Heavy Flavor Physics

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Outline

- Introduction and Motivation
- How do we make theoretical predictions?
- How well can we calculate?
- Conclusions

Introduction and Motivation

Motivation: high scales

- Flavor physics allows access to new physics at scales beyond reach of current colliders
- E.g. $K-\bar{K}$ mixing, $B-\bar{B}$ mixing probe scales above hundreds of TeV
- See Jure Zupan's Pheno 2019 talk

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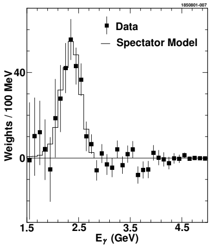
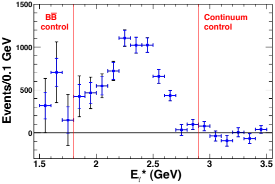
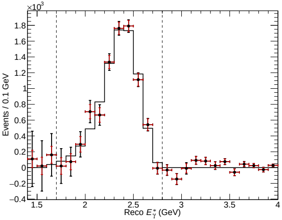
CLEO (2001)



BaBar (2012)



Belle (2016)



- At leading twist the $\bar{B} \to X_s \, \gamma$ photon spectrum is the B-meson pdf

How do we make theoretical predictions?

Effective Hamiltonian

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  For review see [Buras, hep-ph/9806471]

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$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1 Q_1^q + C_2 Q_2^q + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

- C_i calculable in perturbation theory
- Q_i operators with non-perturbative matrix elements

$$\begin{aligned} Q_1^q &= (\bar{q} b)_{V-A} (\bar{s} q)_{V-A} \quad (q = u, c) \\ Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b \end{aligned}$$

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- What kind of objects do we encounter?

- The general matrix element: $\langle f(p_f)|O|i(p_i)\rangle$
  O can be local or non-local; p_i, p_f independent or not
  List options in increased complexity

Non perturbative objects: $\langle f(p_f)|O|i(p_i)\rangle$

1) Decay constant: Local operator, $p_f = 0$

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4) Non-local Form factor: Non-local operator, $p_i - p_f = q$

$$\langle K^{(*)}(p_f)|\bar{s}_L(0)\gamma^\rho \cdots \tilde{G}_{\alpha\beta}b_L(tn)|B(p_i)\rangle$$

[Khodjamirian, Mannel, Pivovarov, Wang, JHEP **09**, 089 (2010)]

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- $\alpha_s(m_b) \sim 0.2$
- $\Lambda_{\rm QCD}/m_b \sim 0.1$

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- $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \, \gamma$
- $|V_{cb}|$ and $\bar{B} \to D^{(*)} \, \ell \, \bar{\nu}_\ell$, $R(D^{(*)})$

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- Many more topics in parallel sessions

Flavor topics in parallel sessions

- Monday May 9, 2022
- t-quark mass: Deepak Sathyan, Sagar Airen
- B meson decays to two Baryons: Mark Farino, Tianping Gu
- $\mathcal{A}_{FB}$ for inclusive semileptonic B decays: Florian Herren
- U-spin in c decays: Margarita Gavrilova
- $K \to \mu^+\mu^-$: Mitrajyoti Ghosh
- ϵ_K at NLL EW: Zachary Polonsky
- Dark showers at Belle II: Elias Bernreuther
- Tuesday May 10, 2022
- Flavor Constraints on BSM: Shiyuan Xu
- New Physics in B Decays: Bhubanjyoti Bhattacharya
- BSM Physics in $\bar{B} \to D^* \ell \bar{\nu}_\ell$:Quinn Campagna
- Belle II results: Lucia Kapitnov
- Heavy QCD Axion at Belle II: Vazha Loladze
- New physics is $B \to K\nu\bar{\nu}$: Rusa Mandal

- **Please let me know if I missed your flavor talk title**

$|V_{cb}|$ and $\bar{B} \to X_c\, \ell\, \bar{\nu}_\ell$

- Semileptonic $b \to c$ transition

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} C_1(\mu) V_{cb}\, \bar{\ell}\gamma_\mu(1-\gamma^5)\nu_\ell\, \bar{c}\gamma^\mu(1-\gamma^5)b$$

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- Using the optical theorem can calculate $\bar{B} \to X_c\, \ell\, \bar{\nu}_\ell$ as an OPE

