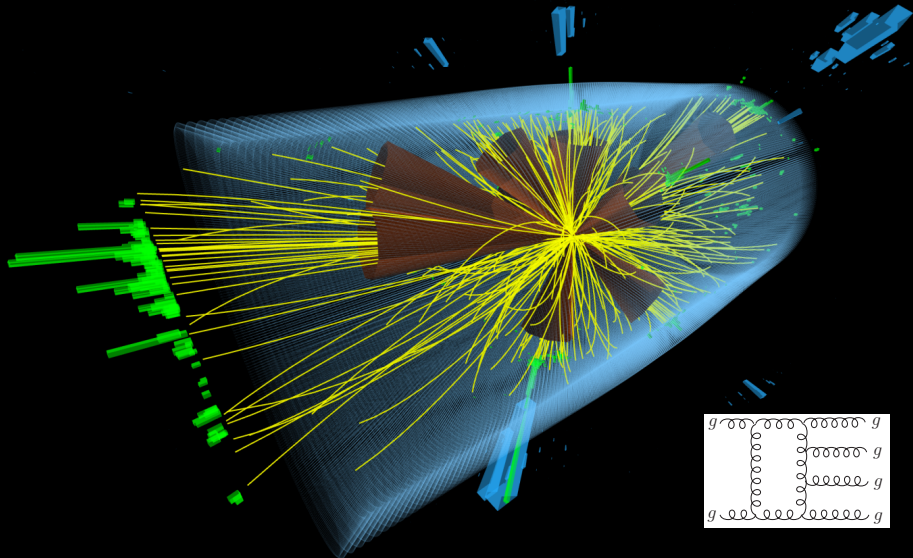


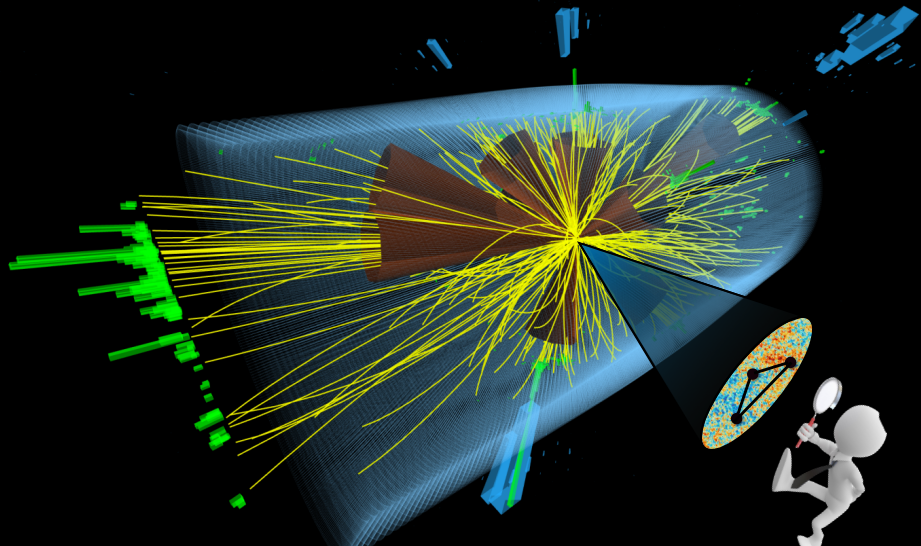
Conformal Colliders Meet the LHC

Ian Moutl
Yale



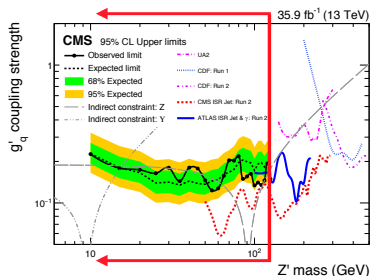
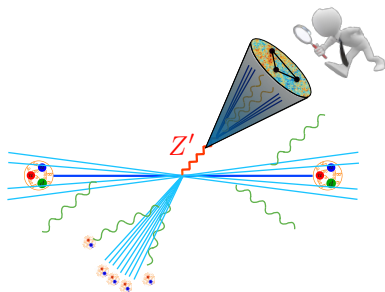


Jet Substructure!



