Hydrostatics and Hydrodynamics of the Tank

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Contents of the Presentation

- Hydrostatic Analysis
- Modal Analysis
- Mathematical Modelling of Sloshing

Static Analysis

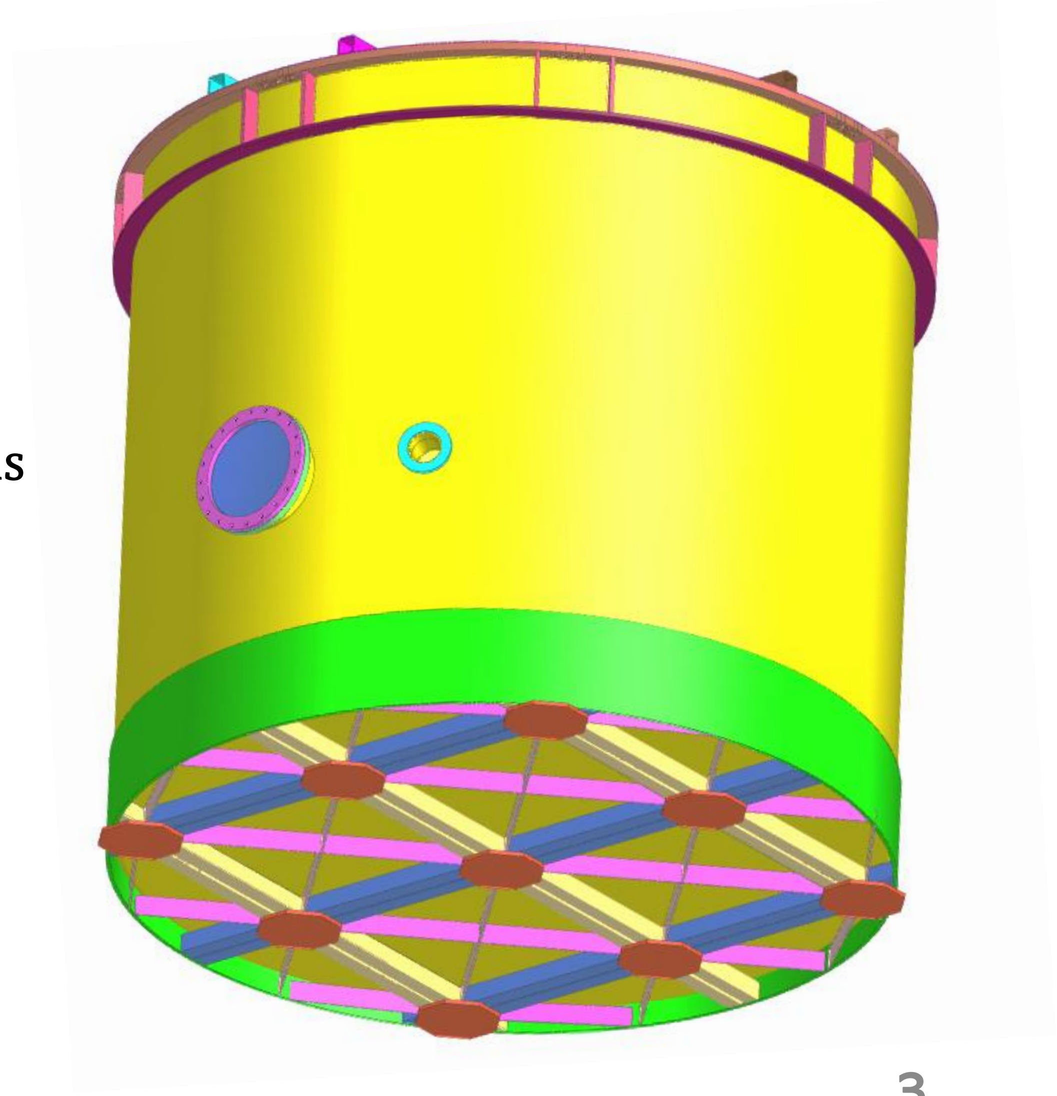
Boundary Condition

The bottom plates are restricted to all dofs.

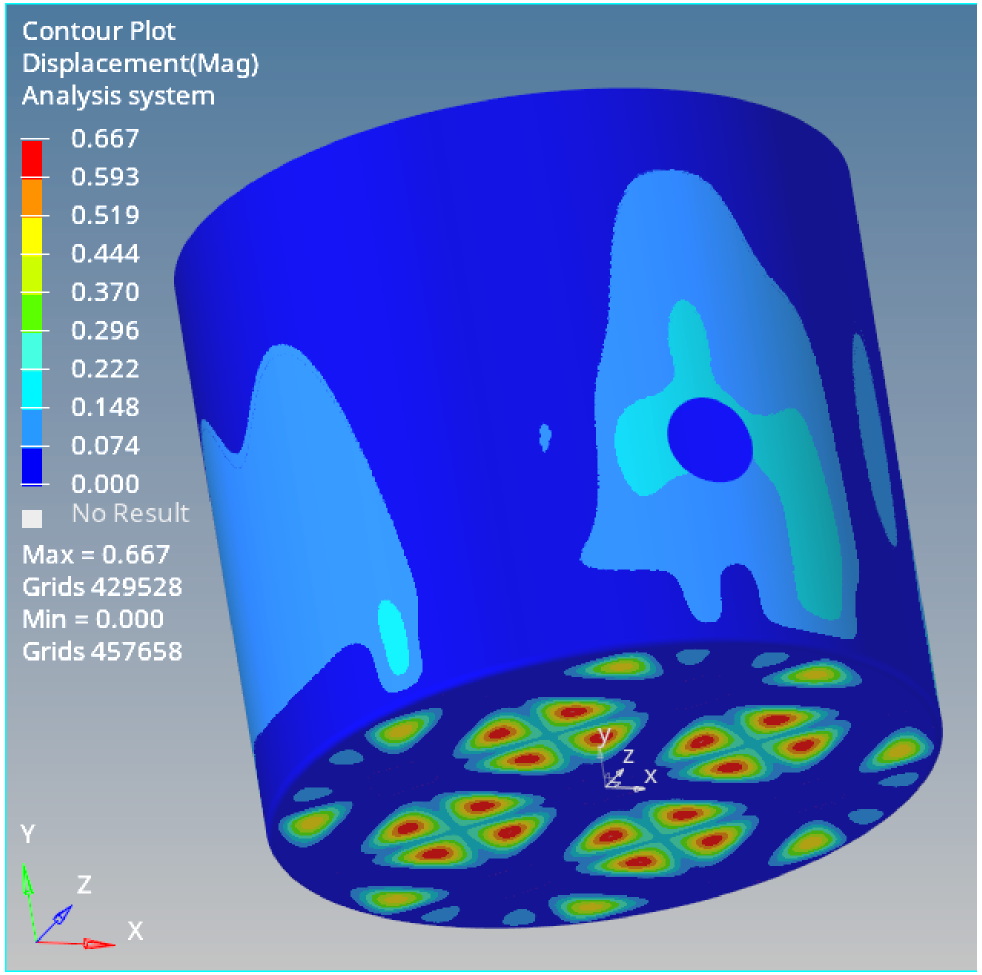
Loads

Self wt.(Gravity)+ Hydrostatic Pressure on tank walls (ht. of water =3.4m) +100 kg (CDS) on top rafters

