

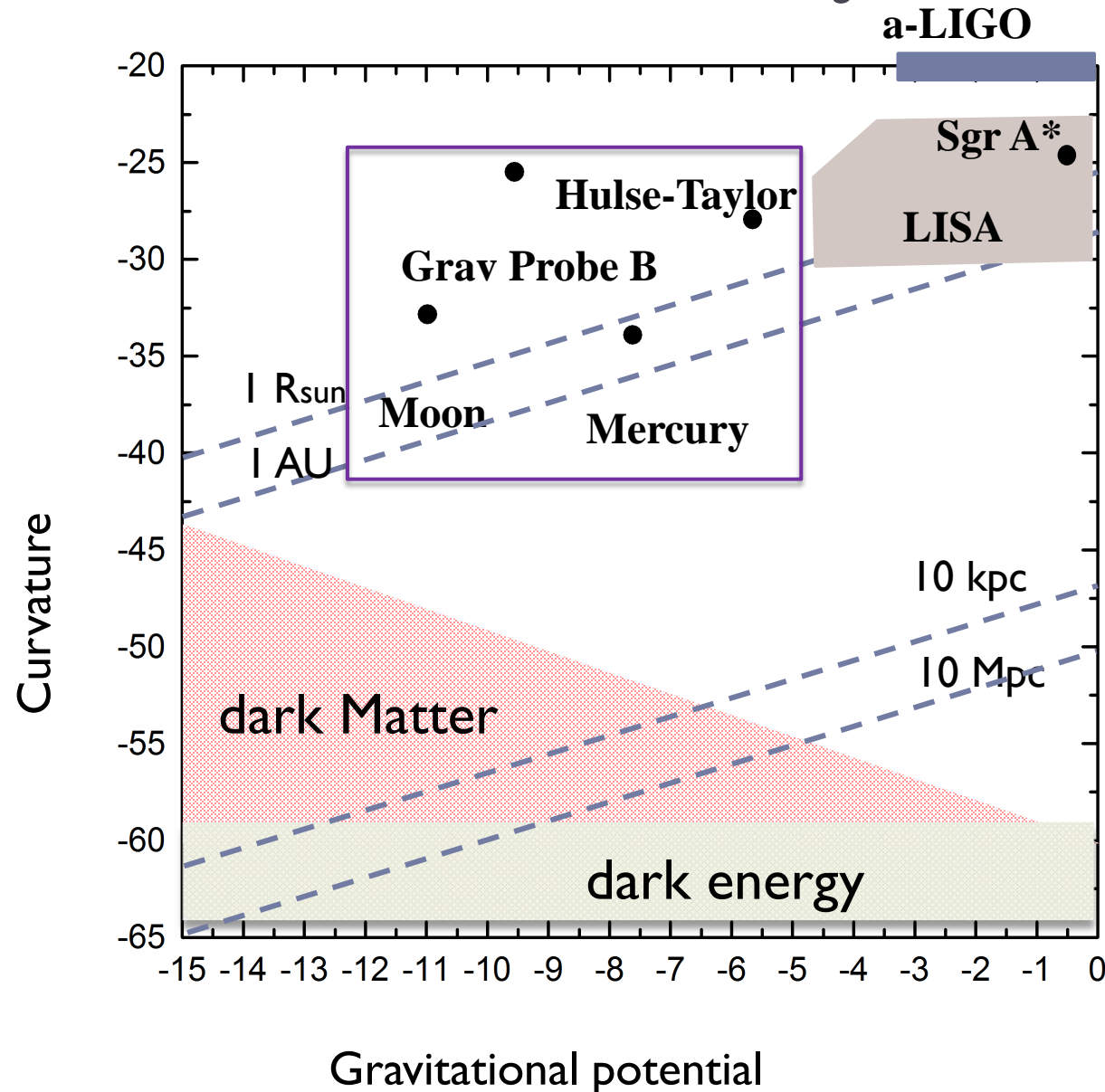
A background image showing a complex network of blue filaments and nodes, representing the cosmic web or dark matter distribution in the universe.