Jet Substructure: Searches

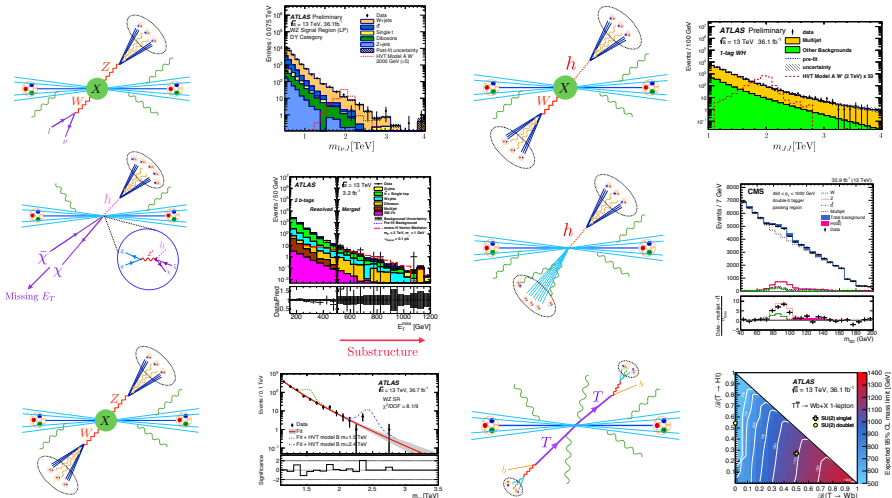
- **Jet Substructure** uses the internal structure of jets to provide **qualitatively new** ways to study physics at the LHC.



- Its introduction in 2008 by **Butterworth, Davison, Rubin and Salam**, along with anti- k_T by **Cacciari, Soyez, Salam** reinvigorated the study of jets in QCD.

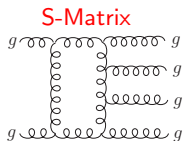
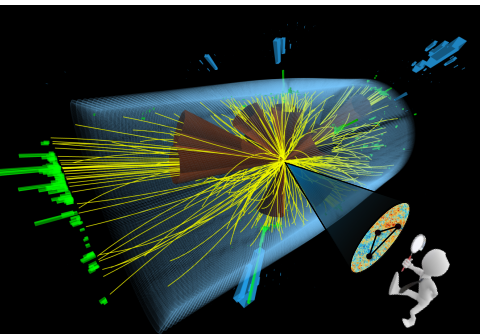
Jet Substructure: Searches

- Large impact on searches, and synergies with Machine Learning.



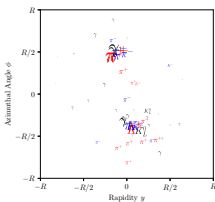
Changing the Perspective

- This changes the problem from studying the production of jets (**S-matrix elements**) to studying the **statistical properties of energy flux within jets**.

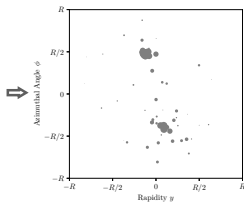


Energy Flux

Full event is a set of particles having momentum and charge/flavor



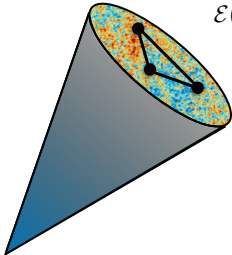
The **energy** flow is unpixelized and ignores charge/flavor information



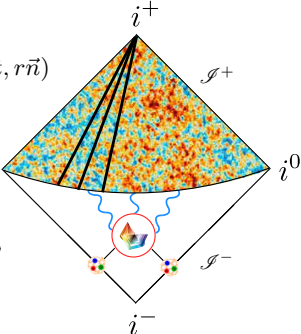
- Requires the development of a **new set of theoretical tools and new ways of thinking about jets**.

Insights from Conformal Field Theory

- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators. [Hofman, Maldacena]



A 3D diagram of a light-cone. The top circular face is filled with a noisy, colorful pattern representing energy flow. Three black lines connect the center of this face to the bottom vertex of the cone, representing light-rays.

