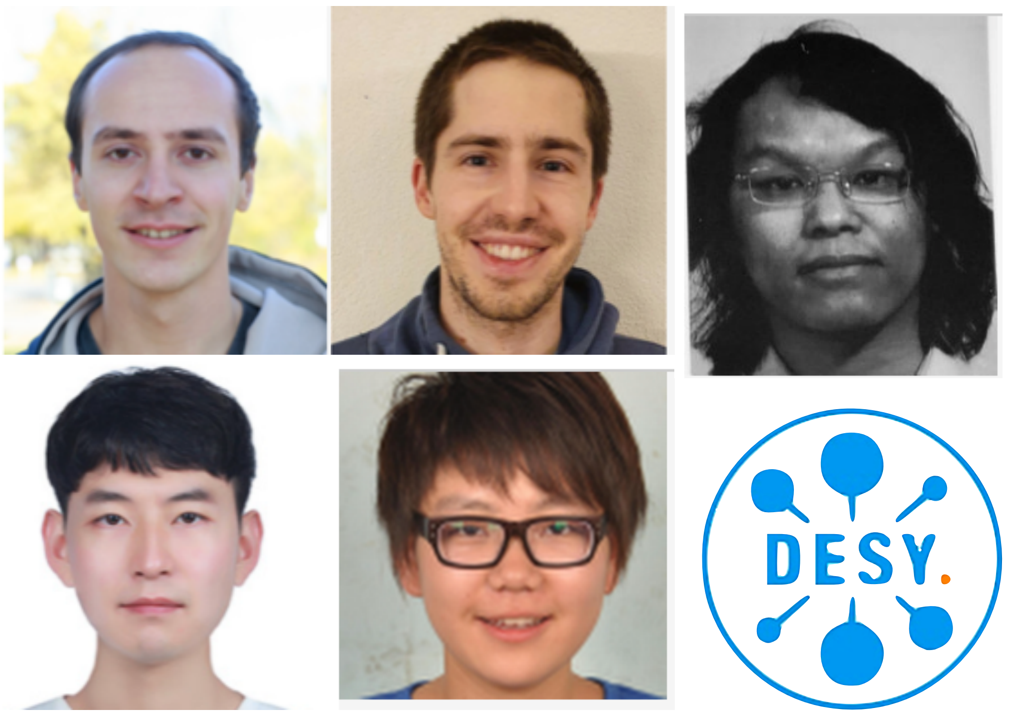
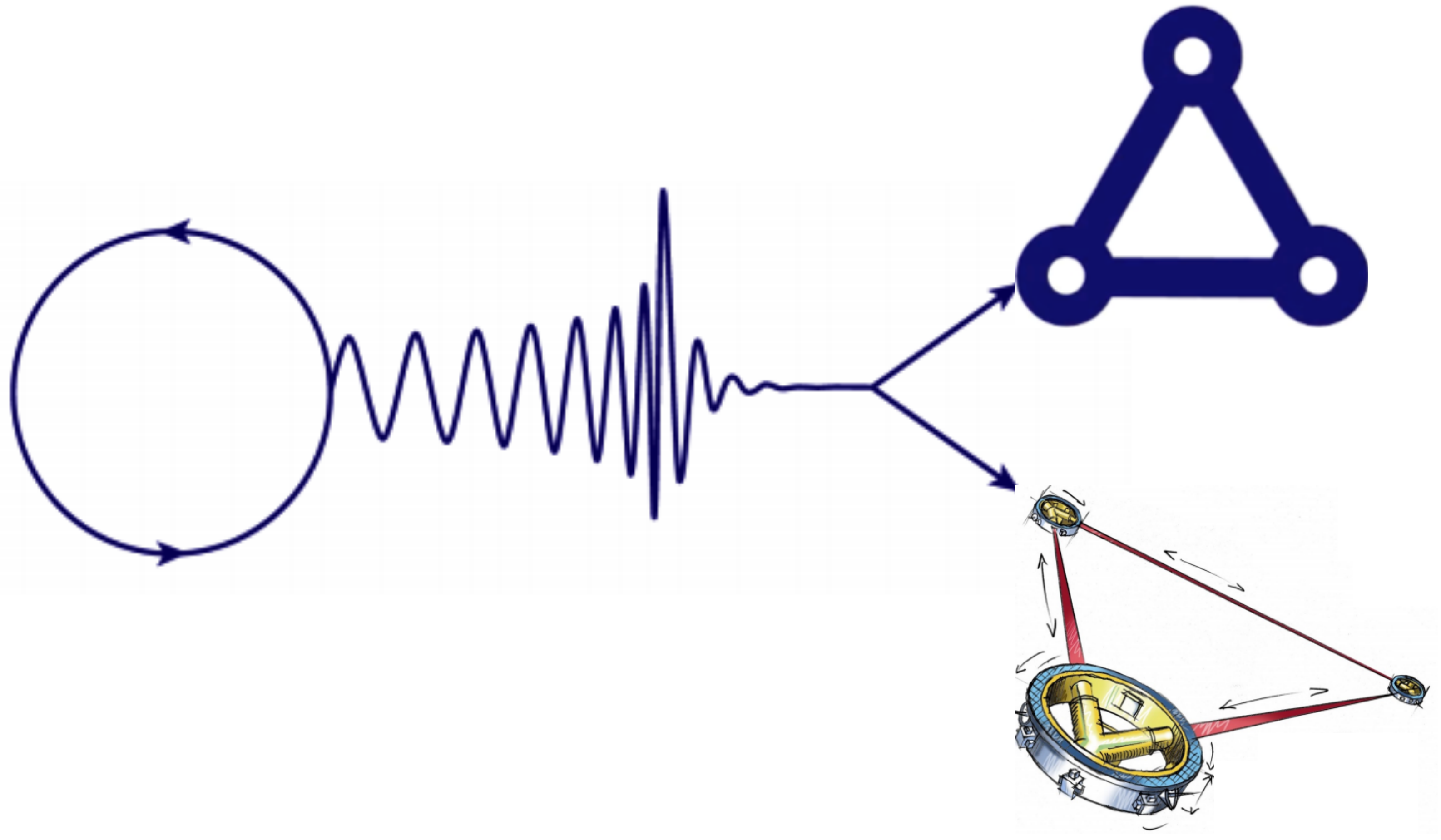
Precision Gravity: From the LHC to LISA and ET

Based on work
in collaboration with
C. Dlapa, G. Kälin, Z. Liu,
G. Cho & Z. Yang



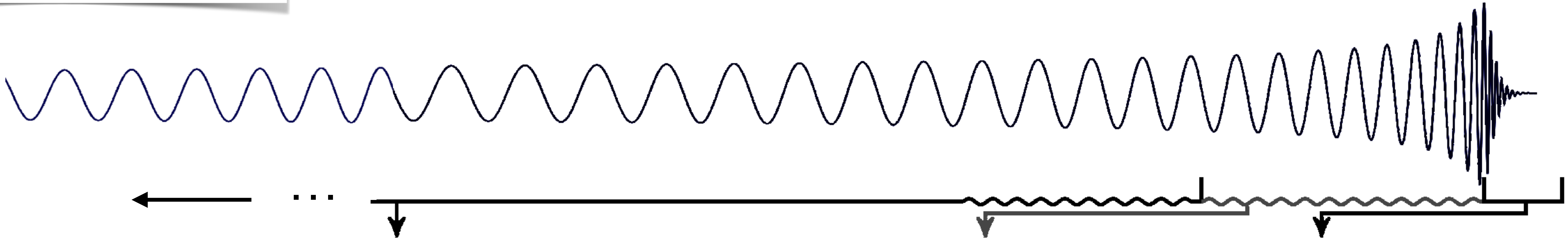


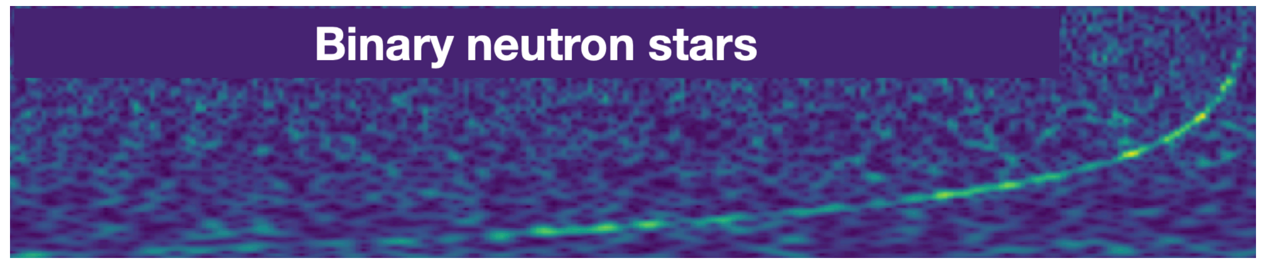
Rafael A. Porto

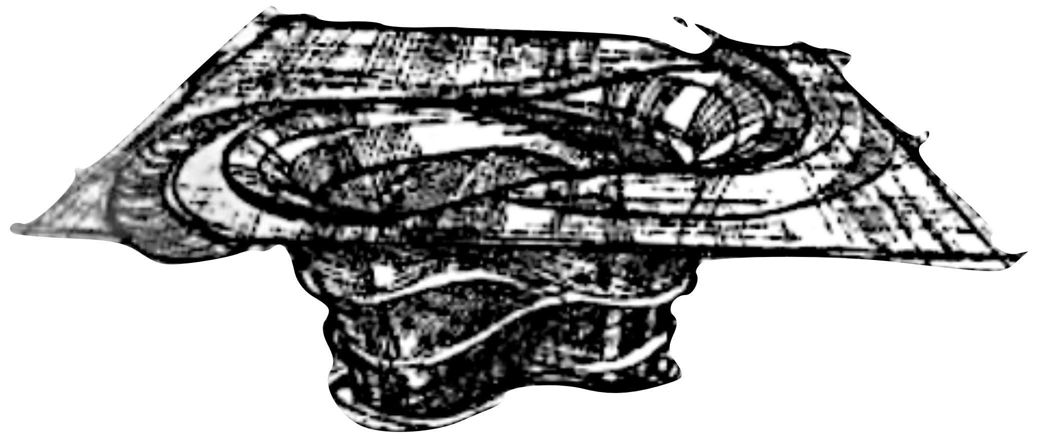

Süddeutsche Zeitung
NEUESTE NACHRICHTEN AUS POLITIK, KULTUR, WIRTSCHAFT UND SPORT
Das Streiflicht
Der Beweis

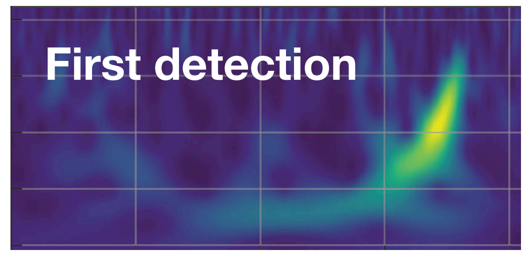
President Obama @POTUS44

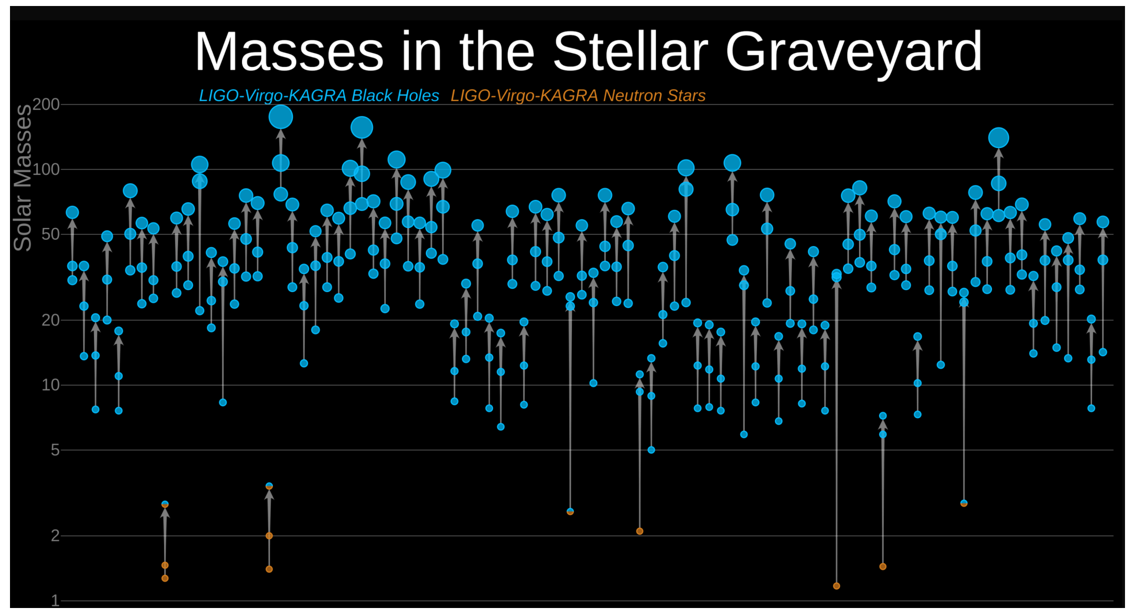
Einstein was right! Congrats to @NSF and @LIGO on detecting gravitational waves - a huge breakthrough in how we understand the universe.





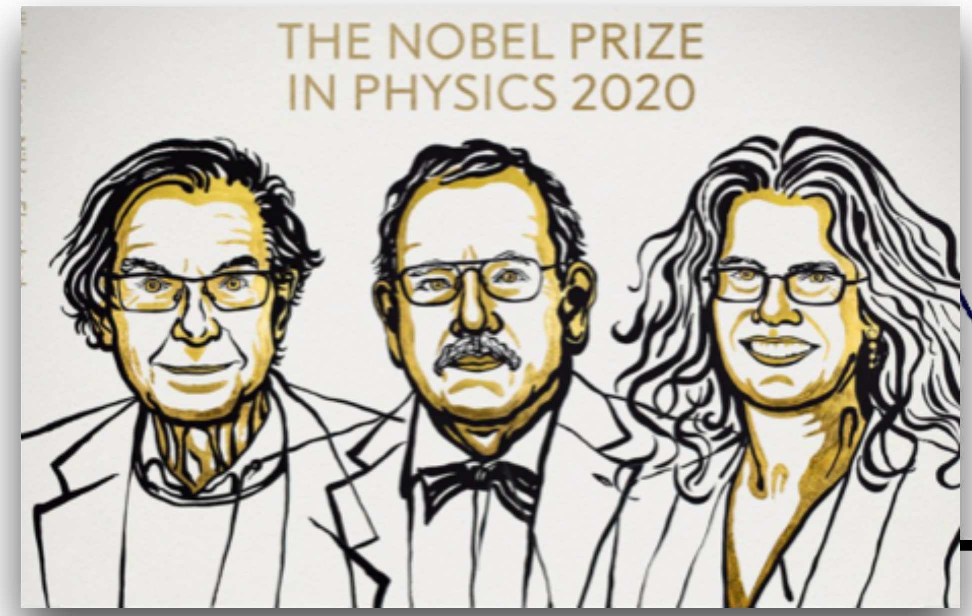


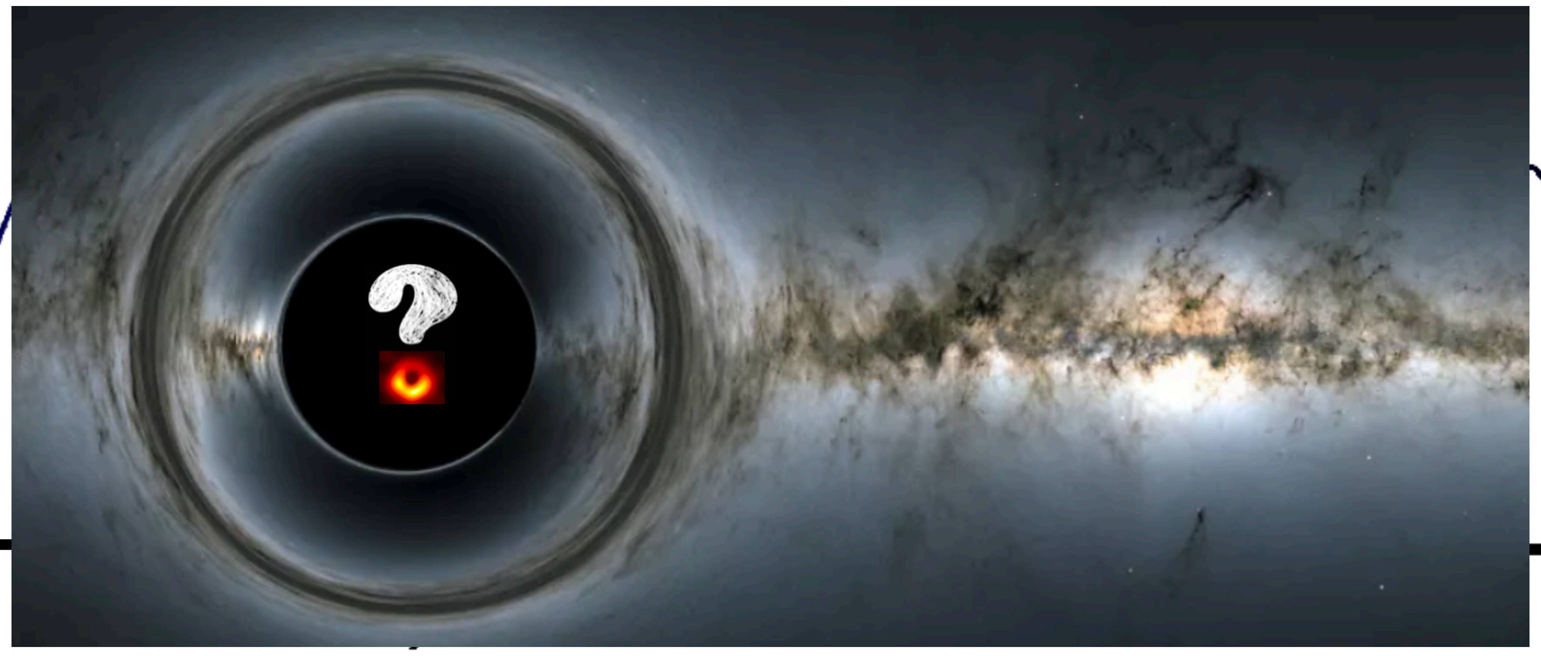


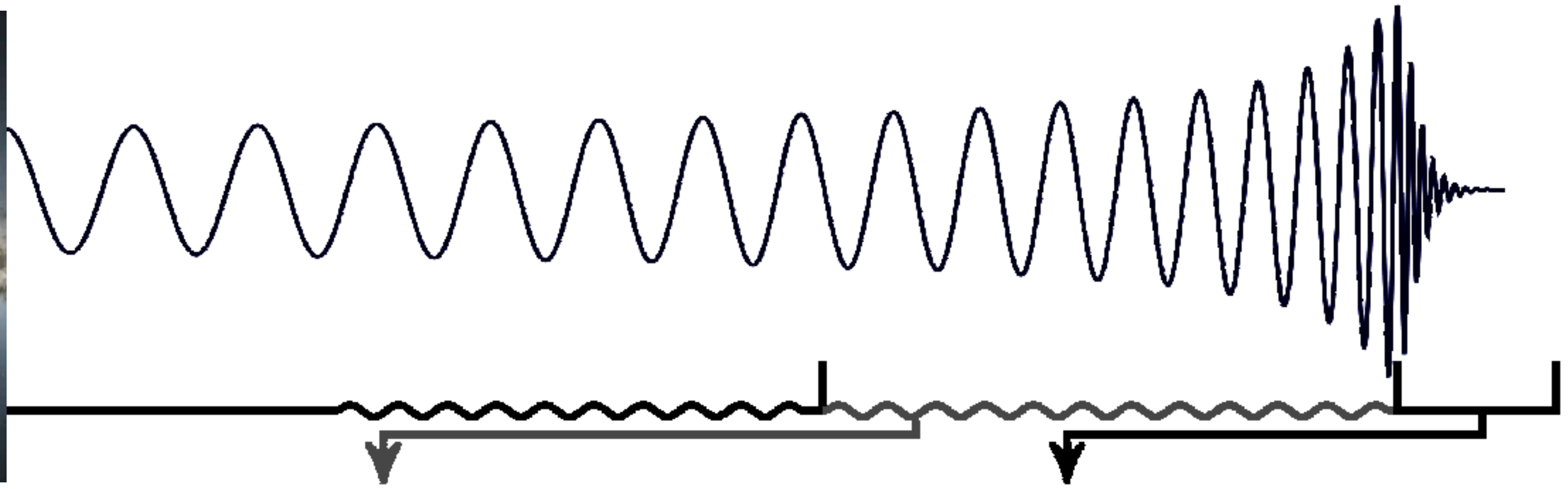


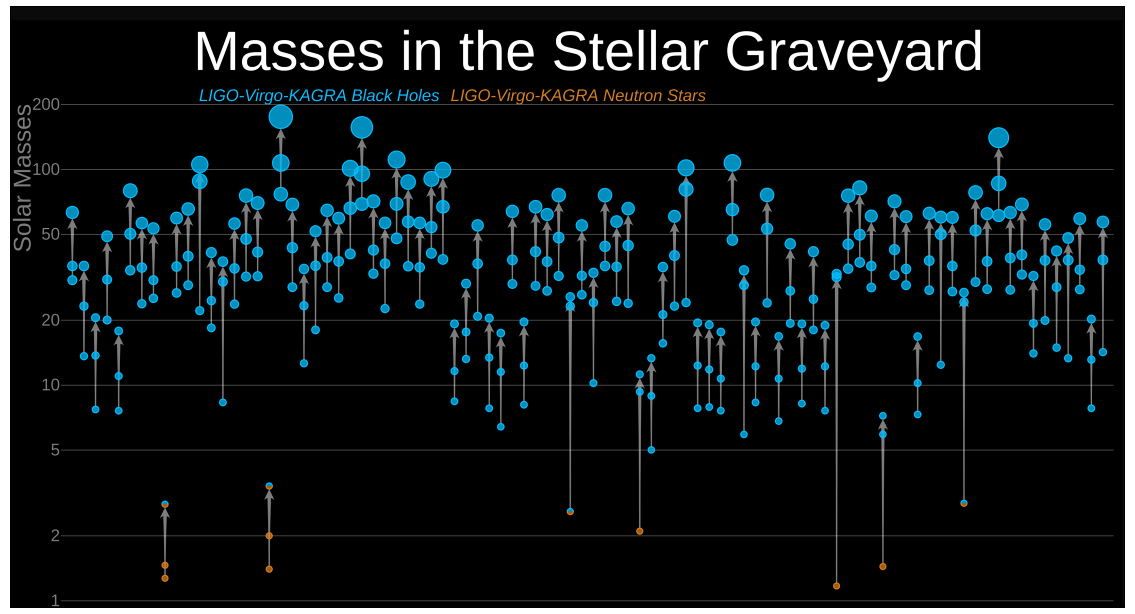
"for the discovery that black hole formation is a robust prediction of the general theory of relativity"

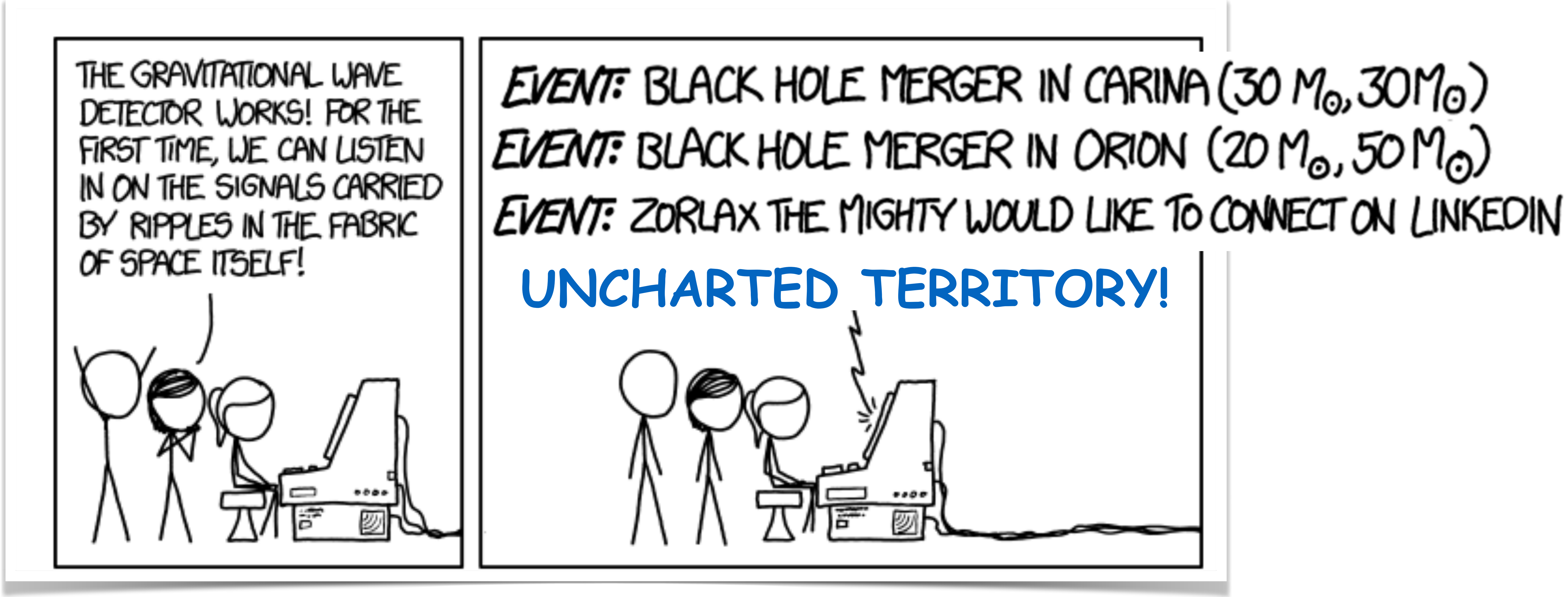
"for the discovery of a supermassive compact object at the centre of our galaxy"











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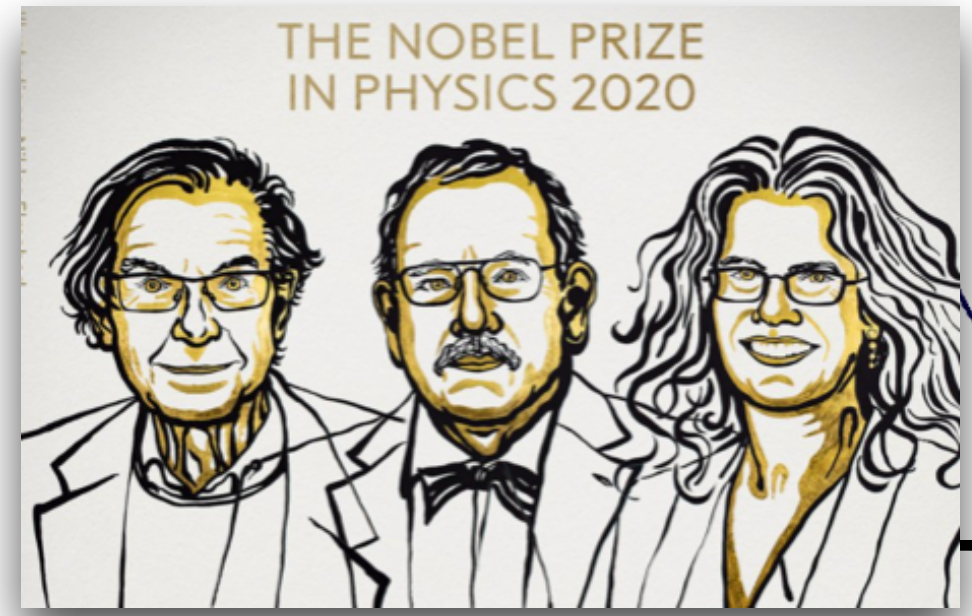
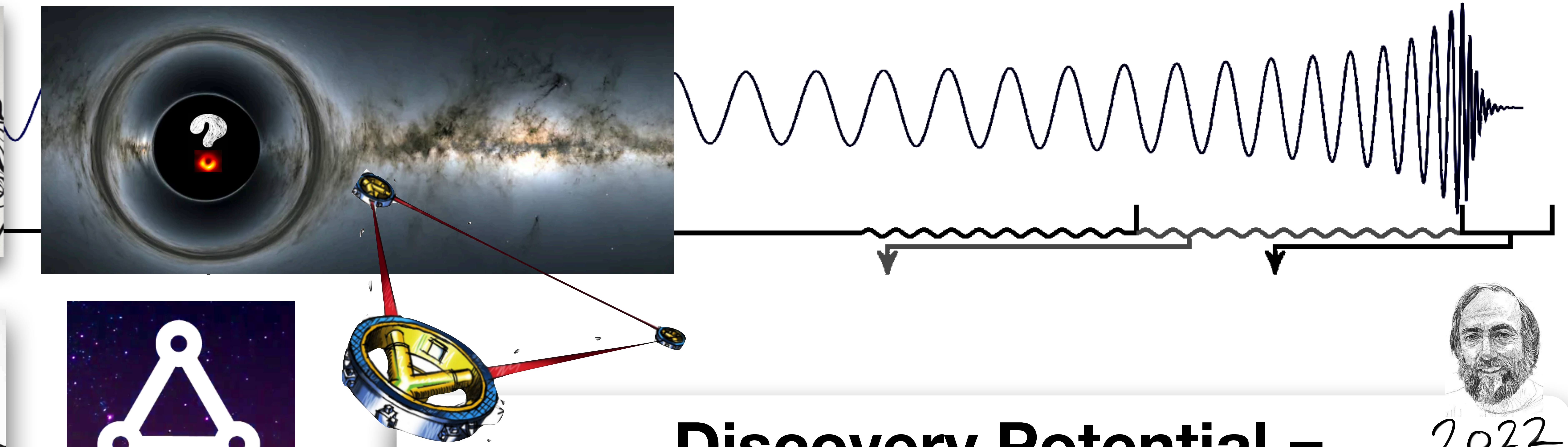
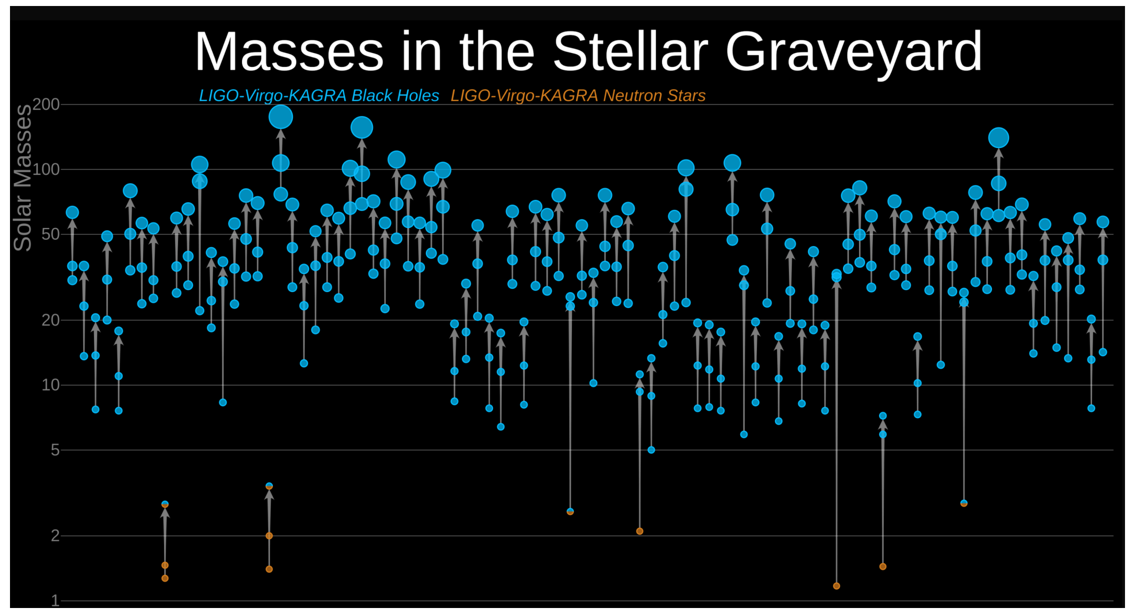