# Observational probes of dark energy/modified gravity

Kazuya Koyama

Institute of Cosmology and Gravitation,  
University of Portsmouth

# Tests of General Relativity



curvature

$$R = \frac{GM}{r^3}$$

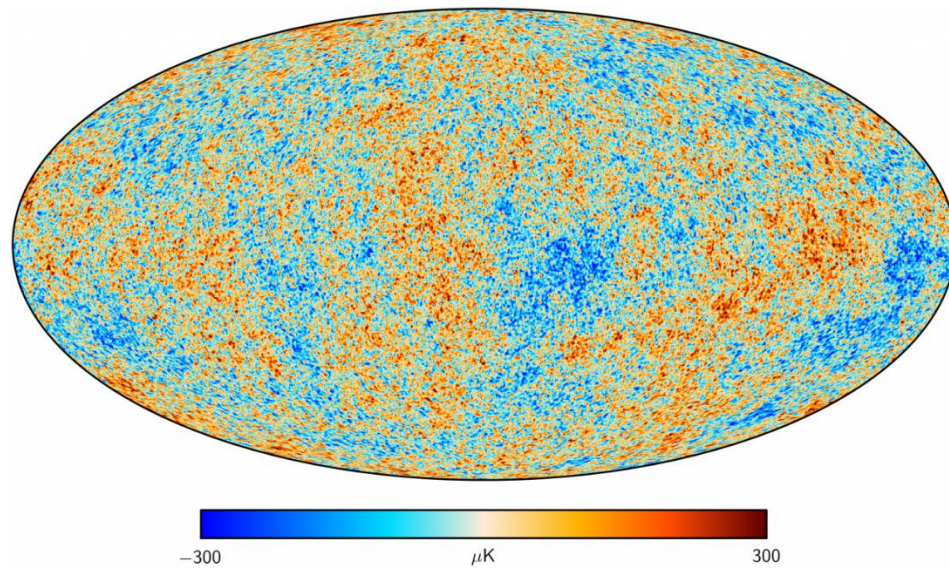
potential

$$\psi = \frac{GM}{r}$$

# Cosmological observations

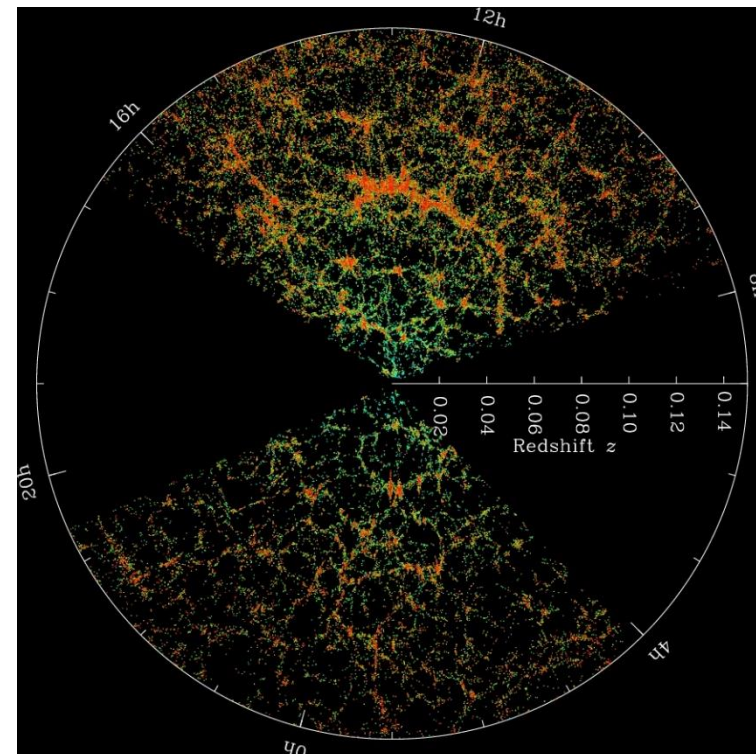
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## Cosmic microwave background (CMB)



<https://www.cosmos.esa.int/web/planck>

## Large scale structure



<https://www.sdss.org/>

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# Standard model of cosmology

## ► Lambda (L) CDM model

Einstein equations and matter conservation

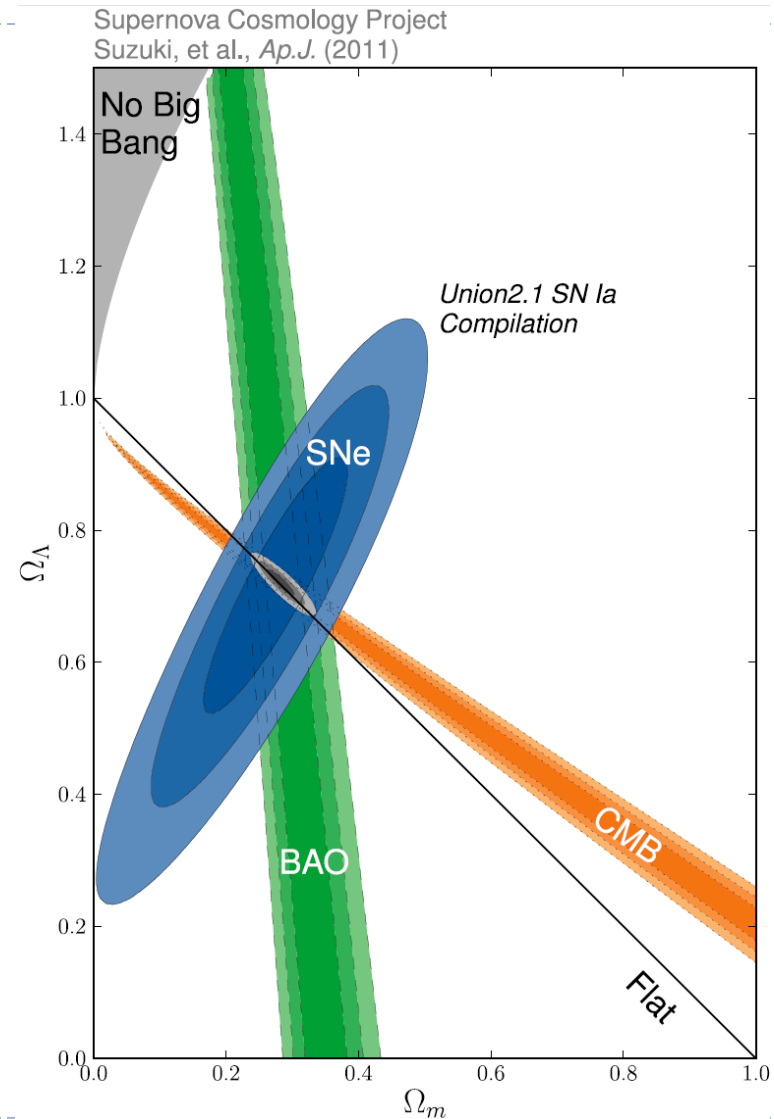
$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}$$

$$\dot{\rho} + 3H(\rho + P) = 0, \quad \rho = \sum_i \rho_i$$

The background expansion history

$$E(z) = \frac{H(z)}{H_0} \quad 1+z = \frac{a_0}{a}$$

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda$$



# Linear perturbations

- ▶ Geometry (FRW metric + perturbations)

$$ds^2 = a(\eta)^2 [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)d\vec{x}^2]$$