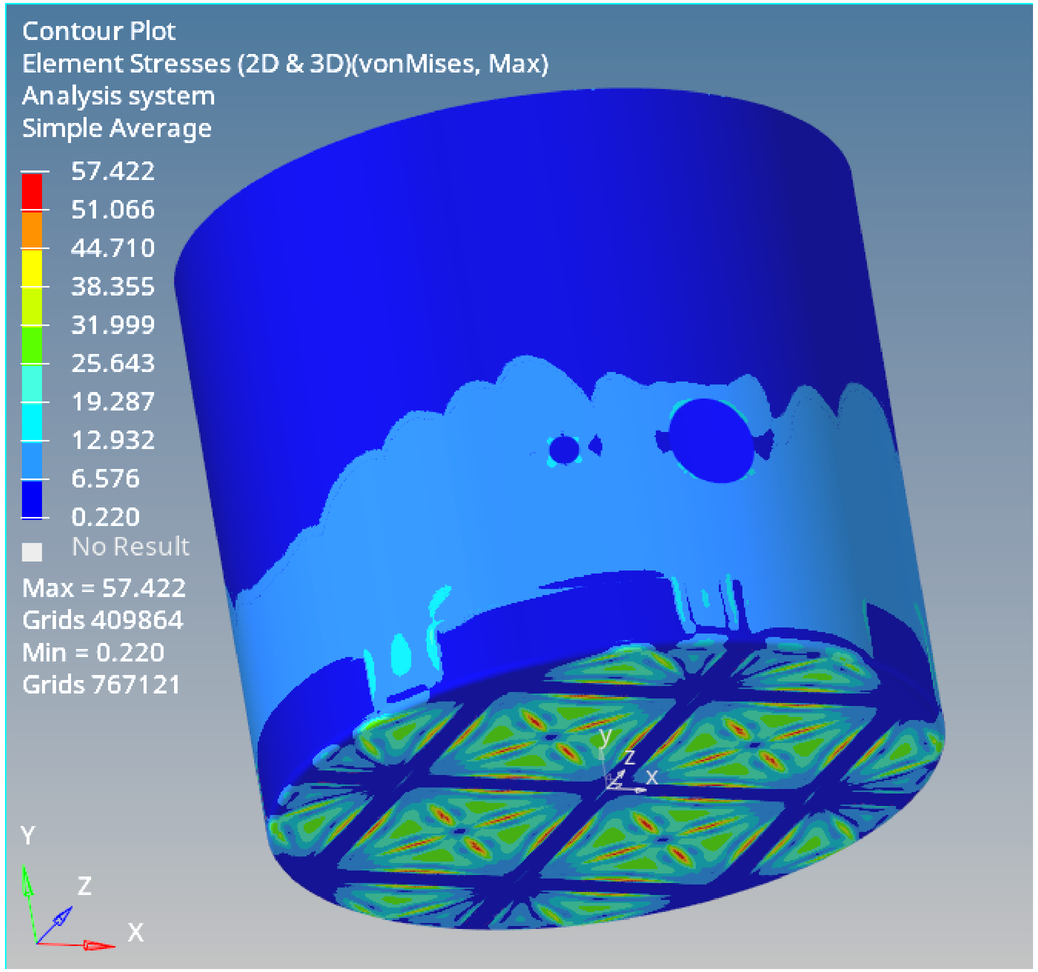
- Wt. of Water = 38.50 tons
- Ht. of Water level = 3400mm
- Tank Wall Thickness = 6mm