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + \cdots$$

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- $\langle O_i \rangle$ are non perturbative, can be extracted from experiment
  - $\langle O_0 \rangle = \langle \bar{B} | \bar{b} b | \bar{B} \rangle = 1$
  - $\langle O_2^{\text{kin.}} \rangle = \langle \bar{B} | \bar{b}(iD)^2 b | \bar{B} \rangle \Rightarrow \mu_\pi^2$
  - $\langle O_2^{\text{mag.}} \rangle = \langle \bar{B} | \bar{b} \, \sigma_{\mu\nu} G^{\mu\nu} b | \bar{B} \rangle \Rightarrow \mu_G^2$ can be extracted from $M_{B^*} - M_B$

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- $1/m_b^2$: Two operators

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Interlude

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- These are unknown but extremely small
  For example: $\alpha_s \left(\Lambda_{\text{QCD}}/m_b\right)^4 \sim 0.2 \cdot (0.1)^4 \sim 10^{-5}$

Power corrections

- $1/m_b^4, 1/m_b^5$ matrix elements extracted from $\bar{B} \to X_c \ell \bar{\nu}_\ell$
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

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Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c(3\ \text{GeV})$	0.987	0.013	r_2	−0.063	0.037
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$\mathbf{10^3 \lvert V_{cb} \rvert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

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- "The higher power corrections have a minor effect on $|V_{cb}|$ ...
There is a -0.25% reduction in $|V_{cb}|$"

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- State of the art Inclusive $|V_{cb}| = 42.16(51) \cdot 10^{-3}$
  [Bordone, Capdevila, Gambino, PLB **822**, 136679 (2021)]

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- Can the theoretical prediction be improved?

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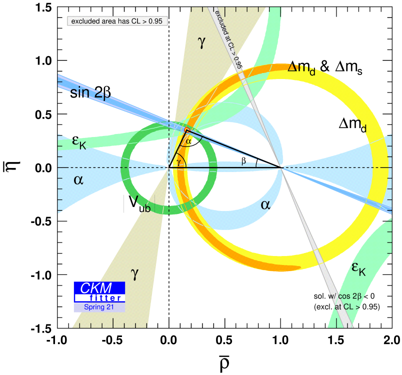

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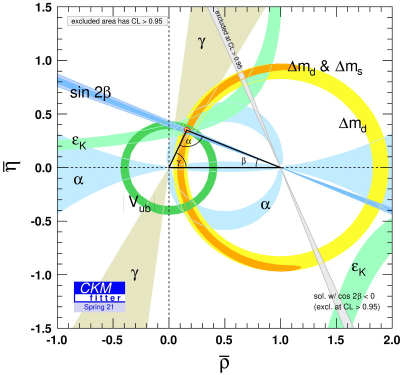

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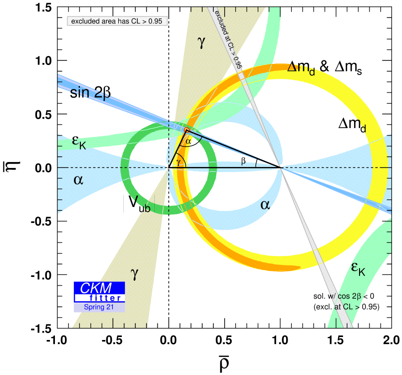

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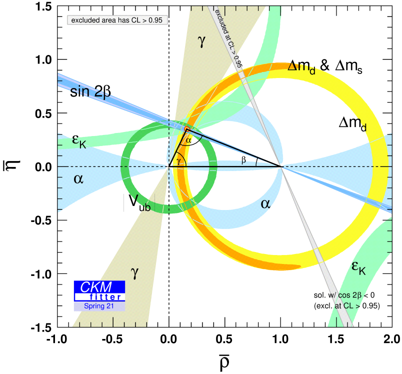

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- **If** we could measure total $\Gamma(\bar{B} \to X_u l \bar{\nu})$ we could use a **local** OPE

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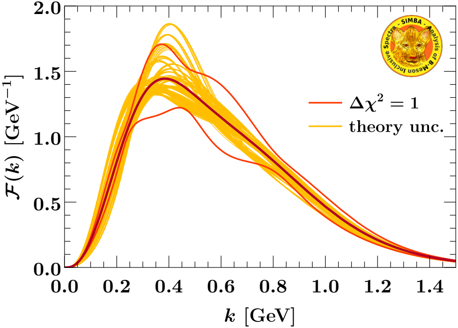

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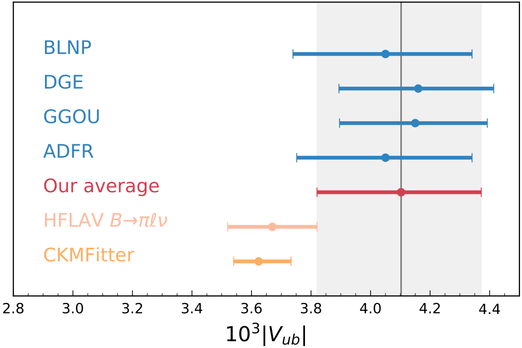