- ▶ Matter

$$T_0^0 = -\rho_m(1 + \delta_m)$$

$$T_i^0 = \rho_m v_{mi}, \quad \partial^i v_{mi} = \theta_m$$

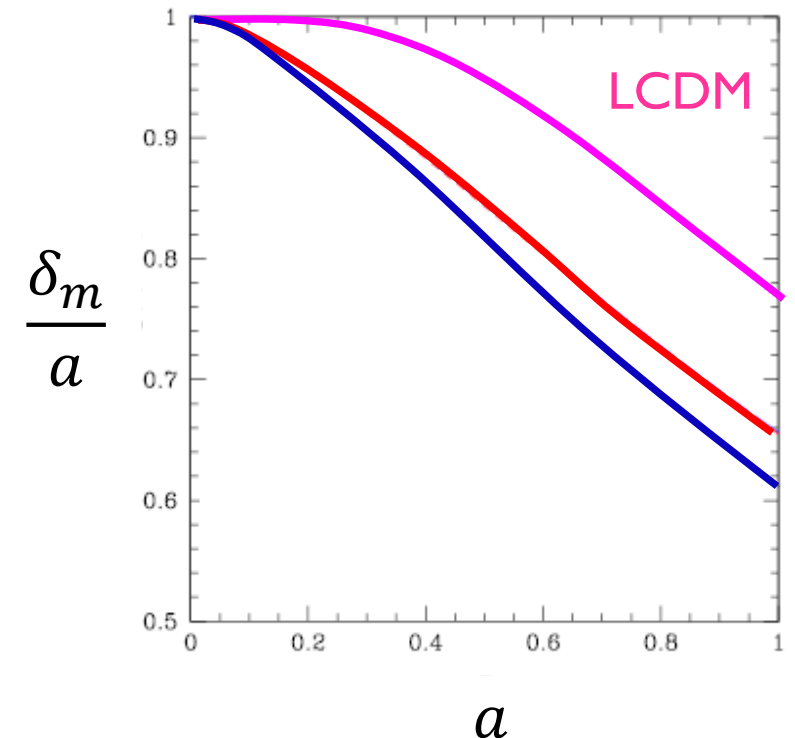
Energy-momentum conservation

$$\dot{\delta}_m - \frac{1}{a}\theta_m - 3\dot{\Phi} = 0$$

$$\dot{\theta}_m + H\theta_m - \frac{k^2}{a^2}\Psi = 0$$

$$\Rightarrow \ddot{\delta}_m + 2H\dot{\delta}_m = \frac{k^2}{a^2}\Psi$$

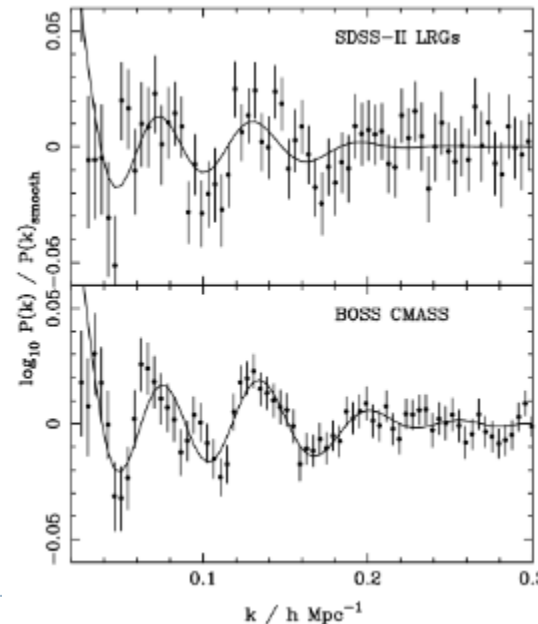
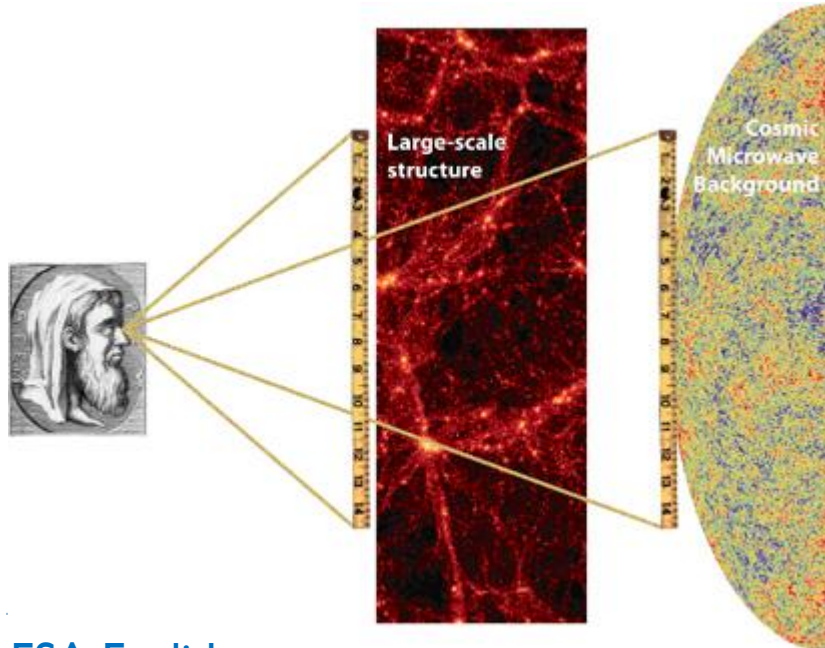
*modified gravity changes the growth of structure formation*



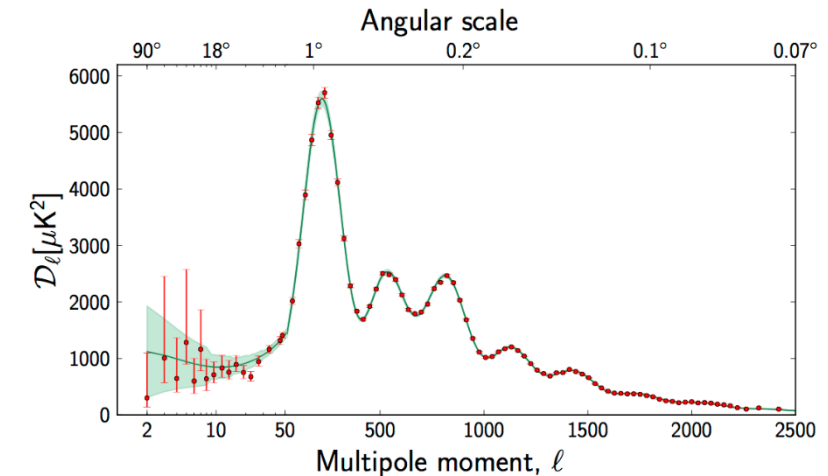


# Observations –background

- ▶ Background  $H(z)$  comoving distance  $r(z) = H_0^{-1} \int_0^z \frac{1}{E(z')} dz'$
- Supernovae: luminosity distance
- CMB/Baryon Acoustic Oscillation (BAO): angular diameter distance



BOSS



ESA Planck

# Observations

## ► Weak lensing

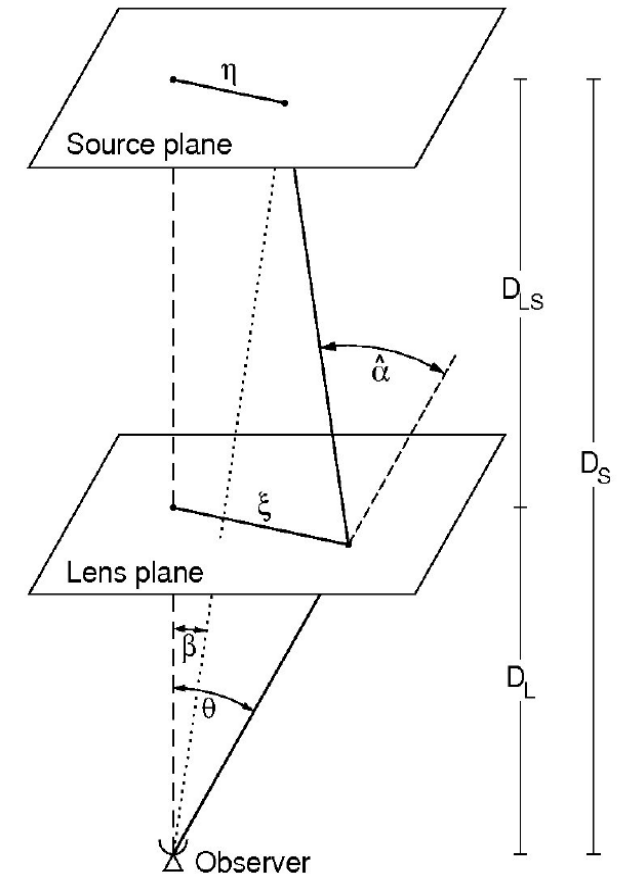
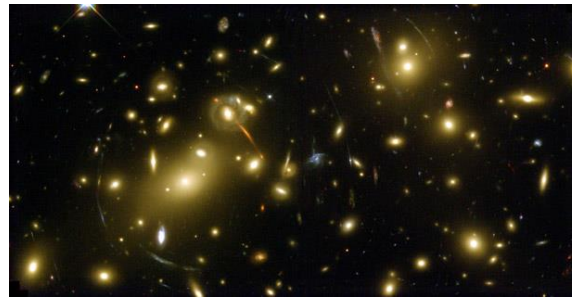
Bartelmann & Schneider astro-ph/9912508

