Heavy Flavor Physics

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Outline

- Introduction and Motivation
- How do we make theoretical predictions?
- How well can we calculate?
- Conclusions

Introduction and Motivation

Motivation: high scales

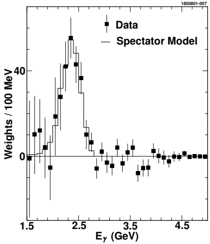
- Flavor physics allows access to new physics at scales beyond reach of current colliders
- E.g. $K-\bar{K}$ mixing, $B-\bar{B}$ mixing probe scales above hundreds of TeV
- See Jure Zupan's Pheno 2019 talk

Motivation: Theoretically Interesting

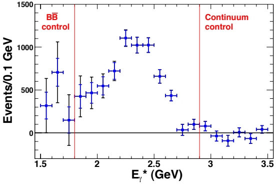
- Theoretically Interesting: test of basic QFT tools
- Factorization theorems
- Operator product expansion
  Example: $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$ OPE is known to
  Perturbative: third order, Non-perturbative: fourth order

- Theoretically Interesting: window to non-perturbative physics

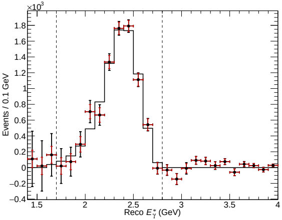
CLEO (2001)



BaBar (2012)



Belle (2016)



- At leading twist the $\bar{B} \to X_s \, \gamma$ photon spectrum is the B-meson pdf

How do we make theoretical predictions?

Effective Hamiltonian

- At energies $\ll m_W, m_Z, m_t$ effective Hamiltonian is known
  For review see [Buras, hep-ph/9806471]
  e.g. $\bar{B} \to X_s \gamma$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1 Q_1^q + C_2 Q_2^q + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

- C_i calculable in perturbation theory
- Q_i operators with non-perturbative matrix elements

$$\begin{aligned} Q_1^q &= (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (q = u, c) \\ Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu} b \end{aligned}$$

Main problem

- Main problem: we know the operators
  but usually cannot calculate the matrix elements

- Strong interaction operators made of quarks and gluons

- Local: e.g. $\bar{q}(0)\cdots q(0)$
- Non-Local: e.g. $\bar{q}(0)\cdots q(tn)$ $\quad n$ light-cone vector

- What kind of objects do we encounter?

- The general matrix element: $\langle f(p_f)|O|i(p_i)\rangle$
  O can be local or non-local; p_i, p_f independent or not
  List options in increased complexity

Non perturbative objects: $\langle f(p_f)|O|i(p_i)\rangle$

1) Decay constant: Local operator, $p_f = 0$

$$\langle 0|\bar{d}\gamma^\mu(1-\gamma_5)u|\pi(p)\rangle = i f_\pi p^\mu$$

Also diagonal local matrix elements: $\langle \bar{B}|\bar{b}\,\vec{\boldsymbol{D}}^{\boldsymbol{2}}\,b|\bar{B}\rangle = 2M_B\mu_\pi^2$

2) Form factor: Local operator, $p_f - p_i = q$

$$\langle N(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^N(q^2)q_\nu\right]u(p_i)$$

Flavor: $\langle D(p_f)|\bar{c}\gamma^\mu b|\bar{B}(p_i)\rangle = f_+(q^2)(p_i+p_f)^\mu + f_-(q^2)(p_i-p_f)^\mu$

3) PDF: Non-local operator, $p_f - p_i = 0$

$$\phi_q(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\xi n\cdot pt}\langle N(p)|\bar{\psi}(0)\,[0,tn]\,\frac{\not{n}}{2}\,\psi(tn)|N(p)\rangle$$

Flavor: $S(\omega) = \frac{1}{2\pi}\frac{1}{2M_B}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle \bar{B}(v)|\bar{b}(0)\,[0,tn]b(tn)|\bar{B}(v)\rangle$

4) Non-local Form factor: Non-local operator, $p_i - p_f = q$

$$\langle K^{(*)}(p_f)|\bar{s}_L(0)\gamma^\rho \cdots \tilde{G}_{\alpha\beta}b_L(tn)|B(p_i)\rangle$$

[Khodjamirian, Mannel, Pivovarov, Wang, JHEP **09**, 089 (2010)]

What to do with Non Perturbative Objects?

- What to do with the Non Perturbative Objects?