Einstein Telescope

Discovery Potential =

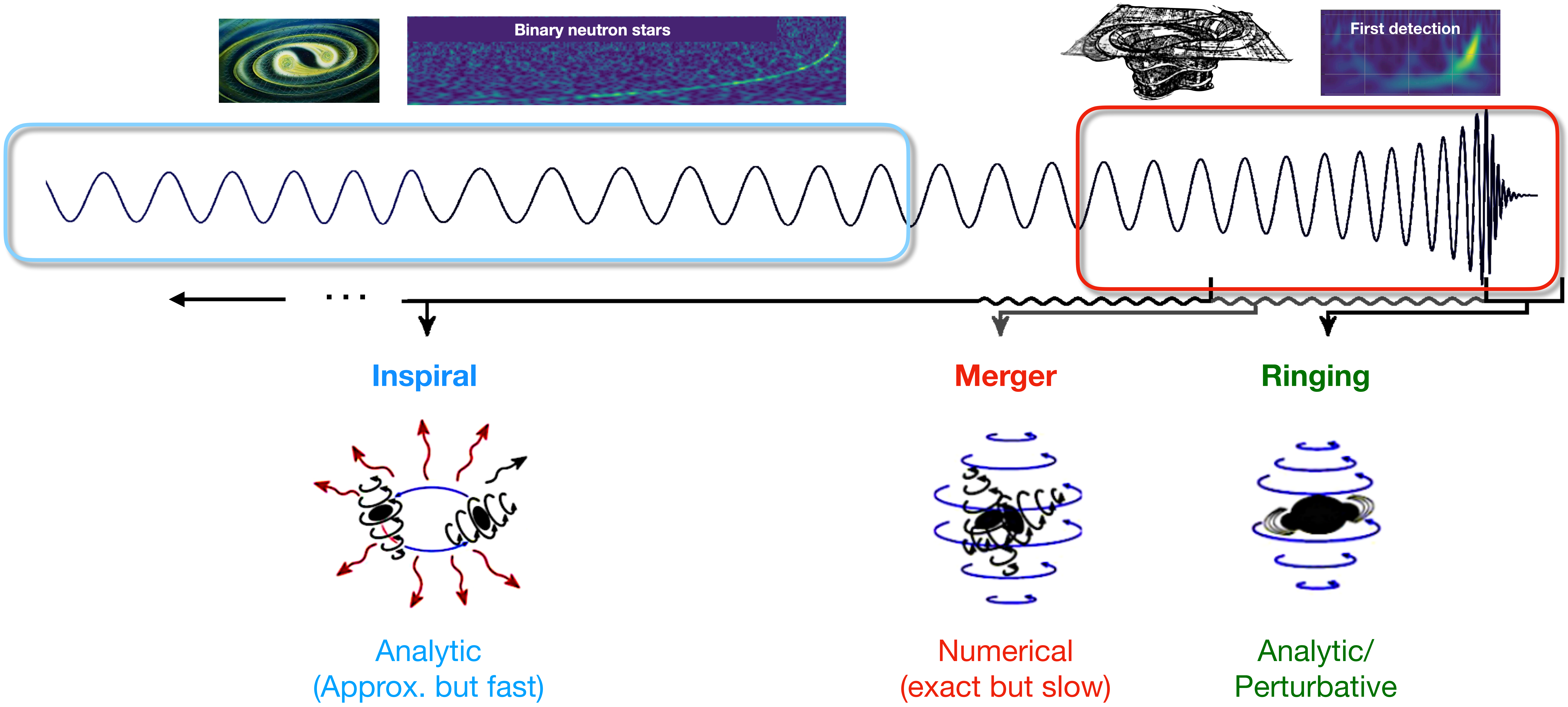
2022 ~~1993~~

Precise Theoretical Predictions

"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the gravitational wave's information"

Challenge in GW Science

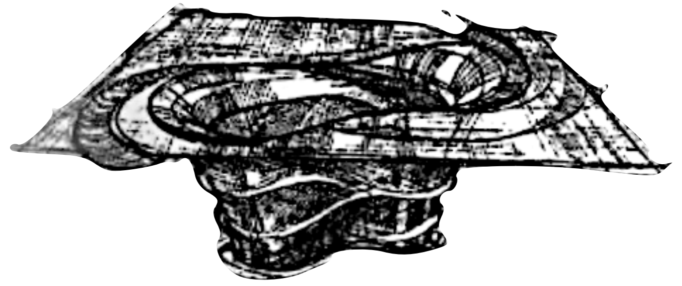
$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{\alpha l} \Gamma^l_{i\alpha}\Gamma^\alpha_{ml} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right)$$

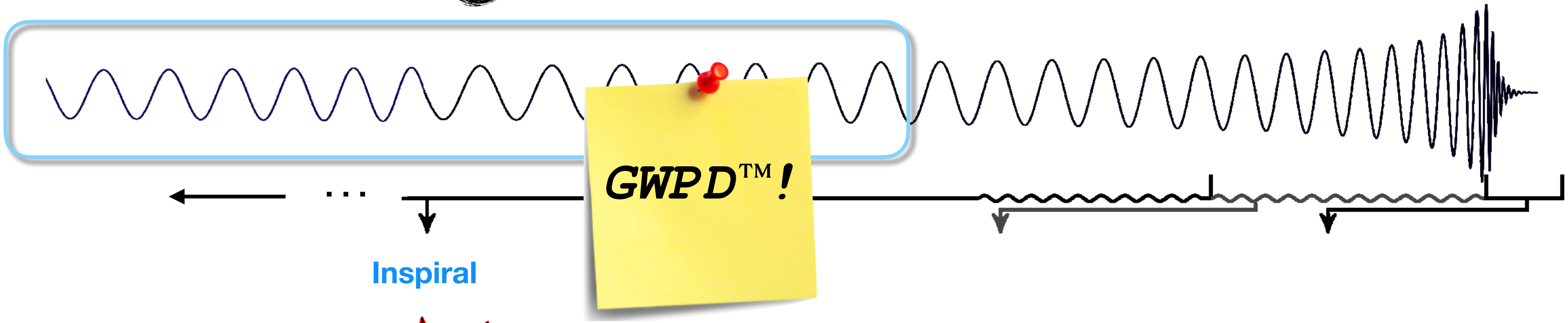


‘GW Precision Data’

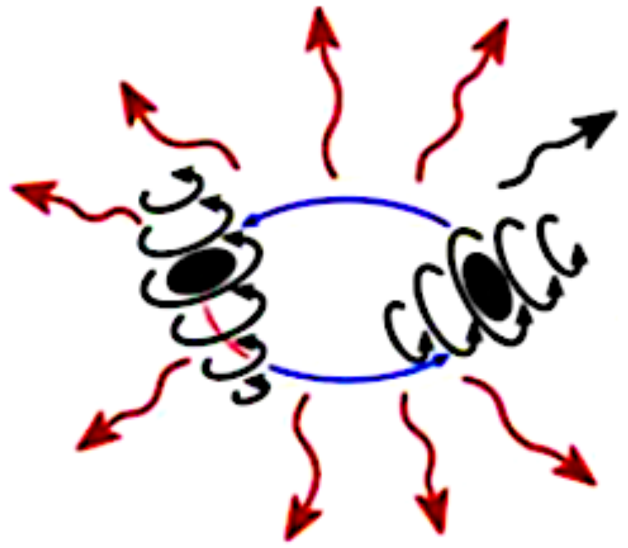
1000+ cycles in band @ Design-Sensitivity

100+ events per year!





Inspiral



Post-Newtonian

$$n\mathrm{PN} = \mathcal{O}(v^{2n})$$

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1000+ cycles in band @ Design-Sensitivity

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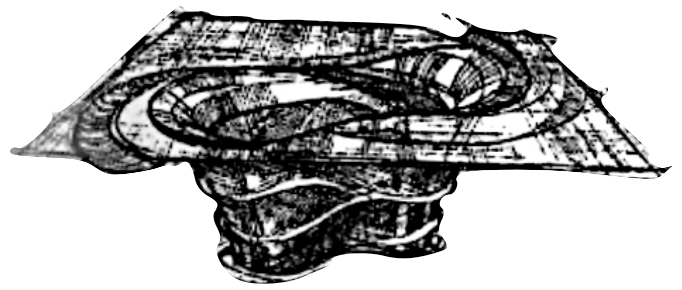
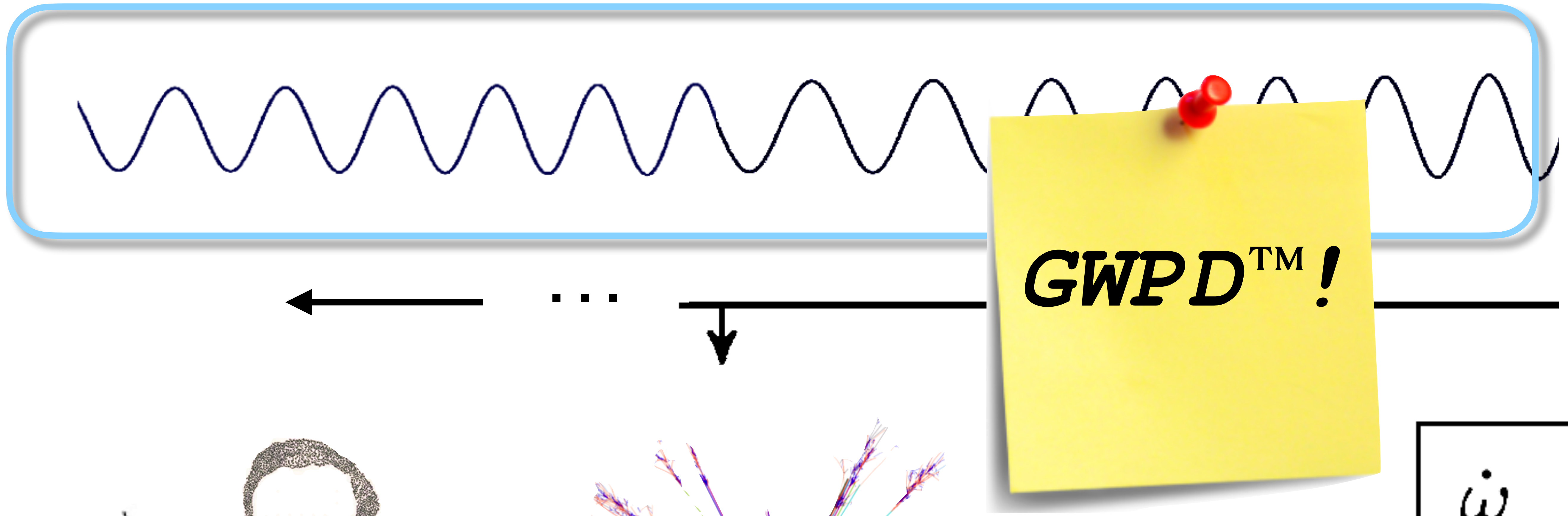
state of the art





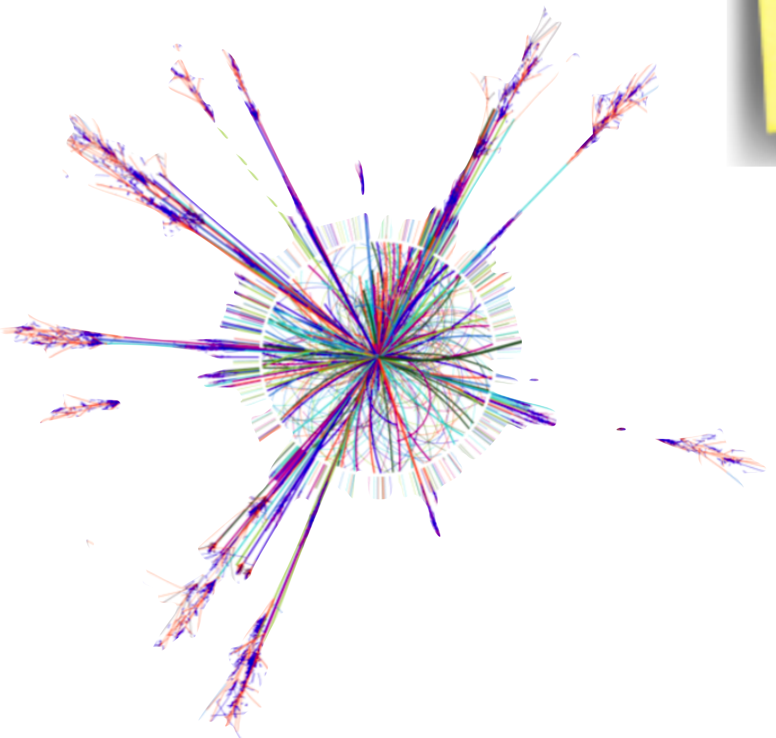
3.5PN order
(almost 4PN)

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$

$$4\pi R^2 \mathcal{F} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right]$$

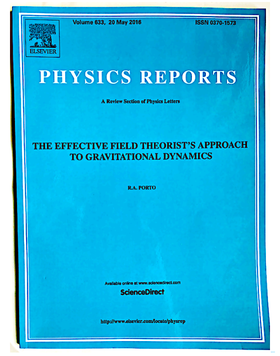


The effective field theorist’s approach to gravitational dynamics

Rafael A. Porto

Physics Reports

Volume 633, 20 May 2016, Pages 1-104



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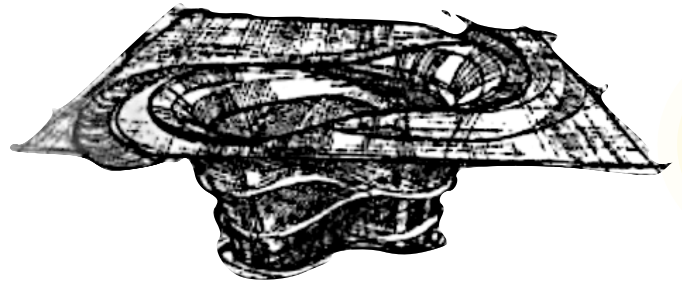
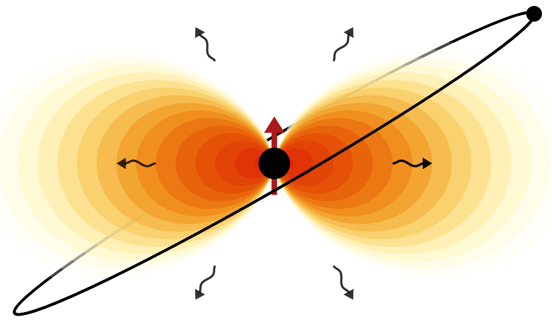
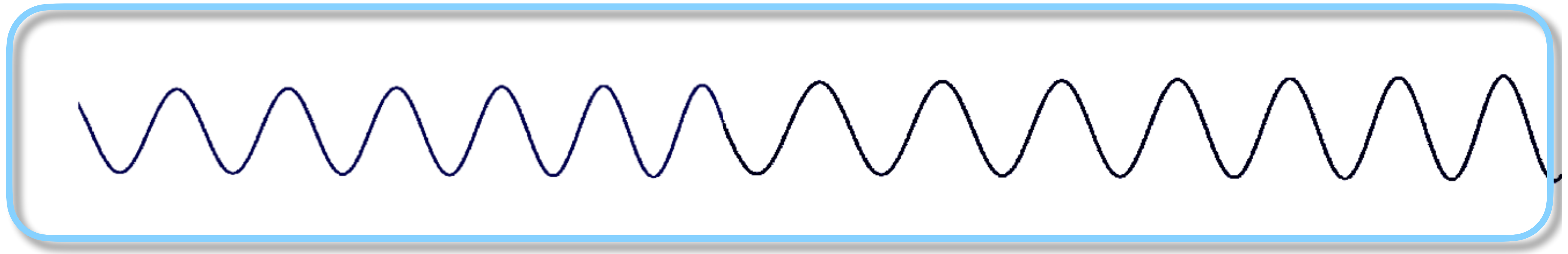
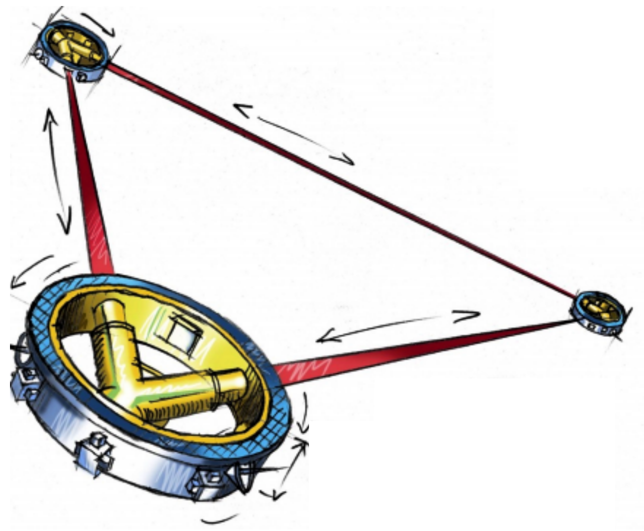




Are we ready for the future?





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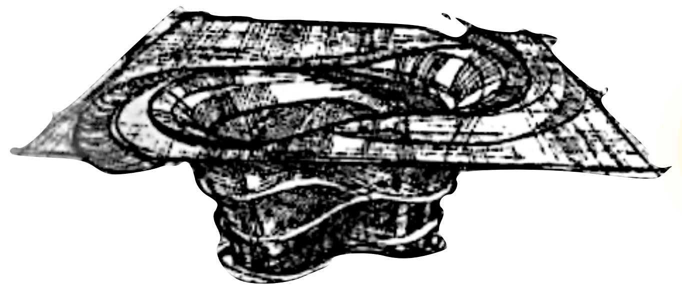
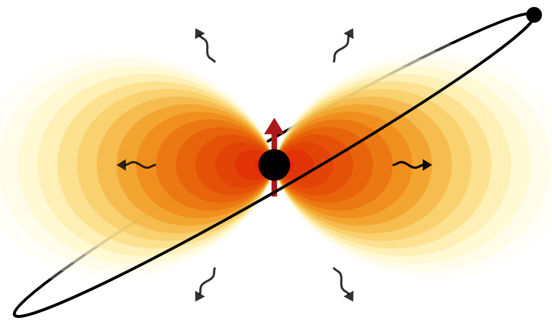
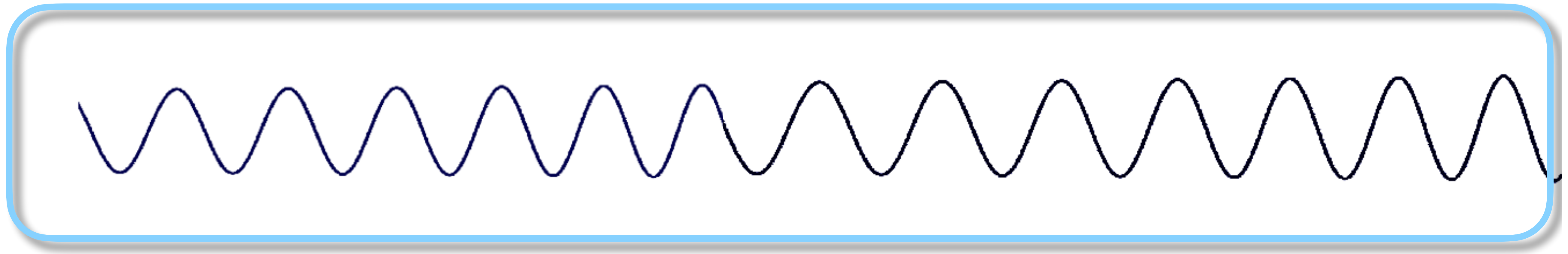
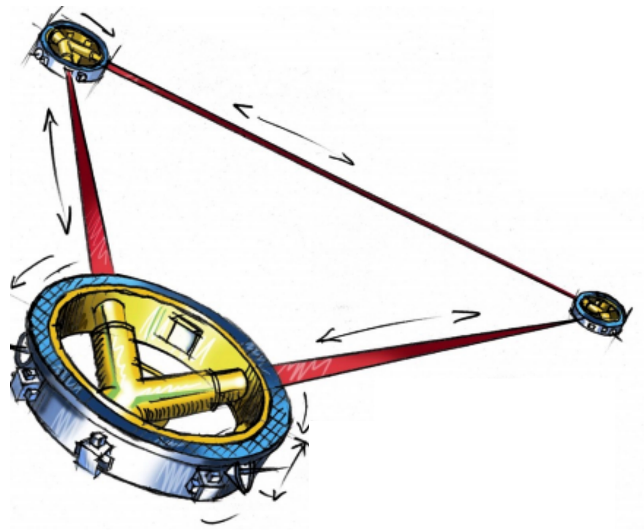
Theoretical uncertainties dominate over planned empirical reach





NOT GOOD ENOUGH



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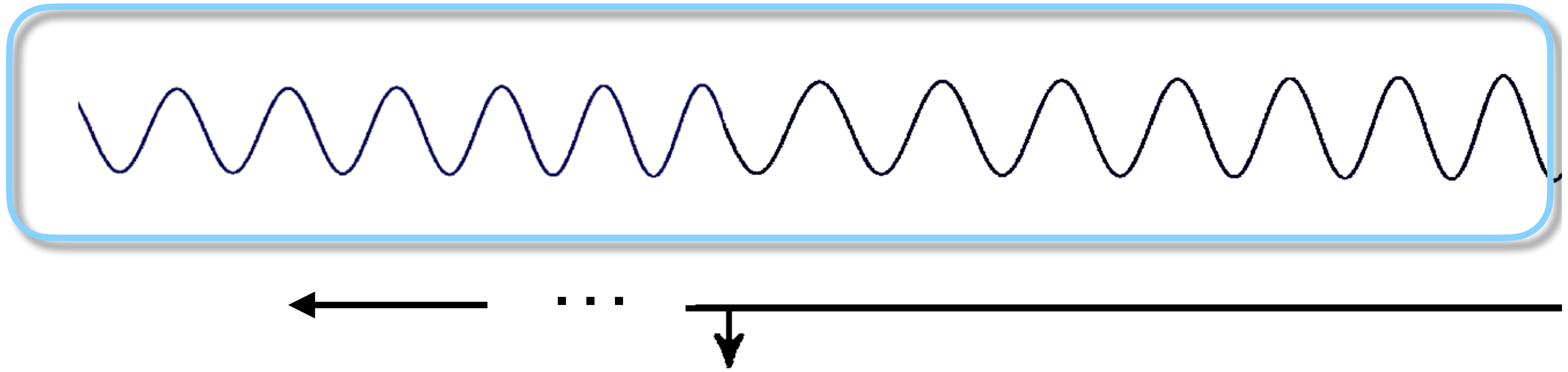
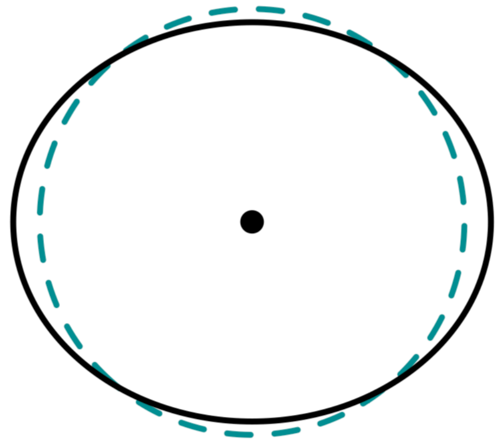
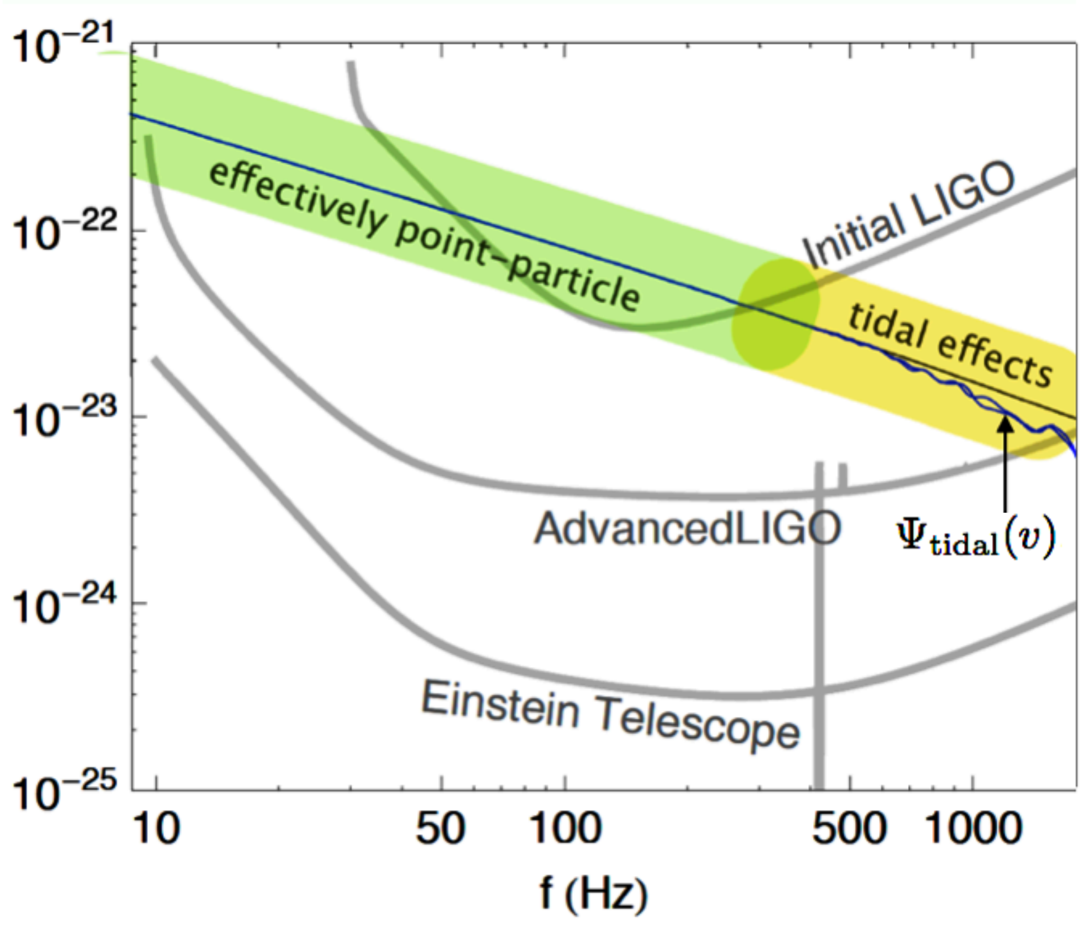
- **Gravitational-wave experiments on ground and in space** require **more accurate waveform** models: new **theoretical challenges** and **opportunities**.

We haven’t reached the analytic precision to distinguish between compact bodies!





