theta\sin\phi\rangle = \frac{1}{4}A_7.$$

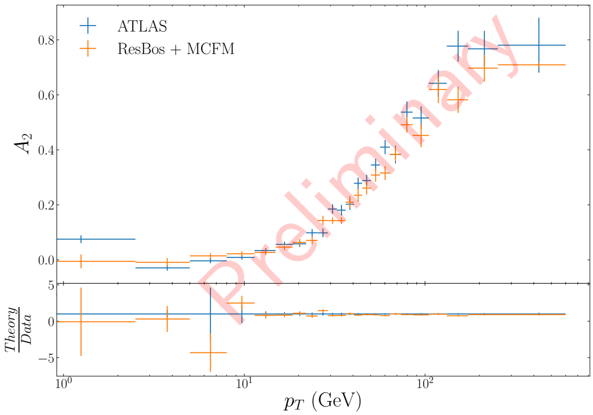
NNLO Angular Coefficients

- Well known issue with angular coefficients in the ResBos code at NNLO (No issue with matching to NLO)
- CDF-II only used the NLO so the angular functions are exact to that order
- ResBos only included NNLO corrections to the total rate, but not to the angular functions
- This is an issue with matching to an incomplete NNLO calculation, and not an issue with the resummation or the matching to fixed order
- Only effects larger p_T ($p_T > 30$ GeV, CDF has a cut of $p_T < 15$ GeV)
- Has been resolved via matching to MCFM (preliminary results next slides)

NNLO Angular Coefficients



NOTE: Uncertainties are purely statistical for ResBos + MCFM



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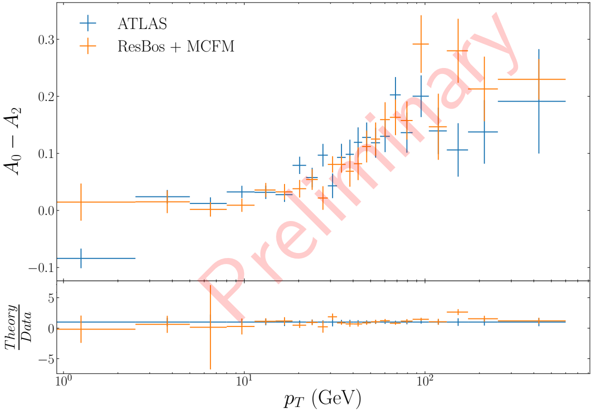

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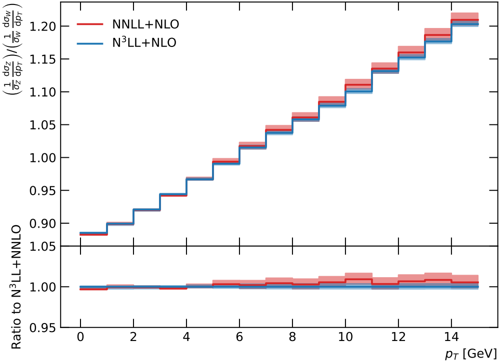
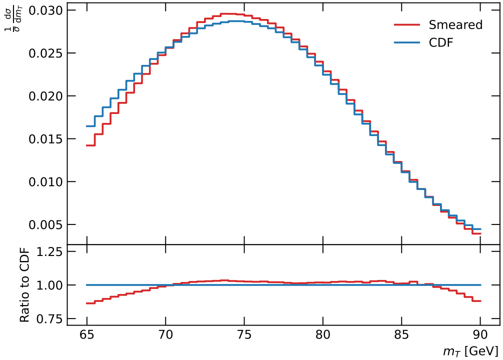
$P_T(Z)/P_T(W)$



- Ratio is stable to higher order corrections at small p_T
- Scale uncertainty only using correlated prediction
- Need to investigate the CDF estimated uncertainty from this ratio

Detector Smearing



Detector Smearing:

- Fit functional form (Smearing 1):
$$\frac{\sigma}{E} = a \oplus \frac{b}{\sqrt{E}} \oplus \frac{c}{E}$$
- Use gaussian with 5%(11%) width for $\ell(\nu)$ (Smearing 2)
- Note results not sensitive to smearing effect if data and theory smeared identically

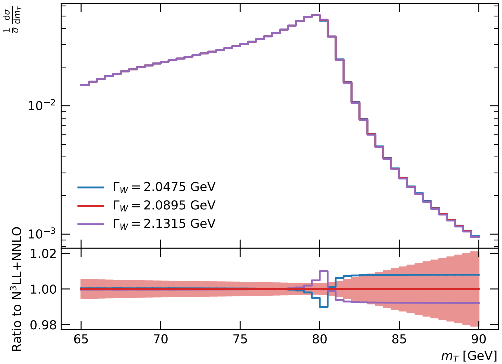
	Mass Shift [MeV]	
Observable	Smearing 1	Smearing 2
m_T	0.2 $\pm$ 1.8 $\pm$ 1.0	1.0 $\pm$ 2.1 $\pm$ 1.3
$p_T(\ell)$	4.3 $\pm$ 2.7 $\pm$ 1.3	4.5 $\pm$ 2.6 $\pm$ 1.4
$p_T(\nu)$	3.0 $\pm$ 3.4 $\pm$ 2.2	3.8 $\pm$ 4 $\pm$ 2.7

Width Effect

Width Effect:

- Central width: $\Gamma_W = 2.0895$ GeV
- NLO width proportional to M_W^3
- Negligible shift

Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5



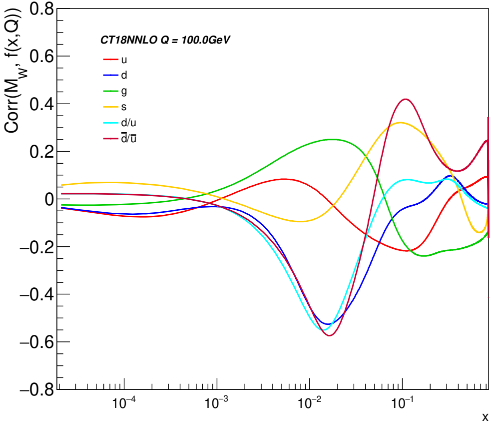
PDF Uncertainties

	m_T		$p_T(\ell)$		$p_T(\nu)$	
PDF Set	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2

- Central value is shift from 80,385 MeV
- Uncertainty is the PDF uncertainty for the given set

- Need to combine to compare to 3.9 MeV from CDF
- Rough estimates say it is consistent with CDF

PDF Correlations



- PDF-induced correlation of M_W and CT18 NNLO error set vs. x at $Q = 100$ GeV
- Region around $x = \frac{M_W}{\sqrt{s}}$ dominated by $\bar{d}/\bar{u}$, d/u and d PDFs