1) Calculate using some non perturbative method, e.g. Lattice
2) Extract carefully from experiment
3) Use symmetries
4) When all else fails, model

- For example

1) f_B calculated from Lattice QCD
2) ϕ_q extracted from fits to DIS
3) $SU(3)$ flavor for $B \to PP$
4) Non-perturbative error for $\bar{B} \to X_s\,\gamma$, $|V_{ub}|$

- Since $m_b \sim 5$ GeV $\Rightarrow$ two expansion parameters for b-quark decays

- $\alpha_s(m_b) \sim 0.2$
- $\Lambda_{\mathrm{QCD}}/m_b \sim 0.1$

How well can we calculate?

How well can we calculate?

- Questions:
- What is the current "state of the art"?
- Can the theoretical prediction be improved?
- Will it lead to smaller error bars?

- Examples:
- $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \, \gamma$
- $|V_{cb}|$ and $\bar{B} \to D^{(*)} \, \ell \, \bar{\nu}_\ell$, $R(D^{(*)})$

- See also "Challenges in Semileptonic B Decays" Workshop (April 2022, Barolo, Italy) `https://indico.cern.ch/event/851900/`

- Many more topics in parallel sessions

Flavor topics in parallel sessions

- Monday May 9, 2022
- t-quark mass: Deepak Sathyan, Sagar Airen
- B meson decays to two Baryons: Mark Farino, Tianping Gu
- $\mathcal{A}_{FB}$ for inclusive semileptonic B decays: Florian Herren
- U-spin in c decays: Margarita Gavrilova
- $K \to \mu^+\mu^-$: Mitrajyoti Ghosh
- ϵ_K at NLL EW: Zachary Polonsky
- Dark showers at Belle II: Elias Bernreuther
- Tuesday May 10, 2022
- Flavor Constraints on BSM: Shiyuan Xu
- New Physics in B Decays: Bhubanjyoti Bhattacharya
- BSM Physics in $\bar{B} \to D^* \ell \bar{\nu}_\ell$:Quinn Campagna
- Belle II results: Lucia Kapitnov
- Heavy QCD Axion at Belle II: Vazha Loladze
- New physics is $B \to K\nu\bar{\nu}$: Rusa Mandal

- **Please let me know if I missed your flavor talk title**

$|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

- Semileptonic $b \to c$ transition

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} C_1(\mu) V_{cb} \, \bar{\ell} \gamma_\mu (1 - \gamma^5) \nu_\ell \, \bar{c} \gamma^\mu (1 - \gamma^5) b$$

- Using the optical theorem can calculate $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$ as an OPE

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + \cdots$$

- $c_0 \langle O_0 \rangle$ is a free quark decay. At tree level same as $\mu \to e \, \bar{\nu}_e \nu_\mu$
- c_i^j perturbative in α_s
- $\langle O_i \rangle$ are non perturbative, can be extracted from experiment
  - $\langle O_0 \rangle = \langle \bar{B} | \bar{b} b | \bar{B} \rangle = 1$
  - $\langle O_2^{\text{kin.}} \rangle = \langle \bar{B} | \bar{b} (iD)^2 b | \bar{B} \rangle \Rightarrow \mu_\pi^2$
  - $\langle O_2^{\text{mag.}} \rangle = \langle \bar{B} | \bar{b} \, \sigma_{\mu\nu} G^{\mu\nu} b | \bar{B} \rangle \Rightarrow \mu_G^2$ can be extracted from $M_{B^*} - M_B$

$|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

- Using the optical theorem can calculate $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$ as an OPE

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + c_3^j \frac{\langle O_3^j \rangle}{m_b^3} + c_4^j \frac{\langle O_4^j \rangle}{m_b^4} + c_5^j \frac{\langle O_5^j \rangle}{m_b^5} + \cdots$$

- $1/m_b^0$: One operator
- $1/m_b$: No operators
- $1/m_b^2$: Two operators
  [Blok, Koyrakh, Shifman, Vainshtein PRD **49**, 3356 (1994)]
  [Manoar, Wise PRD **49**, 1310 (1994)]
- $1/m_b^3$: Two operators
  [Gremm, Kapustin, PRD **55**, 6924 (1997)]
- [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]:
- $1/m_b^4$: Nine operators
- $1/m_b^5$: Eighteen operators
- All above: c_i^j at $\mathcal{O}(\alpha_s^0)$. Are these all the possible operators?