(“susceptibility”)



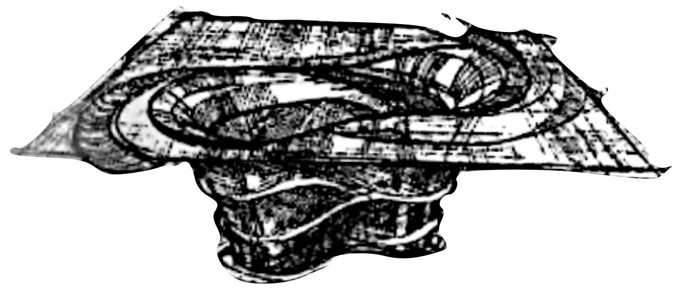
$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

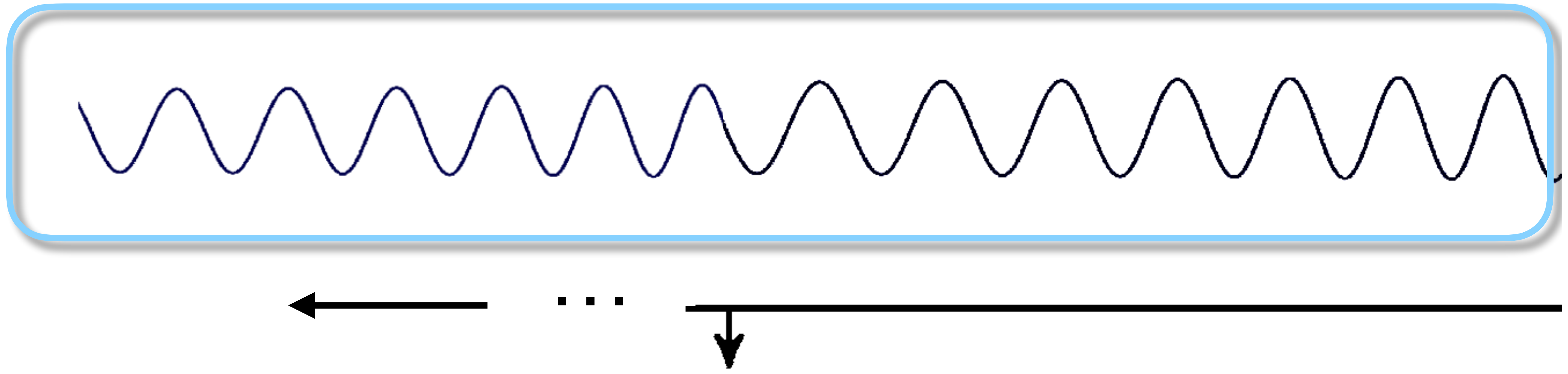
N⁵LO
5PN

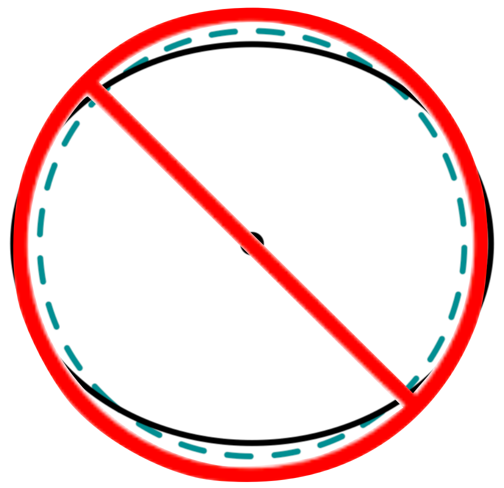
$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

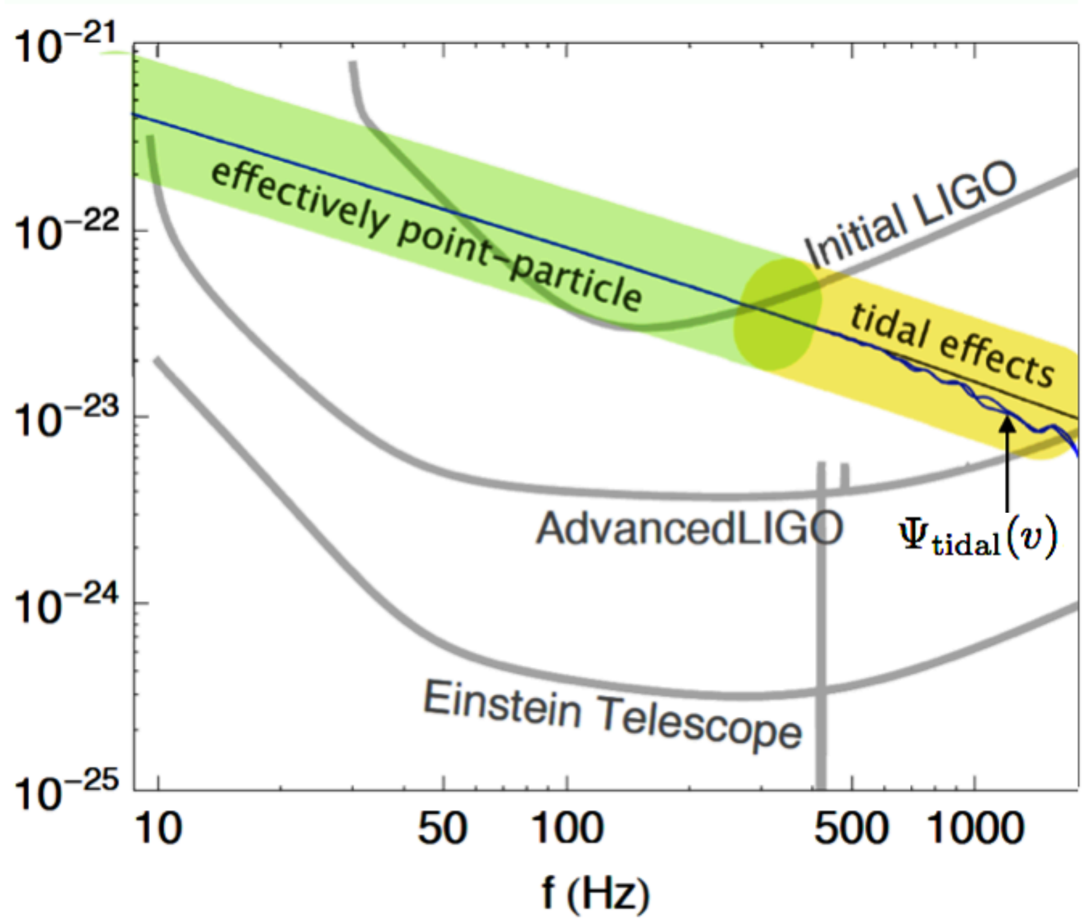
e.g. Equation of State of Neutron Stars

We haven’t reached the analytic precision to distinguish between compact bodies!









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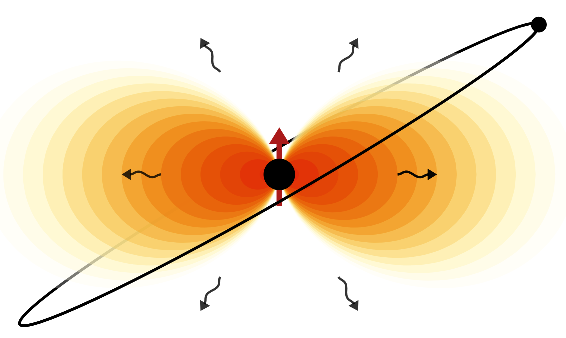
vanishes for black-holes in Einstein’s gravity (4d)

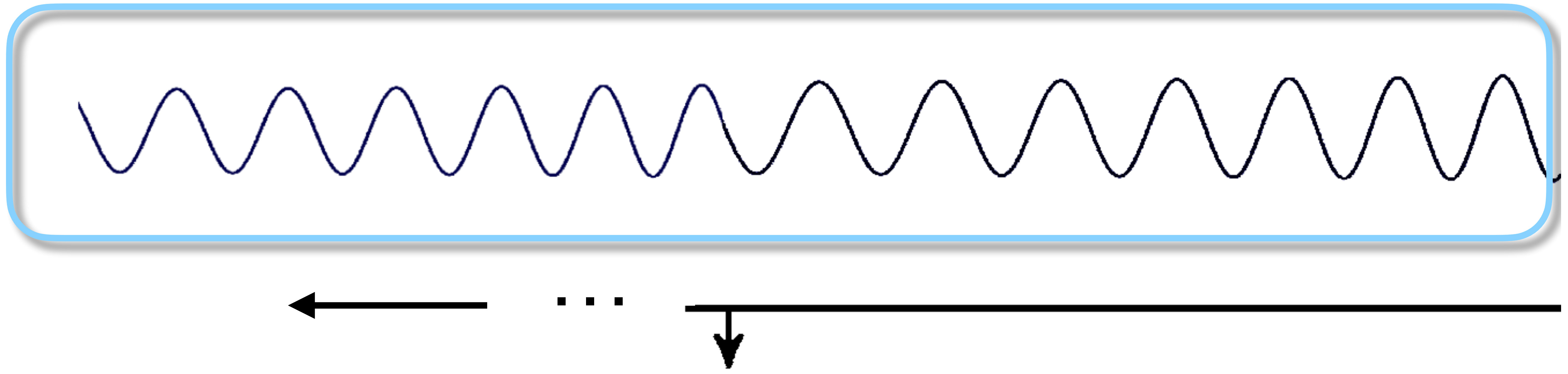
Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

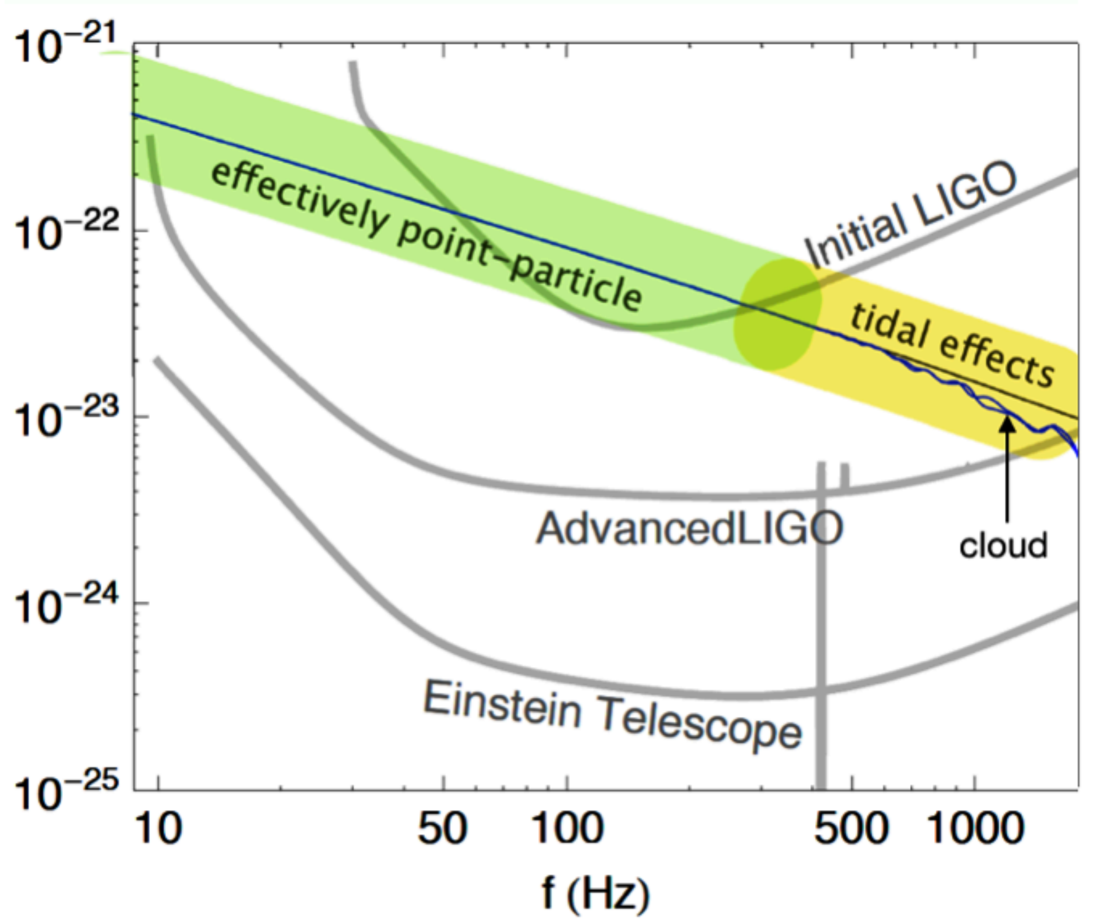
The tune of love and the *nature(ness)* of spacetime

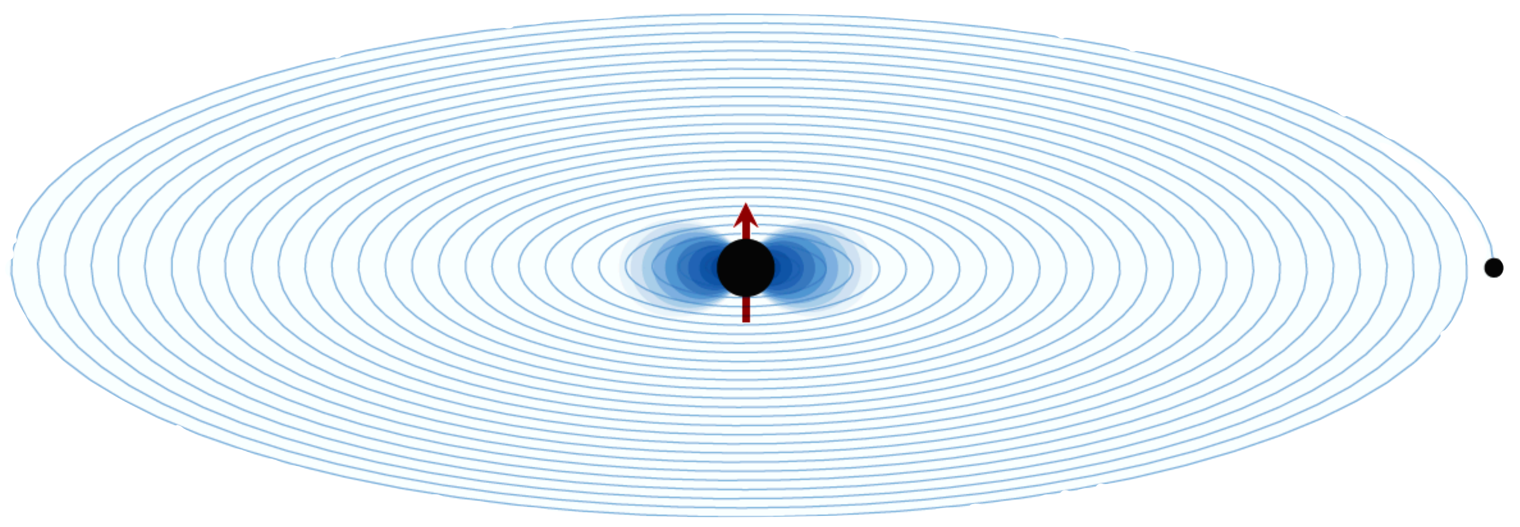
*Rafael A. Porto**

We haven't reached the analytic precision to distinguish between compact bodies!









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'New Physics'
Threshold

'Standard Model' Background!

Probing ultralight bosons with binary black holes
Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto
Phys. Rev. D 99, 044001 (2019)
Published February 4, 2019

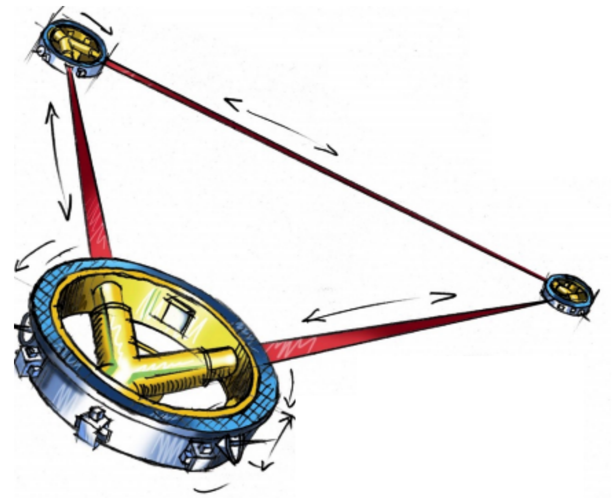
Physics See Synopsis:
Black Holes Could Reveal New Ultralight Particles

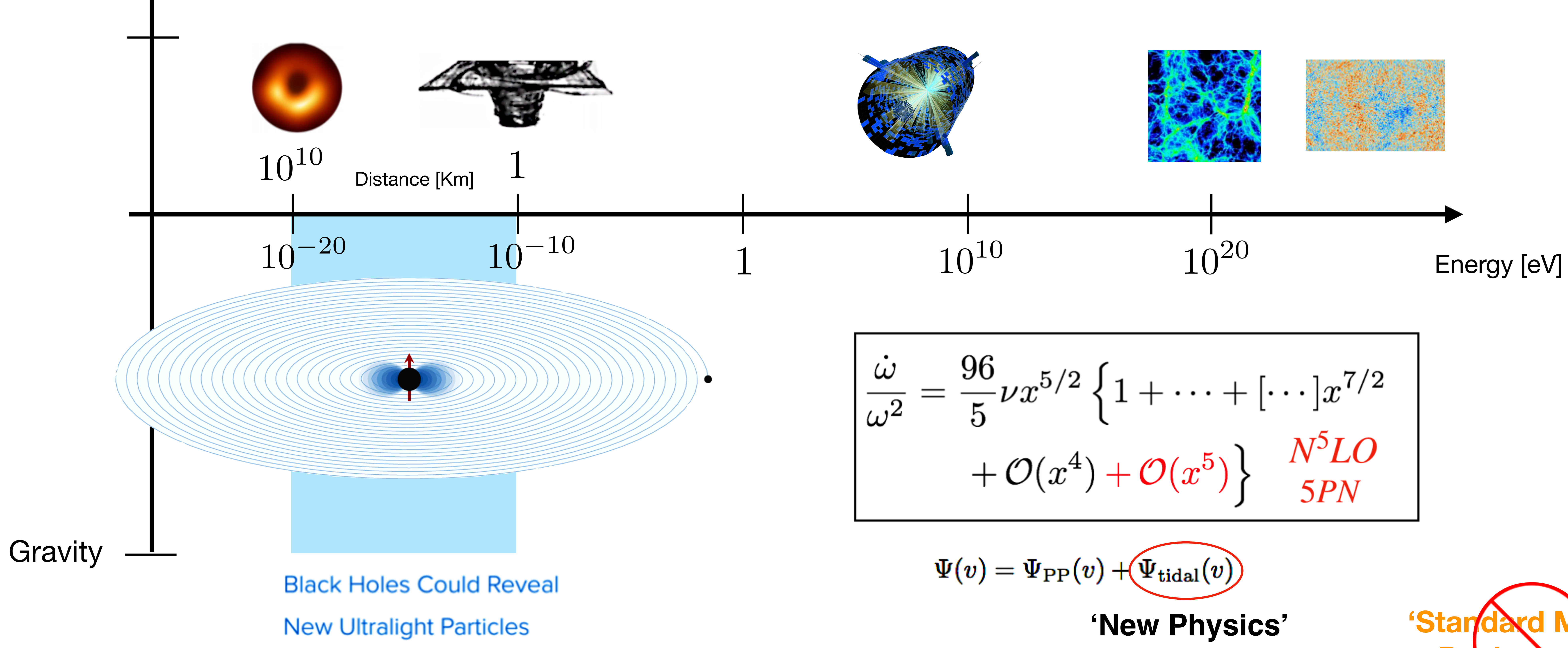
NEW frontier in particle physics

Gravitational Collider Physics

Baumann Chia **RAP** 19’

Baumann Chia **RAP** Stout 20’





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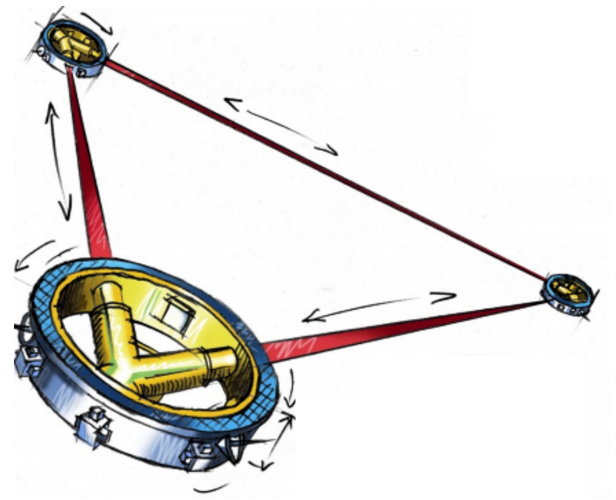
‘New Physics’
Threshold

‘Standard Model’
Background!

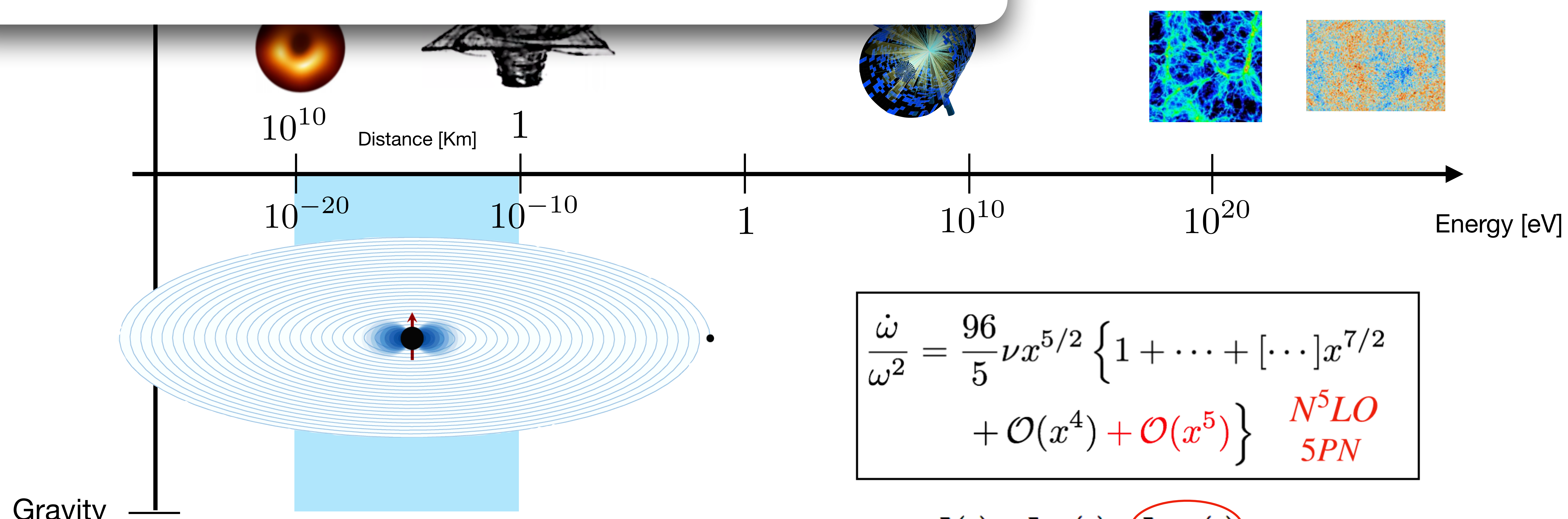
NEW frontier in particle physics



Gravitational Collider Physics

Discovery Potential =

Precise Theoretical Predictions



Black Holes Could Reveal
New Ultralight Particles

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$$\Psi(v) = \Psi_{PP}(v) + \Psi_{tidal}(v)$$

'New Physics'
Threshold

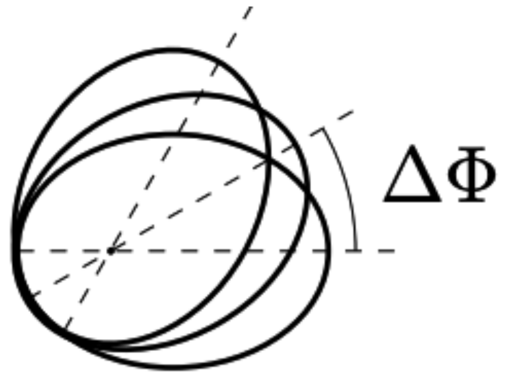
'Standard Model' Background!

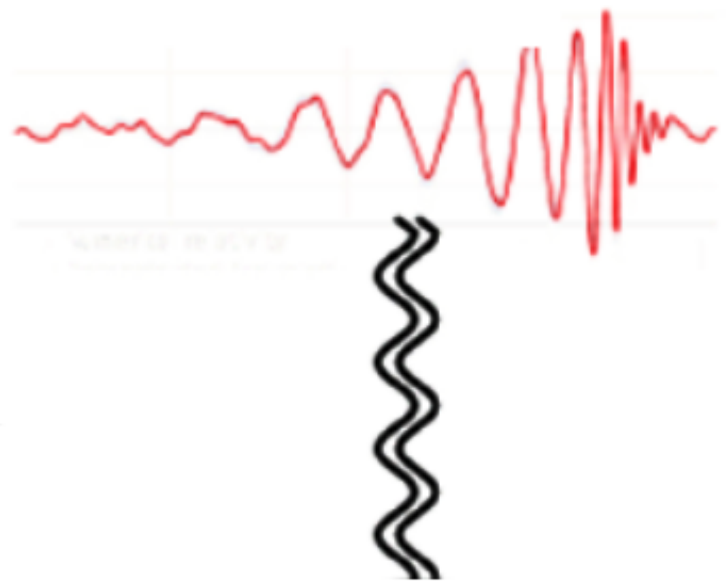
Outline remaining of the talk…

Discovery Potential =

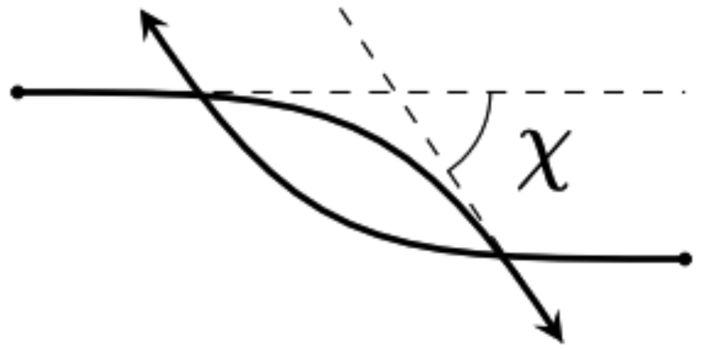
Precise Theoretical Predictions

- **Part I: Bound/Unbound**



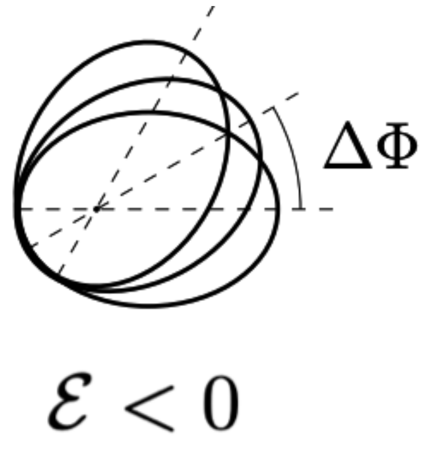


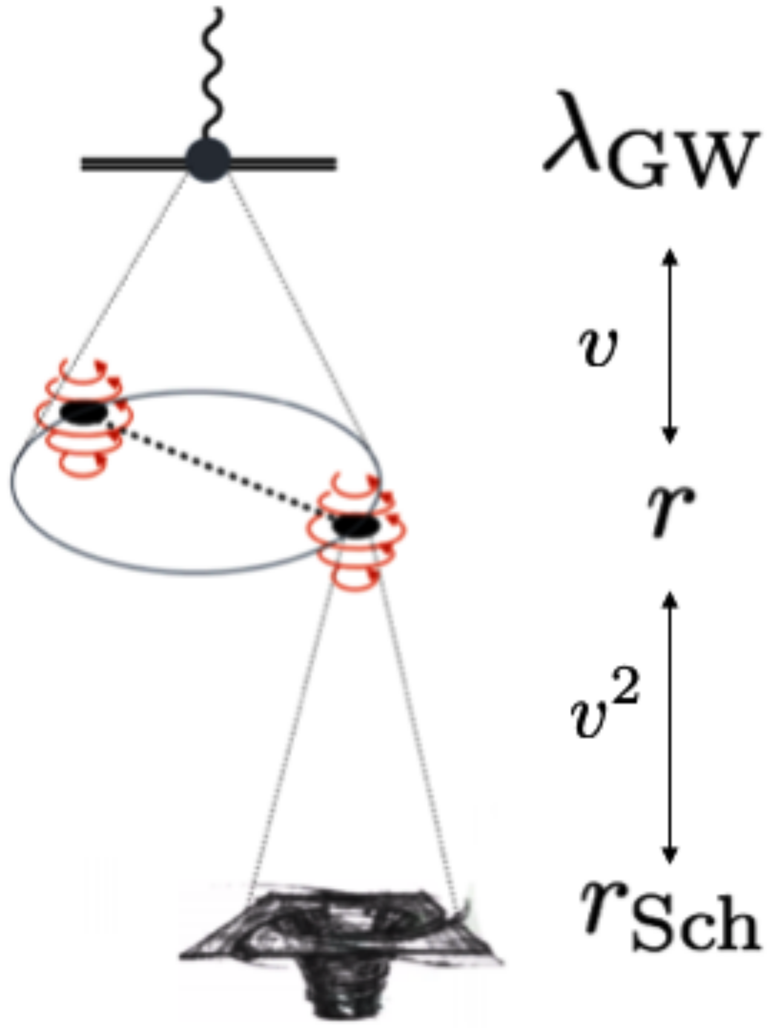
- **Part II: Boundary2Bound**



Goldberger Rothstein (2006)
Porto (2006)
Goldberger Ross (2009)