$$ds^2 = a^2 \left[ -(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$

Convergence

$$\kappa(\vec{n}) = \int d\chi \underbrace{\frac{D_{SL} D_L}{D_S}}_{\text{geometry}} \nabla_{\perp}^2 \phi_W(\eta_0 - \chi, \chi \vec{n}), \quad \phi_W = \frac{1}{2} (\Psi + \Phi)$$

Galaxy shape is determined by shear which can be computed from convergence

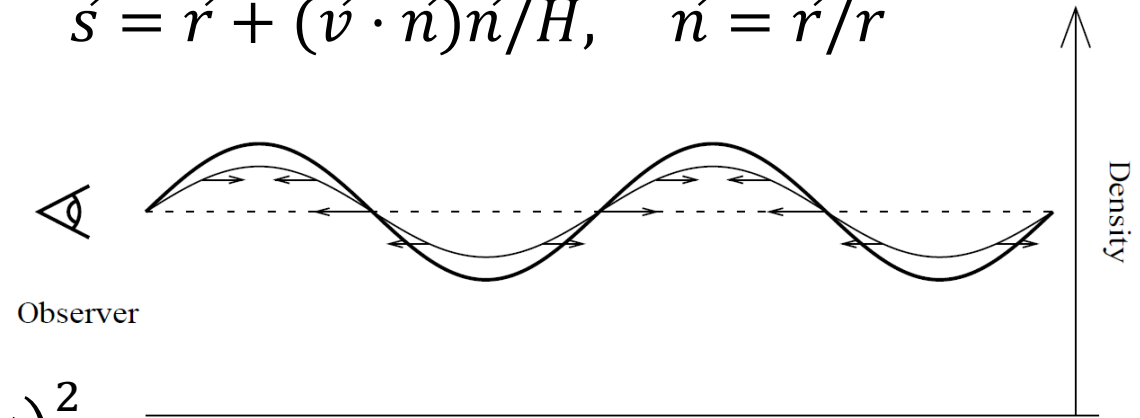


# Observations

## ▶ Redshift distortions

galaxies have peculiar velocities  
 clustering of galaxies in redshift space  
 is enhanced along the line of sight

$$\vec{s} = \vec{r} + (\vec{v} \cdot \vec{n})\vec{n}/H, \quad \vec{n} = \vec{r}/r$$



$$\delta^s(k, \mu) = \delta_m(k) - \mu^2 \theta_m(k), \quad \mu^2 = \frac{(\vec{k} \cdot \vec{n})^2}{k^2}$$

Hamilton astro-ph/9708102

If the continuity equation holds, the velocity divergence is related to the growth rate

$$\delta^s(k, \mu) = \delta_m(k) \left( 1 - \mu^2 \frac{\theta_m(k)}{\delta_m(k)} \right) = \delta_m(k) (1 + \mu^2 f) \quad f = \frac{d \ln \delta_m}{d \ln a}$$

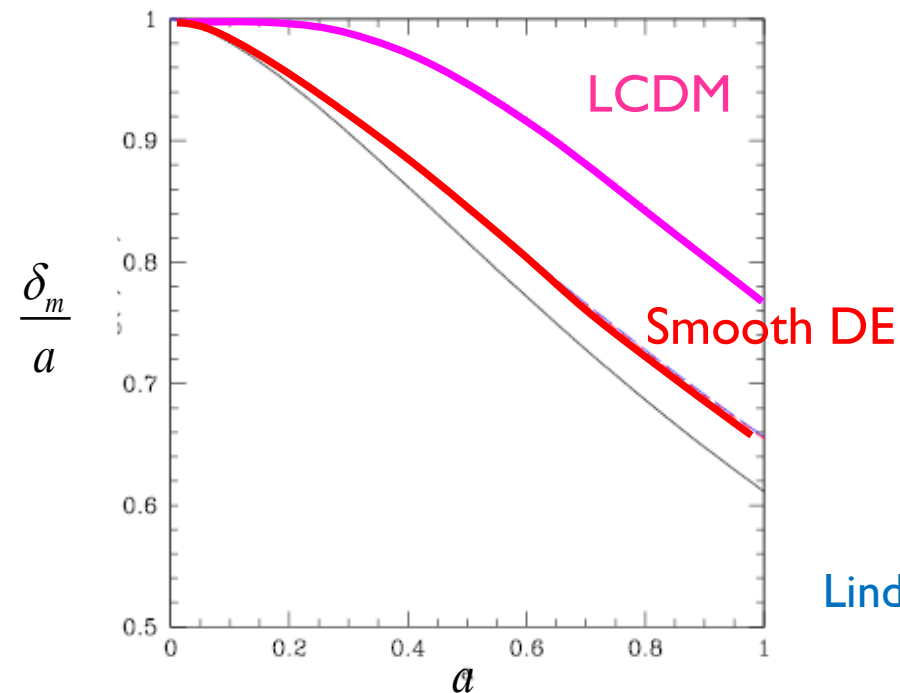
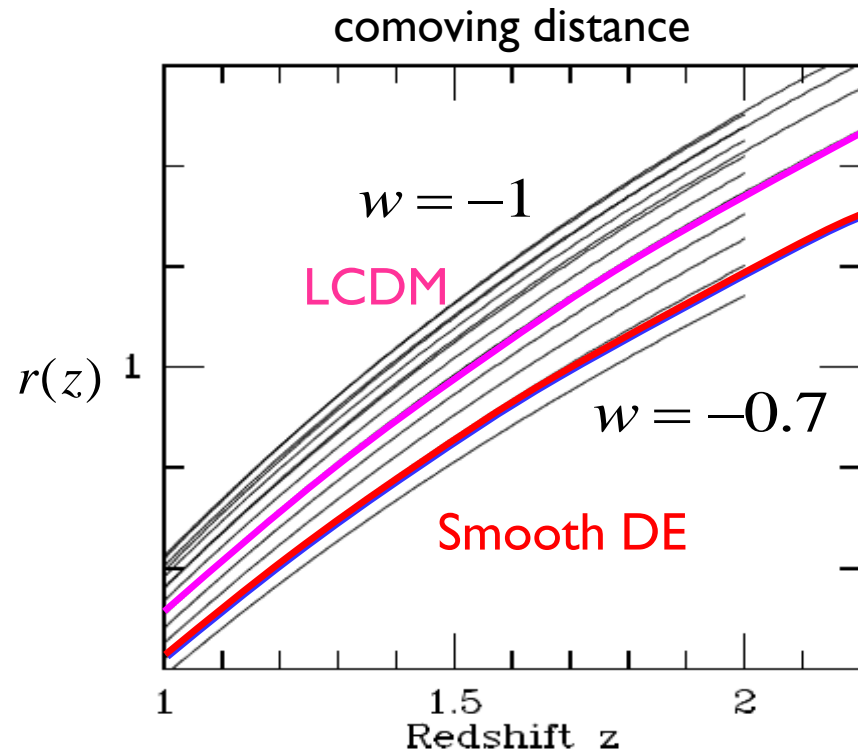