Interlude

- Are these all the possible operators?

- Question answered in [Gunawardna, GP JHEP **1707** 137 (2017)]
- List such operators, in principle, to *arbitrary* dimension
- NRQED and NRQCD bilinear ops., in principle, to *arbitrary* dimension
- See also [Kobach, Pal PLB **772** 225 (2017)] using Hilbert series

- Are these all the possible operators? No.
- For $1/m_b^0$, $1/m_b^2$, $1/m_b^3$ these are all the possible operators
- $1/m_b^4$: 9 operators at $\mathcal{O}(\alpha_s^0) \Rightarrow$ 11 operators at $\mathcal{O}(\alpha_s)$ or higher
- $1/m_b^5$: 18 operators at $\mathcal{O}(\alpha_s^0) \Rightarrow$ 25 operators at $\mathcal{O}(\alpha_s)$ or higher

- These are unknown but extremely small
  For example: $\alpha_s\,(\Lambda_{\rm QCD}/m_b)^4 \sim 0.2 \cdot (0.1)^4 \sim 10^{-5}$

Power corrections

- $1/m_b^4, 1/m_b^5$ matrix elements extracted from $\bar{B} \to X_c \ell \bar{\nu}_\ell$
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c$(3 GeV)	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
ρ_{LS}^3	−0.169	0.097	r_6	0.016	0.025
$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044
$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
$\overline{m}_9$	−0.035	0.044	r_{15}	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3\vert V_{cb}\vert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

- "The higher power corrections have a minor effect on $|V_{cb}|$ ...
  There is a -0.25% reduction in $|V_{cb}|$"

State of the art: $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

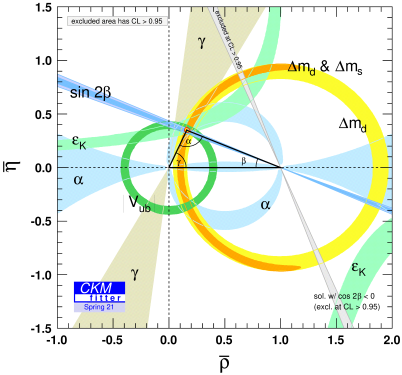
- What is the current "state of the art"? As of 2021

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + c_3^j \frac{\langle O_3^j \rangle}{m_b^3} + c_4^j \frac{\langle O_4^j \rangle}{m_b^4} + c_5^j \frac{\langle O_5^j \rangle}{m_b^5} + \cdots$$

- c_0 known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1), \mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^3)$ for selected observables
- c_2^j known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1)$
- c_3^j known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1)$ for selected observables
- c_4^j known at $\mathcal{O}(\alpha_s^0)$
- c_5^j known at $\mathcal{O}(\alpha_s^0)$

- State of the art Inclusive $|V_{cb}| = 42.16(51) \cdot 10^{-3}$
  [Bordone, Capdevila, Gambino, PLB **822**, 136679 (2021)]
- HFLAV 2021: Exclusive $|V_{cb}| = 38.90(53) \cdot 10^{-3}$
- Exclusive/Inclusive $|V_{cb}|$ puzzle remains
- Can the theoretical prediction be improved?
  Yes, c_3^j at $\mathcal{O}(\alpha_s^1)$ fully differential
- Will it lead to smaller error bars? Probably

$|V_{ub}|$ and $\bar{B} \to X_u\, \ell\, \bar{\nu}_\ell$



- $|V_{ub}|$ plays a role in the unitarity triangle fit
  Like $|V_{cb}|$, $|V_{ub}|$ inclusive is larger than $|V_{ub}|$ exclusive
- PDG August 2021 review

- Inclusive $|V_{ub}| = (4.13 \pm 0.12^{+0.13}_{-0.14} \pm 0.18) \cdot 10^{-3}$
- Exclusive $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \cdot 10^{-3}$

$|V_{ub}|$ and $\bar{B} \to X_u \ell \bar{\nu}_\ell$: Framework

- **If** we could measure total $\Gamma(\bar{B} \to X_u l \bar{\nu})$ we could use a **local** OPE

$$d\Gamma \sim \sum_{i,j} c_i^j \frac{\langle O_i^j \rangle}{m_b^i}$$

c_i^j perturbative, $\langle O_i^j \rangle$ non-perturbative **numbers**

- Since $\Gamma(\bar{B} \to X_c \ell \bar{\nu}_\ell) \gg (\bar{B} \to X_u \ell \bar{\nu}_\ell)$ total rate **cannot** be measured
Need to cut the charm background: e.g. $M_X^2 < M_D^2 \sim m_b \Lambda_{\text{QCD}}$