EFT approach to GW physics *PN*





virial theorem

$$v^2 \sim \frac{Gm}{r}$$

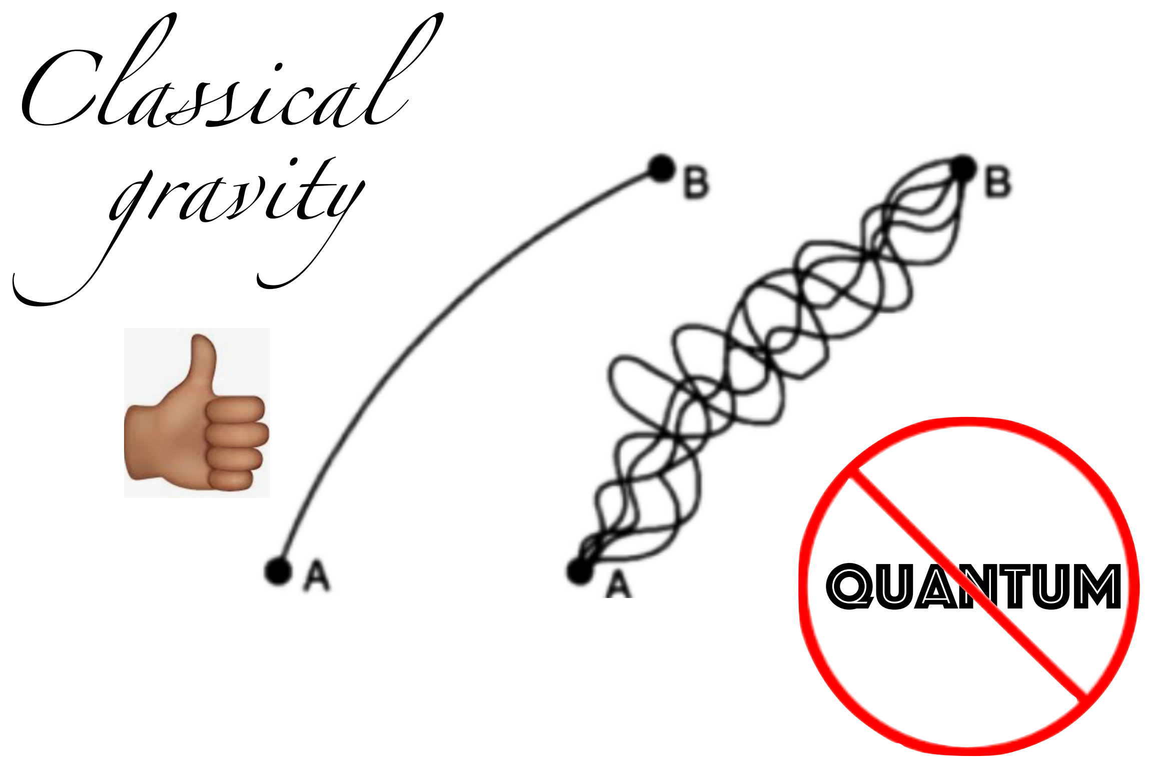
- **Separation of Scales for PN sources:**

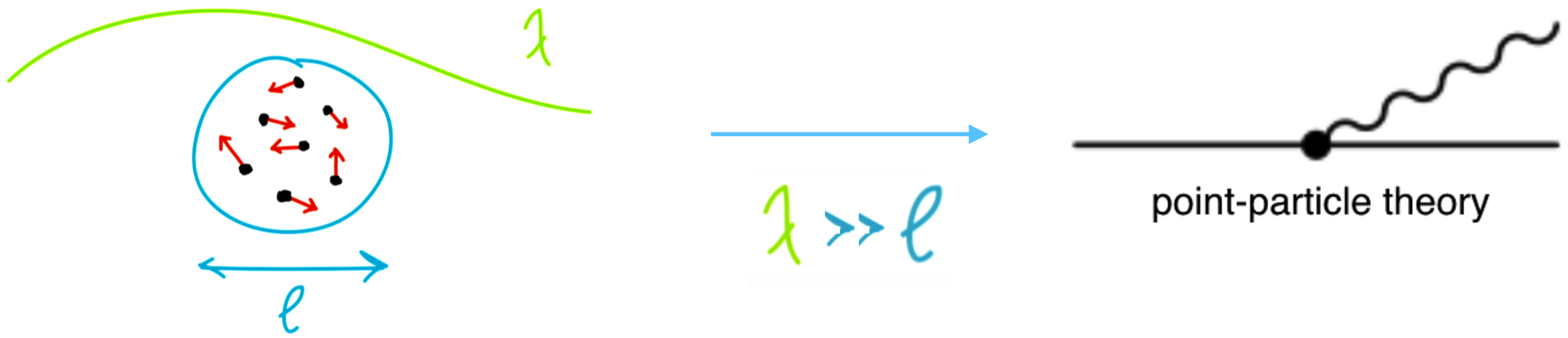
$$r_{\rm Sch} \ll r \ll \lambda_{\rm GW}$$

- **Effective Field Theory:**
  One scale at a time ("method of regions")

$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

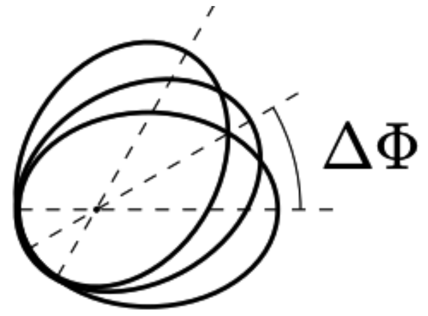
Radiation Modes | Potential Modes | Finite Size

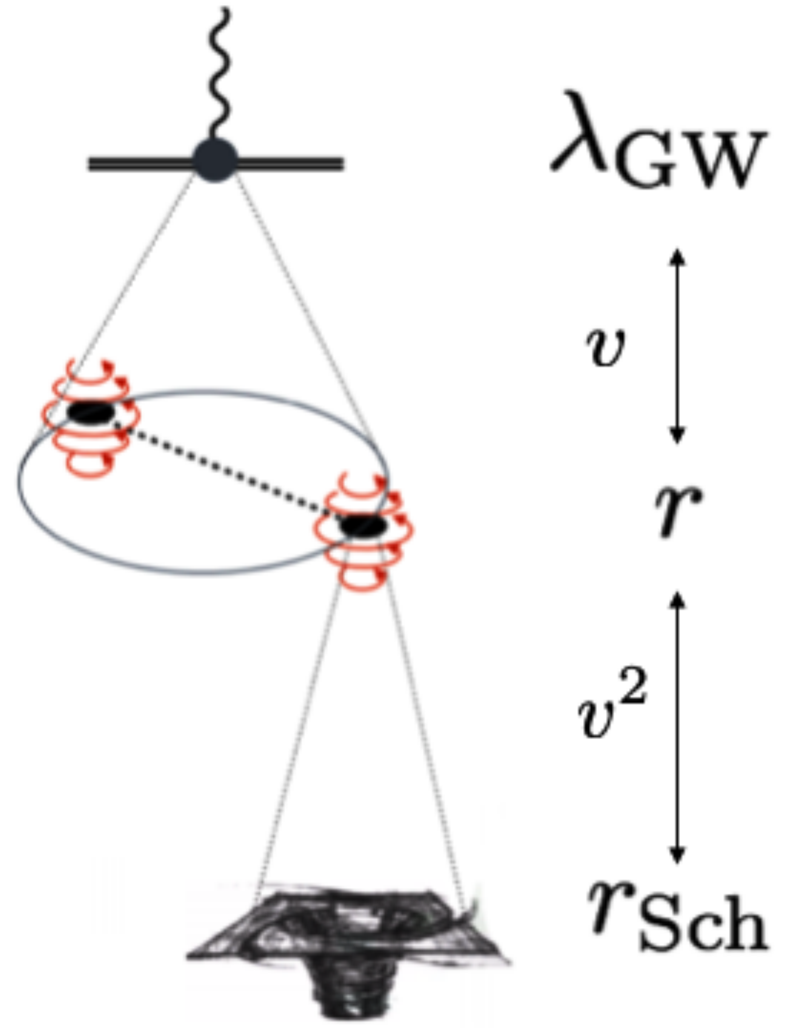




** Conservative non-spinning for simplicity*

EFT approach to GW physics *PN*



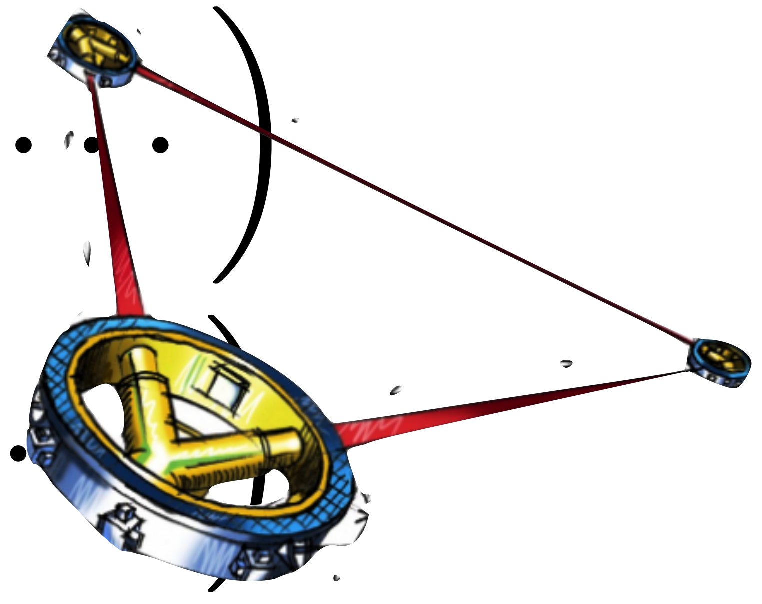


virial theorem

$$v^2 \sim \frac{Gm}{r}$$

Halley Hooke **Newton** (16XX)	Droste EIH (1917)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa et al.** (2015-19')	'New Physics Threshold'	
0PN	1PN	2PN	3PN	4PN	5PN	6PN

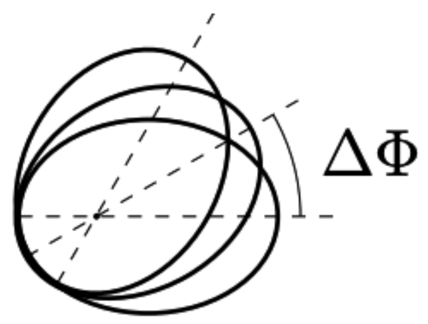
$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right)$$
$$G^2\left(1+v^2+v^4+v^6+v^8+\cdots\right)$$
$$G^3\left(1+v^2+v^4+v^6+\cdots\right)$$
$$G^4\left(1+v^2+v^4+\cdots\right)$$
$$G^5\left(1+v^2+\cdots\right)$$
$$\vdots$$



Einstein Telescope

Cho **RAP** Yang 2201.05138 **(4PN spin including fluxes!)**

EFT approach to GW physics *PN*



	0PN	1PN	2PN	3PN	4PN	5PN	6PN
	Halley Hooke **Newton** (16XX)	Droste EIH (1917)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa et al.** (2015-19')		

$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right)$$

$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right)$$

$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right)$$

$$G^4\left(1 + v^2 + v^4 + \cdots\right)$$

$$G^5\left(1 + v^2 + \cdots\right)$$

$$\vdots$$

Lorentz-Droste potential
(aka Einstein-Infeld-Hoffmann)

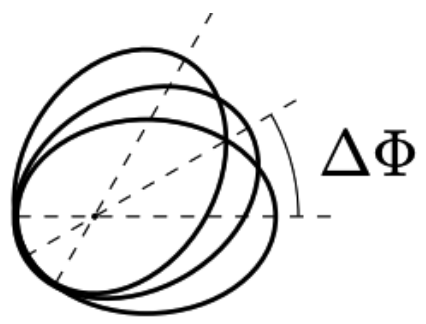
$$V(v,r) \sim -\frac{Gm^2}{r}\left(1 + v^2 + \frac{Gm}{r} + \cdots\right)$$

TABLE OF SURFACE INTEGRALS FOR $\int_0^1 \Lambda_{ms} n_s \, dS$

No.	Expression	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	Result	Remarks
1	$\frac{1}{m}g_{,s}\dot\eta^s\dot\eta^m$	$-\frac{16}{3}$				$-\frac{4}{3}$		$-\frac{8}{3}$	$-\frac{4}{15}$		$\frac{8}{15}$	$\frac{4}{15}$				$\frac{4}{5}$			-8	$\bar g_{,s} = -2\overset{2}{m}\frac{\partial_r^1}{\partial\eta^s}$
2	$\frac{1}{m}\bar g\ddot\eta^m$	-2						-4	$-\frac{4}{3}$			$-\frac{29}{3}$	3	$\frac{11}{3}$	$-\frac{5}{3}$	$\frac{2}{3}$	$\frac{32}{3}$	$-\frac{22}{3}$	-8	$\bar g = -\frac{2\overset{2}{m}}{r}; \ddot\eta^m = -\frac{1}{2}\bar g_{,m}$
3	$\frac{1}{m}\bar g_{,m}\dot\eta^s\dot\eta^s$	1					$-\frac{4}{3}$	$-\frac{4}{5}$			$\frac{8}{5}$	$\frac{4}{5}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{15}$			2	$\bar g_{,m} = -2\overset{1}{m}\ddot\eta^m$
4	$\frac{1}{m}\bar g_{,m}\dot\zeta^s\dot\zeta^s$											2	$\frac{1}{3}$	1	$-\frac{1}{3}$				3	
5	$\frac{1}{m}\bar g_{,m}\bar f$	$\frac{4}{3}$				2	$\frac{2}{3}$					$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$				5	$\bar g_{,m}\bar f = -\bar g\bar f_{,m}; \bar f = -\frac{2\overset{1}{m}}{r}$
6	$\frac{1}{m}\overset{2}{m}\bar r_{,ssm}$											-2							-2	$\bar r_{,ssm} = (\overset{2}{\bar r}_{,ssm})$ for $x^s = \eta^s$ *
7	$\frac{1}{m}\bar g_{,s}\dot\zeta^m\dot\eta^s$	$\frac{16}{5}$				$\frac{8}{3}$	$\frac{4}{5}$	$\frac{4}{3}$											8	
8	$\frac{1}{m}\bar g_{,s}\dot\zeta^s\dot\eta^m$	$\frac{16}{5}$					$-\frac{8}{15}$	4	$\frac{4}{3}$	-2									6	
9	$\frac{1}{m}\bar g_{,m}\dot\eta^s\dot\zeta^s$	$-\frac{32}{15}$		$-\frac{16}{3}$	$-\frac{8}{3}$		$\frac{4}{5}$		$\frac{4}{3}$										-8	
10	$\frac{1}{m}\bar g_{,s}\dot\zeta^s\dot\zeta^m$	$-\frac{8}{3}$				-4	$-\frac{4}{3}$												-8	

Goldberger Rothstein (2006)

EFT approach to GW physics *PN*



GR accounts for the
'anomalous' precession
(Precision measurements are key!)

0PN	1PN	2PN	3PN	4PN	5PN	6PN
Halley Hooke **Newton** (16XX)	Droste EIH (1917)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa et al.** (2015-19')		

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$$\vdots$$

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(aka Einstein-Infeld-Hoffmann)

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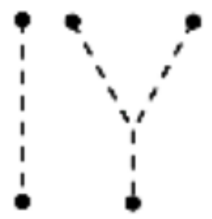
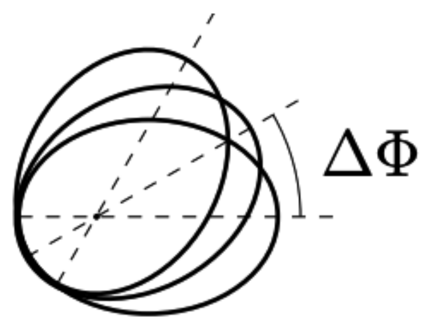


$$\frac{1}{p_0^2 - \boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1 + \frac{p_0^2}{\boldsymbol{p}^2} + \cdots\right).$$

Classical 'loop' =
iterated Green's function
+ point-like sources

EFT approach to GW physics *PN*



Galley Leibovich
RAP Ross
1511.07379

RAP Rothstein
1703.06433

Foffa **RAP**
Rothstein Sturani
1903.05118

0PN	1PN	2PN	3PN	4PN	5PN	6PN
Halley Hooke **Newton** (16XX)	Droste EIH (1917)	Chandra, Ohta et al. (70's)	Blanchet, Damour, et al. (00')	Damour et al., Blanchet et al. **Foffa et al.** (2015-19')		

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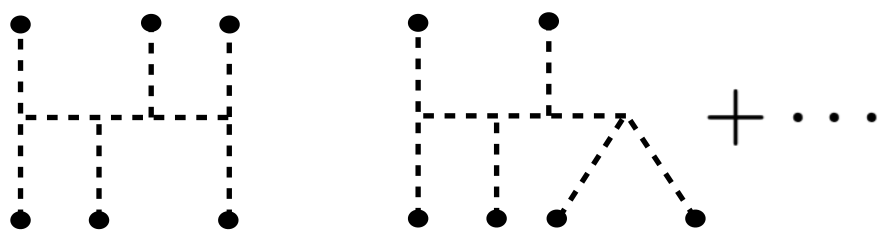
$$G^2\left(1+v^2+v^4+v^6+v^8+\cdots\right)$$

$$G^3\left(1+v^2+v^4+v^6+\cdots\right)$$

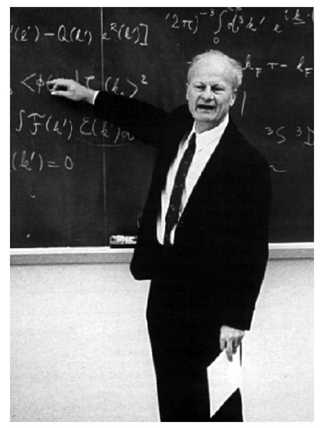
$$G^4\left(1+v^2+v^4+\cdots\right) \longleftarrow \log v$$

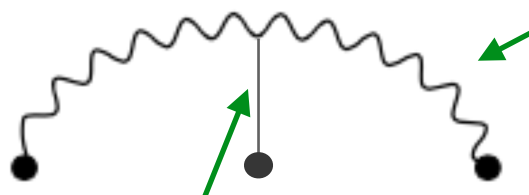
$$G^5\left(1 + \right.$$

"Tail effect"
(scattering off the geometry sourced by the binary)



$$\equiv \int_{k_1,k_2,k_3,k_4} \frac{N_{49}}{k_1^2\, p_2^2\, k_3^2\, p_4^2\, k_{12}^2\, k_{13}^2\, k_{23}^2\, k_{24}^2\, k_{34}^2},$$



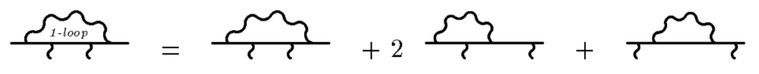


PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

EFT approach to Atomic physics



$$(e^2/4\pi)\left(\frac{1}{2m}(qa-aq)+\frac{4q^2}{3m^2}a\left(\ln\frac{m}{\lambda_{min}}-\frac{3}{8}\right)\right), \quad (24)$$

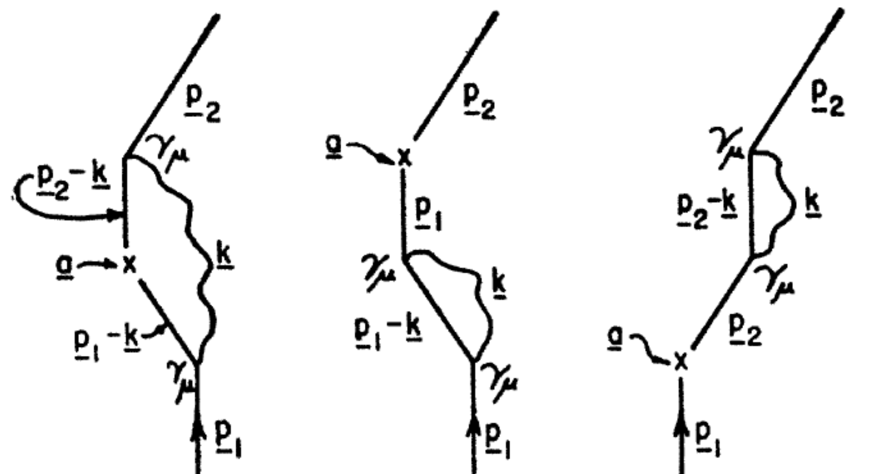
which shows the change in magnetic moment and the Lamb shift as interpreted in more detail in B.[13]

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{max}-1=\ln\lambda_{min}$ used by the author should have been $\ln 2k_{max}-5/6=\ln\lambda_{min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf, The author feels unhappily responsible for the very considerable delay in the publication of French's result occasioned by this error. This footnote is appropriately numbered.

Space-Time Approach to Quantum Electrodynamics

R. P. FEYNMAN
Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)



H. A. Bethe, The electromagnetic shift of energy levels, Phys. Rev. **72**, 339 (1947).
F. J. Dyson, The electromagnetic shift of energy levels, Phys. Rev. **73**, 617 (1948).
J. B. French and V. F. Weisskopf, The electromagnetic shift of energy levels, Phys. Rev. **75**, 1240 (1949).
N. M. Kroll and W. E. Lamb, On the self-energy of a bound electron, Phys. Rev. **75**, 388 (1949).

PHYSICAL REVIEW D **96**, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

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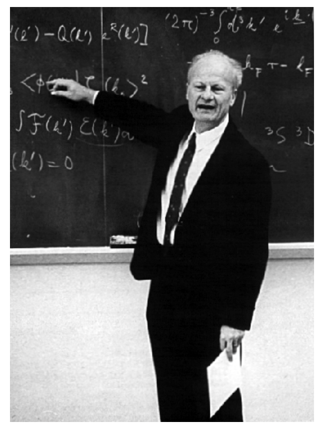

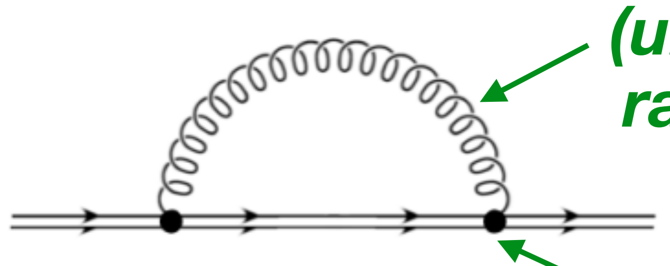

Galley Leibovich
RAP Ross
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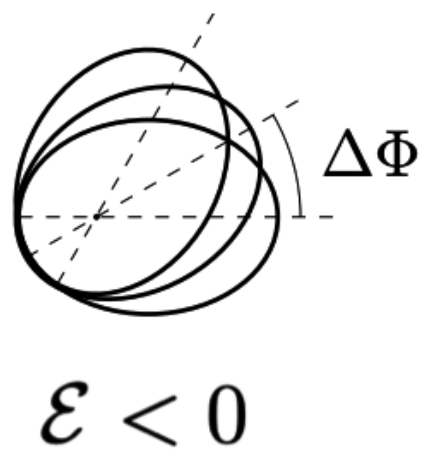
Foffa **RAP**
Rothstein Sturani
1903.05118

Bluemlein
Marquard Meier
2110.13822

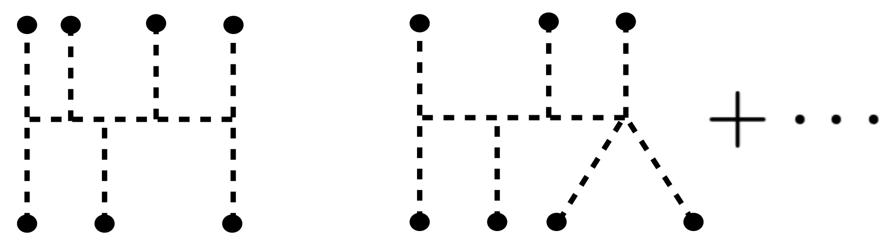
Foffa Sturani
2110.14146

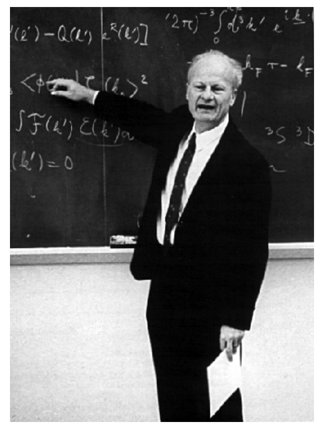
EFT approach to GW physics *PN*



0PN	1PN	2PN	3PN	4PN	5PN	6PN

$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right)$$
$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right)$$
$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right)$$
$$G^4\left(1 + v^2 + v^4 + \cdots\right)$$
$$G^5\left(1 + v^2 + \cdots\right)$$
$$1$$

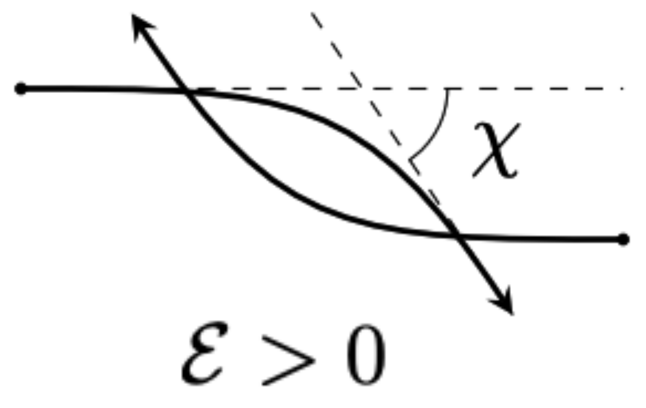




non-linear memory effect (scattering off the earlier radiation)

Kalin **RAP**
2006.01184

EFT approach to GW physics *PM*



0PN	1PN	2PN	3PN	4PN	5PN	6PN

$$G\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) \quad \text{1PM}$$

$$G^2\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) \quad \text{2PM}$$

$$G^3\left(1 + v^2 + v^4 + v^6 + \cdots\right) \quad \text{3PM}$$

$$G^4\left(1 + v^2 + v^4 + \cdots\right) \quad \text{4PM}$$

$$G^5\left(1 + v^2 + \cdots\right) \quad \text{5PM}$$

$$\vdots$$

Fully (special) relativistic integration problem!