# Expansion history v structure growth

## ► LCDM/Smooth DE

There is a one-to-one correspondence between background expansion history and growth of structure



$$w(z) = \frac{P_{DE}}{\rho_{DE}}$$

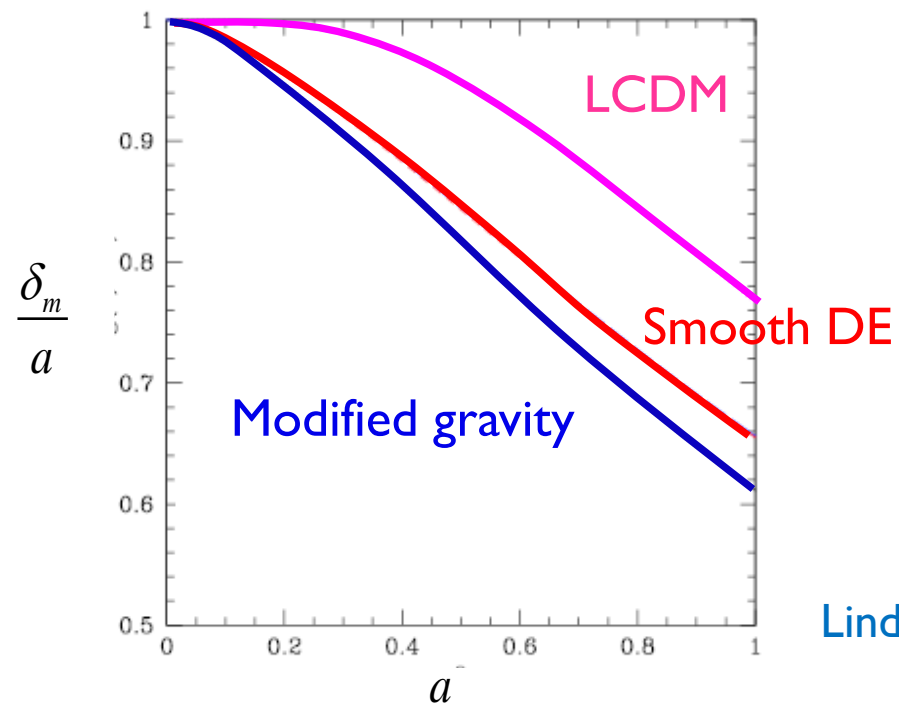
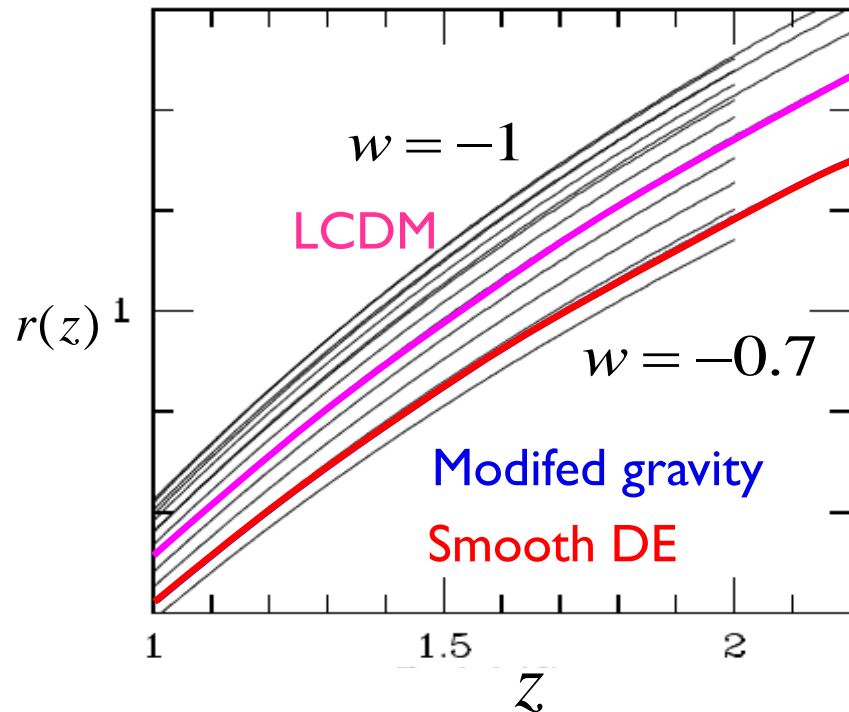
Linder astro-ph/0507263

# Expansion history v structure growth

## ► Modified gravity

Modified gravity changes the growth of structure formation

Even if it has the same expansion history as smooth DE, structure growth is different