- Not inclusive enough for local OPE, but non-local OPE still possible

$M_X^2 \sim m_b^2$	local OPE	("OPE region")
$M_X^2 \sim m_b \Lambda_{\text{QCD}}$	Non local OPE	("end point region")
$M_X^2 \sim \Lambda_{\text{QCD}}^2$	No inclusive description	("resonance region")

$|V_{ub}|$ and $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$: Framework

Need to cut the charm background: e.g. $M_X^2 < M_D^2 \sim m_b \Lambda_{\rm QCD}$

- Not inclusive enough for local OPE, but non-local OPE still possible

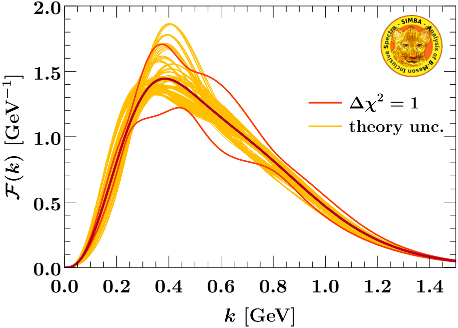
$$d\Gamma \sim H \cdot J \otimes S + \mathcal{O}\left(\frac{1}{m_b}\right)$$

- Can factorize perturbative coefficient into hard H and jet J functions
- S is a non-perturbative "shape function" (B-meson PDF)

- At leading power in $\Lambda_{\rm QCD}/m_b$, S is $\bar{B} \to X_s \, \gamma$ photon spectrum

Recent work: SIMBA Collaboration

- At leading power in $\Lambda_{\mathrm{QCD}}/m_b$, S is $\bar{B} \to X_s\,\gamma$ photon spectrum
- Recent extraction by the SIMBA (Analysis of B-Meson Inclusive Spectra) Collaboration [Bernlochner, Lacker, Ligeti, Stewart, F. Tackmann, K. Tackmann, PRL **127**, 102001 (2021)]



$|V_{ub}|$ and $\bar{B} \to X_u \ell \bar{\nu}_\ell$: Framework

- At subleading power in $\Lambda_{\rm QCD}/m_b$

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \cdots$$

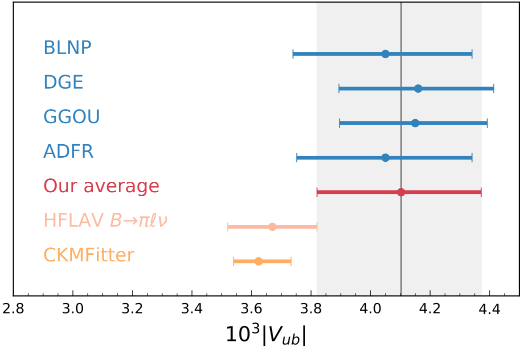
- Several subleading shape functions (SSF) appear (s_i) ("higher twist")
- Different linear combinations for $\bar{B} \to X_u \ell \bar{\nu}_\ell$ and $\bar{B} \to X_s \gamma$
- $\bar{B} \to X_s \gamma$ has unique SSF ("resolved photon contributions")

- Shape functions moments are related to universal matrix elements: E.g. leading shape function: 1^{st} moment $\leftrightarrow m_b$, 2^{nd} moment $\leftrightarrow \mu_\pi^2$

- Different theoretical frameworks for $|V_{ub}|$ extractions:
- Use similar perturbative inputs, currently $\mathcal{O}(\alpha_s)$
- Differ in how they extract (or model) S
- Differ in how they treat power corrections

Recent work: Inclusive $|V_{ub}|$ from Belle data

- Current extractions used
- BLNP [Lange, Neubert, GP, PRD **72**, 073006, (2005)]
- DGE [Andersen, Gardi, JHEP **01**, 097, (2006)]
- GGOU [Gambino, Giordano, Ossola, Uraltsev, JHEP **10**, 058, (2007)]
- ADFR [Aglietti, Di Lodovico, Ferrera, Ricciardi, EPJC **59**, 831, (2009)]
- Recent work: Inclusive $|V_{ub}|$ from Belle data
  [L. Cao *et al.* [Belle], PRD **104**, 012008 (2021)]