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt n^i T_{0i}(t, r\vec{n})$$


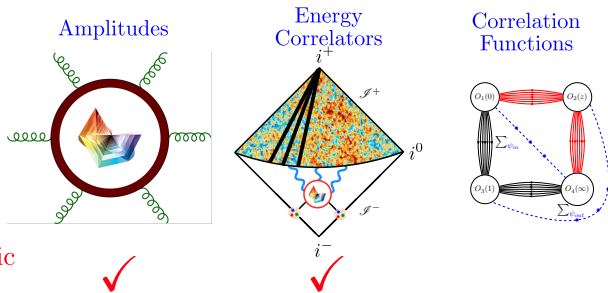
A Penrose diagram of a spacetime region. The top vertex is labeled i^+ , the bottom vertex is i^- , and the right boundary is i^0 . The upper and lower regions are labeled \mathcal{I}^+ and \mathcal{I}^- respectively. A small circular region in the lower part of the diagram contains a colorful pattern, with blue wavy lines extending upwards towards the top vertex.

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

- From the perspective of QFT, jet substructure is the study of correlation functions of energy flow operators.

Energy Correlators

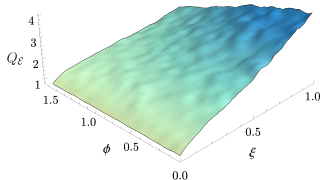
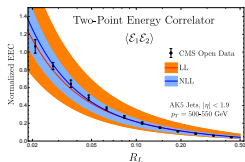
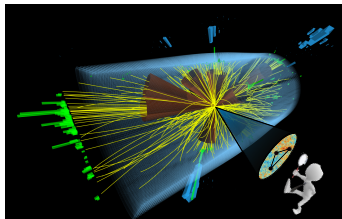
- Correlation functions of energy flow operators take an interesting intermediate position between amplitudes and correlation functions.



- Despite their physical importance, much less explored.

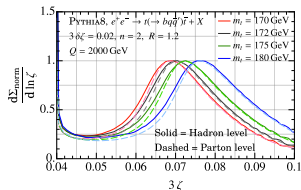
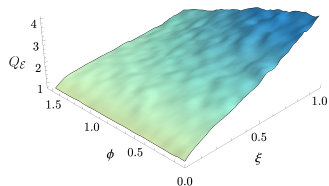
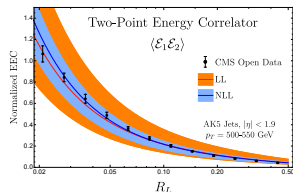
Conformal Colliders Meet the LHC

- Progress in the understanding of lightray operators allows the calculation and measurement of the **shapes and scalings of multipoint correlators**, inside high energy jets at the LHC.

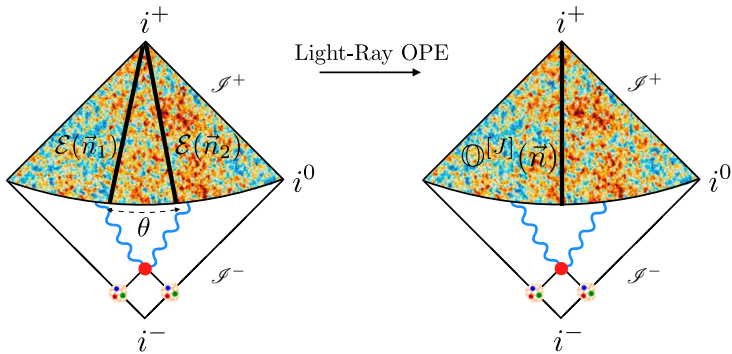


Outline

- Scaling Behavior in Jet Substructure
- Non-Gaussianities in Energy Flux
- Weighing the Top Quark



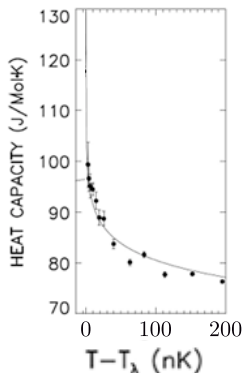
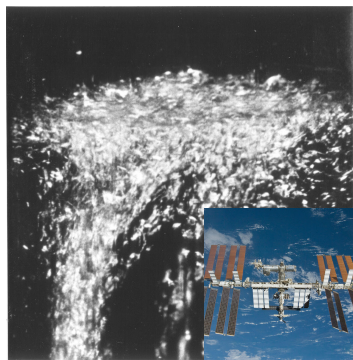
Scaling Behavior in Jet Substructure



Scaling Behavior in QFT

- QFTs exhibit universal behavior as operators are brought together.
- For local operators, this is captured by the **operator product expansion (OPE)** \implies scaling behavior!