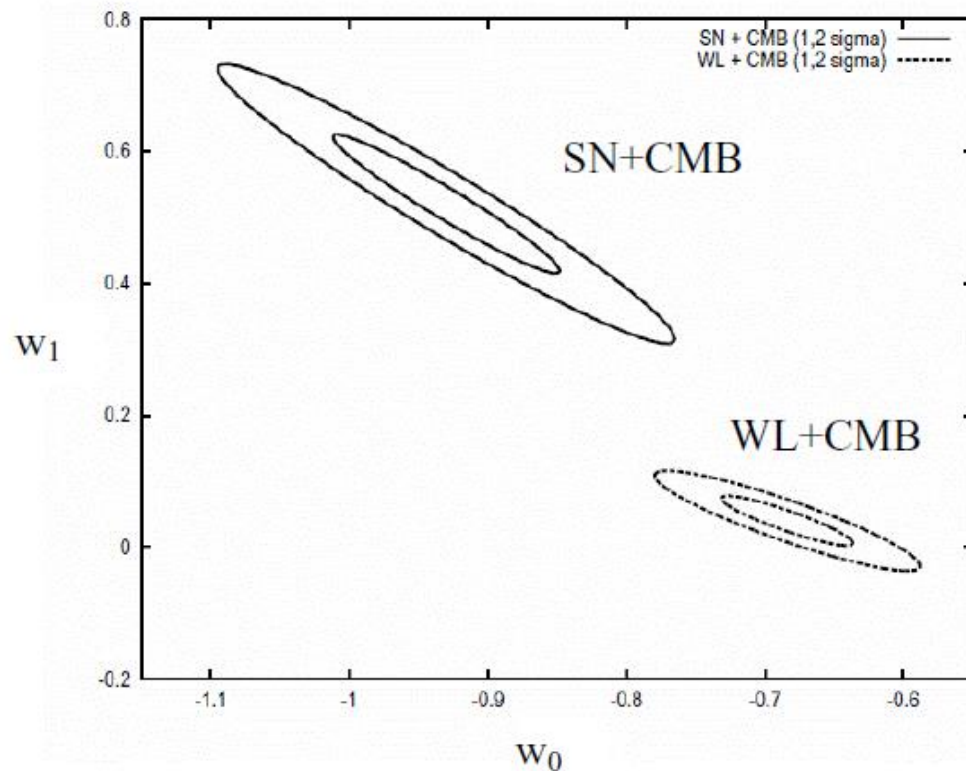


Linder astro-ph/0507263

# Consistency test

Assume that the Universe is described by a modified gravity model but we still try to fit the data using smooth DE

$$w(z) = w_0 + w_1 z,$$



SNe+CMB  
SNe+weak lensing

$$w(z) = \frac{P_{DE}}{\rho_{DE}}$$

Inconsistent!

Ishak et.al. astro-ph/0507184

# Consistency relation

- ▶ In GR, gravitational equations are given by

$$H^2 = \frac{8\pi G}{3} \rho_T, \quad \rho_T = \sum_i \rho_i$$

$$\frac{k^2}{a^2} \Phi = 4\pi G a^2 \rho_T \delta_T, \quad \rho_T \delta_T = \sum_i \rho_i \delta_i$$

- ▶ Consistency relation

$$\alpha(k, t) = \frac{2k^2}{3a^2 H^2} \frac{(\Phi + \Psi) - \Psi}{\delta_T} = 1$$

↙ background
↓ Weak lensing
↘ Galaxy distribution
↗ Redshift Space Distortion

$k^2 \Psi = \frac{d(a\theta_m)}{dt}$

$\delta_g = b_T \delta_T$

*We have just enough number of observations to check the relation*

# Parametrisation

Amendola et.al JCAP 0804 (2008) 013  
Zhao et.al. Phys. Rev. Lett. 103 (2009) 241301

## ► Background

$$F(H^2) = \frac{8\pi G}{3} \rho_m \quad \Rightarrow \quad H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE})$$

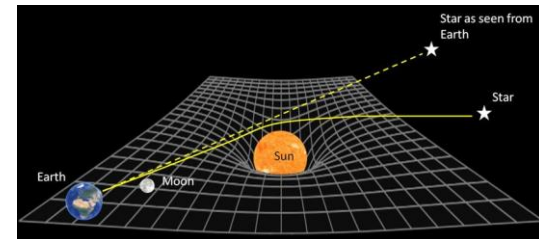
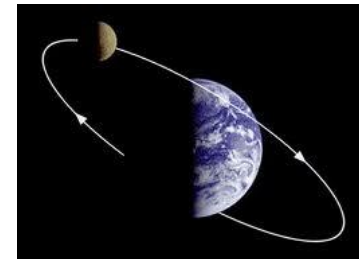
Equation of state  $w_{DE}(z) = \frac{P_{DE}}{\rho_{DE}}$  can be ill-defined for modified gravity as  $\rho_{DE}$  can vanish

Instead, we can parametrise the effective dark energy density directly  $\Omega_{DE}(z) = \frac{\rho_{DE}(z)}{\rho_{crit}}$

## ► Perturbations

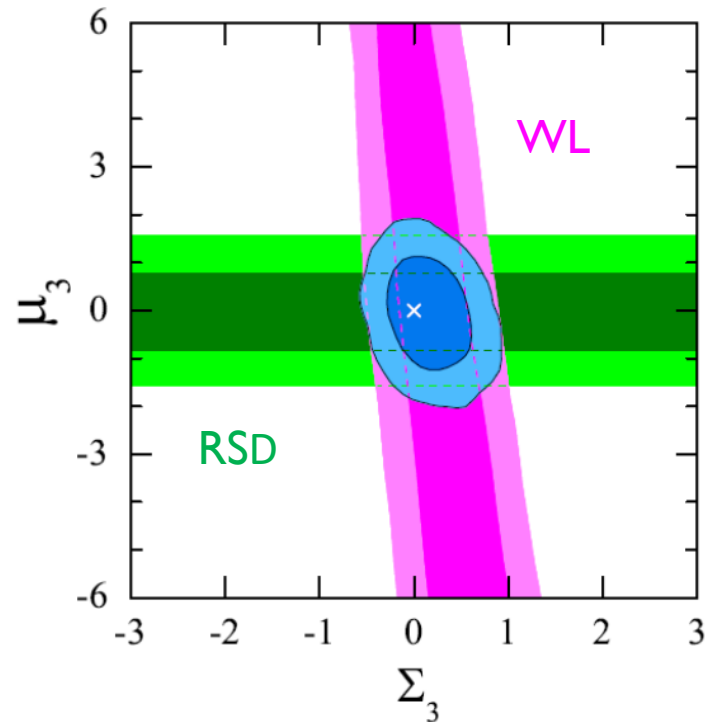
$$k^2 \Psi = -4\pi G a^2 \mu(z, k) \rho_m \delta_m \quad : \text{Newton potential}$$

$$k^2 (\Psi + \Phi) = -8\pi G a^2 \Sigma(z, k) \rho_m \delta_m \quad : \text{lensing potential}$$