Recent work: Inclusive $|V_{ub}|$ from Belle data

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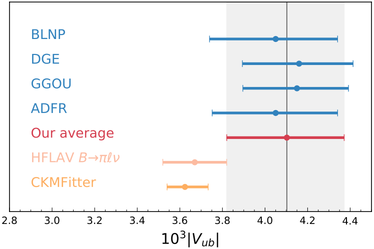
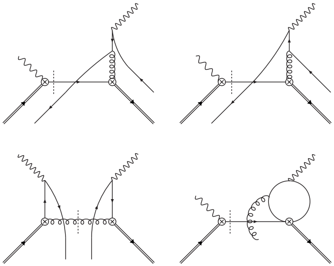


- State of the art: theoretical framework developed before 2010
- Can the theoretical prediction be improved?
- Yes, many NNLO calculations are known:
- H, J at $\mathcal{O}(\alpha_s^2)$, j_i/m_b at $\mathcal{O}(\alpha_s)$, resolved photon contributions
- Not fully combined yet
  [Gunawardana, Lange, Mannel, Olschewsky, Vos, GP, *to appear*]
- Will it lead to smaller error bars? Not necessarily

$\bar{B} \to X_s\,\gamma$

- $\bar{B} \to X_s\,\gamma$ BSM probe. PDG 2021: $\text{Br} = (3.49 \pm 0.19) \cdot 10^{-4}$
- 2015 SM prediction of branching ratio $(3.36 \pm 0.23) \cdot 10^{-4}$
  [M. Misiak *et al.*, PRL **114**, 221801 (2015)]
- Largest uncertainty $\sim 5\%$ is non-perturbative from "resolved photons"
  At Λ_{QCD}/m_b [Benzke, Lee, Neubert, GP JHEP **1008**, 099 (2010)]:



  Top line $Q_{7\gamma} - Q_{8g}$, Bottom left: $Q_{8g} - Q_{8g}$, Bottom right: $Q_1 - Q_{7\gamma}$
- SM CP asymmetry dominated by $Q_1^q - Q_{7\gamma}$: $-0.6\% < \mathcal{A}^{\text{SM}}_{X_s\gamma} < 2.8\%$
  [Benzke, Lee, Neubert, GP PRL **106**, 141801 (2011)]
  PDG 2021: $\mathcal{A}_{X_s\gamma} = 1.5\% \pm 1.1\%$. Can we improve this?

$\bar{B} \to X_s \gamma$

- At $\Lambda_{\mathrm{QCD}}/m_b$: resolved photons from $Q_{7\gamma} - Q_{8g}$, $Q_{8g} - Q_{8g}$, $Q_1 - Q_{7\gamma}$
  - $Q_{7\gamma} - Q_{8g}$ constrained by isospin asymmetry $\bar{B}^{0/\pm} \to X_s \gamma$ uncertainty reduced by a Belle measurement [Watanuki *et al.* [Belle Collaboration] PRD **99**, 032012 (2019)]
  - $Q_{8g} - Q_{8g}$ is hard to improve
  - $Q_1 - Q_{7\gamma}$ depends on a non-perturbative function $g_{17}(\omega, \omega_1)$ whose moments can be extracted from $\bar{B} \to X_c \ell \bar{\nu}_\ell$ OPE
- 2010 analysis only had 2 non-zero moments [Benzke, Lee, Neubert, GP, JHEP **1008**, 099 (2010)]

$$\langle \omega^0 \omega_1^0 g_{17} \rangle = 0.237 \pm 0.040 \text{ GeV}^2, \quad \langle \omega^1 \omega_1^0 g_{17} \rangle = 0.056 \pm 0.032 \text{ GeV}^3$$

- 2019 analysis added 6 non-zero moments [Gunawardna, GP JHEP **11** 141 (2019)]

$$\langle \omega^0 \omega_1^2 g_{17} \rangle = 0.15 \pm 0.12 \text{ GeV}^4, \quad \langle \omega^2 \omega_1^0 g_{17} \rangle = 0.015 \pm 0.021 \text{ GeV}^4$$
$$\langle \omega^3 \omega_1^0 g_{17} \rangle = 0.008 \pm 0.011 \text{ GeV}^5, \quad \langle \omega^1 \omega_1^1 g_{17} \rangle = 0.073 \pm 0.059 \text{ GeV}^4$$
$$\langle \omega^2 \omega_1^1 g_{17} \rangle = -0.034 \pm 0.016 \text{ GeV}^5, \quad \langle \omega^1 \omega_1^2 g_{17} \rangle = 0.027 \pm 0.014 \text{ GeV}^5.$$