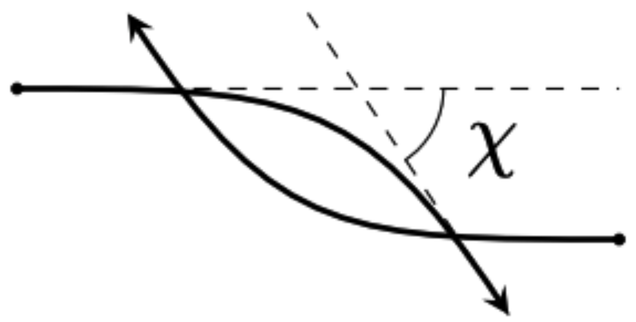
$$\frac{Gm}{b} \ll 1$$

Post-Minkowskian expansion

$$e^{iW} = \int \underbrace{D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]}_{\text{classical 'soft' region}} D[r_s^{-1}]\, e^{iS_{\rm full}}$$

classical
'soft' region

EFT approach to GW physics *PM*



$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$E = M(1+\nu\mathcal{E})$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)}\,,$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN		
	$G\big(1$	$+\,v^2$	$+\,v^4$	$+\,v^6$	$+\,v^8$	$+\,v^{10}$	$+\cdots\big)$	1PM
		$G^2\big(1$	$+\,v^2$	$+\,v^4$	$+\,v^6$	$+\,v^8$	$+\cdots\big)$	2PM
			$G^3\big(1$	$+\,v^2$	$+\,v^4$	$+\,v^6$	$+\cdots\big)$	3PM
				$G^4\big(1$	$+\,v^2$	$+\,v^4$	$+\cdots\big)$	4PM
					$G^5\big(1$	$+\,v^2$	$+\cdots\big)$	5PM

Westphal (1985)

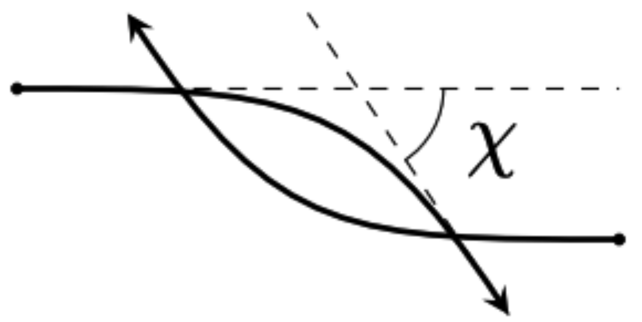
Cheung et al (2019)

Kalin **RAP** (2020)

$$\frac{\chi_b^{(1)}}{\Gamma} = \frac{2\gamma^2-1}{\gamma^2-1}\,,$$

$$\frac{\chi_b^{(2)}}{\Gamma} = \frac{3\pi}{8}\,\frac{5\gamma^2-1}{\gamma^2-1}\,,$$

EFT approach to GW physics *PM*



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$$E = M(1+\nu\mathcal{E})$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)}\,,$$

0PN 1PN 2PN 3PN 4PN 5PN 6PN

Westphal (1985)

Cheung et al (2019)

Kalin **RAP** (2020)

$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right) \quad \text{1PM}$$

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Bern et al (2019)

Kalin Liu **RAP** (2020)

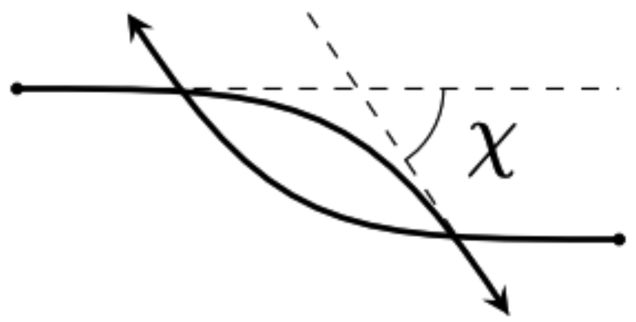
$$G^3\left(1+v^2+v^4+v^6+\cdots\right) \quad \text{3PM}$$

$$G^4\left(1+v^2+v^4+\cdots\right) \quad \text{4PM}$$

$$G^5\left(1+v^2+\cdots\right) \quad \text{5PM}$$

$$\chi_b^{(3)} = \frac{\Gamma^3}{(\gamma^2-1)^{3/2}}\left[\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^{3/2}} - \frac{4}{3}\frac{\nu}{\Gamma^2}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25)\right.$$
$$\left. -8\frac{\nu}{\Gamma^2}(4\gamma^4-12\gamma^2-3)\,\text{arcsinh}\sqrt{\frac{\gamma-1}{2}}\right].$$

EFT approach to GW physics *PM*



'PM-bootstrapping two-body problem' =

*D*ifferential Equations + boundary conditions from **PN!**

$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}} = \frac{1+x^2}{2x}$$

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\, \vec{h}(x,\epsilon)$$

canonical to N2LO!

can be solved in terms of Polylogarithms

2007.04977
Kalin Liu **RAP** (2020)

$$G^3\left(1+v^2+v^4+v^6\ +\cdots\right) \quad \textbf{3PM}$$

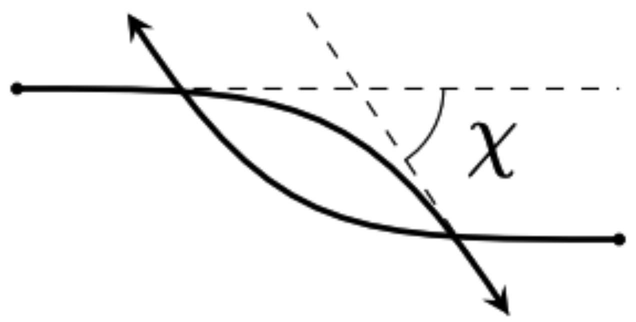
$$G^4\left(1+v^2+v^4\ +\cdots\right) \quad \textbf{4PM}$$

$$G^5\left(1+v^2\ +\cdots\right) \quad \textbf{5PM}$$

$$\chi_b^{(3)} = \frac{\Gamma^3}{(\gamma^2-1)^{3/2}}\left[\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^{3/2}} - \frac{4}{3}\frac{\nu}{\Gamma^2}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25)\right.$$
$$\left. -8\frac{\nu}{\Gamma^2}(4\gamma^4-12\gamma^2-3)\,\text{arcsinh}\sqrt{\frac{\gamma-1}{2}}\right].$$

$\log x$

EFT approach to GW physics *PM*



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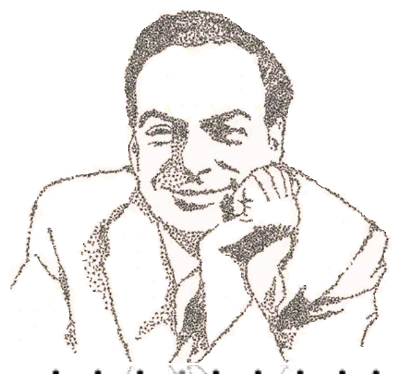
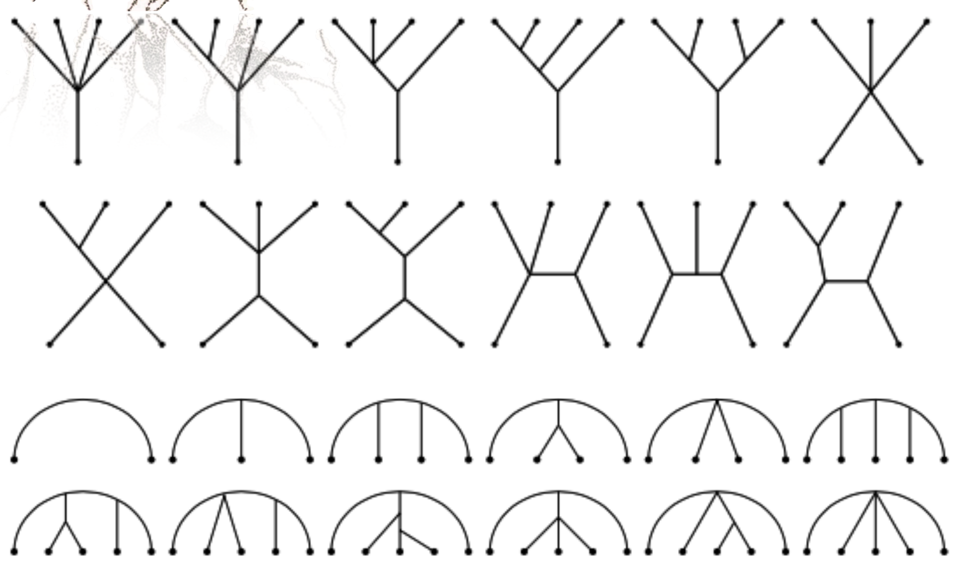
$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$

<u>**Not canonical**</u> at N3LO

Introduces elliptic integrals!





2007.04977

Kalin Liu **RAP** (2020)

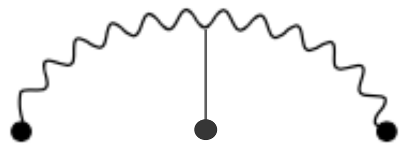
$$G^3\left(1+v^2+v^4+v^6\ +\cdots\right) \quad \text{3PM}$$

Dlapa Kalin Liu **RAP** (2021)

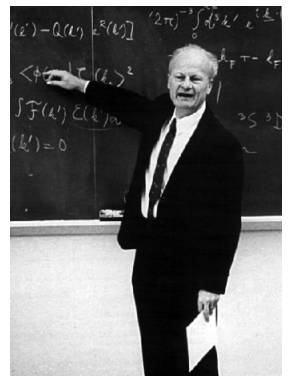
2112.11296

$$G^4\left(1+v^2+v^4\ +\cdots\right) \quad \text{4PM}$$

See also Bern et al. (2112.10750)



"Tail effect"



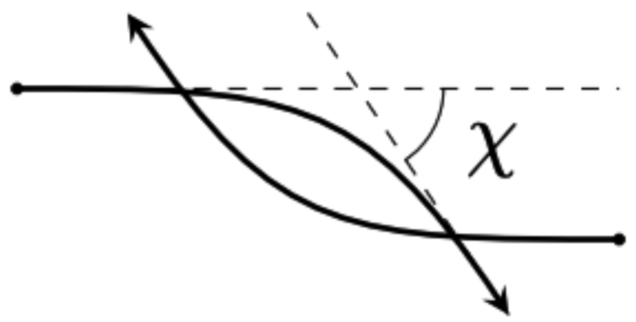
"Bethe logarithm"

$\log v$

$$G^5\left(1+v^2\ +\cdots\right) \quad \text{5PM}$$

$$\frac{\chi^{(4)}_{b\,(\text{comb})}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + 2\chi_{2\epsilon}(x)\log(1-x)\Big),$$

EFT approach to GW physics *PM*



$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$\gamma = \frac{1+x^2}{2x}$$

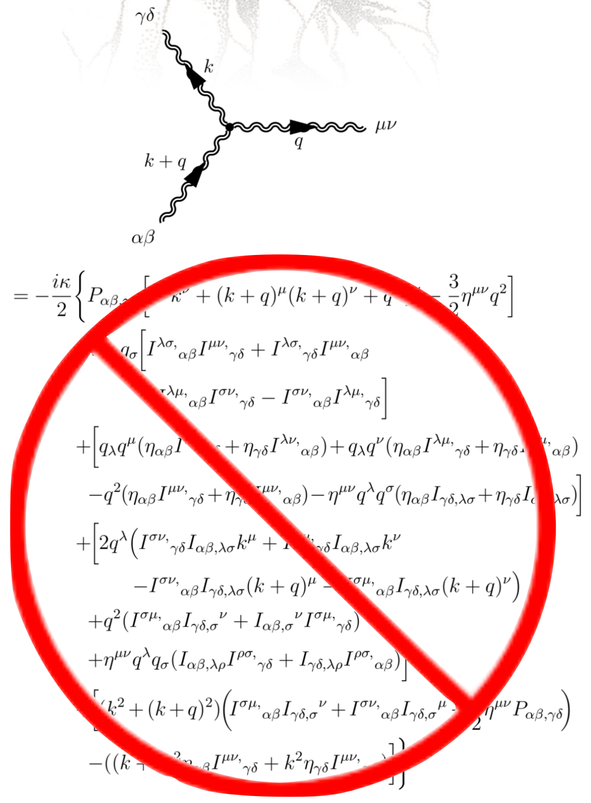
'PM-bootstrapping two-body problem' =

*D*ifferential Equations + boundary conditions from **PN!**

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$

Lots of redundancy

don't panic!



2007.04977
Kalin Liu **RAP** (2020)

$$G^3\left(1+v^2+v^4+v^6\ +\cdots\right) \quad \text{3PM}$$

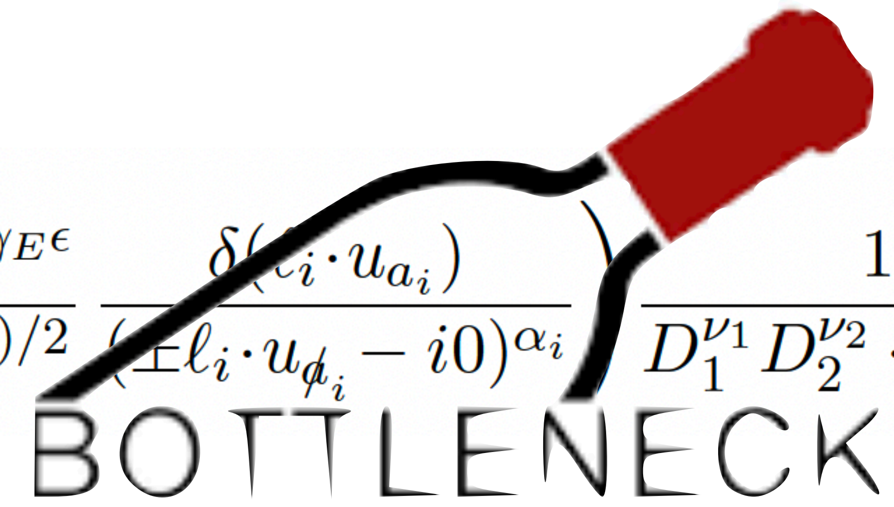
Dlapa Kalin Liu **RAP** (2021)
2112.11296

$$G^4\left(1+v^2+v^4\ +\cdots\right) \quad \text{4PM}$$

$$G^5\left(1+v^2\ +\cdots\right) \quad \text{5PM}$$

$$\vdots$$

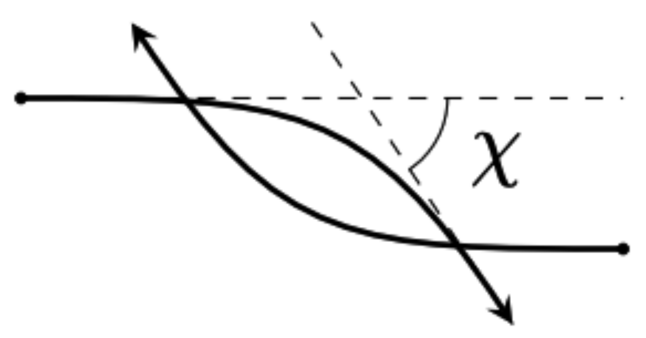
$$\int \left(\prod_{i=1}^{n} \frac{d^D\ell_i\, e^{\gamma_E \epsilon}}{\pi^{(D-1)/2}} \frac{\delta(\ell_i\cdot u_{a_i})}{(\pm \ell_i \cdot u_{\phi_i} - i0)^{\alpha_i}}\right) \frac{1}{D_1^{\nu_1} D_2^{\nu_2}\cdots D_N^{\nu_N}},$$



Dlapa Kalin Liu **RAP**

2106.08276
2112.11296

EFT approach to GW physics *PM*



$$\gamma \equiv u_1 \cdot u_2 = \frac{1}{\sqrt{1-v^2}}$$

$$\gamma = \frac{1+x^2}{2x}$$

Combined (pot+tail) result **at 4PM** includes ***logarithms***, ***dilogarithms***

and ***elliptic integrals*** of the first and second kind

$$\frac{\chi^{(4)}_{b\,(\mathrm{comb})}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + 2\chi_{2\epsilon}(x)\log(1-x)\Big),$$

$$\chi_s(x) = \frac{105 h_1(x)}{128\left(x^2-1\right)^4},$$

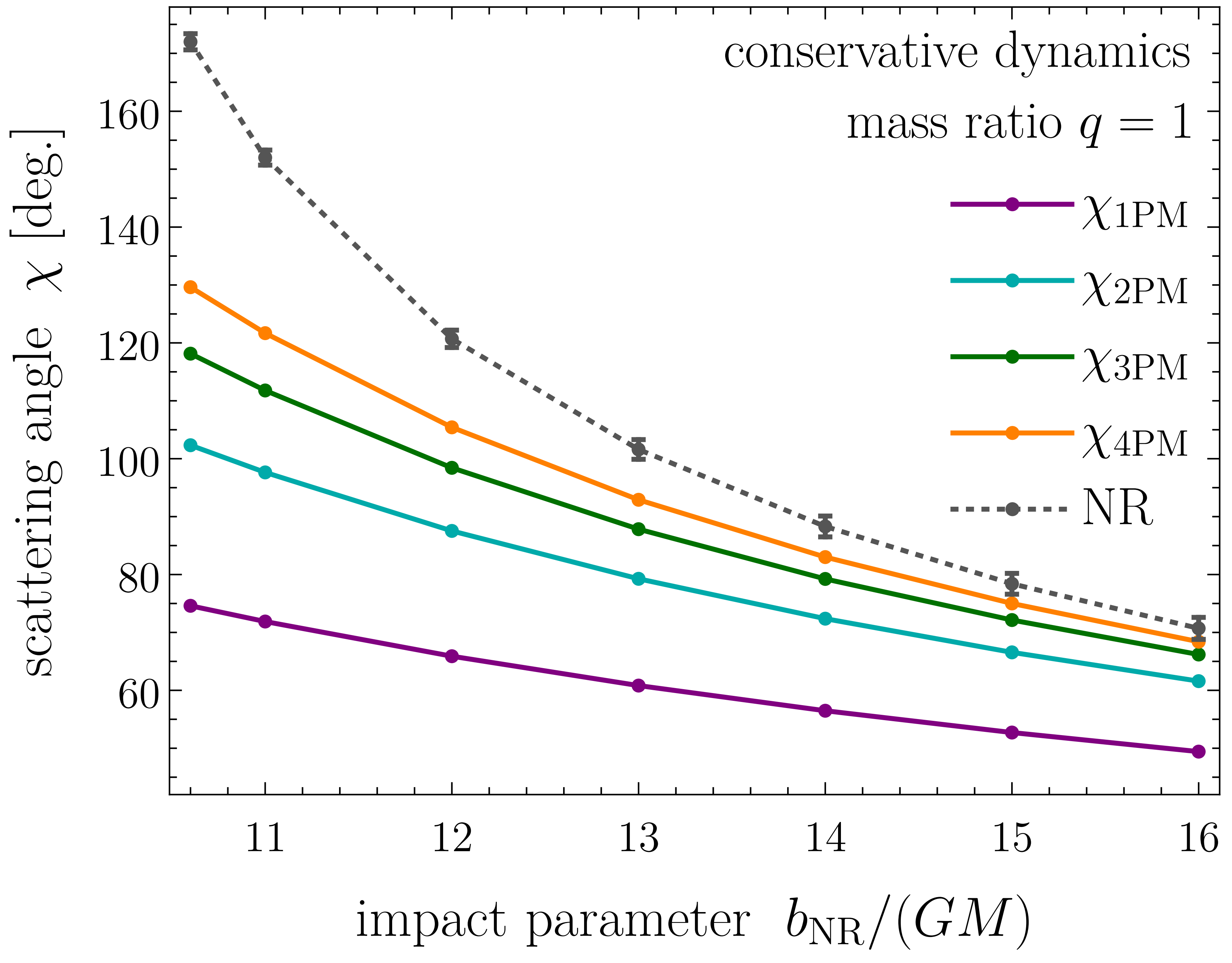
$$\chi_{2\epsilon}(x) = -\frac{3h_2(x)\log(x)}{32x\left(x^2-1\right)^5} + \frac{3h_3(x)\log\left(\frac{x+1}{2}\right)}{32x^2\left(x^2-1\right)^2} + \frac{h_4(x)}{64x^2\left(x^2-1\right)^4},$$

$$\begin{aligned}\chi_c(x) = &-\frac{21h_6(x)\mathrm{E}^2\left(1-x^2\right)}{8\left(x^2-1\right)^4} + \frac{3h_7(x)\mathrm{K}\left(1-x^2\right)\mathrm{E}\left(1-x^2\right)}{8\left(x^2-1\right)^4} - \frac{15h_8(x)\mathrm{K}^2\left(1-x^2\right)}{16\left(x^2-1\right)^4} - \frac{h_{16}(x)\log\left(x^2+1\right)}{32x^3\left(x^2-1\right)^4}\\ &+ \frac{3h_{19}(x)\mathrm{Li}_2\left(-\frac{(x-1)^2}{(x+1)^2}\right)}{128x^4\left(x^2-1\right)^2} + \frac{\pi^2 h_{35}(x)}{512(x-1)^3x^4(x+1)^5} + \frac{3h_{36}(x)\log^2(2)}{16x^2\left(x^2-1\right)^2} + \frac{3h_{37}(x)\log(2)\log(x)}{8\left(x^2-1\right)^5} - \frac{3h_{38}(x)\log(2)\log(x+1)}{16x^2\left(x^2-1\right)^2}\\ &+ \frac{3h_{39}(x)\log(2)}{16x^2\left(x^2-1\right)^4} + \frac{3h_{40}(x)\log^2(x)}{256x^4\left(x^2-1\right)^8} - \frac{3h_{41}(x)\log(x)\log(x+1)}{128x^4\left(x^2-1\right)^5} + \frac{h_{42}(x)\log(x)}{64x^3\left(x^2-1\right)^7} - \frac{3h_{43}(x)\log^2(x+1)}{2x\left(x^2-1\right)^2}\\ &+ \frac{h_{44}(x)\log(x+1)}{32x^3\left(x^2-1\right)^4} + \frac{3h_{45}(x)\left(\mathrm{Li}_2\left(\frac{x-1}{x}\right) - \mathrm{Li}_2(-x)\right)}{128(x-1)^3x^4(x+1)^5} - \frac{3h_{46}(x)\mathrm{Li}_2\left(\frac{x-1}{x+1}\right)}{64(x-1)^2x^4} + \frac{h_{47}(x)}{384x^3\left(x^2-1\right)^6\left(x^2+1\right)^7}.\end{aligned}$$

$$\mathrm{Li}_2(z) \equiv \int_z^0 dt\,\frac{\log(1-t)}{t}, \quad \mathrm{K}(z) \equiv \int_0^1 \frac{dt}{\sqrt{\left(1-t^2\right)\left(1-zt^2\right)}},$$

$$\mathrm{E}(z) \equiv \int_0^1 dt\,\frac{\sqrt{1-zt^2}}{\sqrt{1-t^2}},$$

Mohammed Khalil (AEI & UMD)

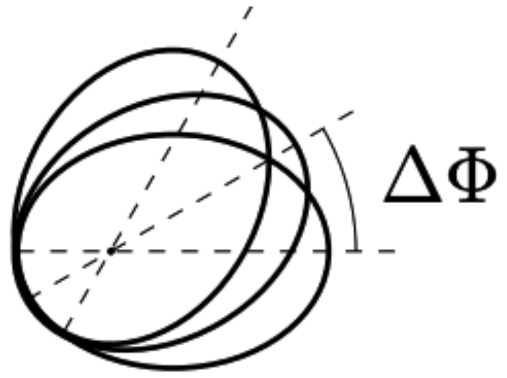


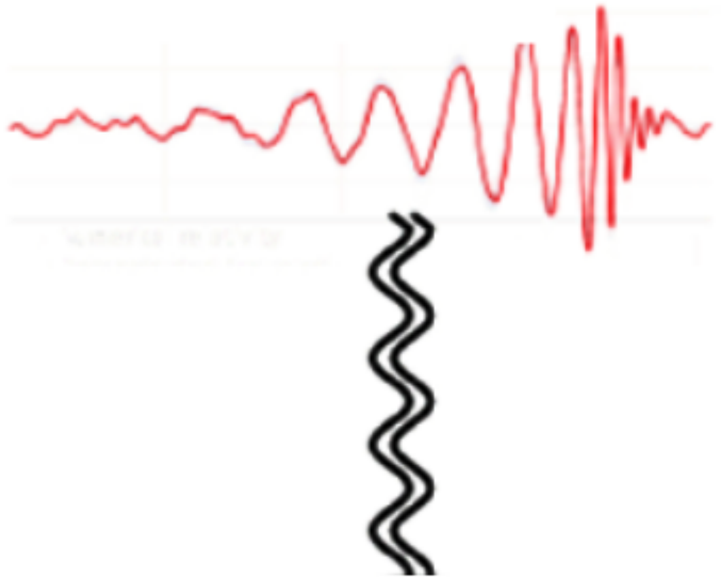
Comparison with numerical simulations
(M. Khalil et al., to appear)

Discovery Potential =

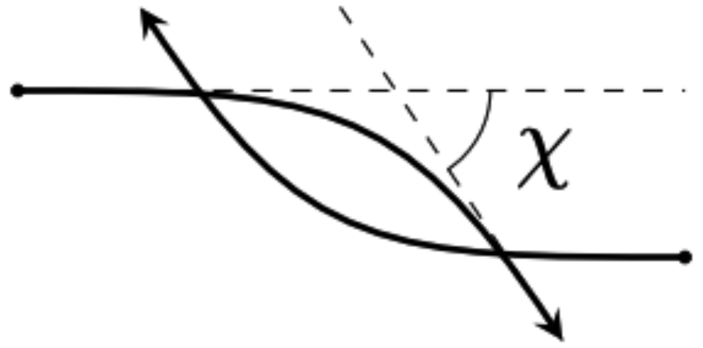
Precise Theoretical Predictions

- Part I: Bound/Unbound





- **Part II: Boundary2Bound**

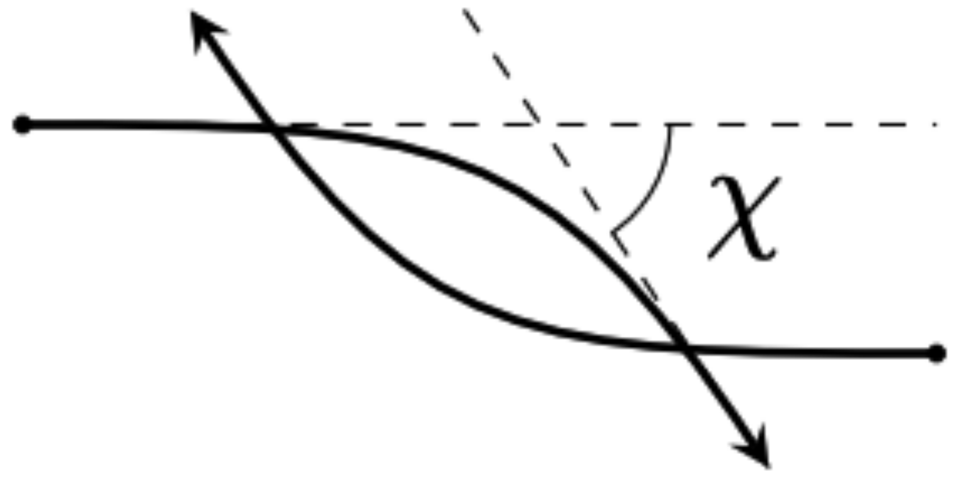


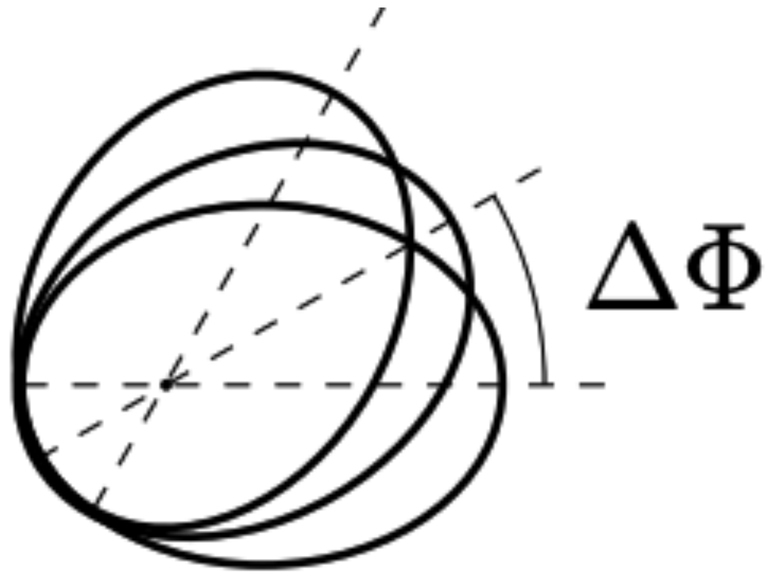
Kalin **RAP**

1910.03008

1911.09130

How do we compute *bound* observables from *boundary* data?





$$\chi_b^{(3)} = \frac{\Gamma^3}{(\gamma^2-1)^{3/2}}\left[\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^{3/2}} - \frac{4}{3}\frac{\nu}{\Gamma^2}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) - 8\frac{\nu}{\Gamma^2}(4\gamma^4 - 12\gamma^2 - 3)\,\mathrm{arcsinh}\sqrt{\frac{\gamma-1}{2}}\right].$$

$$\frac{\chi^{(4)}_{b\,(\mathrm{comb})}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + 2\chi_{2\epsilon}(x)\log(1-x)\Big),$$

$$\chi_s(x) = \frac{105 h_1(x)}{128\left(x^2-1\right)^4},$$

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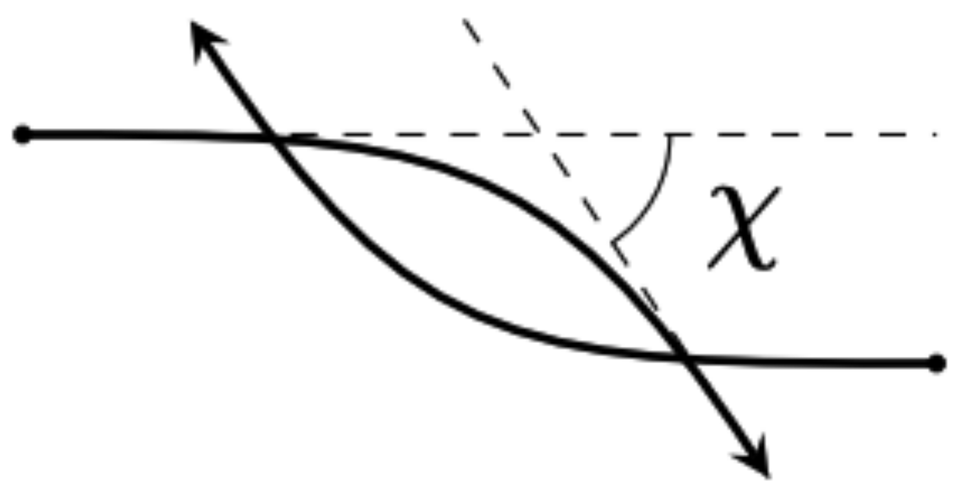
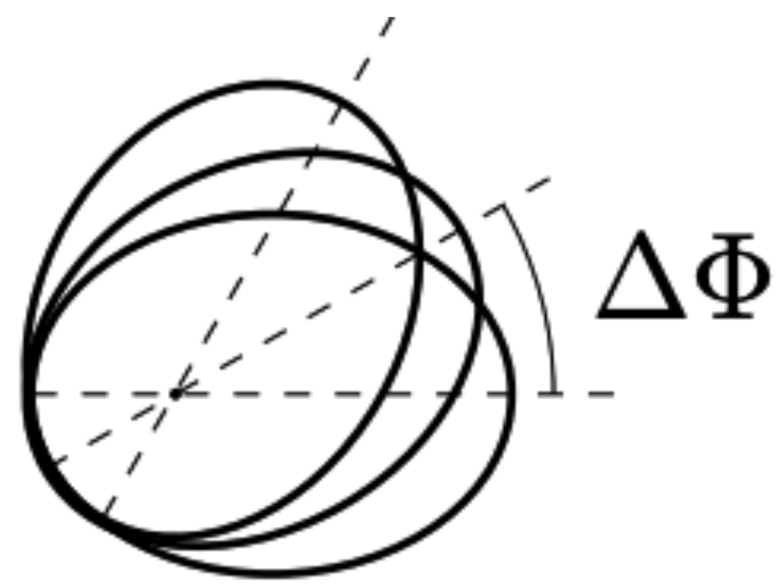
How do we compute *bound* observables from *boundary* data?