Tank Results



Max. Displacement = 0.67 mm

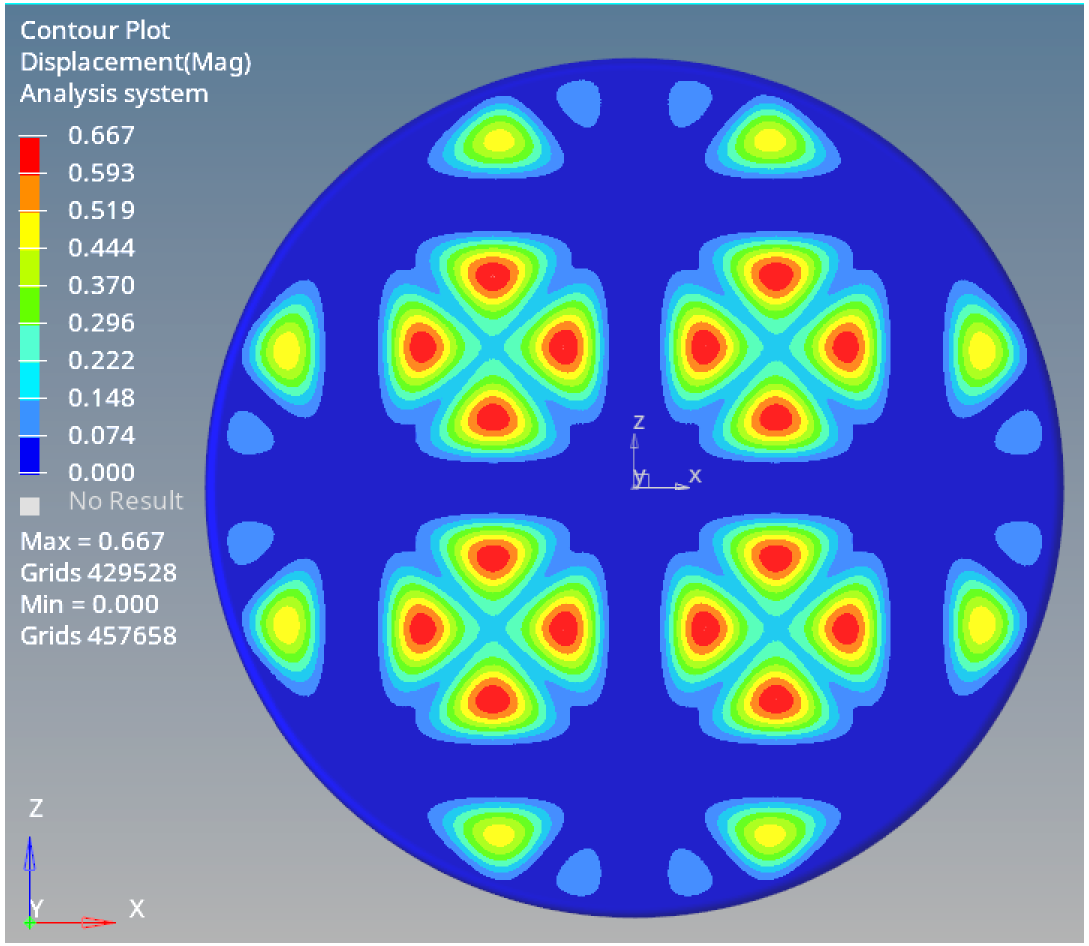


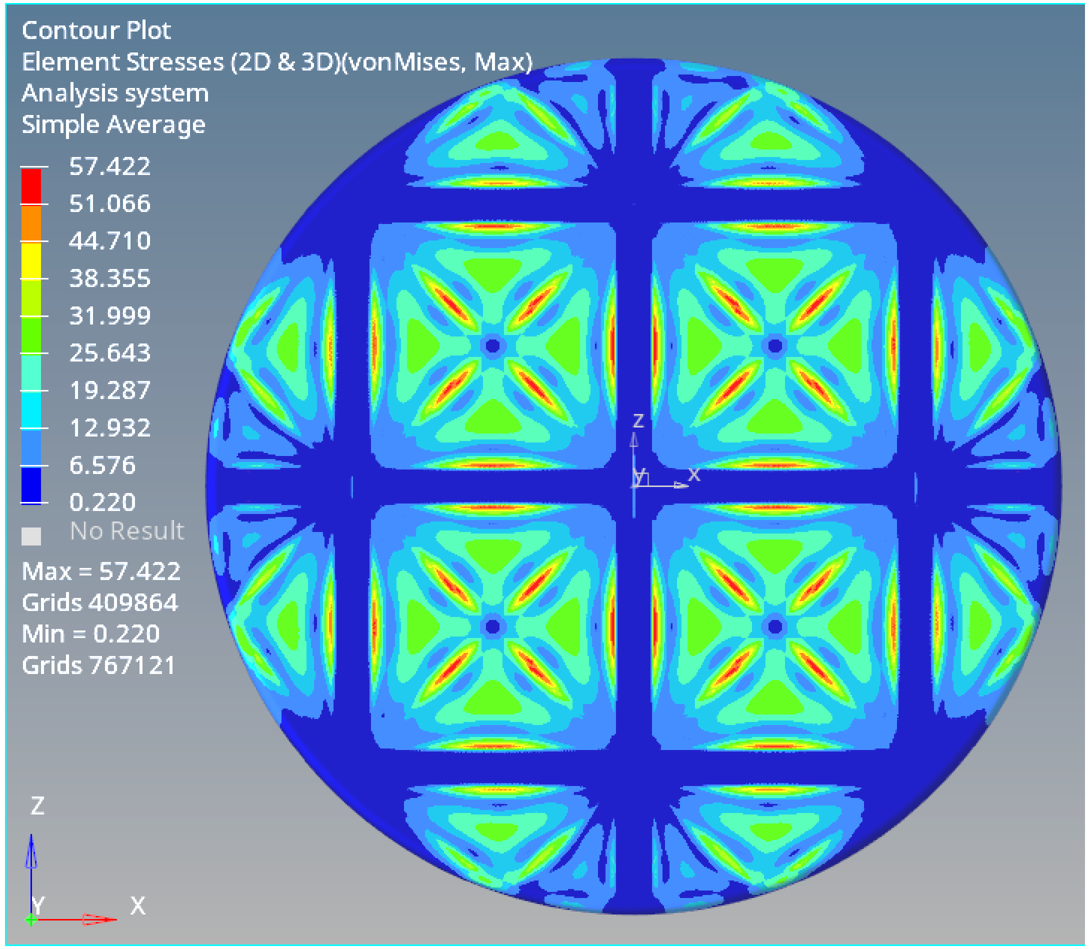
Max. Stress = 57.42 MPa

Hydrostatic Analysis

Tank Bottom View

Tank Results

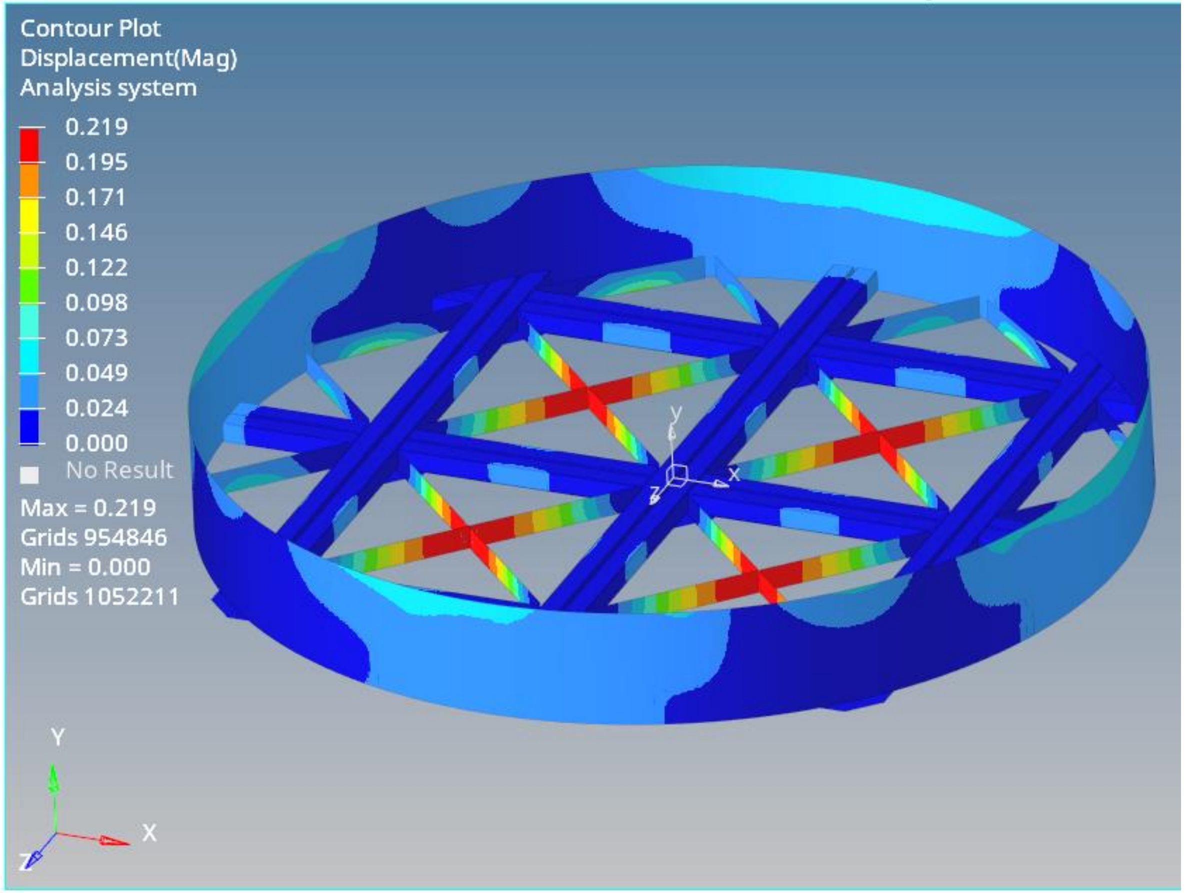


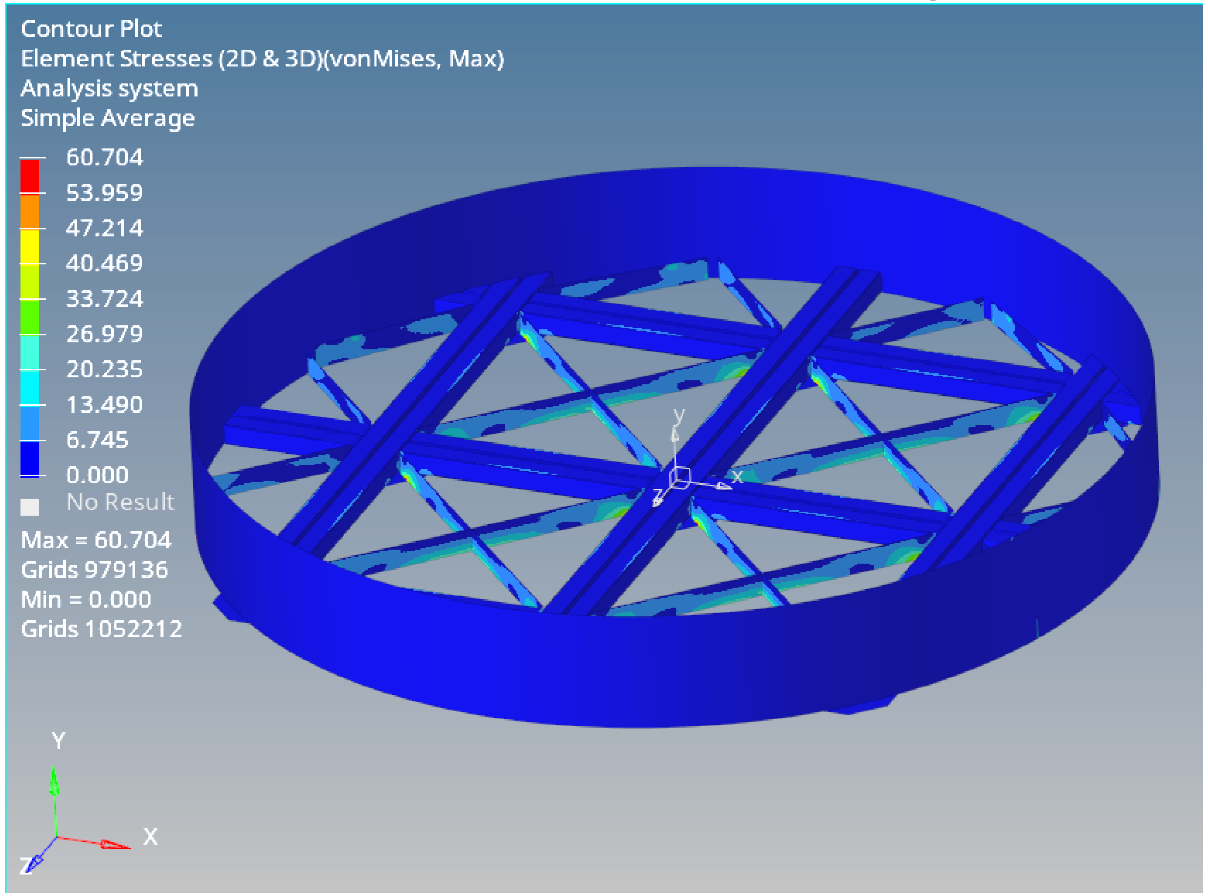


Max. Displacement = 0.67 mm

Max. Stress = 57.42 MPa

Base Results

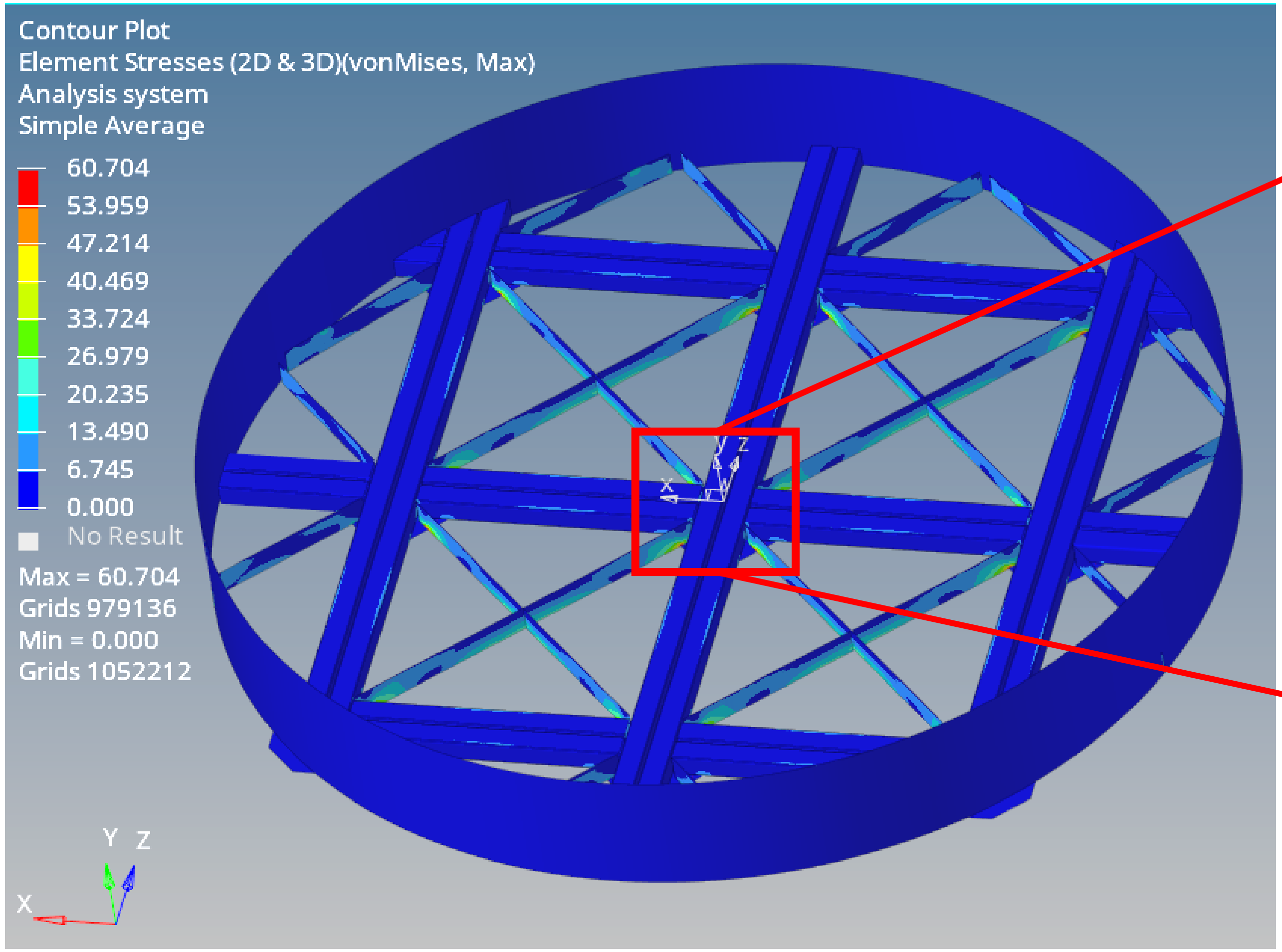


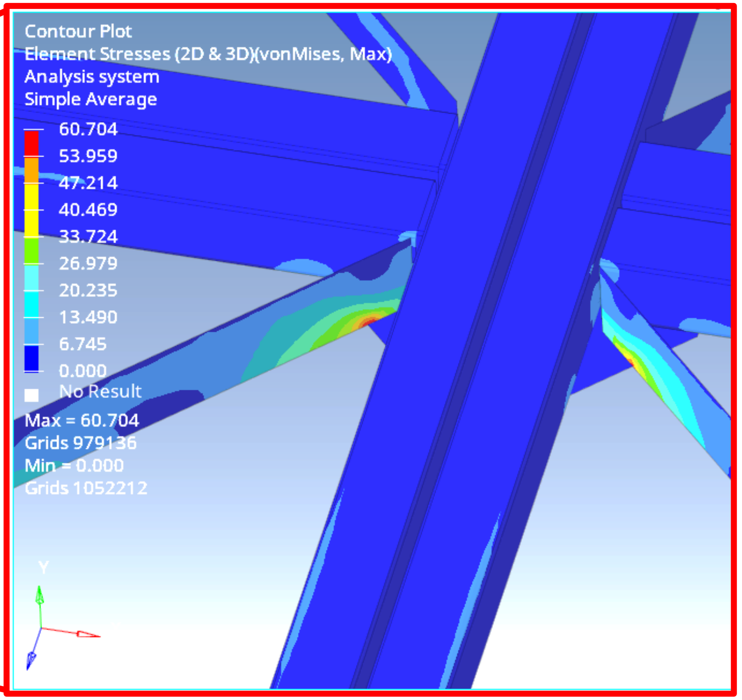


Max. Displacement = 0.22 mm

Max. Stress = 60.70 MPa