Data from [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

$\bar{B} \to X_s \gamma$

- Using moments model $Q_1 - Q_{7\gamma}$ resolved photon
- New estimate of uncertainty: Total rate $\downarrow$ 50%, CP asymmetry $\uparrow$ 33%

  [Gunawardna, GP JHEP **11** 141 (2019)]

  See Ayesh Gunawardna Pheno 2019 talk
- 2015 SM prediction of branching ratio $(3.36 \pm 0.23) \cdot 10^{-4}$

  [M. Misiak *et al.*, PRL **114**, 221801 (2015)]
- 2020 SM prediction of branching ratio $(3.40 \pm 0.17) \cdot 10^{-4}$

  [Misiak, Rehman, Steinhauser, JHEP **06**, 175 (2020)]
- Using different models, including *some* $\Lambda_{\text{QCD}}^2/m_b^2$ corrections and larger m_c range, a smaller reduction was found in

  [Benzke, Hurth PRD **102** 114024 (2020)]
- Can the theoretical prediction be improved?

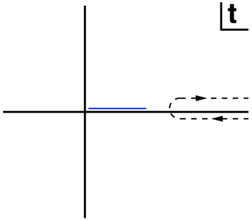
  Yes, m_c can be better controlled by an NLO analysis of $Q_1 - Q_{7\gamma}$
- Will it lead to smaller error bars? Not necessarily

$|V_{cb}|$ and $\bar{B} \to D^{(*)}\,\ell\,\bar{\nu}_\ell$, $R(D^{(*)})$: Form factors

- For exclusive decays, e.g., $\bar{B} \to D^{(*)}\,\ell\,\bar{\nu}_\ell$, we need form factors

$$\langle D(p_f)|\bar{c}\gamma^\mu b|\bar{B}(p_i)\rangle = f_+(q^2)(p_i+p_f)^\mu + f_-(q^2)(p_i-p_f)^\mu$$

- Unknown functional form, but known analytic structure in $t=q^2$:



[Richard J. Hill, FPCP 2006 proceedings (hep-ph/0606023)]

- For H, L **mesons**:

- $H \to L$ semileptonic data: $0 \le t \le (m_H - m_L)^2$ (blue line)
- Singularity starts at $\bar{H}L$ threshold $t = (m_H + m_L)^2$ (dashed curves)

Outside the cut the form factor is analytic

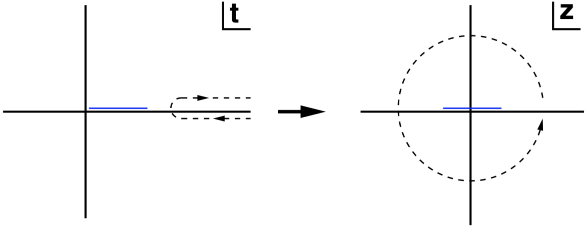
$|V_{cb}|$ and $\bar{B} \to D^{(*)}\,\ell\,\bar{\nu}_\ell$, $R(D^{(*)})$: z expansion

- Form factor analytic outside a cut $t \in [t_{\rm cut}, \infty]$

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $z(t_0, t_{\rm cut}, t_0) = 0$

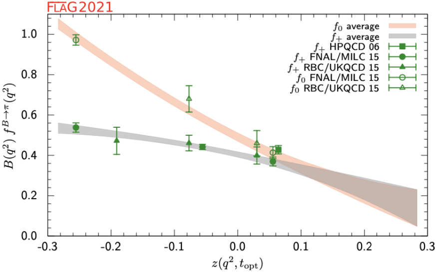


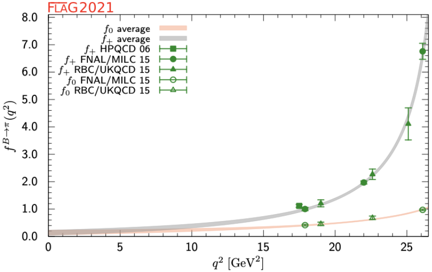
- Expand form factor in a Taylor series in z: $f(q^2) = \sum_{k=0}^{\infty} a_k\,[z(q^2)]^k$

$|V_{cb}|$ and $\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell$, $R(D^{(*)})$: z expansion