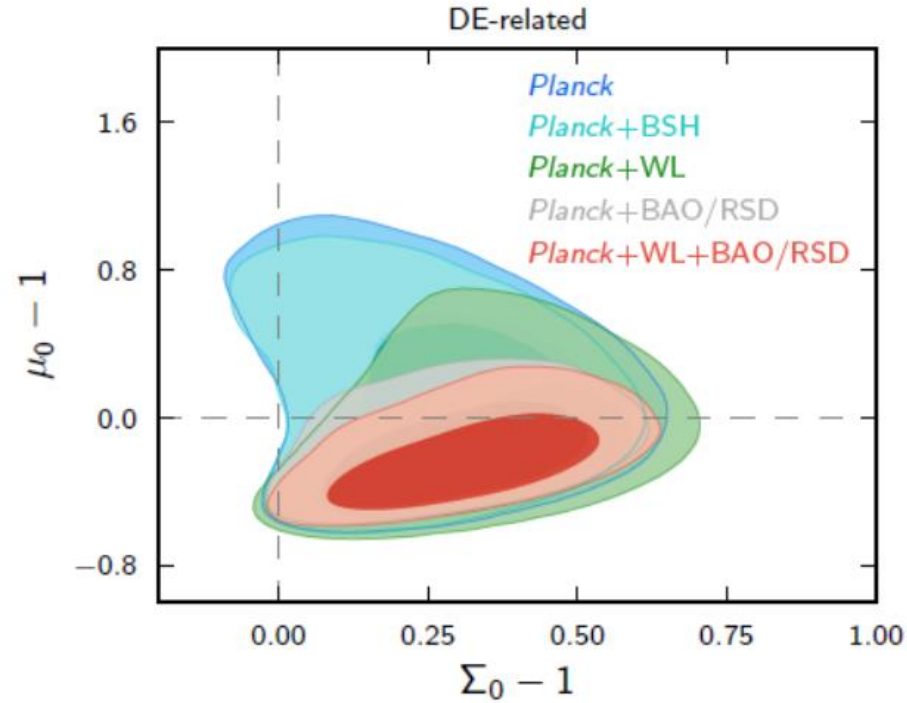
# Current constraints

## ► Weak Lensing +Redshift distortion



$$\mu(a) = \mu_s(1 + a^s), \Sigma(a) = \Sigma_s(1 + a^s)$$

Song et.al. PRD84 (2011) 083523



$$\mu(a, k) = \mu_0 \Omega_L(a), \quad \Sigma(a, k) = \Sigma_0 \Omega_L(a)$$

Planck 2015 “Modified gravity and dark energy”

► RSD: Redshift space distortions

WL: Weak lensing



# From theory to data

Peirone et.al. arXiv:1712.00444

Espejo et.al. arXiv:1809.01121

## ► Effective theory of dark energy [Gubitosi et.al. arXiv:1210.0201](#)

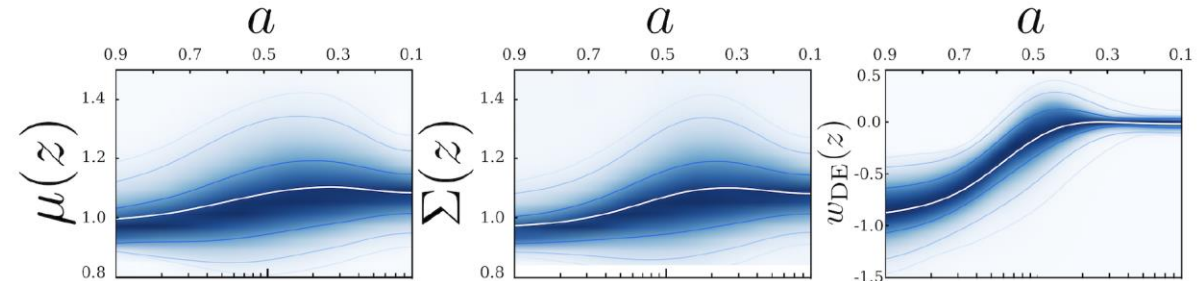
General description of the background and linear perturbations in a scalar-tensor theory

$$\begin{aligned} \mathcal{S} = & \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} \Omega(t) R + \Lambda(t) - c(t) a^2 \delta g^{00} \right. \\ & + \frac{M_2^4(t)}{2} (a^2 \delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2} a^2 \delta g^{00} \delta K^\mu{}_\mu \\ & + \frac{\bar{M}_2^2(t)}{2} \left[ (\delta K^\mu{}_\mu)^2 - \delta K^\mu{}_\nu \delta K^\nu{}_\mu - \frac{a^2}{2} \delta g^{00} \delta \mathcal{R} \right] + \\ & \left. + S_m[g_{\mu\nu}, \chi_m], \right\} \end{aligned}$$

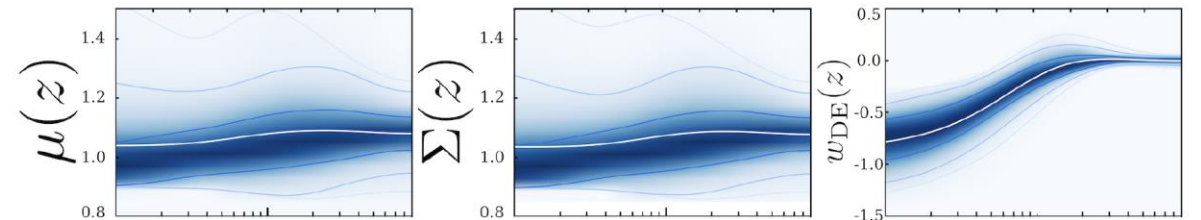
a random sample of these functions

$$f(a) = \frac{\sum_{n=0}^N \alpha_n (a - a_0)^n}{1 + \sum_{m=1}^M \beta_m (a - a_0)^m}$$