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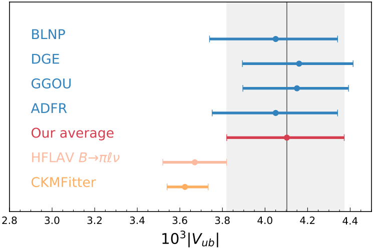

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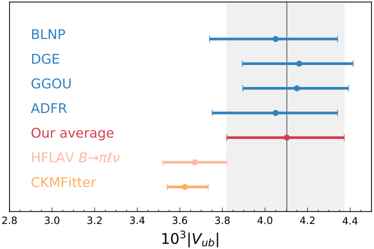

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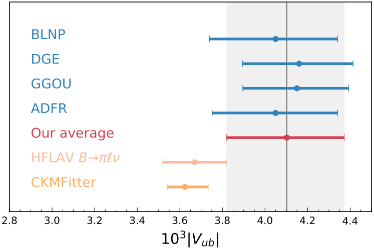

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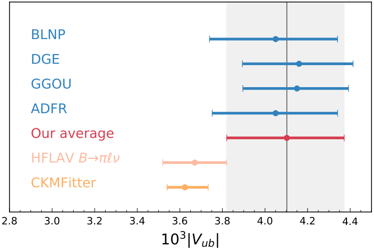

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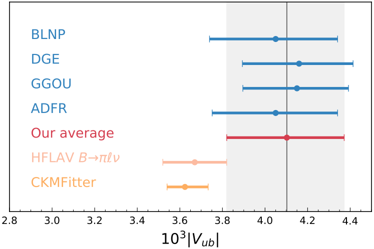

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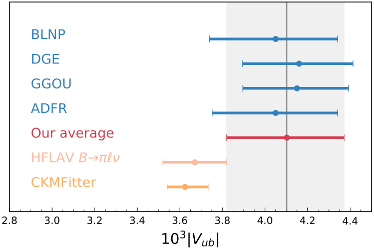

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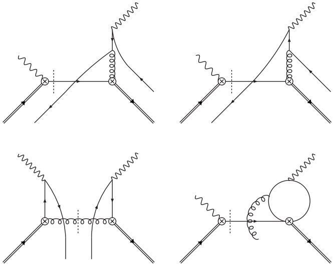

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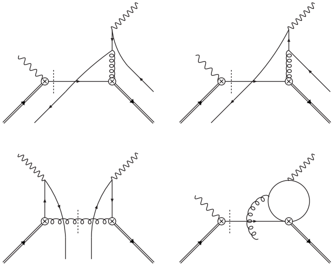

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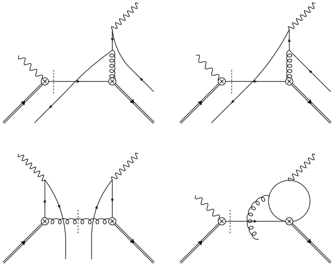

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$$\langle \omega^2 \omega_1^1 g_{17} \rangle = -0.034 \pm 0.016 \text{ GeV}^5, \quad \langle \omega^1 \omega_1^2 g_{17} \rangle = 0.027 \pm 0.014 \text{ GeV}^5.$$

Data from [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

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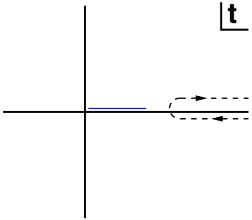

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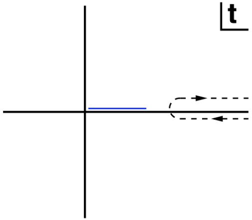

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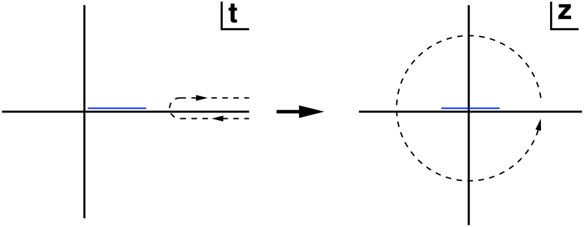
where $z(t_0, t_{\text{cut}}, t_0) = 0$