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}$$

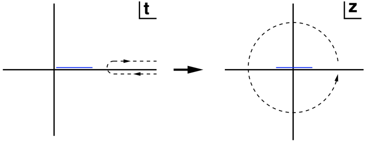




[Y. Aoki *et al.*,[FLAG Review 2021], arXiv:2111.09849 (hep-lat)

- Data in z has less "structure": can extract only few coefficients

$|V_{cb}|$ and $\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell$, $R(D^{(*)})$: Unitarity bound



- Expand form factor in a Taylor series in z: $f(q^2) = \sum_{k=0}^{\infty} a_k \, [z(q^2)]^k$
- For H, L **mesons**:
  - $H \to L$ semileptonic data: $0 \le t \le (m_H - m_L)^2$ (blue line)
  - Singularity starts at $\bar{H}L$ threshold $t = (m_H + m_L)^2$ (dashed curves)
  - Crossing symmetry and unitarity: meson unitarity bound $\sum_{k=0}^{\infty} |a_k|^2 \le 1$, e.g. [Boyd, Grinstein, Lebed, NPB **461**, 493 (1996)]

- Bounds ensure form factor extraction is model-independent

- For H, L **baryons**:
  - $H \to L$ semileptonic data: $0 \le t \le (m_H - m_L)^2$ (blue line)
  - Singularity starts at meson threshold, no unitarity bound [Hill, GP PRD **82**, 113005 (2010)]

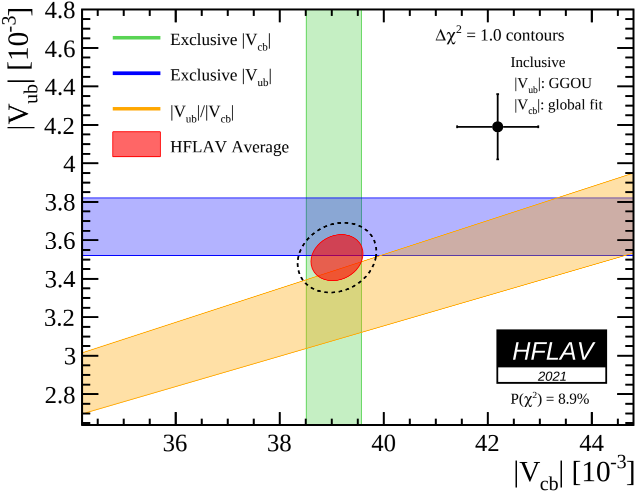
$|V_{cb}|$ and $\bar{B} \to D^{(*)}\,\ell\,\bar{\nu}_\ell$, $R(D^{(*)})$: Unitarity bound

- State of the art: unitarity bounds not imposed for Lattice extraction of $\bar{B} \to D^{(*)}\,\ell\,\bar{\nu}_\ell$ form factors, e.g. [A. Bazavov *et al.* [Fermilab Lattice and MILC], arXiv:2105.14019 (hep-lat)]

	Lattice QCD	Lattice + BaBar	Lattice + Belle	Lattice + both
a_0	0.0330(12)	0.0331(12)	0.0325(10)	0.0320(10)
a_1	−0.155(55)	−0.089(40)	−0.160(44)	−0.148(31)
a_2	−0.12(98)	−0.16(21)	−0.70(94)	−0.60(22)
b_0	0.01229(23)	0.01229(22)	0.01238(22)	0.01246(22)
b_1	−0.003(12)	0.0123(69)	0.015(10)	0.0038(46)
b_2	0.07(53)	0.36(17)	−0.30(24)	0.02(12)
c_1	−0.0058(25)	−0.0008(11)	0.0010(17)	0.00008(94)
c_2	−0.013(91)	0.054(46)	0.035(57)	0.080(36)
c_3		−0.12(83)	−0.34(76)	−1.11(56)
d_0	0.0509(15)	0.0516(15)	0.0521(15)	0.0526(14)
d_1	−0.327(67)	−0.197(50)	−0.179(49)	−0.194(43)
d_2	−0.03(96)	0.19(92)	−0.01(90)	−0.004(898)
χ^2/dof	0.64/3	9.28/5	111/81	126/84
$\sum_i^N a_i^2$	0.04(24)	0.035(71)	0.5(1.3)	0.39(27)
$\sum_i^N (b_i^2 + c_i^2)$	0.005(70)	0.15(18)	0.21(48)	1.2(1.3)
$\sum_i^N d_i^2$	0.110(61)	0.08(35)	0.035(25)	0.040(15)
$\lvert V_{cb}\rvert \times 10^3$		39.66(91)	38.18(82)	38.40(74)