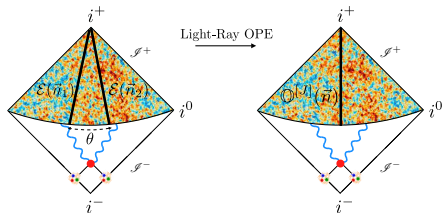
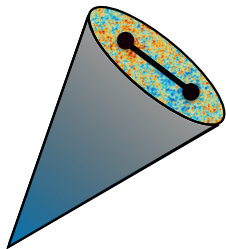
λ -point of Helium



$$\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i$$

The OPE Limit of Lightray Operators

- Energy flow operators also admit an OPE!
- Jet Substructure is the study of the OPE limit of lightray operators.



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i-4} \mathcal{O}_i(\hat{n}_1)$$

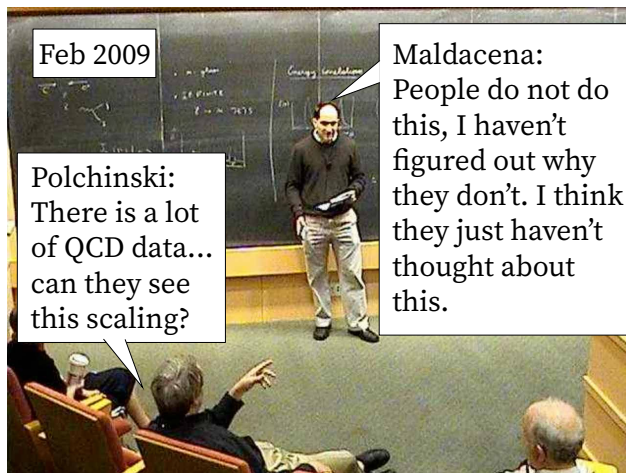
[Hofman, Maldacena]

[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

- Allows a complete new approach to jet substructure as the study of the symmetry and OPE structure of these operators.

Theory-Experiment Gap

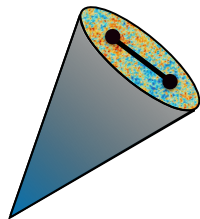
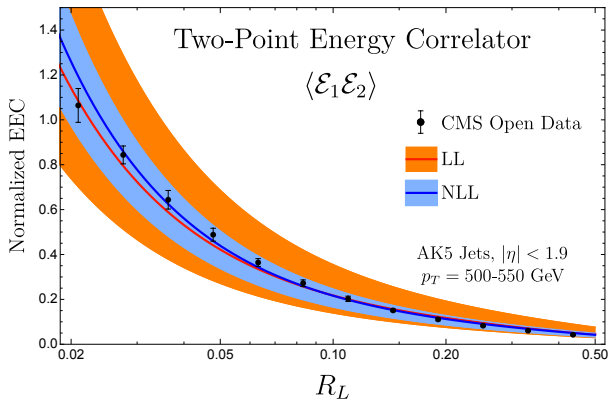
- OPE scaling is the most basic prediction of QFT for jet substructure.



- Shockingly, still true as of 2022... lets change that...

The OPE Limit in Data

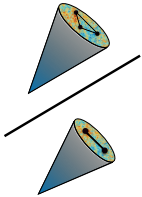
- The $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$ OPE inside high-energy jets!

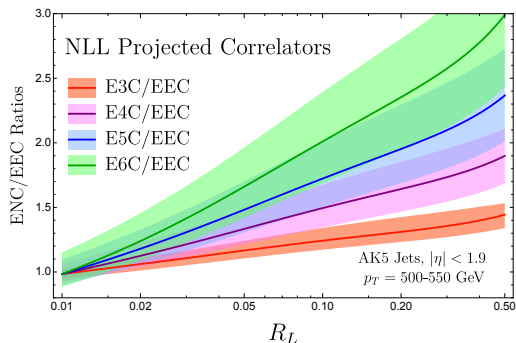


- Beautiful illustration of the universality of the OPE limit in QFT!
- Universality allows calculations in the complicated LHC environment.

Higher Point Scaling

- A remarkable prediction of the light-ray OPE is that at the **quantum level**, N -point correlators develop an **anomalous scaling** that depends on N .