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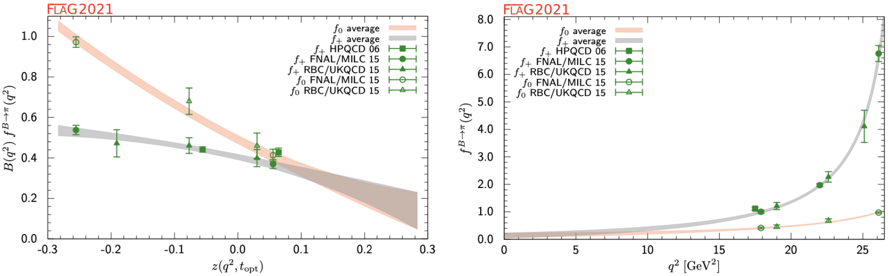

[Y. Aoki *et al.*,[FLAG Review 2021], arXiv:2111.09849 (hep-lat)

- Data in z has less "structure": can extract only few coefficients

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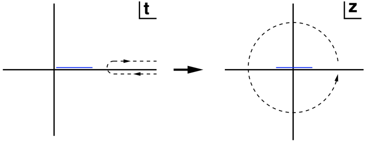

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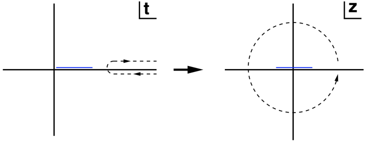

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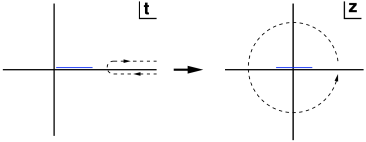

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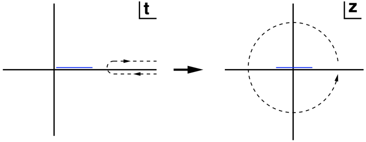

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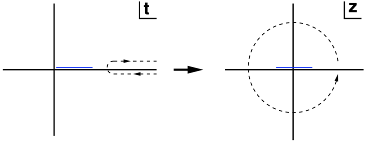

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- For H, L **baryons**:
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- Singularity starts at meson threshold, no unitarity bound [Hill, GP PRD **82**, 113005 (2010)]

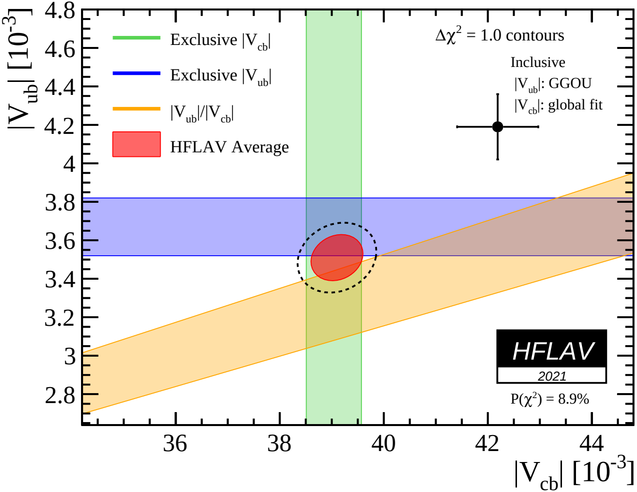
$|V_{cb}|$ and $\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell$, $R(D^{(*)})$: Unitarity bound

- State of the art: unitarity bounds not imposed for Lattice extraction of $\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell$ form factors, e.g. [A. Bazavov *et al.* [Fermilab Lattice and MILC], arXiv:2105.14019 (hep-lat)]

	Lattice QCD	Lattice + BaBar	Lattice + Belle	Lattice + both
a_0	0.0330(12)	0.0331(12)	0.0325(10)	0.0320(10)
a_1	−0.155(55)	−0.089(40)	−0.160(44)	−0.148(31)
a_2	−0.12(98)	−0.16(21)	−0.70(94)	−0.60(22)
b_0	0.01229(23)	0.01229(22)	0.01238(22)	0.01246(22)
b_1	−0.003(12)	0.0123(69)	0.015(10)	0.0038(46)
b_2	0.07(53)	0.36(17)	−0.30(24)	0.02(12)
c_1	−0.0058(25)	−0.0008(11)	0.0010(17)	0.00008(94)
c_2	−0.013(91)	0.054(46)	0.035(57)	0.080(36)
c_3		−0.12(83)	−0.34(76)	−1.11(56)
d_0	0.0509(15)	0.0516(15)	0.0521(15)	0.0526(14)
d_1	−0.327(67)	−0.197(50)	−0.179(49)	−0.194(43)
d_2	−0.03(96)	0.19(92)	−0.01(90)	−0.004(898)
χ^2/dof	0.64/3	9.28/5	111/81	126/84
$\sum_i^N a_i^2$	0.04(24)	0.035(71)	0.5(1.3)	0.39(27)
$\sum_i^N (b_i^2 + c_i^2)$	0.005(70)	0.15(18)	0.21(48)	1.2(1.3)
$\sum_i^N d_i^2$	0.110(61)	0.08(35)	0.035(25)	0.040(15)
$\lvert V_{cb}\rvert \times 10^3$		39.66(91)	38.18(82)	38.40(74)