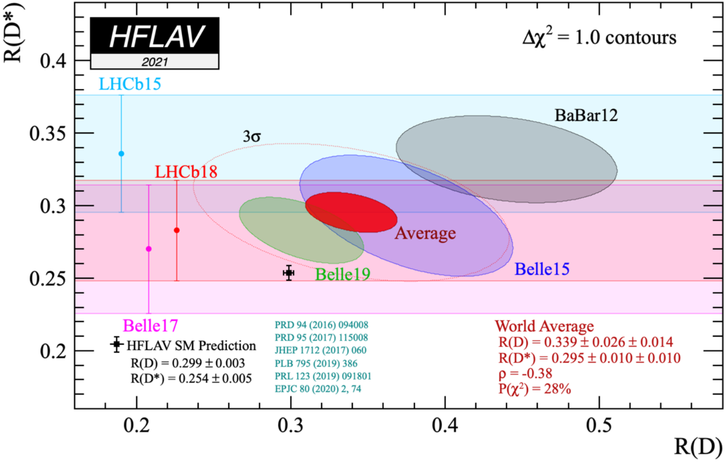
- Unitarity bounds not imposed but respected within errors
- "Unitary-required priors" on coefficients are used
- Imposing a unitarity bound does not change $|V_{cb}|$ and $R(D^{(*)})$

$|V_{ub}|$ & $|V_{cb}|$ state of the art



$R(D)$ and $R(D^*)$: state of the art

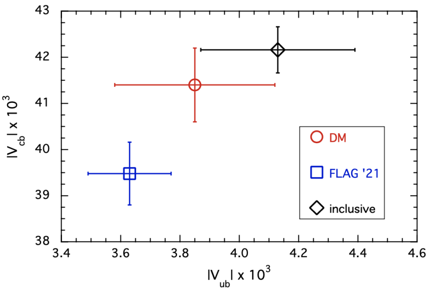
$$R(D^{(*)}) \equiv \frac{\mathrm{Br}(\bar{B} \to D^{(*)}\, \tau\, \bar{\nu}_\tau)}{\mathrm{Br}(\bar{B} \to D^{(*)}\, \ell\, \bar{\nu}_\ell)}, \quad \ell = e, \mu$$

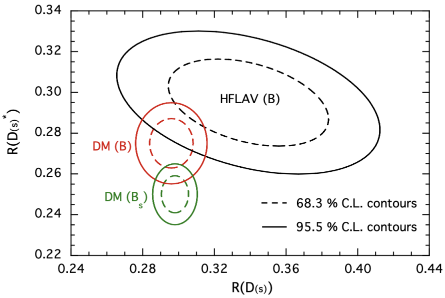


$|V_{cb}|$ and $R(D^{(*)})$: Dispersion Matrix approach

- Silvano Simula talk "Challenges in Semileptonic B Decays" Workshop (April 2022, Barolo, Italy) `https://indico.cern.ch/event/851900/`

- Use Dispersion Matrix (DM) approach [Martinelli, Simula and Vittorio, PRD **105**, 034503 (2022), arXiv:2109.15248(hep-ph)], [Martinelli, Naviglio, Simula, Vittorio, arXiv:2204.05925 (hep-ph)]





Conclusions

Conclusions

- Flavor physics probes very high scales and advanced theoretical tools

- This decade will be very exciting with, e.g., LHCb and Belle II data

- Puzzles and tensions motivate further theoretical work

- A big challenge is controlling non-perturbative effects

- Discussed "state of the art" of
  - $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$
  - $|V_{ub}|$ and $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$
  - $\bar{B} \to X_s \, \gamma$
  - $|V_{cb}|$ and $\bar{B} \to D^{(*)} \, \ell \, \bar{\nu}_\ell$, $R(D^{(*)})$

- Future: improve theory, but not necessarily smaller error bars

- More work to do!

Pheno

Precision measurement of $|V_{ub}|$

Gil Paz

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Newman Laboratory for Elementary-Particle Physics,

Cornell University

In Collaboration with

Stefan W. Bosch, Björn O. Lange, Matthias Neubert

(hep-ph/0402094, hep-ph/0403223)