$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$



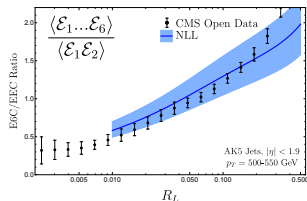
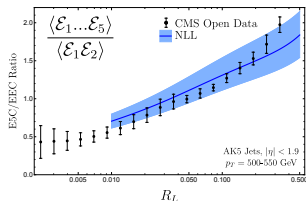
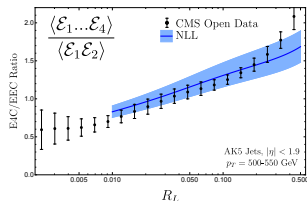
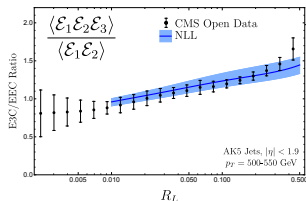
- Directly probes the spectrum of (twist-2) light-ray operators in QCD.

Higher Point Scaling

[Chen, Moulton, Zhang, Zhu]

[Lee, Mecaj, Moulton]

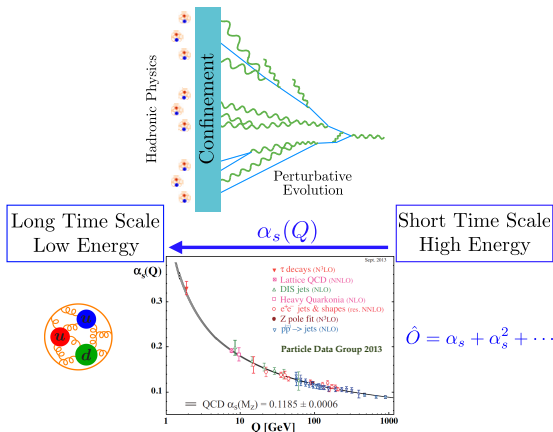
- The remarkable LHC dataset allows these scalings to be measured at the quantum level.



- Fundamentally new probes of jets at colliders!!

The Confinement Transition

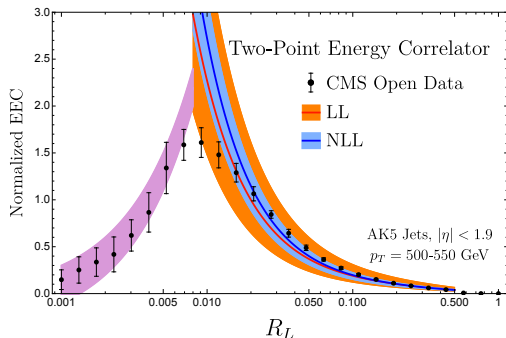
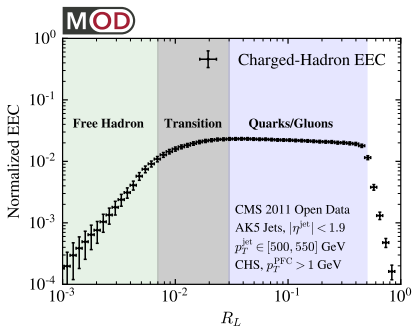
- Jets exhibit a transition from weakly coupled quarks and gluons to freely propagating hadrons.



- Can it be directly imaged?

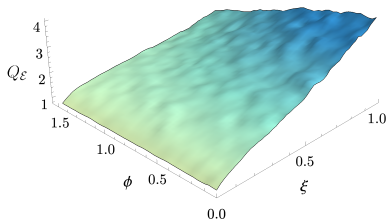
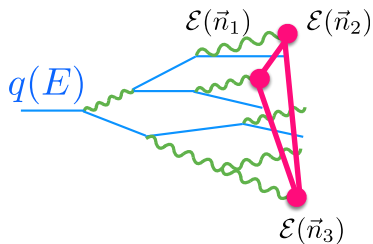
The Confinement Transition

- Distinct scalings associated with **interacting quarks and gluons** and **free hadrons** clearly visible!



- Precision measurements of the confinement transition possible.
- <https://www.youtube.com/watch?v=ORwDv1KTB5U>

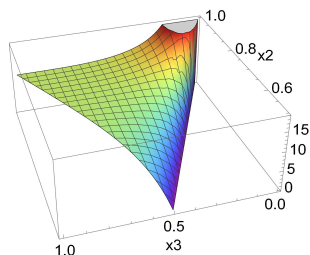
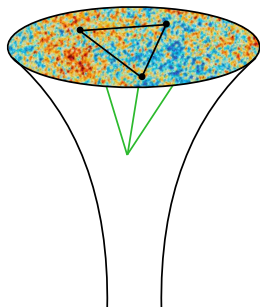
Non-Gaussianities in Energy Flux



[Chen, Moutl, Thaler, Zhu]

Non-Gaussianities

- Higher-point correlators probe more detailed aspects of interactions.
- e.g. Non-Gaussianities allow one to distinguish models of inflation.
- Three-point function, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$, first computed by Maldacena.