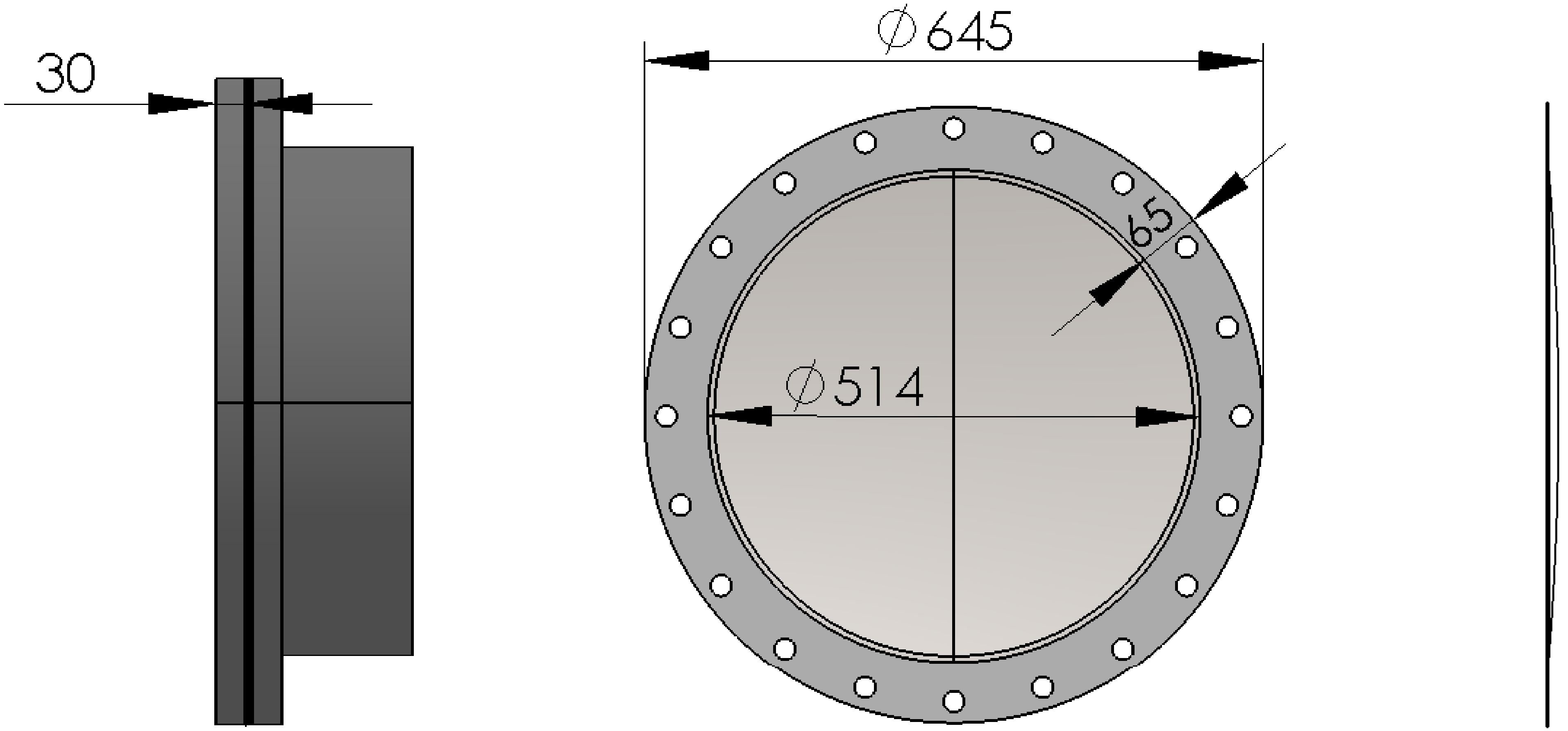
Base Results





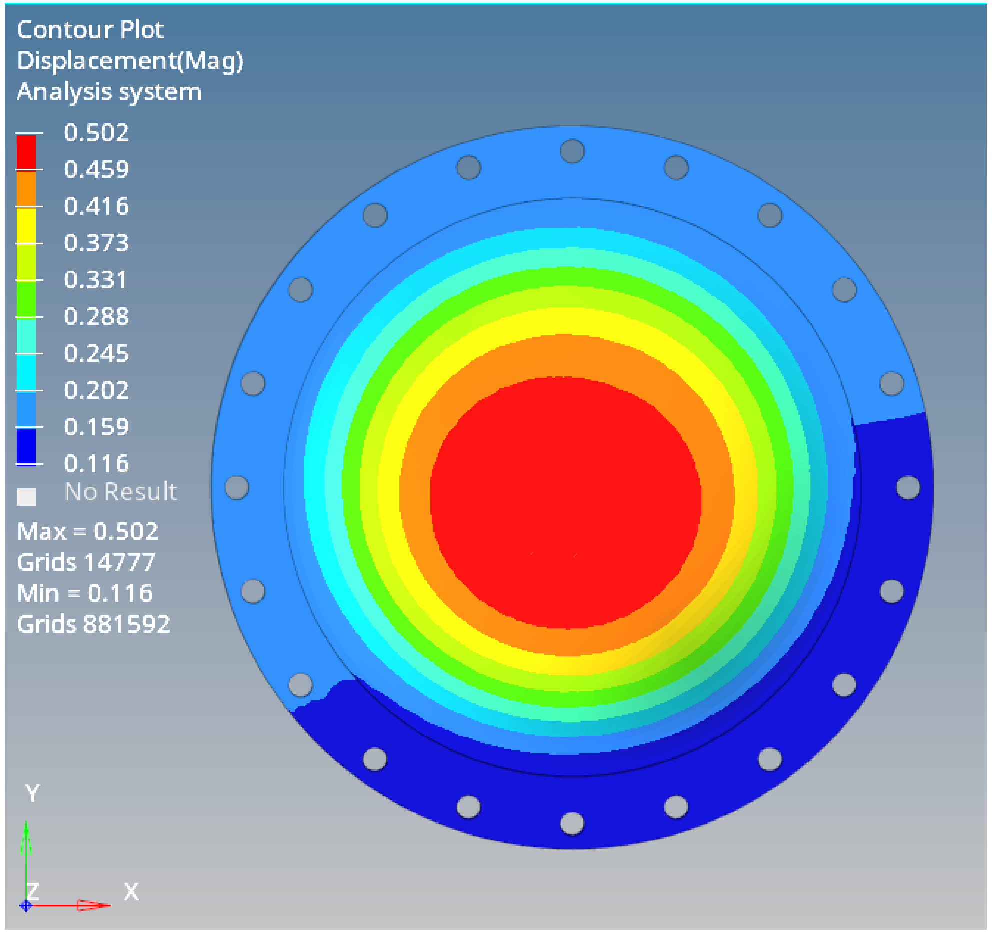
Max. Stress = 60.70 MPa

Beam Window Details

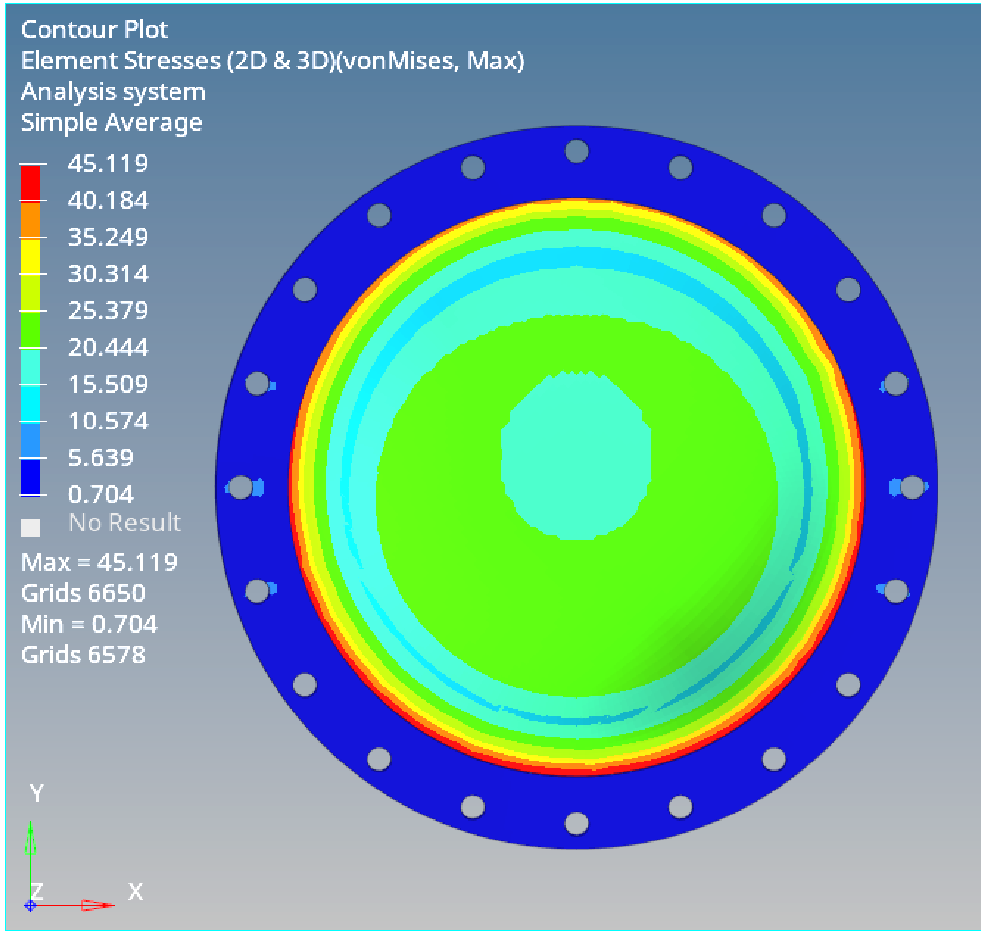


2 mm Concave

Tertiary Beam Window



Max. Displacement = 0.50 mm



Max. Stress = 45.20 MPa

Conclusion

From the results we can infer that

- The maximum stress and displacement due to Hydrostatic Pressure on the Tank assembly occurs on the Tank base of **60.70 MPa** .
- The maximum displacement is obtained on the tertiary beam i.e., **0.50mm**
- In the study, the model appears to be safe by the factor of safety **3.46** (w.r.t to yield stress of 207 MPa)

Tank		Tertiary Beam Window		Base Mesh		Factor of Safety
Max. VonMises stress (MPa)	Max. Displacement (mm)	Max. VonMises stress (MPa)	Max. Displacement (mm)	Max. VonMises stress (MPa)	Max. Displacement (mm)	
57.42	0.67	45.20	0.50	60.70	0.22	3.46

Modal Analysis

Boundary Condition

The bottom plates are restricted to all dofs.