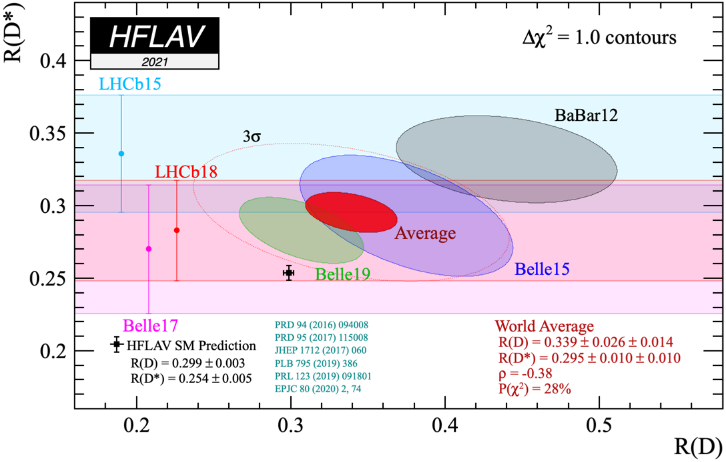
- Unitarity bounds not imposed but respected within errors
- "Unitary-required priors" on coefficients are used
- Imposing a unitarity bound does not change $|V_{cb}|$ and $R(D^{(*)})$

$|V_{ub}|$ & $|V_{cb}|$ state of the art



$R(D)$ and $R(D^*)$: state of the art

$$R(D^{(*)}) \equiv \frac{\text{Br}(\bar{B} \to D^{(*)}\, \tau\, \bar{\nu}_\tau)}{\text{Br}(\bar{B} \to D^{(*)}\, \ell\, \bar{\nu}_\ell)}, \quad \ell = e, \mu$$

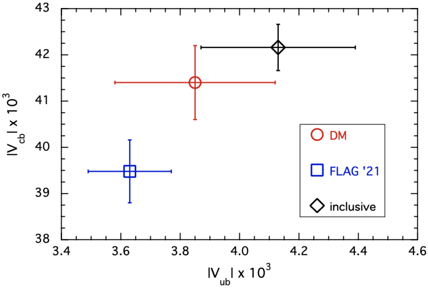


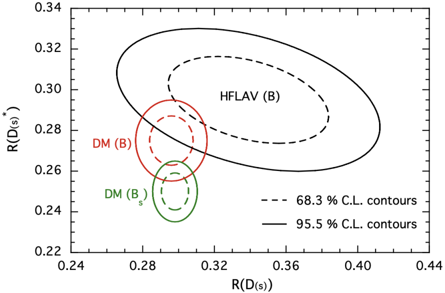
$|V_{cb}|$ and $R(D^{(*)})$: Dispersion Matrix approach

- Silvano Simula talk "Challenges in Semileptonic B Decays" Workshop (April 2022, Barolo, Italy) `https://indico.cern.ch/event/851900/`

$|V_{cb}|$ and $R(D^{(*)})$: Dispersion Matrix approach

- Silvano Simula talk "Challenges in Semileptonic B Decays" Workshop (April 2022, Barolo, Italy) `https://indico.cern.ch/event/851900/`
- Use Dispersion Matrix (DM) approach [Martinelli, Simula and Vittorio, PRD **105**, 034503 (2022), arXiv:2109.15248(hep-ph)], [Martinelli, Naviglio, Simula, Vittorio, arXiv:2204.05925 (hep-ph)]





Conclusions

Conclusions

- Flavor physics probes very high scales and advanced theoretical tools
- This decade will be very exciting with, e.g., LHCb and Belle II data
- Puzzles and tensions motivate further theoretical work
- A big challenge is controlling non-perturbative effects
- Discussed "state of the art" of

- $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$
- $|V_{ub}|$ and $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$
- $\bar{B} \to X_s \, \gamma$
- $|V_{cb}|$ and $\bar{B} \to D^{(*)} \, \ell \, \bar{\nu}_\ell$, $R(D^{(*)})$

- Future: improve theory, but not necessarily smaller error bars
- More work to do!

Pheno

Precision measurement of $|V_{ub}|$

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Newman Laboratory for Elementary-Particle Physics,

Cornell University

In Collaboration with

Stefan W. Bosch, Björn O. Lange, Matthias Neubert

(hep-ph/0402094, hep-ph/0403223)

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