[Cabass, Pajer, Stefanyshyn, Supel]

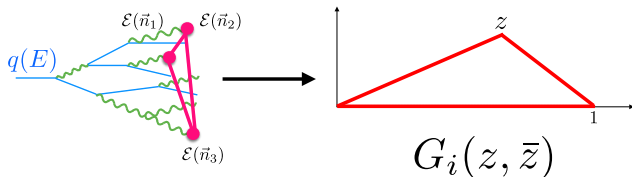
- Can we compute higher-point functions of energy flux?

Multipoint Correlators

- The only explicit results for correlators with $N > 2$ are the remarkable strong coupling results of [Hofman and Maldacena](#):

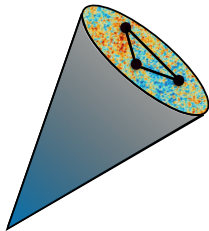
$$\langle \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) \rangle = \left(\frac{q}{4\pi} \right)^n \left[1 + \sum_{i < j} \frac{6\pi^2}{\lambda} [(\vec{n}_i \cdot \vec{n}_j)^2 - \frac{1}{3}] + \frac{\beta}{\lambda^{3/2}} \left[\sum_{i < j < k} (\vec{n}_i \cdot \vec{n}_j)(\vec{n}_j \cdot \vec{n}_k)(\vec{n}_i \cdot \vec{n}_k) + \cdots \right] + o(\lambda^{-2}) \right]$$

- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



Multi-point Correlators at Weak Coupling

- Turn out to have an elegant perturbative structure. e.g. in $\mathcal{N} = 4$



[Chen, Luo, Moul, Yang, Zhang, Zhu]

$$\begin{aligned}
 G_{\mathcal{N}=4}(z) = & \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\
 & - (1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\
 & + \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)
 \end{aligned}$$

- Here Φ and D_2^+ are polylogarithmic functions

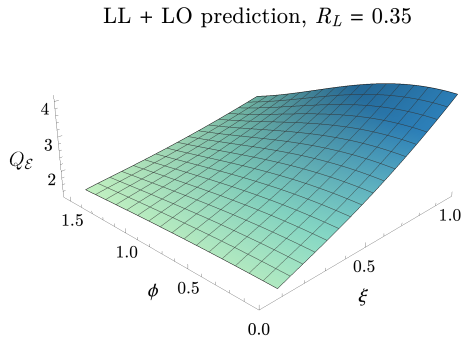
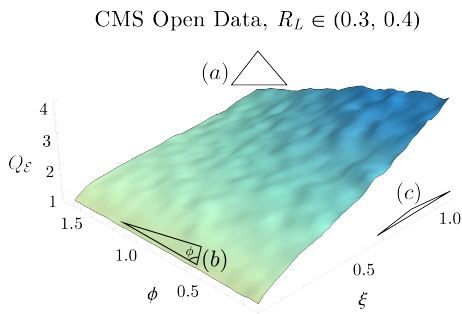
$$\Phi(z) = \frac{2}{z-\bar{z}} \left(\text{Li}_2(z) - \text{Li}_2(\bar{z}) + \frac{1}{2} (\log(1-z) - \log(1-\bar{z})) \log(z\bar{z}) \right)$$

$$D_2^+(z) = \text{Li}_2(1-|z|^2) + \frac{1}{2} \log(|1-z|^2) \log(|z|^2)$$

- Real world QCD involves more complicated polynomials, but is otherwise similar.

Shape Dependence of Non-Gaussianity in Data

- Can directly study non-gaussianities inside high energy jets.

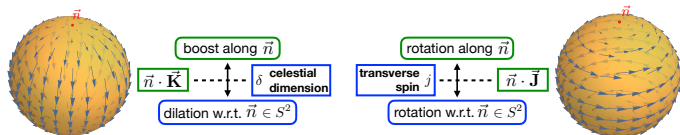


- Illustrates theoretical control over multi-point correlations!