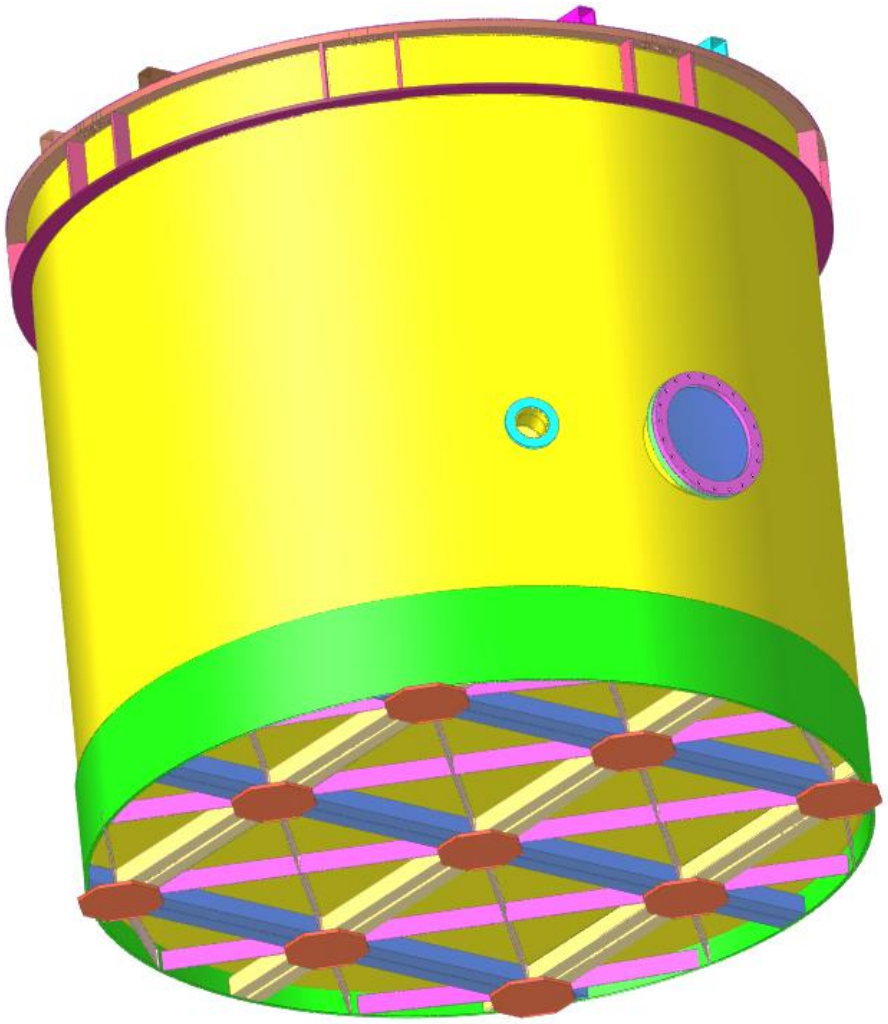
Loads

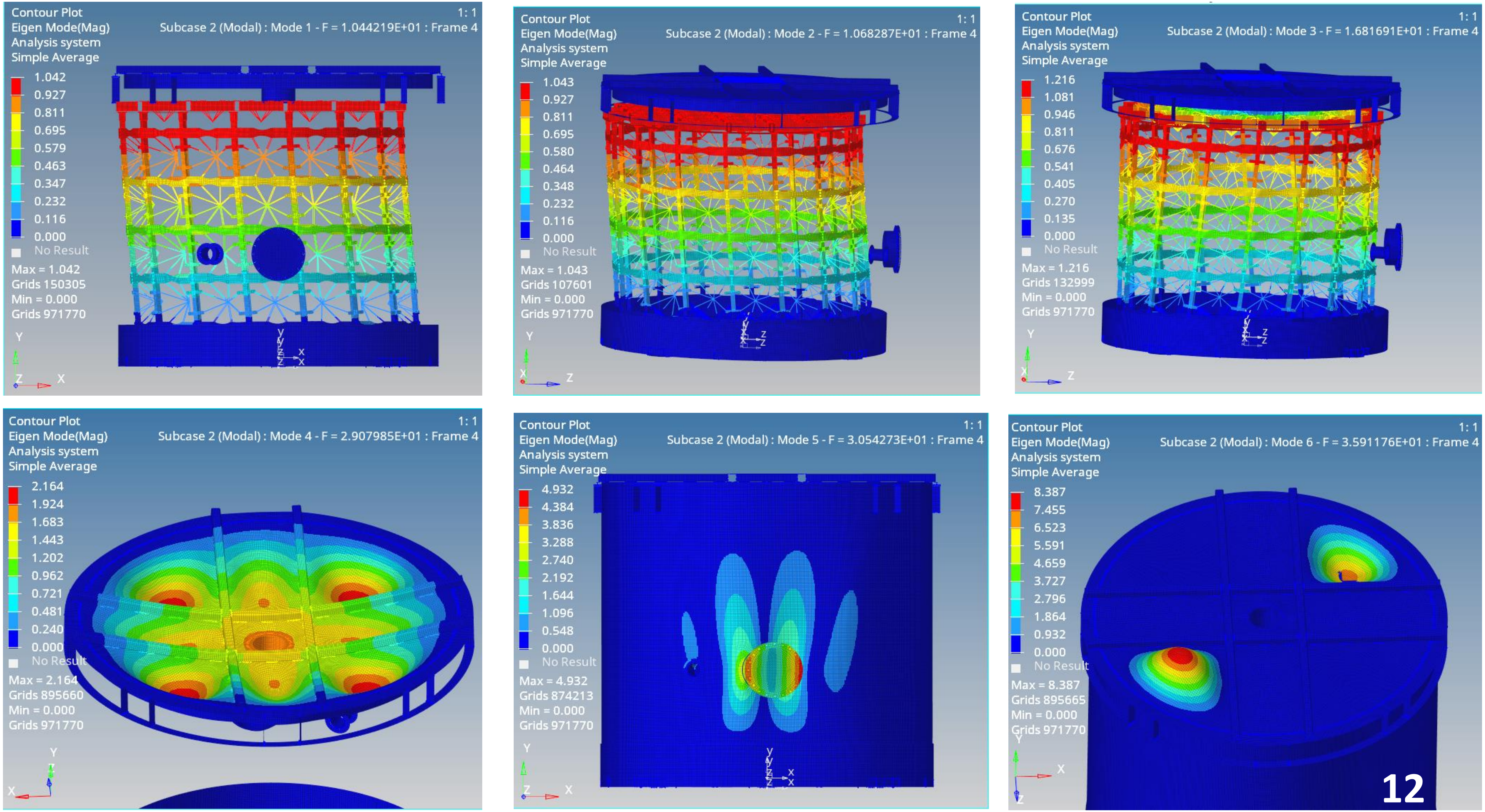
Self wt.(Gravity)+ Hydrostatic Pressure on tank walls