Conservative effects



$$\chi_b^{(3)} = \frac{\Gamma^3}{(\gamma^2-1)^{3/2}}\left[\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^{3/2}} - \frac{4}{3}\frac{\nu}{\Gamma^2}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25)\right.$$
$$\left. -8\frac{\nu}{\Gamma^2}(4\gamma^4-12\gamma^2-3)\,\mathrm{arcsinh}\sqrt{\frac{\gamma-1}{2}}\right].$$

Gravitational interaction is UNIVERSAL!

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian: $H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

Newton in here

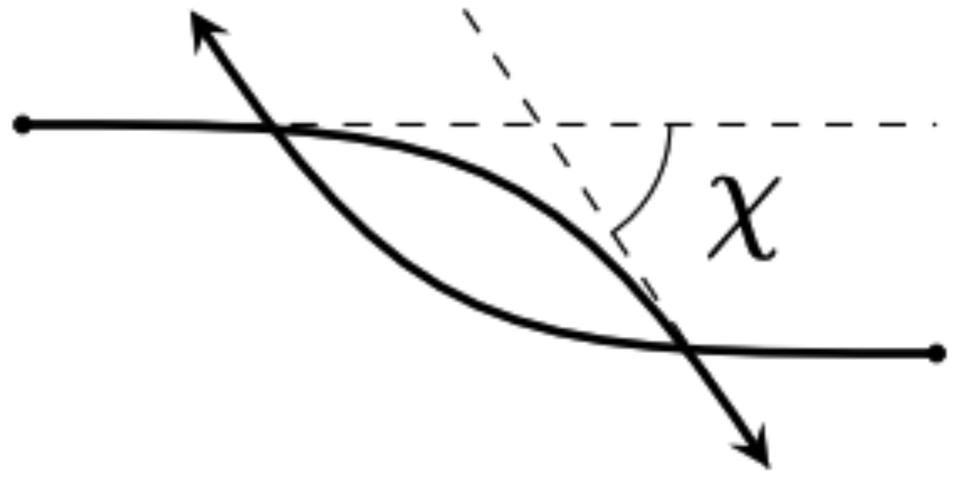
$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\left[\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2}$$
$$\left. + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right],$$

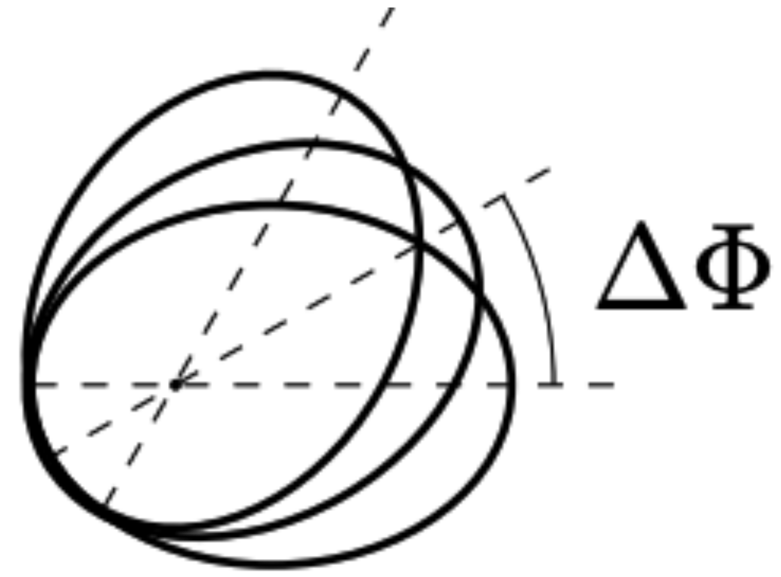
$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

Kalin **RAP**
1910.03008
1911.09130

How do we compute *bound* observables from *boundary* data?



Conservative effects ⟷



$$\chi_b^{(3)} = \frac{\Gamma^3}{(\gamma^2-1)^{3/2}}\left[\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^{3/2}} - \frac{4}{3}\frac{\nu}{\Gamma^2}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) - 8\frac{\nu}{\Gamma^2}(4\gamma^4-12\gamma^2-3)\,\mathrm{arcsinh}\sqrt{\frac{\gamma-1}{2}}\right].$$

Nothing wrong, but…

IN THE ON-SHELL SPIRIT…

Do we really need the —much more cumbersome and gauge-dependent! — Hamiltonian?

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian: $H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$

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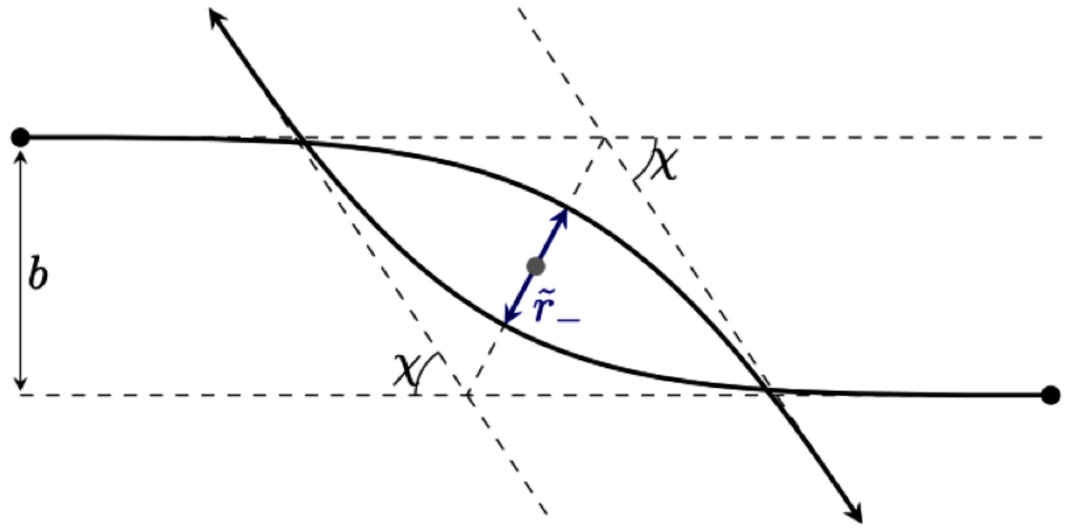
$$m = m_A+m_B, \qquad \mu = m_Am_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1E_2/E^2, \qquad E = E_1+E_2, \qquad \sigma = p_1\cdot p_2/m_1m_2,$$

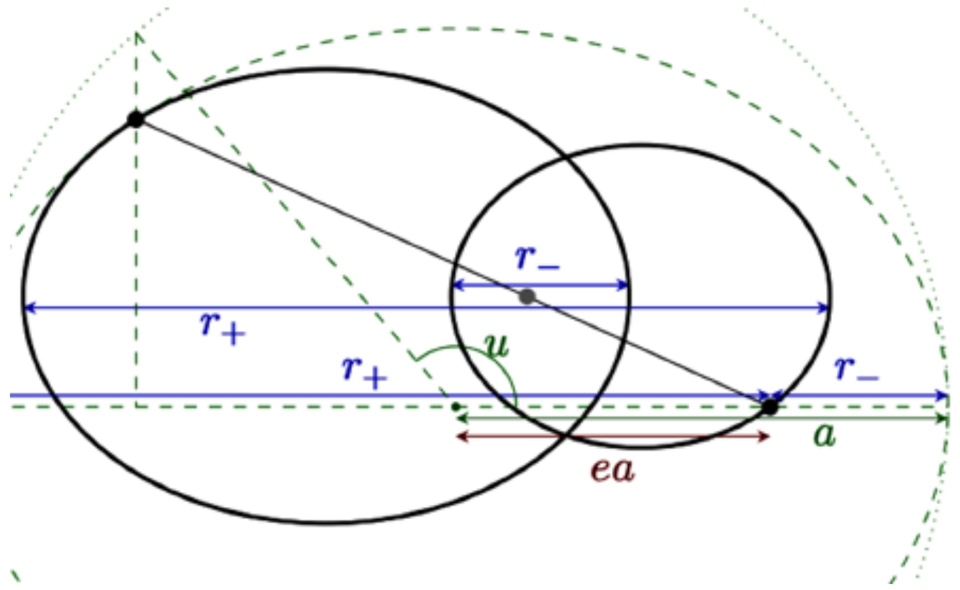
Kalin **RAP**

1910.03008
1911.09130

B2B correspondence



Conservative effects



$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

Scattering angle

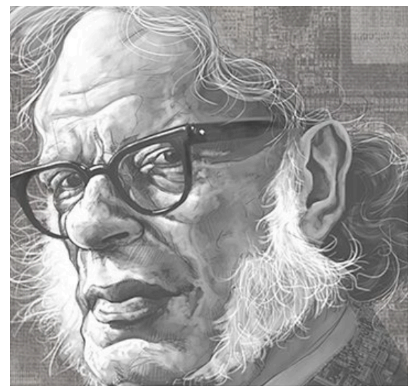
$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

Periastron advance

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

endpoints related by analytic continuation!

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."
—Isaac Asimov



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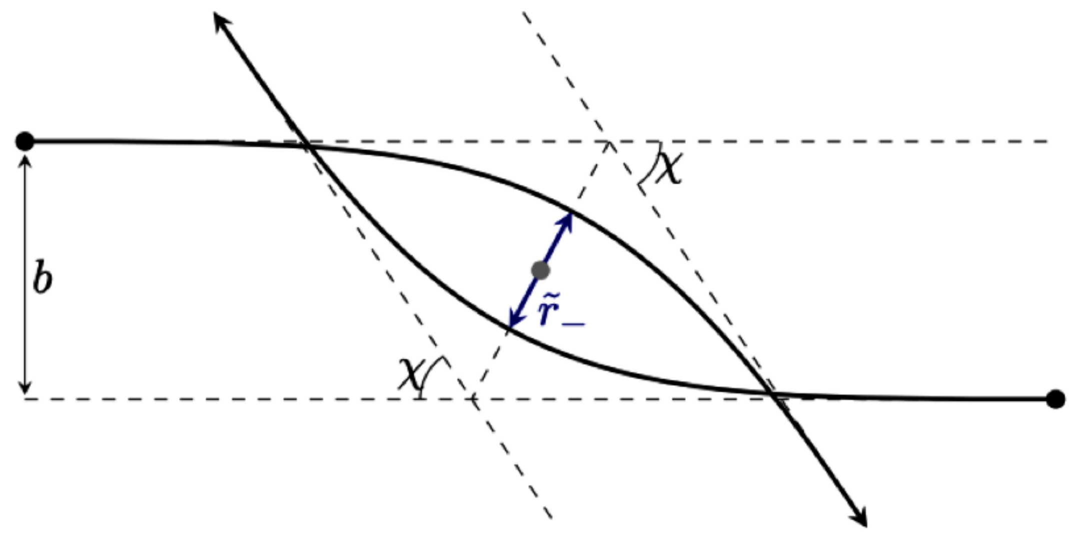
B2B correspondence



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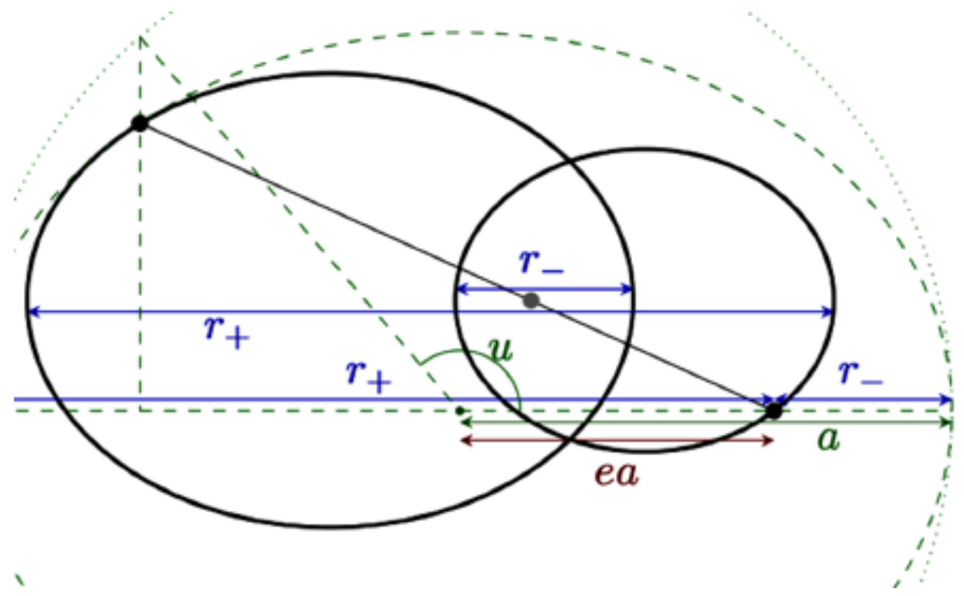
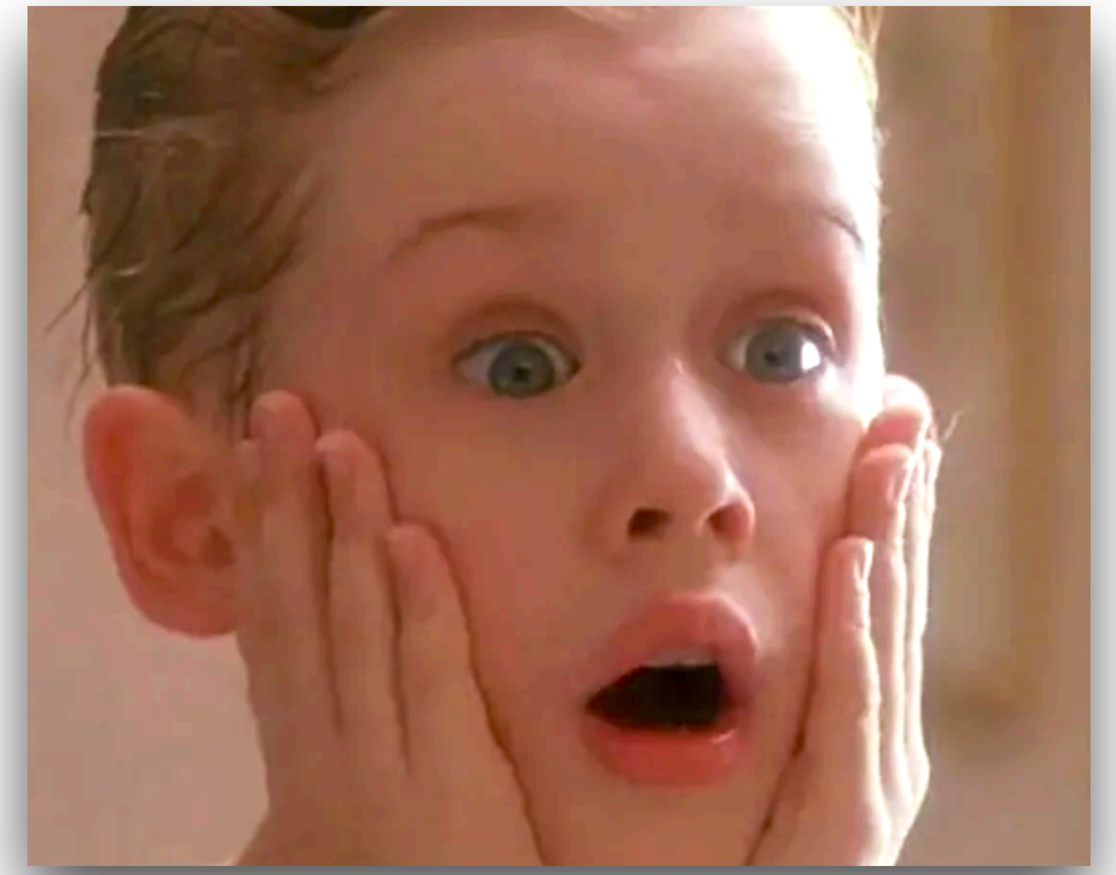


$$\underbrace{\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty}}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r\,;$$

Scattering angle

$$\underbrace{\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})}}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r$$

Periastron advance



$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

LOOP AROUND INFINITY!

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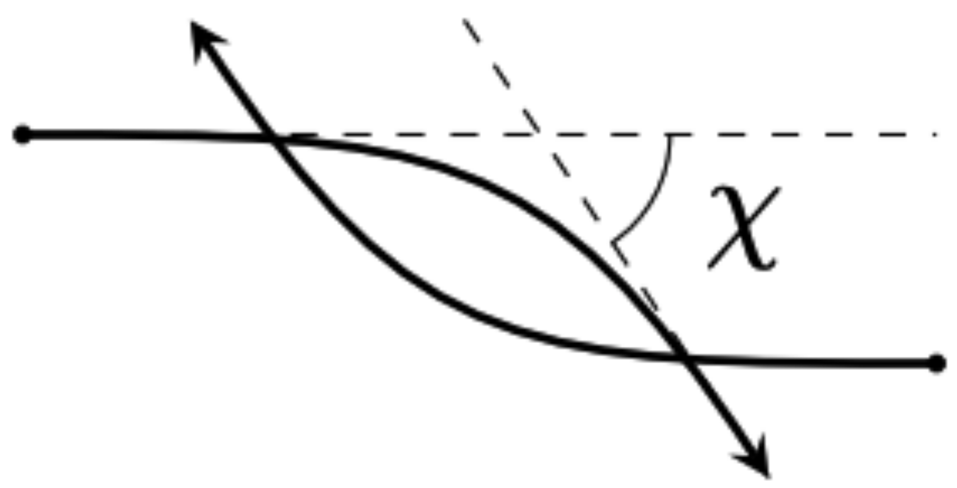
1910.03008

1911.09130

B2B correspondence



Conservative effects

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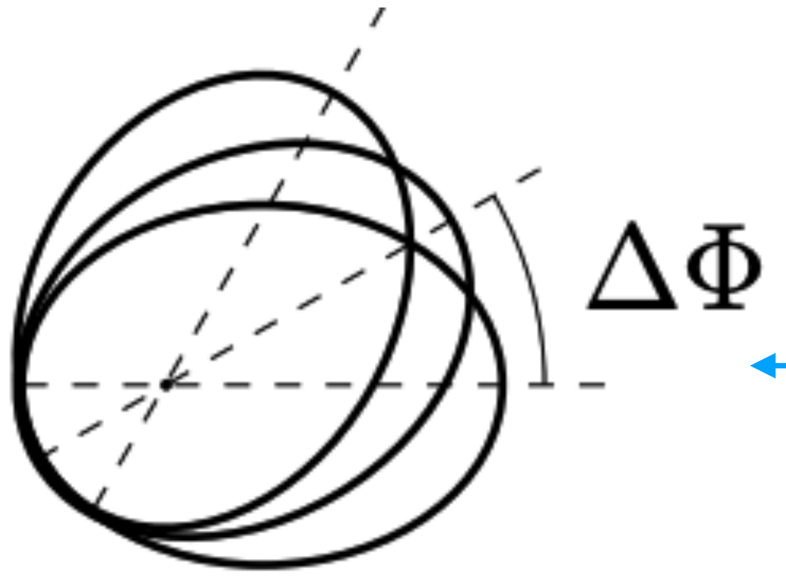


Analytic continuation

$\mathcal{E} < 0$

At the level of the radial action:

$$i_r^{(\mathrm{bound})}(\mathcal{E} < 0,\ J\,) = i_r^{(\mathrm{unbound})}(\mathcal{E} < 0,\ J\,) - i_r^{(\mathrm{unbound})}(\mathcal{E} < 0, -J\,)$$

Central object for the **bound** problem:

$$(GM\mu\,\times)\ \ \delta i_r^{(\mathrm{bound})}(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

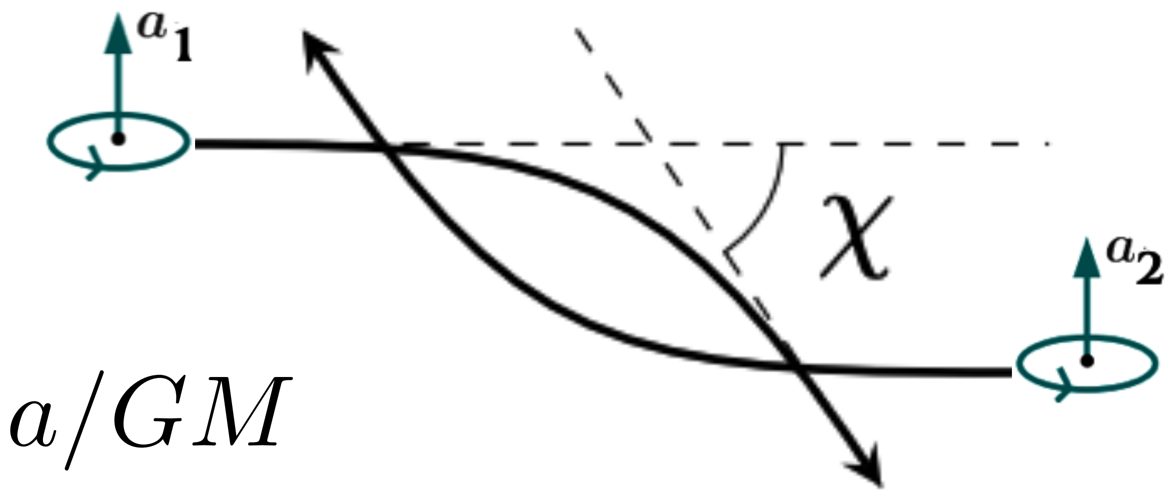
ALL conservative observables!

Kalin **RAP**
1910.03008
1911.09130

Liu **RAP** Yang
2102.10059

B2B correspondence

valid for (planar) aligned-spin

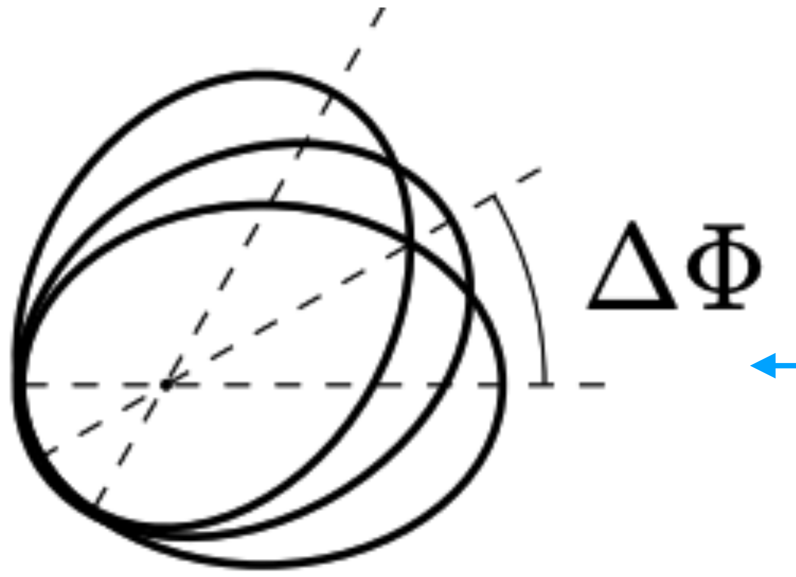


$\tilde{a} = a/GM$

Conservative effects

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

J total (canonical) angular momentum



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$$(GM\mu\,\times)\;\; \delta i_r^{(\text{bound})}(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

ALL conservative observables!

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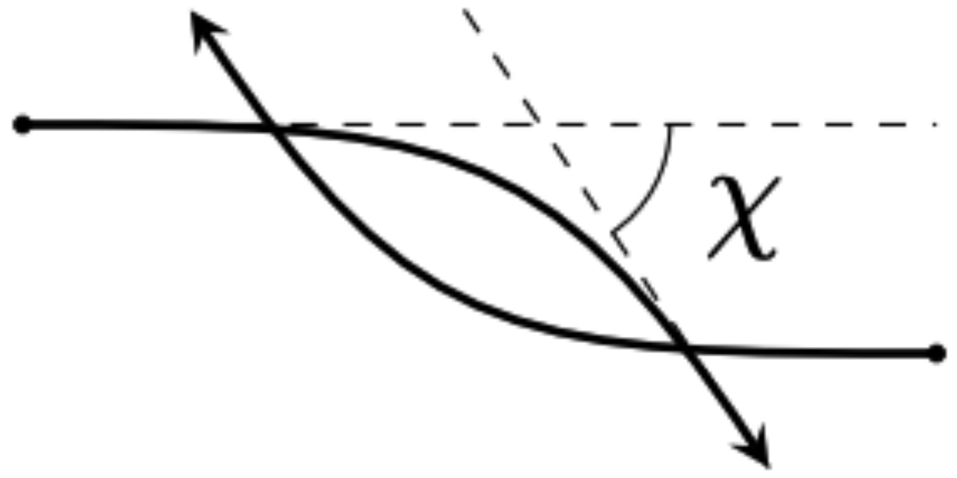

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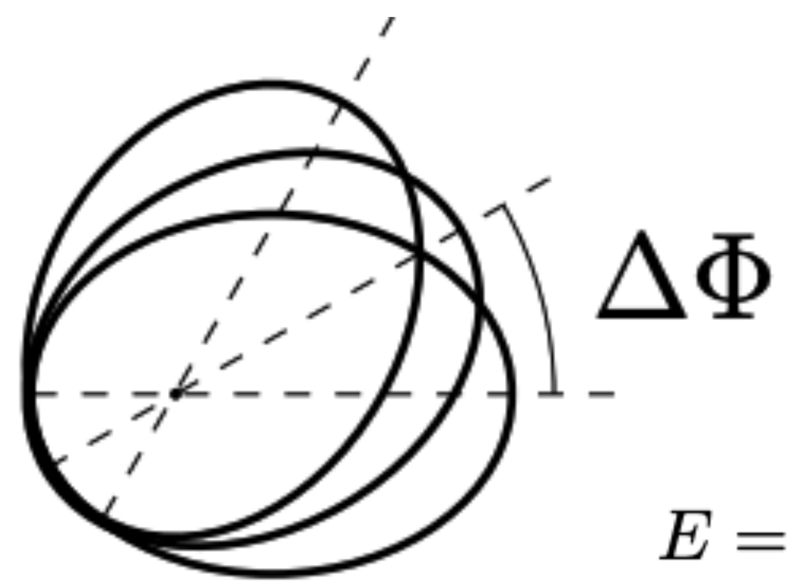
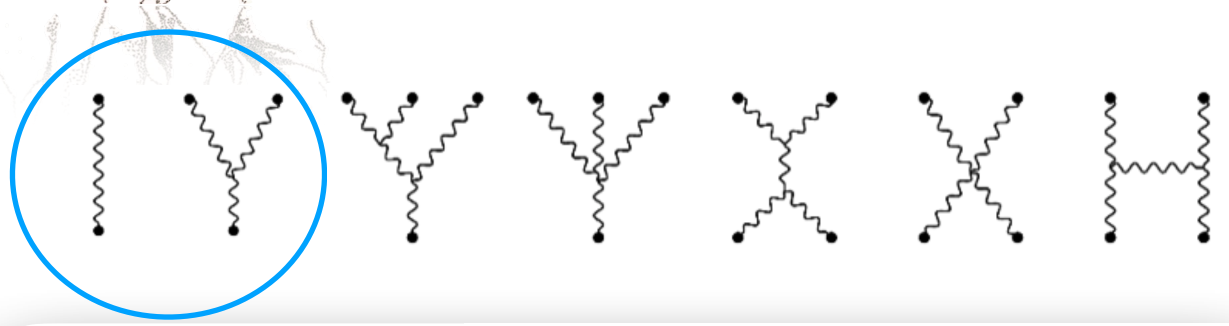


$$E = M(1+\nu\mathcal{E})$$

$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)}\,,$$

1PN, 2PN, 2PN, 3PN, 4PN, 5PN

$$\begin{aligned}\frac{\Delta\Phi}{2\pi} &= \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10-4\nu + \frac{194-184\nu+23\nu^2}{j^2}\right)\mathcal{E} \\ &+ \frac{3}{4j^2}\left(5-5\nu+4\nu^2 + \frac{3535-6911\nu+3060\nu^2-375\nu^3}{10j^2}\right)\mathcal{E}^2 \\ &+ \frac{3}{4j^2}\left((5-4\nu)\nu^2 + \frac{35910-126347\nu+125559\nu^2-59920\nu^3+7385\nu^4}{140j^2}\right)\mathcal{E}^3 \\ &+ \frac{3}{4j^2}\left((5-20\nu+16\nu^2)\,\frac{\nu^2}{4}\right)\mathcal{E}^4 + \cdots\,,\end{aligned}$$

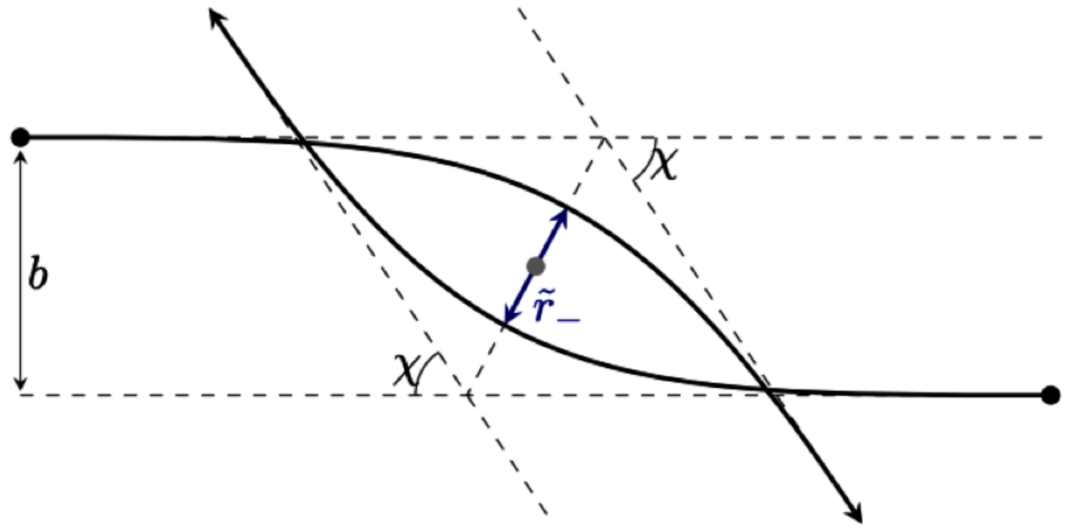


ONE-LOOP EXACT!