Celestial Partial Waves

- An honest to goodness correlation function living on the detector!
- Beautiful theoretical properties in a measurable observable!
- Exhibits a Celestial Block decomposition:

[Chang, Simmons-Duffin], [Chen, Moulton, Sandor, Zhu]



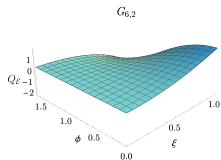
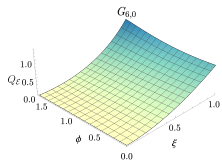
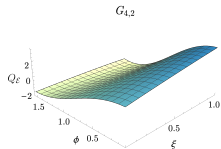
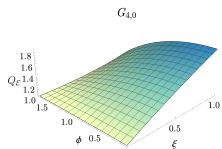
$$g(z, \bar{z}) = \sum_{\delta, j} c_{\delta, j} g_{\delta, j}(z, \bar{z})$$

$$g_{\delta, j}(z, \bar{z}) = \frac{1}{1 + \delta_{j, 0}} (k_{\delta-j}(z) k_{\delta+j}(\bar{z}) + k_{\delta+j}(z) k_{\delta-j}(\bar{z}))$$

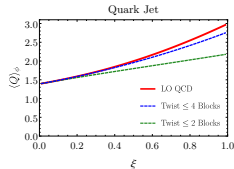
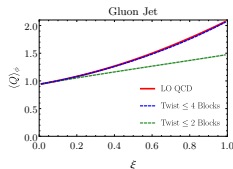
$$k_{\beta}(x) = x^{\beta/2} {}_2F_1\left(\frac{\beta}{2}, \frac{\beta}{2} - 1, \beta, x\right)$$

Celestial Partial Waves

- These are partial waves living on the detector:

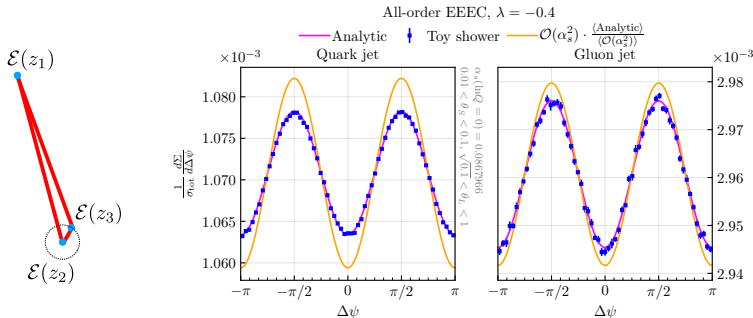


- Celestial block expansion converges rapidly. Genuinely new approach!



Parton Shower Development

- Illustrates complete control of three-point correlations in jets.
- Crucial for validating implementations of higher order effects in parton showers. e.g. **Spin Correlations (transverse spin operators)**



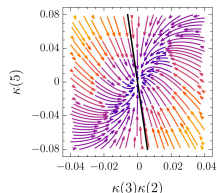
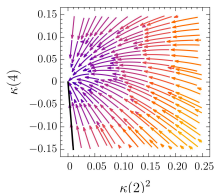
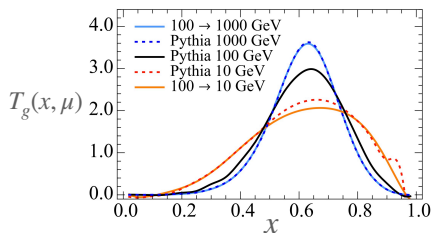
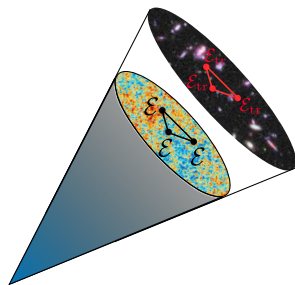
[Karlberg, Salam, Scyboz, Verheyen]

- Full incorporation of higher-point correlations in parton showers will play an important role in enhancing the LHC search program.