(free surface ht. =3.4m) +100 kg (CDS) on top rafters

Analysis up to 50 Natural Modes

Structure and Tank are not coupled in this Study




Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 1 - F = 1.044219E+01 : Frame 4
1.042
0.927
0.811
0.695
0.579
0.463
0.347
0.232
0.116
0.000
No Result
Max = 1.042
Grids 150305
Min = 0.000
Grids 971770
Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 2 - F = 1.068287E+01 : Frame 4
1.043
0.927
0.811
0.695
0.580
0.464
0.348
0.232
0.116
0.000
No Result
Max = 1.043
Grids 107601
Min = 0.000
Grids 971770
Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 3 - F = 1.681691E+01 : Frame 4
1.216
1.081
0.946
0.811
0.676
0.541
0.405
0.270
0.135
0.000
No Result
Max = 1.216
Grids 132999
Min = 0.000
Grids 971770
Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 4 - F = 2.907985E+01 : Frame 4
2.164
1.924
1.683
1.443
1.202
0.962
0.721
0.481
0.240
0.000
No Result
Max = 2.164
Grids 895660
Min = 0.000
Grids 971770
Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 5 - F = 3.054273E+01 : Frame 4
4.932
4.384
3.836
3.288
2.740
2.192
1.644
1.096
0.548
0.000
No Result
Max = 4.932
Grids 874213
Min = 0.000
Grids 971770
Contour Plot
Eigen Mode(Mag)
Analysis system
Simple Average
1: 1
Subcase 2 (Modal) : Mode 6 - F = 3.591176E+01 : Frame 4
8.387
7.455
6.523
5.591
4.659
3.727
2.796
1.864
0.932
0.000
No Result
Max = 8.387
Grids 895665
Min = 0.000
Grids 971770

Results

MODAL PARTICIPATION FACTORS FOR SUBCASE 3

RIGID BODY MODES BASED ON REFERENCE POINT AT ORIGIN OF BASIC COORDINATE SYSTEM

Mode	Frequency	X-TRANS	Y-TRANS	Z-TRANS	X-ROTAT	Y-ROTAT	Z-ROTAT
1	1.044E+01	-1.077E+00	3.128E-07	5.367E-02	1.464E+02	1.731E-01	2.888E+03
2	1.068E+01	-5.391E-02	9.424E-05	-1.068E+00	-2.878E+03	-1.190E-01	1.463E+02
3	1.682E+01	7.458E-05	2.006E-03	-4.195E-05	-1.119E-01	1.656E+03	-2.466E-01
4	2.872E+01	-9.483E-02	-3.873E-02	3.779E-02	5.492E+01	-2.873E+02	5.444E+01
5	2.908E+01	5.511E-03	-8.250E-01	9.631E-03	2.172E+01	1.516E+01	-3.505E+00
6	3.505E+01	2.656E-02	-1.340E-02	2.890E-01	6.916E+02	1.880E+01	-5.038E+01
7	3.591E+01	-2.413E-02	2.247E-03	4.619E-04	2.521E+02	-1.622E+00	3.417E+02
8	3.607E+01	1.763E-02	3.327E-04	1.722E-03	2.932E+02	-3.510E-01	-2.911E+02
9	3.613E+01	-5.311E-03	-4.411E-03	-3.133E-04	7.264E-01	-8.039E-01	7.323E+01
10	3.669E+01	1.778E-03	-3.551E-03	-3.214E-04	4.105E-01	-1.148E+01	-8.333E-02

Critical frequency of tank with structure is:

X dir. = 10.44 Hz

Y dir. = 29.08 Hz

Z dir. = 10.68 Hz

These frequencies will be required to perform the water sloshing analysis

Mathematical Model for Water Sloshing

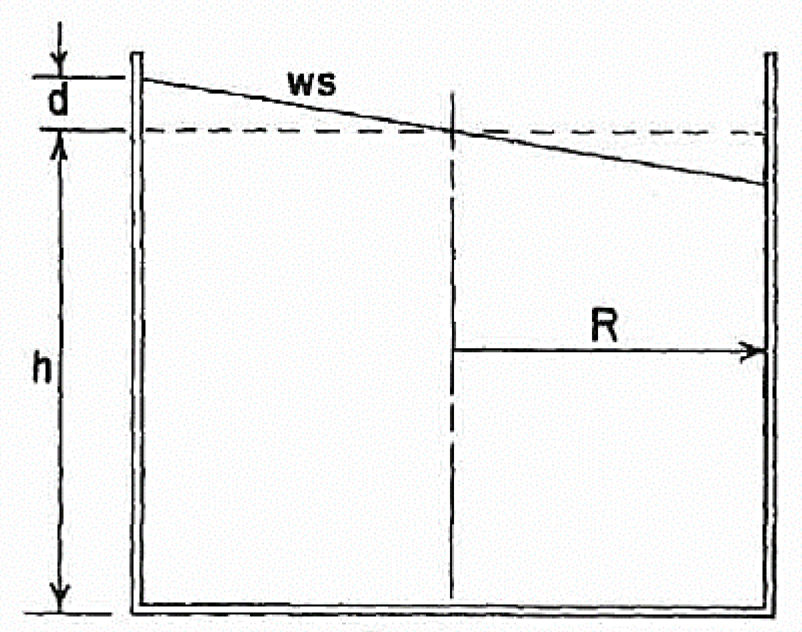


FIG. 1a

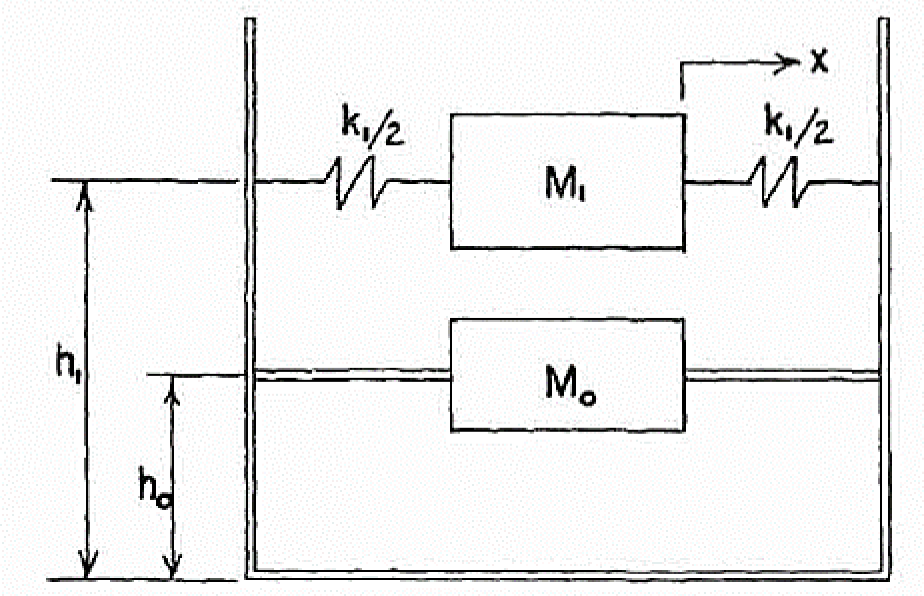


FIG. 1b

$$A_1 = v * \frac{T}{2\pi}$$

$$d = \frac{0.63\, A_1 \left(\frac{k_1\, R}{M_1\, g}\right)}{1 - 0.85\, \frac{A_1}{R} \left(\frac{k_1\, R}{M_1\, g}\right)^2}$$