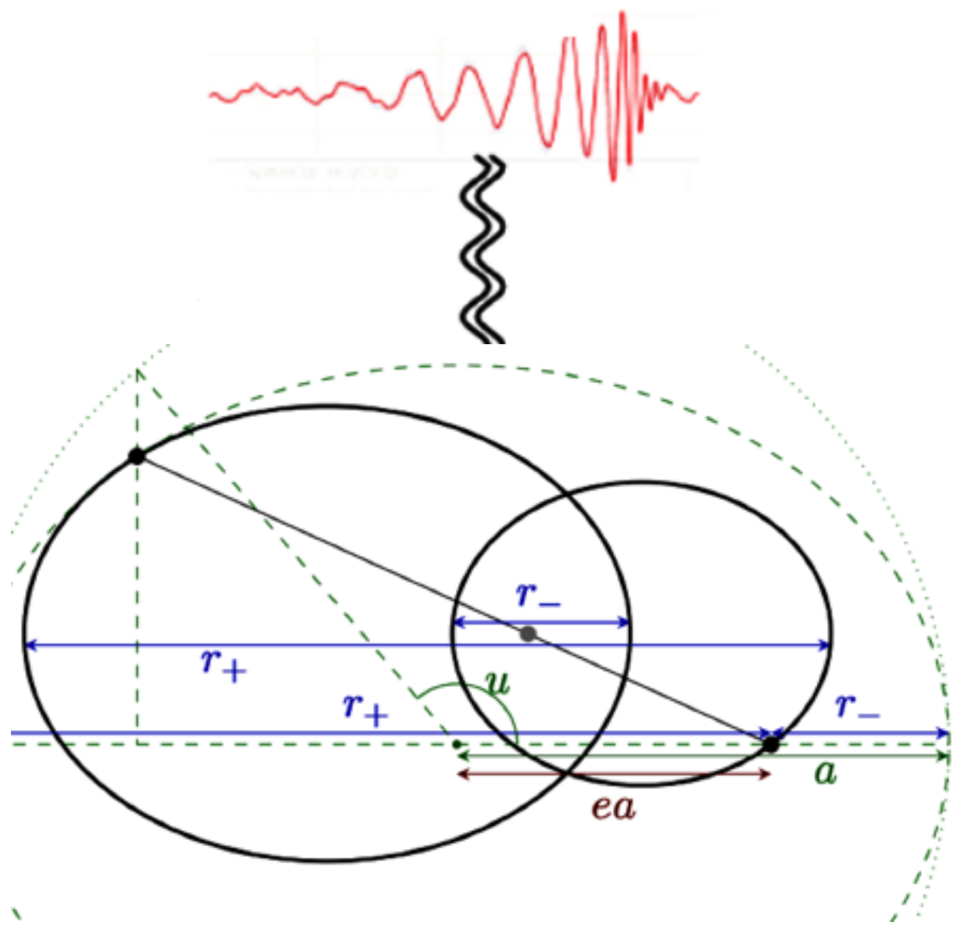
$$\frac{\chi_b^{(2)}}{\Gamma} = \frac{3\pi}{8}\,\frac{5\gamma^2-1}{\gamma^2-1}$$

Cho Kalin **RAP**
2112.03976

B2B correspondence



Radiative effects?!



$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

$$\Delta E_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt\,\frac{dE}{dt} \qquad \longleftrightarrow \qquad \Delta E_{\rm ell}(J,\mathcal{E}) = \oint dt\,\frac{dE}{dt}$$

Aligned-spin configurations

Adiabatic Approx.

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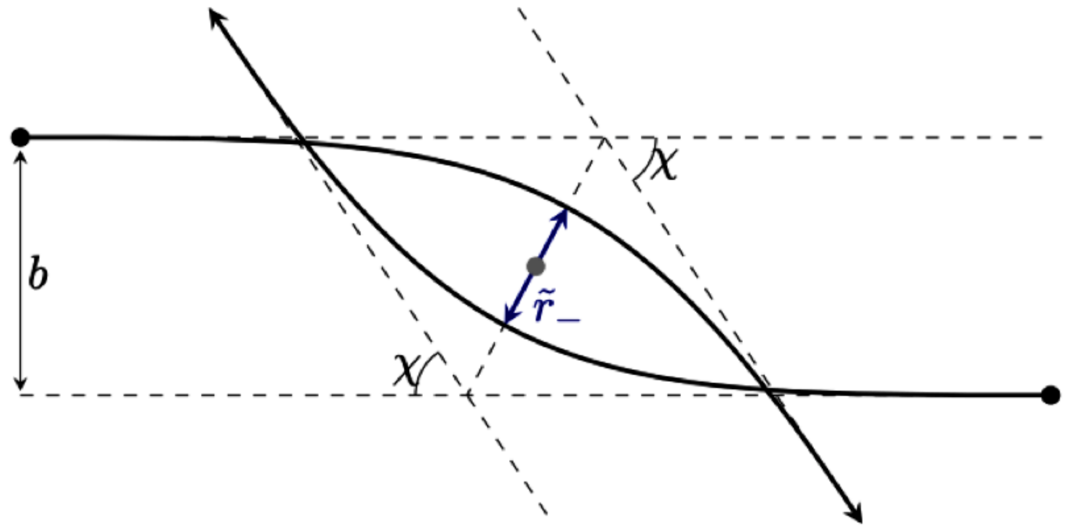
2112.03976

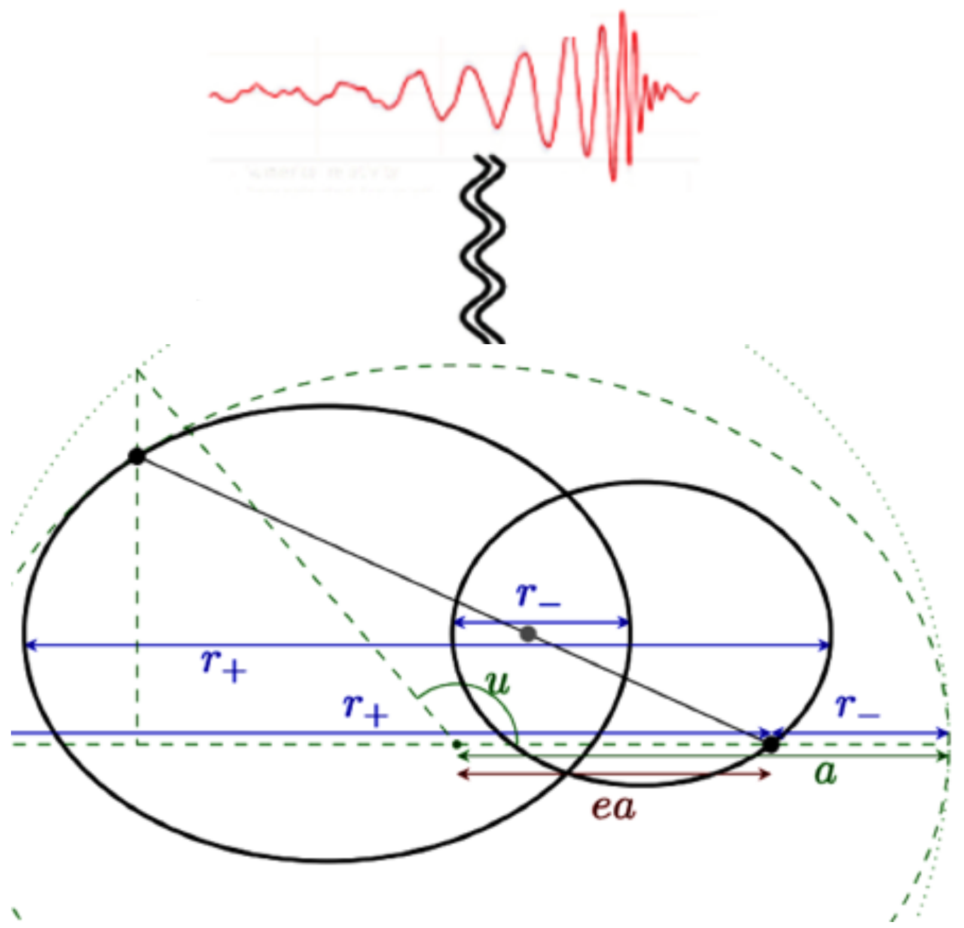
B2B correspondence



valid for (planar) aligned-spin

Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



$$\Delta E_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dE}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}}\frac{dE}{dt}(r,J,\mathcal{E})$$

$$\frac{dE}{dt}(r,J,\mathcal{E}) = \frac{dE}{dt}(r,-J,\mathcal{E})$$

$$\Delta E_{\rm ell}(J,\mathcal{E}) = \oint dt \frac{dE}{dt}$$

$$2\int_{r_-}^{r_+} \frac{dr}{\dot{r}}\frac{dE}{dt}(r,J,\mathcal{E})$$

Similar to radial action: **Loop-around!**

$$\boxed{\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})} \qquad \mathcal{E}<0$$

Aligned-spin configurations

Adiabatic Approx.

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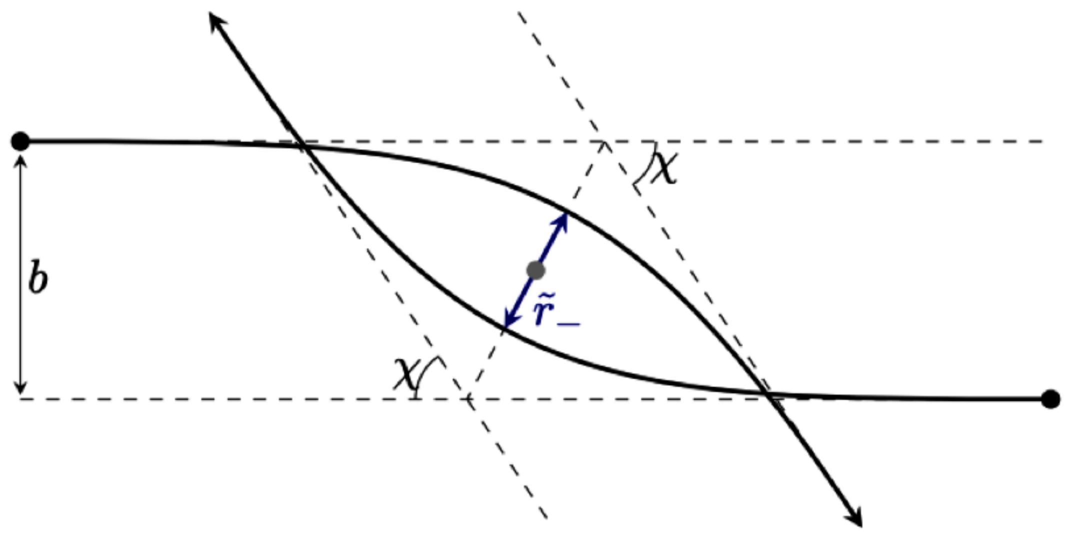
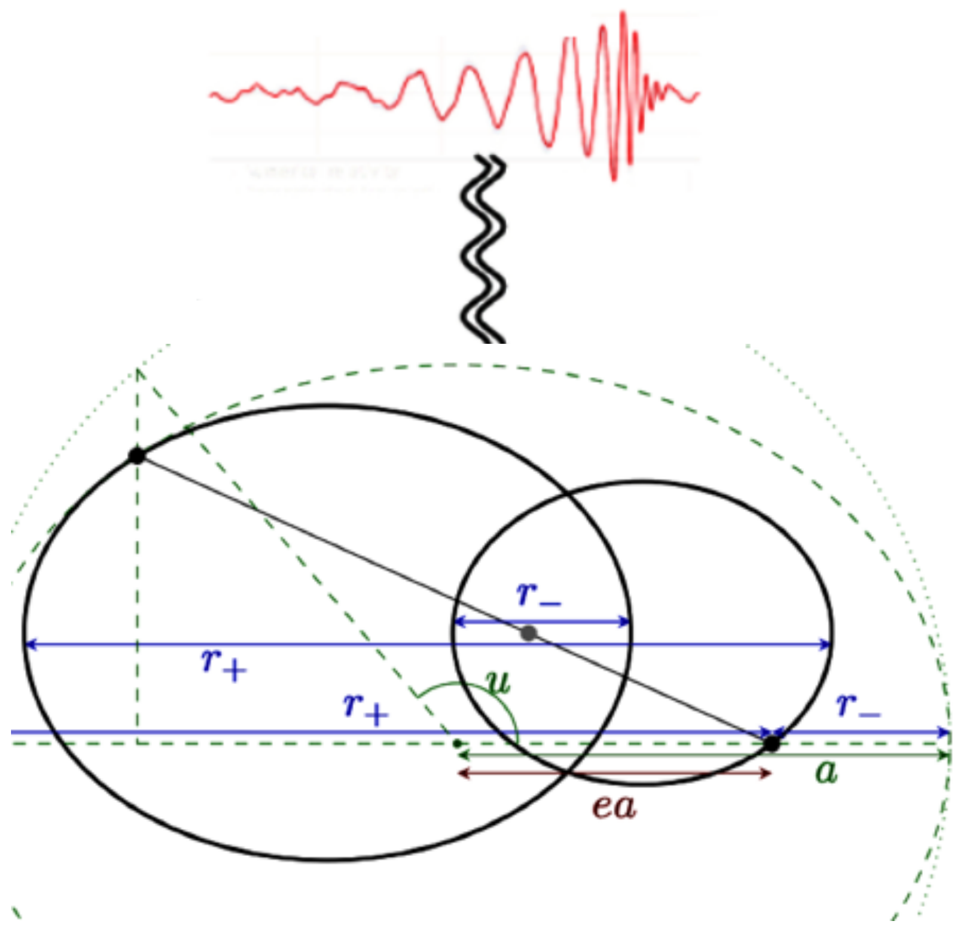
B2B correspondence



valid for (planar) aligned-spin

Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



$$\Delta J_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dJ}{dt} \qquad \Delta J_{\rm ell}(J,\mathcal{E}) = \oint dt \frac{dJ}{dt}$$

$$\frac{dJ}{dt}(r,J,\mathcal{E}) = -\frac{dJ}{dt}(r,-J,\mathcal{E})$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{dJ}{dt}(r,J,\mathcal{E}) \qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{dJ}{dt}(r,J,\mathcal{E})$$

Similar to radial action: **Loop-around!**

$$\Delta J_{\rm ell}(J,\mathcal{E}) = \Delta J_{\rm hyp}(J,\mathcal{E}) + \Delta J_{\rm hyp}(-J,\mathcal{E}) \qquad \mathcal{E}<0$$

Sign flips
Similar to periastron to angle

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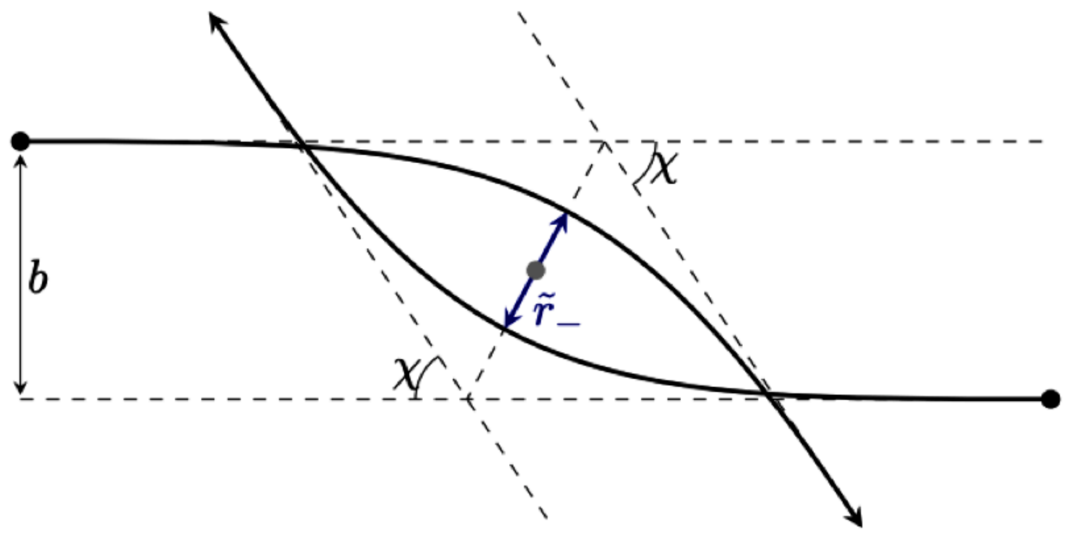
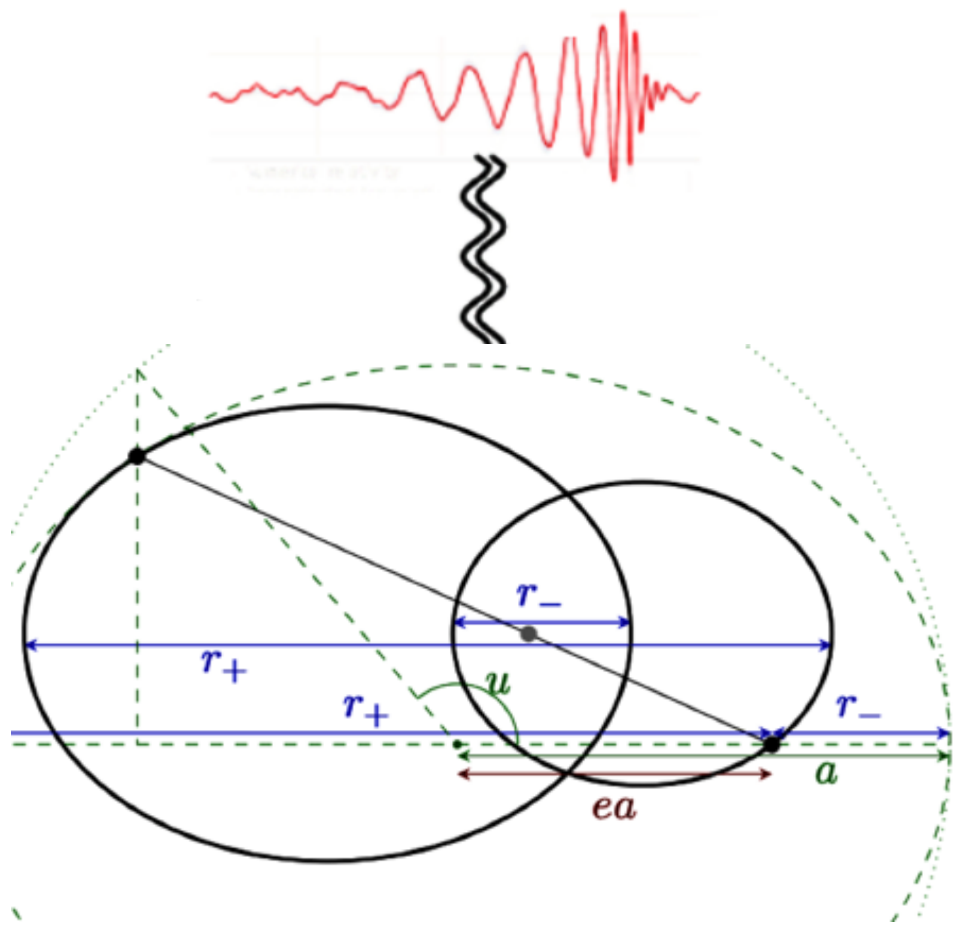
B2B correspondence



Radiative effects

$$\cos^{-1}\left(\frac{1}{e}\right) + \cos^{-1}\left(-\frac{1}{e}\right) = \pi$$



$$e = \frac{r_+ - r_-}{r_+ + r_-}$$

$$\Delta E_{\rm hyp}(j,\mathcal{E}) = \frac{M\,\nu^2}{15}\left[\frac{850\sqrt{2}\sqrt{\mathcal{E}}}{j^6} + \frac{2692\sqrt{2}\mathcal{E}^{3/2}}{3j^4} + \left(\frac{850}{j^7} + \frac{1464\mathcal{E}}{j^5} + \frac{296\mathcal{E}^2}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)\right.$$
$$+ \frac{\sqrt{2}\mathcal{E}^{5/2}(2506431 - 3009160\nu)}{105(1+2\mathcal{E}j^2)j^4} + \frac{\mathcal{E}^{3/2}(182337 - 140480\nu)}{3\sqrt{2}(1+2\mathcal{E}j^2)j^6} - \frac{7\sqrt{\mathcal{E}}(-5763 + 3220\nu)}{2\sqrt{2}(1+2\mathcal{E}j^2)j^8}$$
$$- \frac{2\sqrt{2}\mathcal{E}^{7/2}(-89907 + 156380\nu)}{35(1+2\mathcal{E}j^2)j^2} + \left(\frac{\mathcal{E}\left(\frac{33885}{2} - 15900\nu\right)}{j^7} + \frac{\mathcal{E}^2\left(\frac{46617}{7} - 10464\nu\right)}{j^5}\right.$$
$$\left. + \frac{\frac{40341}{4} - 5635\nu}{j^9} + \frac{\mathcal{E}^3\left(\frac{4786}{7} - 888\nu\right)}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)$$

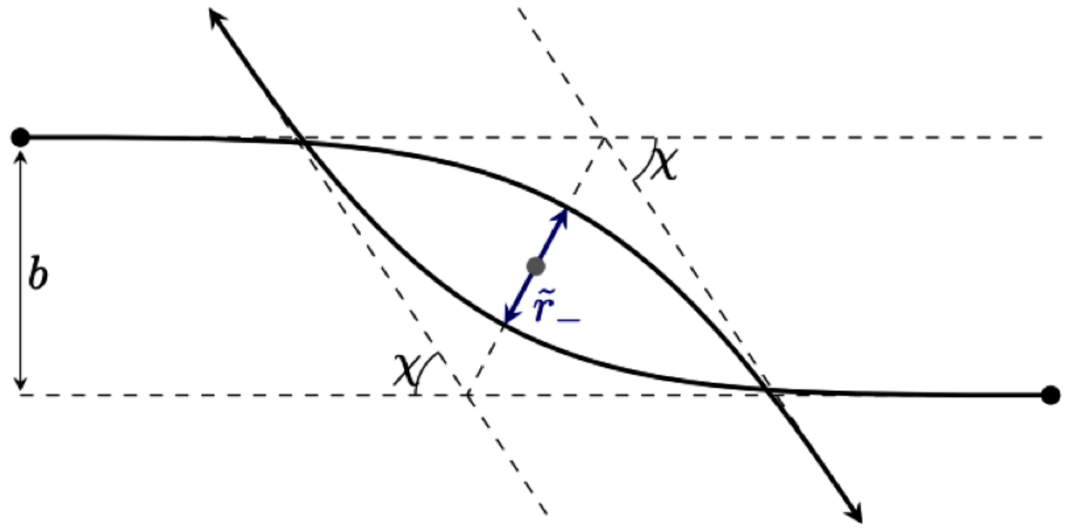
$$\Delta E_{\rm ell}(j,\mathcal{E}) = \frac{M\,\nu^2}{15}\left[\frac{850\pi}{j^7} + \frac{1464\mathcal{E}\pi}{j^5} + \frac{296\mathcal{E}^2\pi}{j^3} + \frac{\mathcal{E}^2\pi}{j^5}\left(\frac{46617}{7} - 10464\nu\right)\right.$$
$$+ \frac{7\pi(5763 - 3220\nu)}{4j^9} + \frac{15\mathcal{E}\pi(2259 - 2120\nu)}{2j^7} + \frac{\mathcal{E}^3\pi}{j^3}\left(\frac{4786}{7} - 888\nu\right)$$

$$\Delta E_{\rm ell}(e,\mathcal{E}) = \Delta E_{\rm hyp}(e,\mathcal{E}) - \Delta E_{\rm hyp}(-e,\mathcal{E})$$

only odd terms survive

Cho Kalin **RAP**
2112.03976

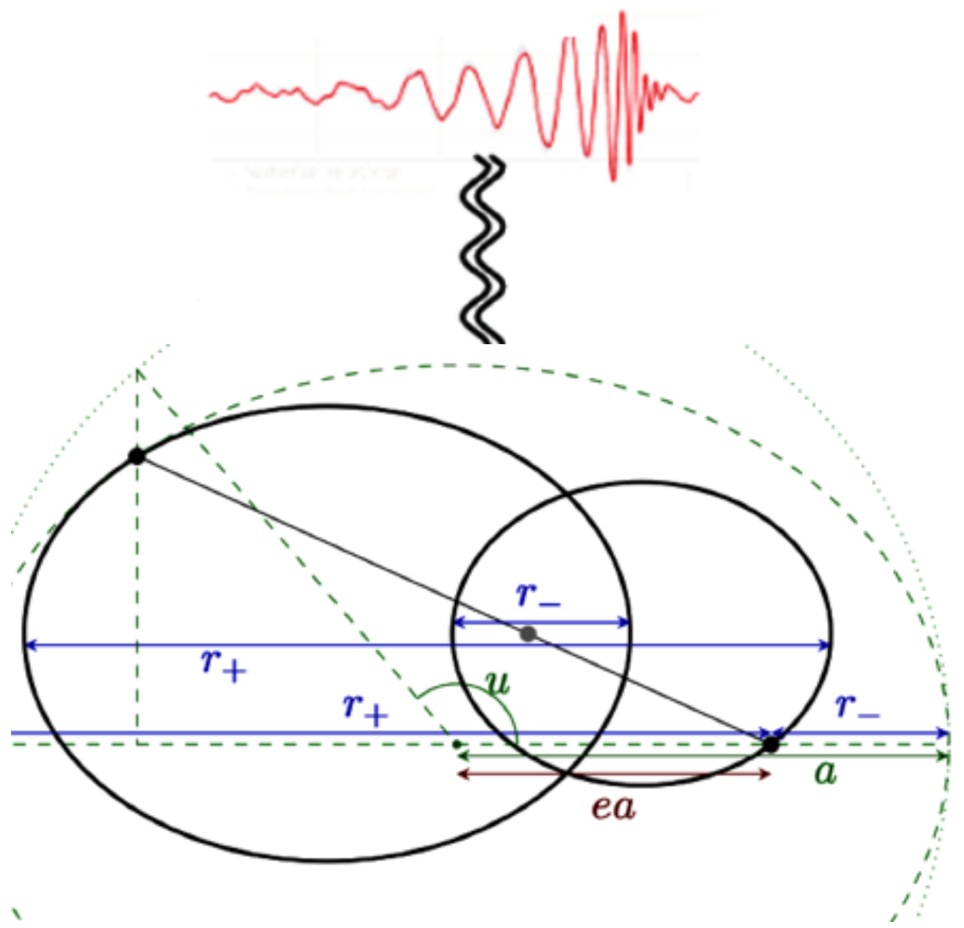
B2B correspondence



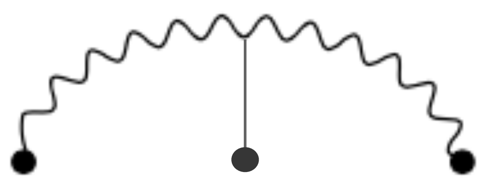
Conservative!

Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



What about the tail Hamiltonian? **Loop around again!**



(Local & Non-Local)

$$H_{\rm tail}(r,\mathcal{E},j) = H_{\rm tail}(r,\mathcal{E},-j)$$

$$\int_{\tilde{r}_-}^{\infty} \frac{dr}{p_r} H_{\rm tail} \longleftrightarrow \int_{r_-}^{r_+} \frac{dr}{p_r} H_{\rm tail}$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

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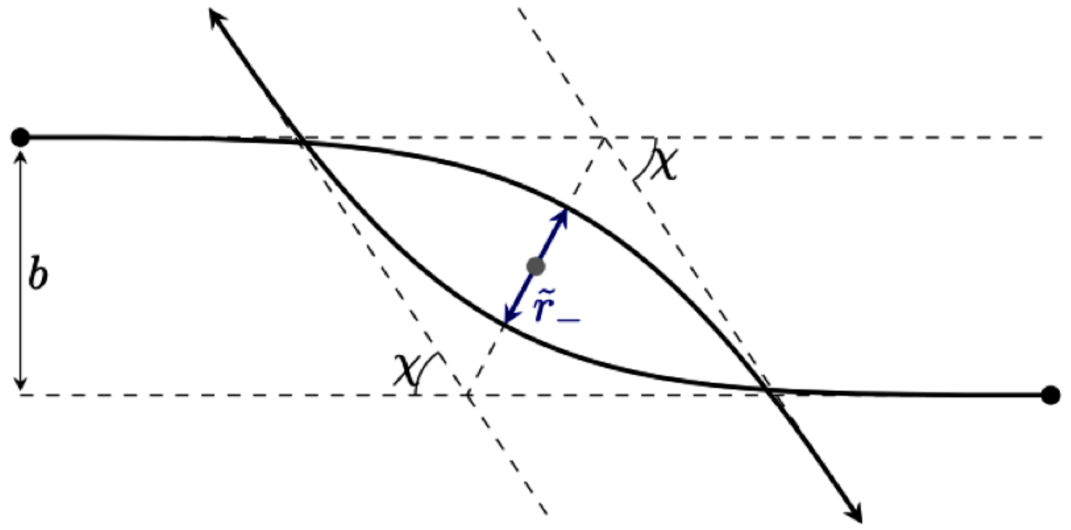
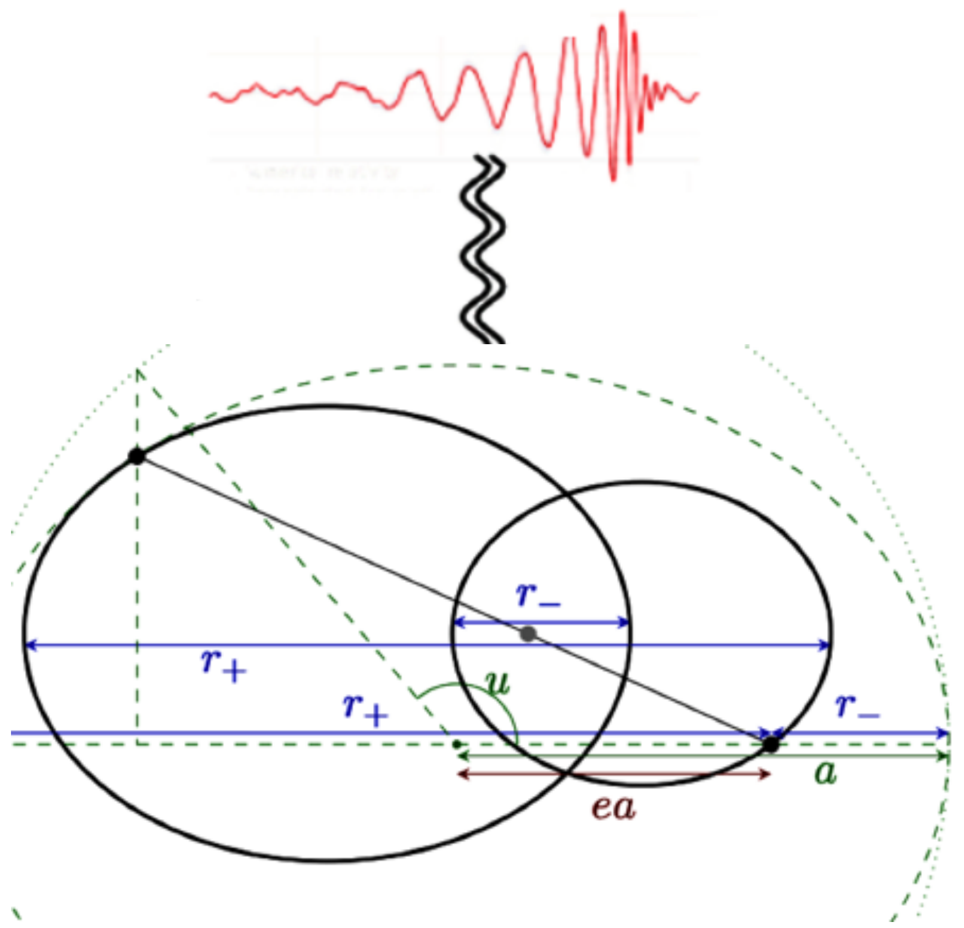
2112.03976

B2B correspondence

Conservative!



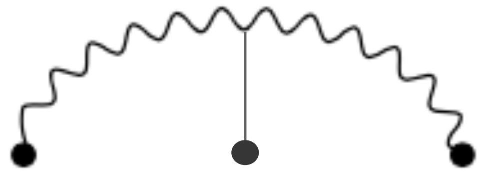


Radi… …ffects

$r_-(J,\mathcal{E}$ … $(J,\mathcal{E})$ … $\mathcal{E} < 0\,.$

$r_+(J$ … $r_-(-J,\mathcal{E})$ … J … $< 0\,,$

What about … tail Hamilt…an? **Loop … …nd again!**



Local & Bethe Logs
for *generic orbits!*

$H_{\rm tail}(r,\mathcal{E},j)$ … $H_{\rm tail}(r,\mathcal{E},-j)$

$\int_{\tilde{r}_-}^{\infty}\frac{dr}{p}$ …   … $\int_{r_-}\frac{dr}{\ldots}H_{\rm tail}$

“large-j” limit ONLY

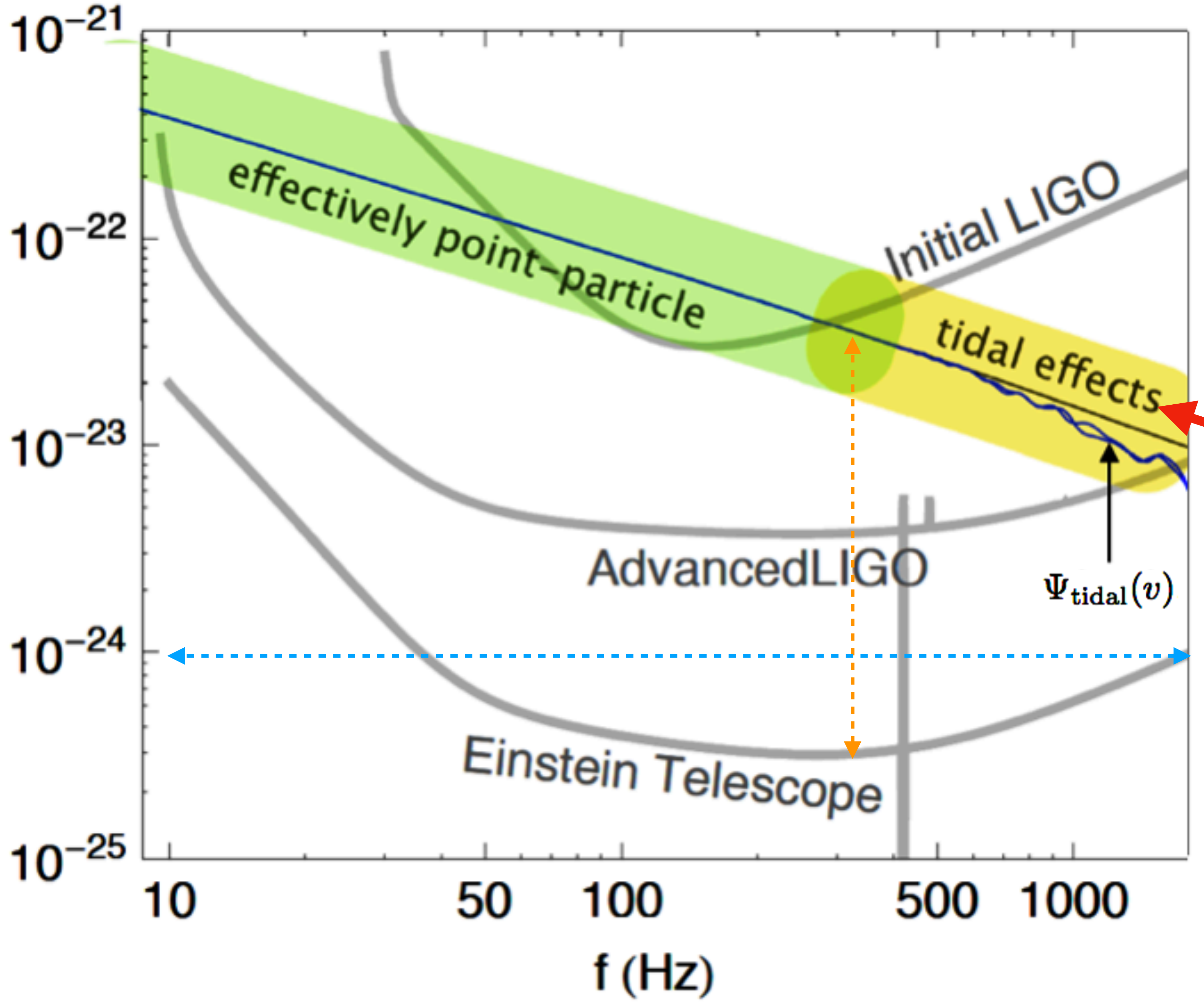
WARNING

$i_r^{bound}(\ldots\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unboun\ldots}(-j,\mathcal{E})$

“Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW’s information”

Kip Thorne ‘Last 3 minutes’ ~~1993~~ 2022

More ‘luminosity/sensitivity’ at ‘short/long distances’

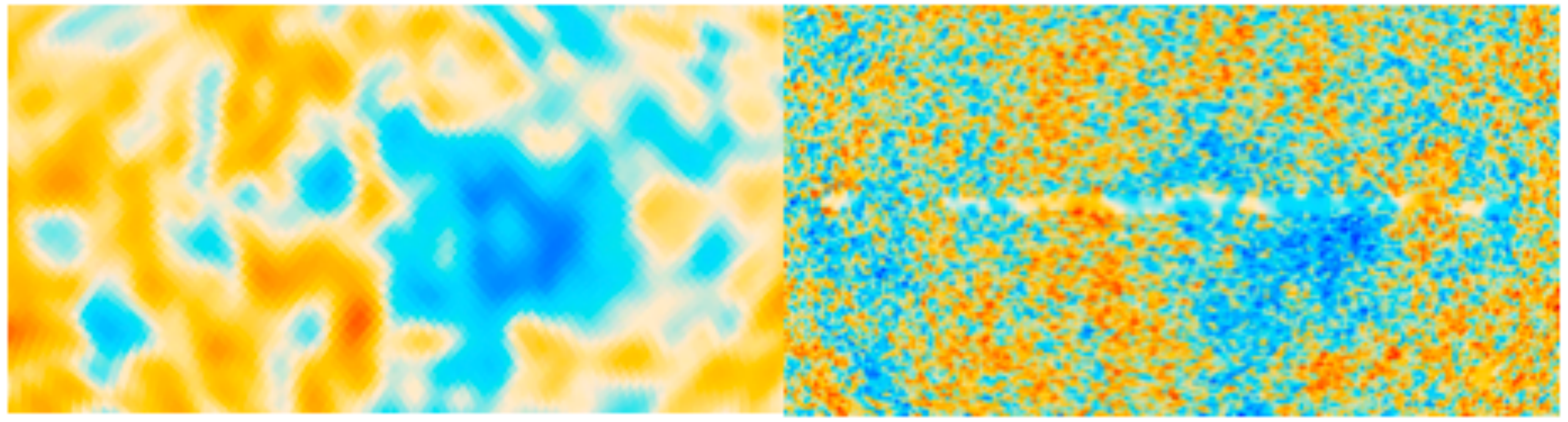


‘New Physics Threshold’

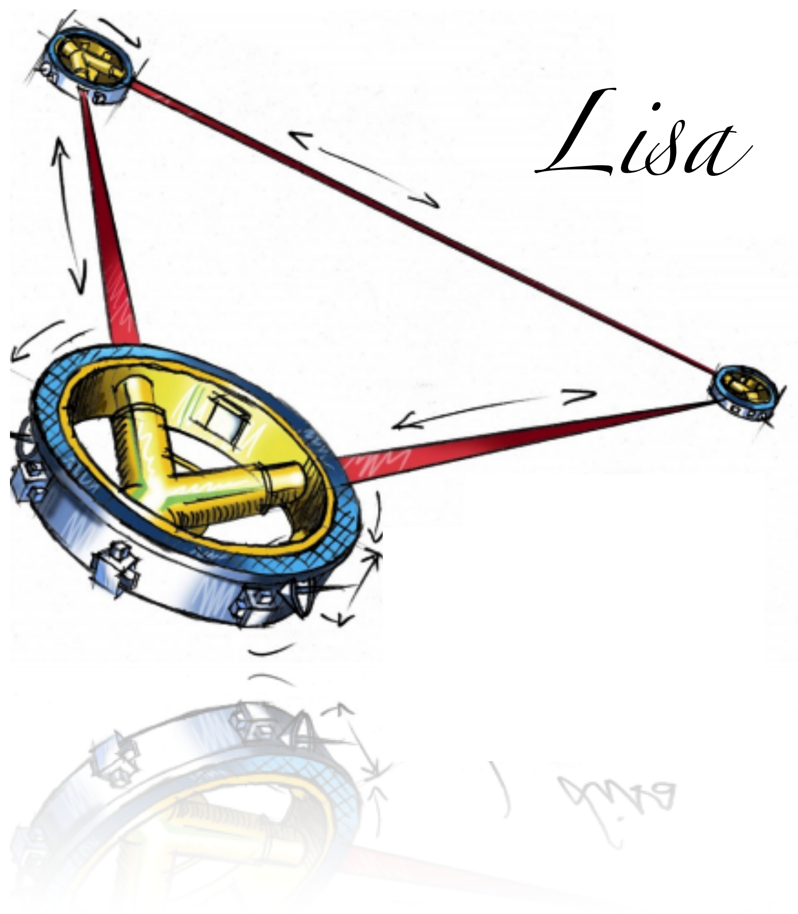
- **Energy/Frequency Frontier**
- **Luminosity Frontier**

also 2G+!

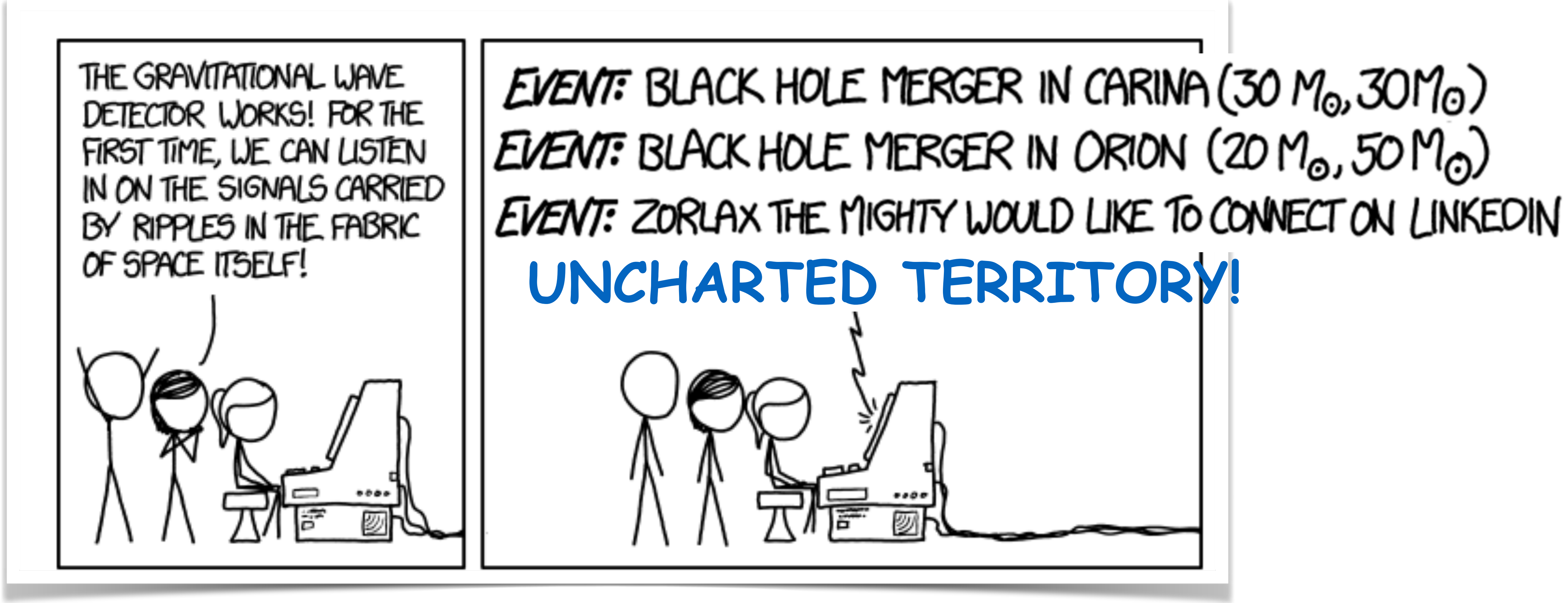
‘Ligo/Virgo’ **‘LISA/ET’** (+20)



Cobe (92) Planck (13)

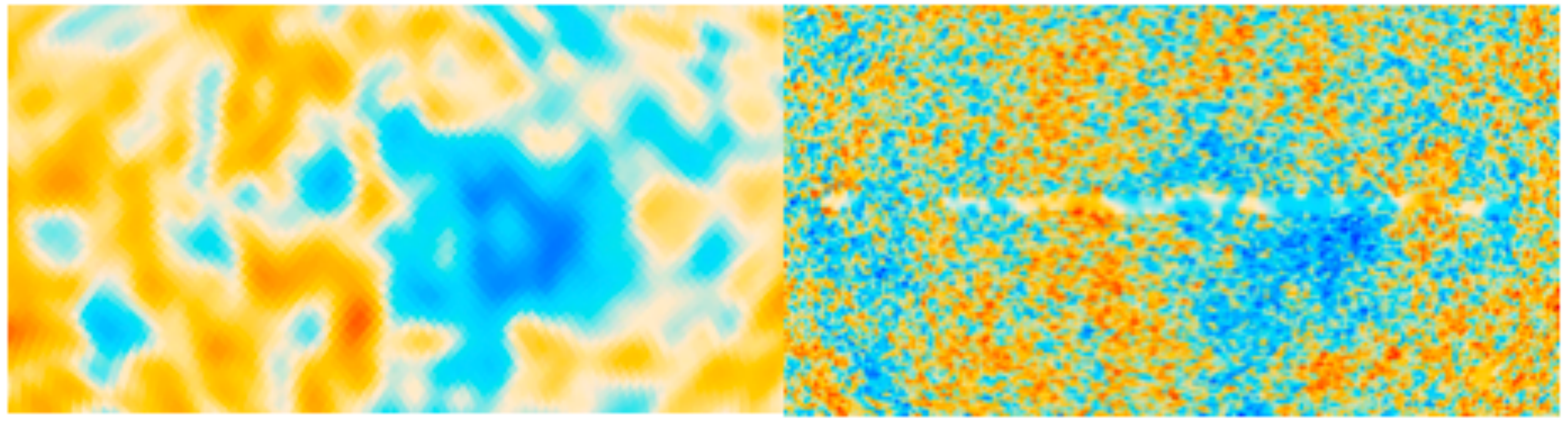


Einstein Telescope

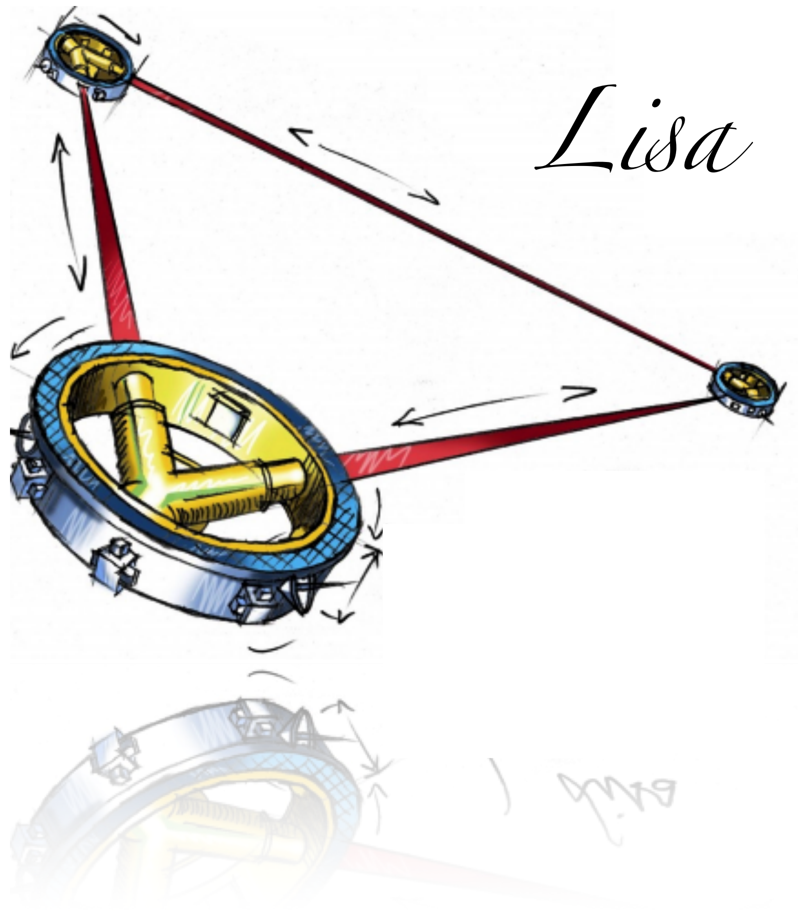


Are we ready for the future?

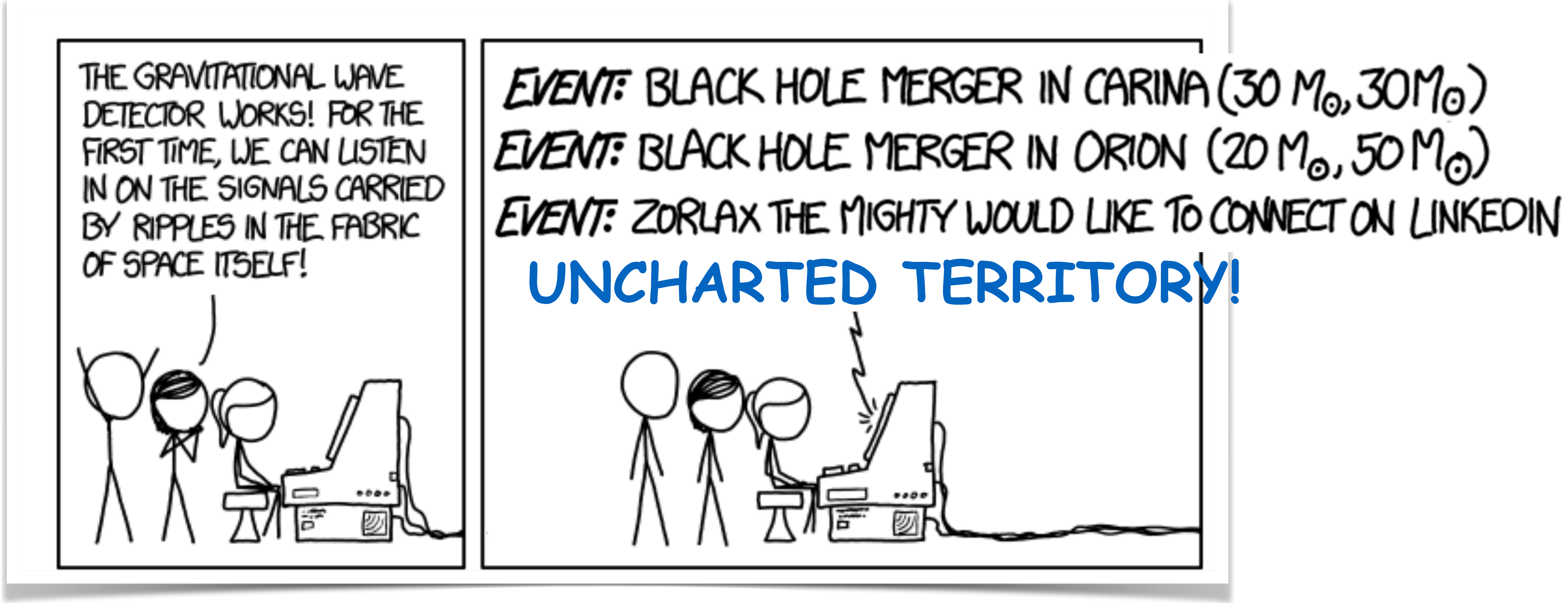
‘Ligo/Virgo’ **‘LISA/ET’** (+20)



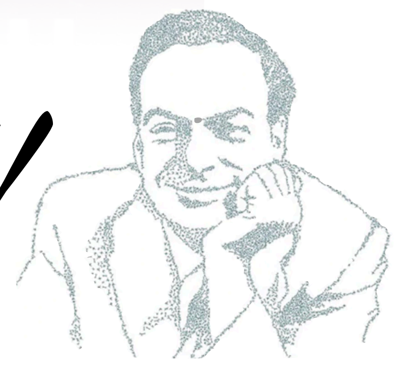
Cobe (92) Planck (13)



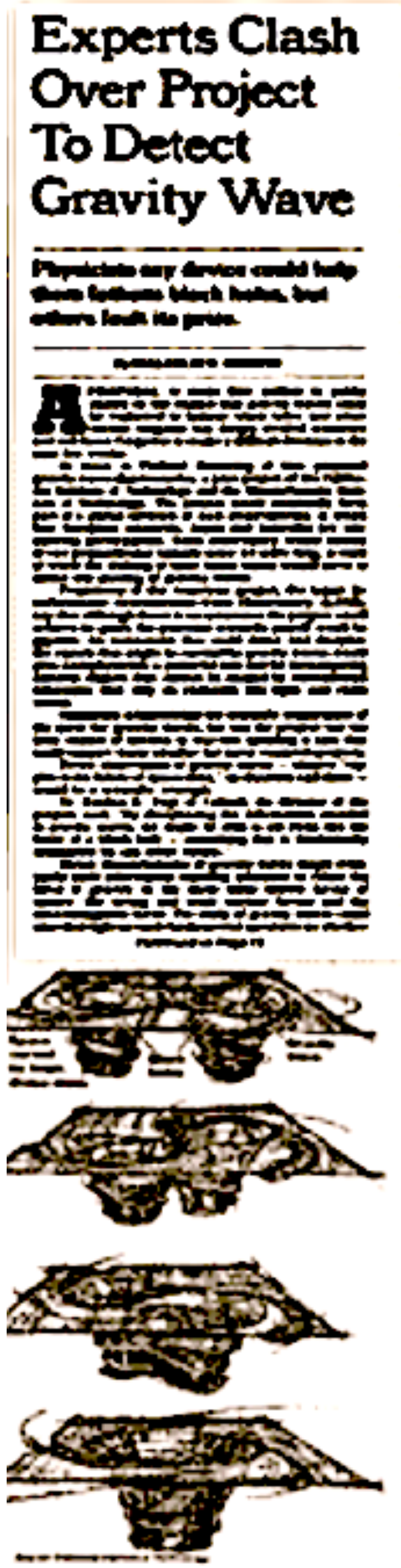
Einstein Telescope



Are we ready for the future?



NYT 1991



Experts Clash Over Project To Detect Gravity Wave

Die Ziet

no.203.078 01.01.203X

EinsTein reloaded!

New era of foundational investigations established through GW Precision Data.

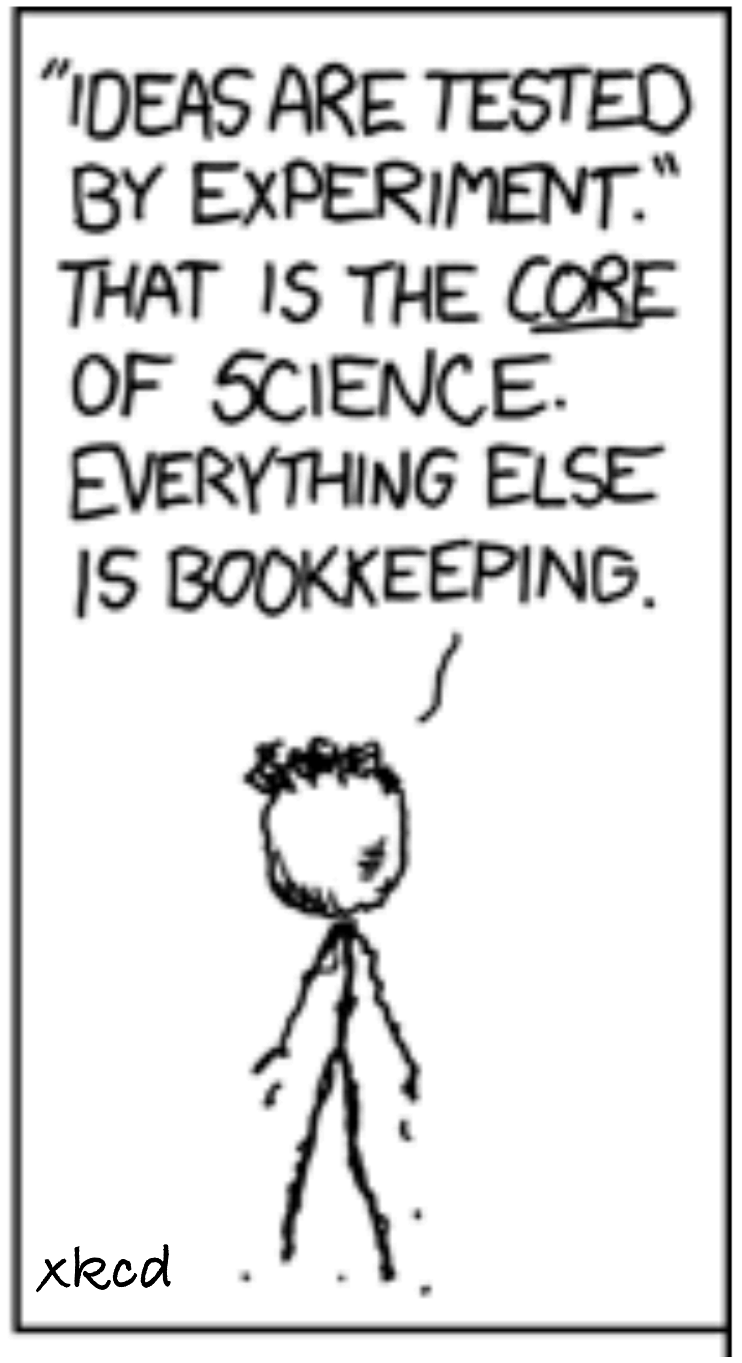
New particles discovered!

Black Holes unveiled!

Was Einstein right?!

erc

Thank you!



*This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 817791).