Track Functions

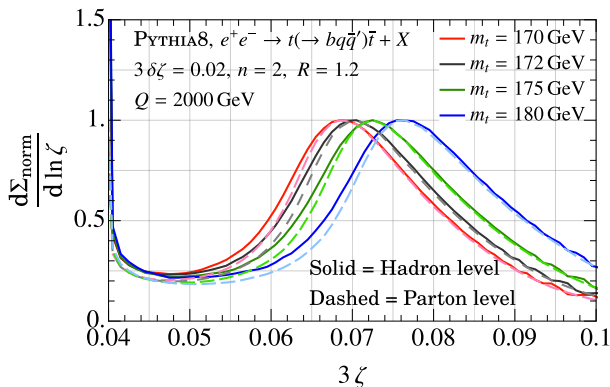
[Chang, Procura, Thaler, Waalewijn]
[Li, Moul, Van Velzen, Waalewijn, Zhu]
[Jaarsma, Li, Moul, Waalewijn, Zhu]

- A key in the ability to study higher point correlations has been the development of QFT formalisms for performing calculations on charged particles (tracks).



- Described by non-perturbative track functions satisfying non-linear RG evolution, encoding correlations in the hadronization process.

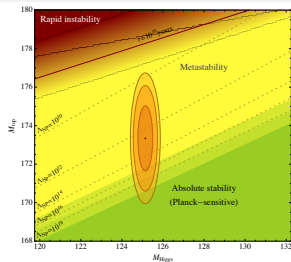
Weighing the Top Quark



[Holguin, Moutl, Pathak, Procura]

Top Quark Mass

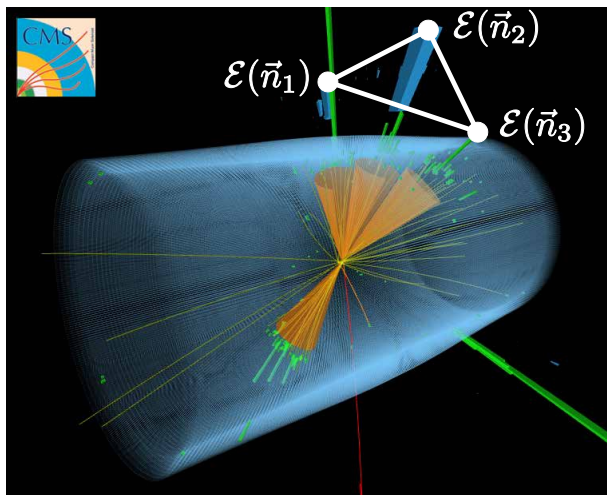
- The **top quark mass** is one of the most important parameters of the SM. e.g. electroweak vacuum stability/criticality, electroweak fits, etc.



- A leading uncertainty ($\gtrsim 1$ GeV) is that it is not understood what has been measured: $m_T^{mc} = 172.26 \pm 0.61$ GeV.
- Need simple observables with top mass sensitivity that can be computed from first principles field theory.

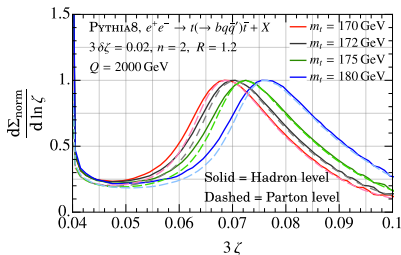
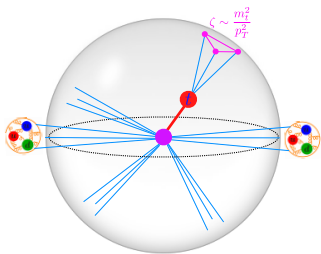
A Top Event

- Massive particles leave clear imprints in correlation functions:



Top Quark Mass Measurement

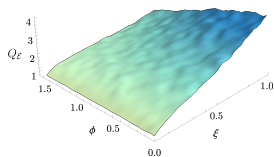
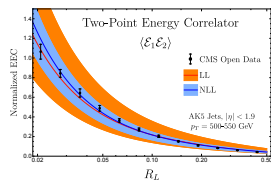
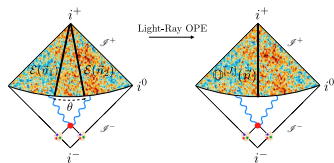
- Massive particles imprint their existence at a characteristic angular scale $\zeta \sim m^2/Q^2$



- Optimistic for a precision ($\lesssim 1$ GeV) top mass extraction at LHC from jet substructure!
- Motivates further understanding of the mathematical structure of higher point correlators.

Summary

- Insights from formal theory are transforming the way we think about jet substructure.
- Jet Substructure provides a physical realization of the OPE limit of lightray operators \implies direct bridge between recent field theory developments and QCD phenomenology.
- Opens the door to a precision physics program using jet substructure, and many new opportunities to search for new physics!



Thanks!