$$M_1 = M(0.6)\, \frac{\tanh 1.8\, h/R}{1.8\, h/R}$$

$$k_1 = 5.4\, \frac{M_1^{\,2}}{M}\, \frac{gh}{R^2}$$

$$T_w = 2\pi \sqrt{\frac{M_1}{k_1}} = \text{a period of vibration}$$

M = total mass of water in the tank

M_1 = 7130 Kg

k_1 = 65.74 kN/m

T_w = 2.06 s *

Natural slosh frequency (f_w) = 0.5 Hz*

*For first mode of vibration

Ref: *"George Housner" ,The dynamic behavior of water tanks*

Wave Height vs Velocity

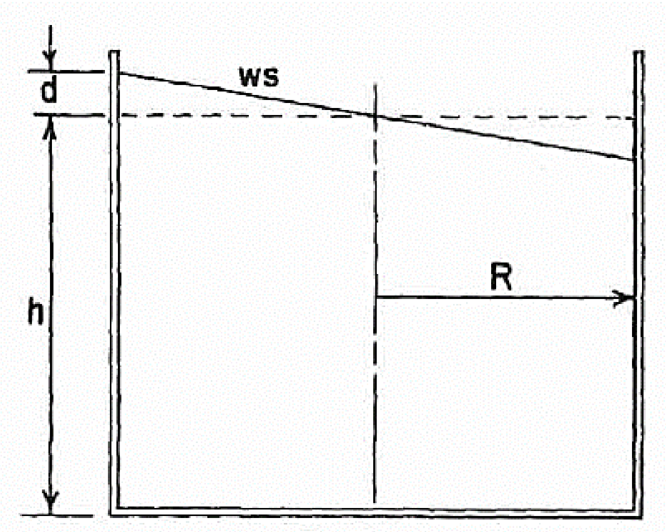
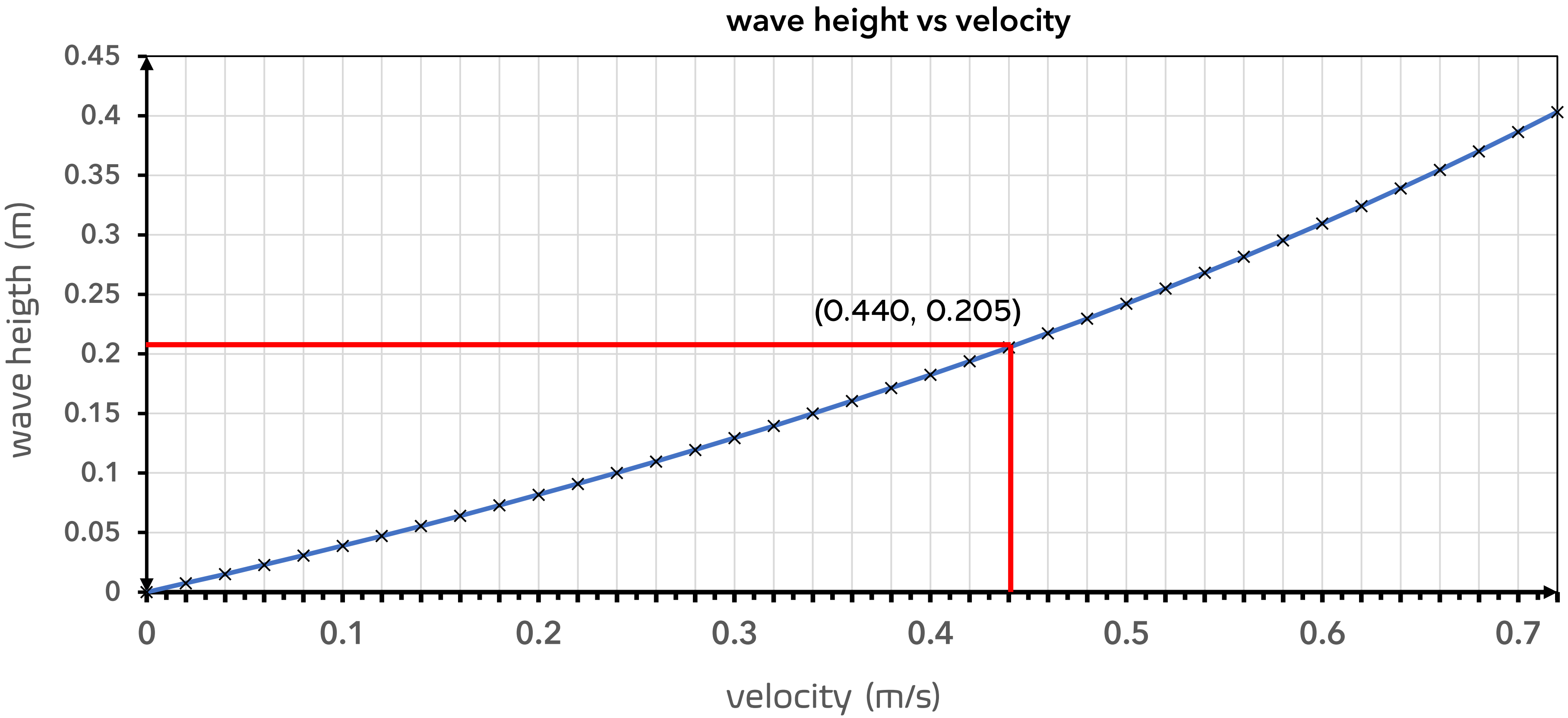
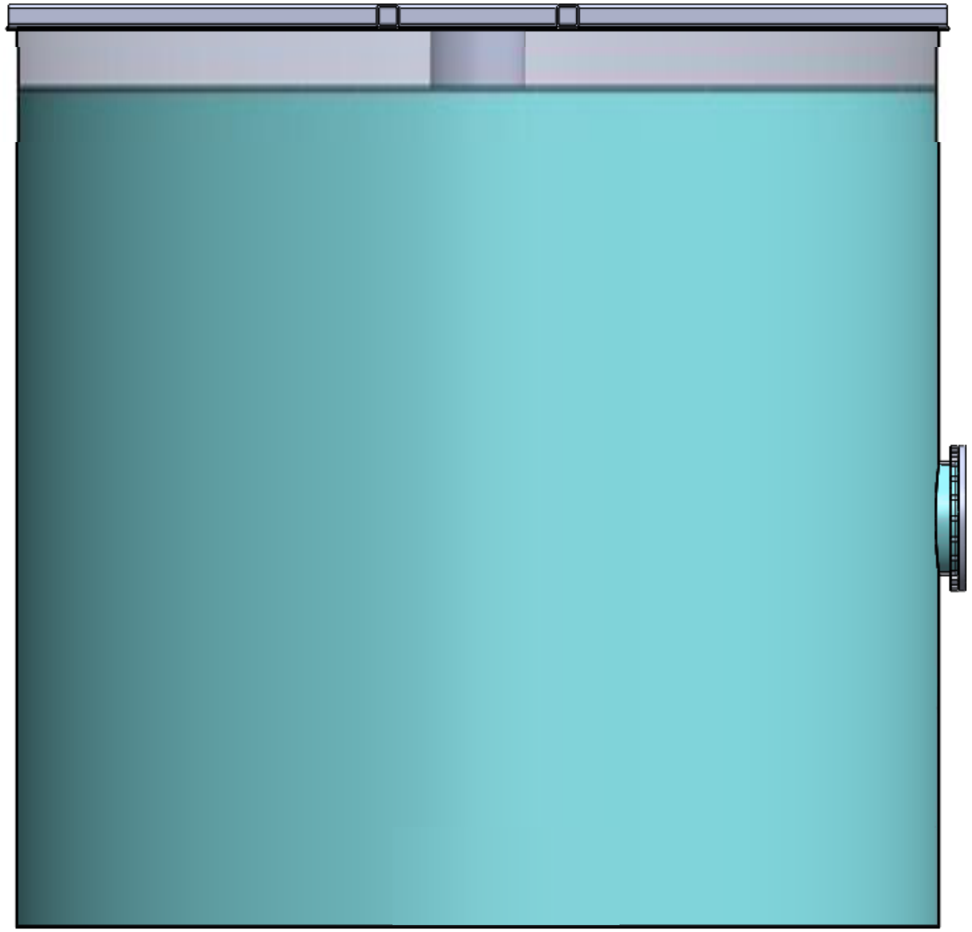


FIG. 1a



Conclusion

- The natural frequency of the water sloshing of the designed tank is **0.5 Hz**.
- The velocity of water corresponding to wave height of 200mm is **0.44 m/s**.
- The initial acceleration of the tank to attain velocity of water 0.44 m/s is under study.



Thank You