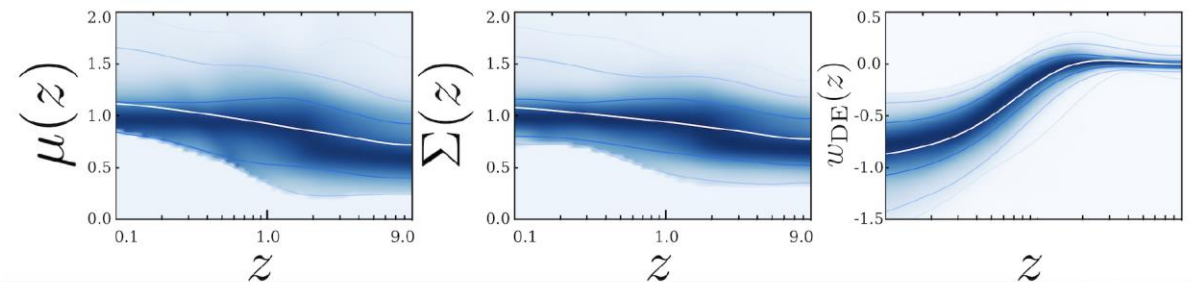
GBD



H<sub>S</sub>



HOR

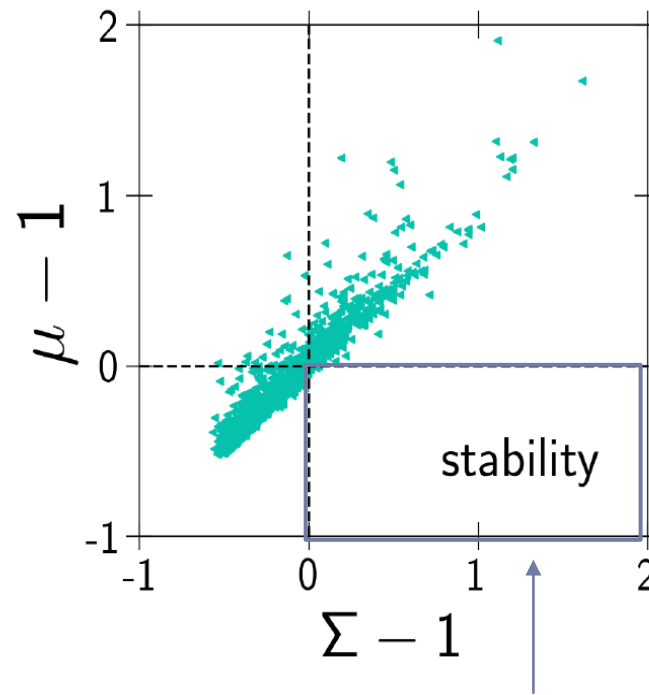
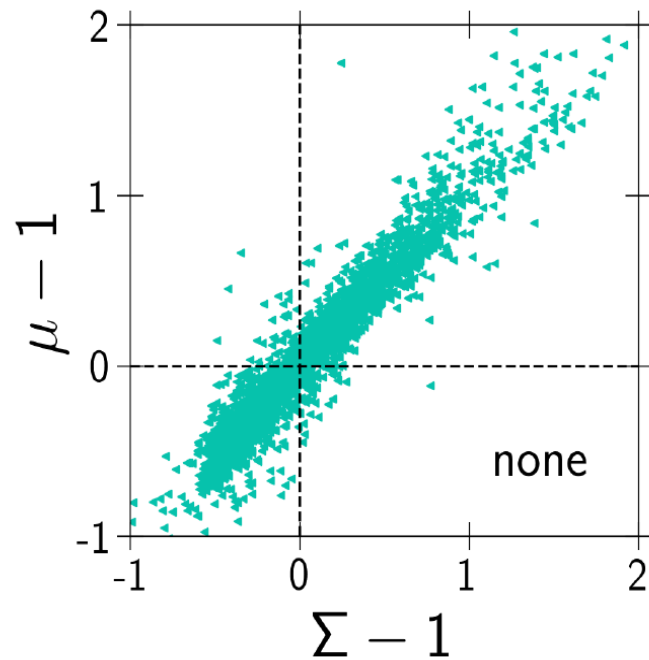


# The role of stability and observational prior

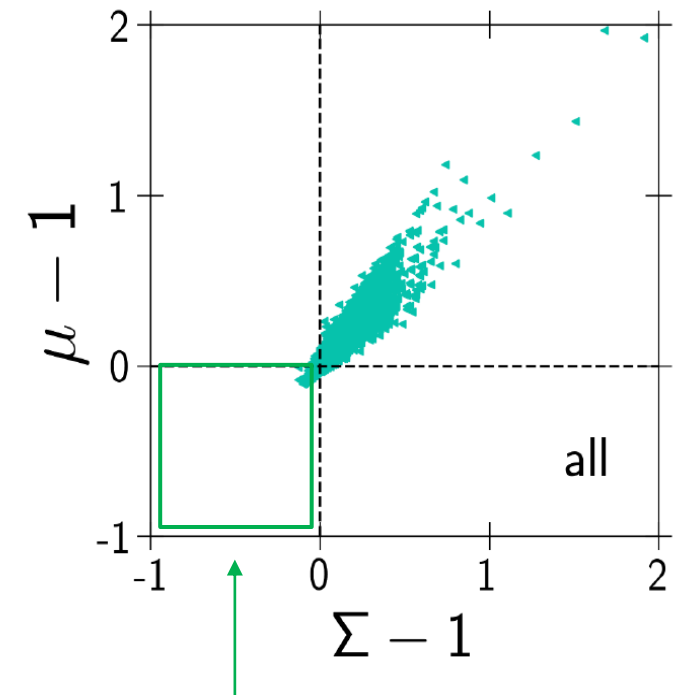
Peirone et.al. arXiv:1712.00444

- ▶ Horndeski theory (the most general scalar tensor theory with 2<sup>nd</sup> order e.o.m)  
gravitational wave constraints

$$-3 \times 10^{-15} < \frac{c_{\text{GW}}}{c} - 1 < 7 \times 10^{-16}$$



Stability condition removes  
this part



Observational prior removes  
this part

# “Tensions” with LCDM – Hubble constant

## ▶ Local measurement of Hubble constant

$$\begin{aligned} m &= M + 25 + 5 \log_{10} D_L(z), \\ &= -5a + 5 \log_{10} c \hat{d}_L(z) \end{aligned}$$

$$\begin{aligned} 5a &= -(M + 25 - 5 \log_{10} H_0) \\ \hat{d}_L(z) &= H_0 D_L(z) / c \end{aligned}$$

m: apparent magnitude, M: absolute magnitude

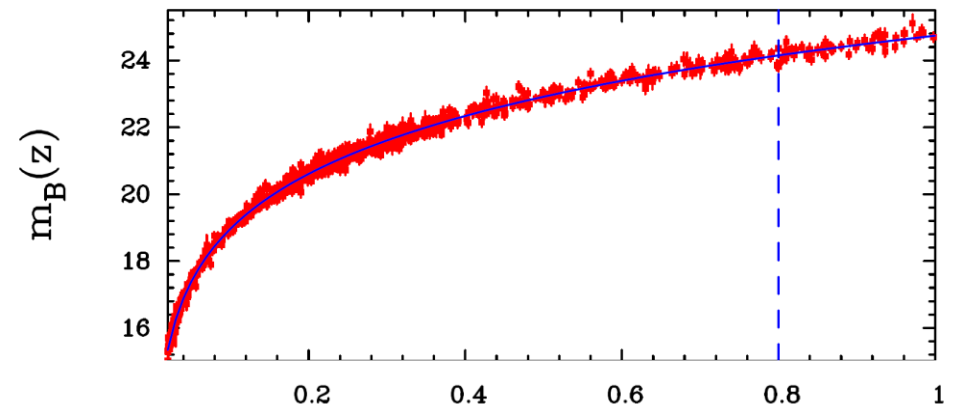
## ▶ Pantheon SNe [Efstathiou arXiv:2103.08723](#)

$$a_B = 0.71273 \pm 0.00176$$

## ▶ Local distance ladder

$$M_B^0 = -19.253 \pm 0.027 \text{ mag}$$

$$\Rightarrow H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$$



SH0ES collaboration<sub>z</sub>

Riess et.al. [arXiv:2112.04510](#)

**5 sigma tension with LCDM + Planck**

$H_0 = 67.4 \pm 0.5$  (Planck Collaboration et al. 2020)

# “Tensions” with LCDM – weak lensing

## ► Weak lensing

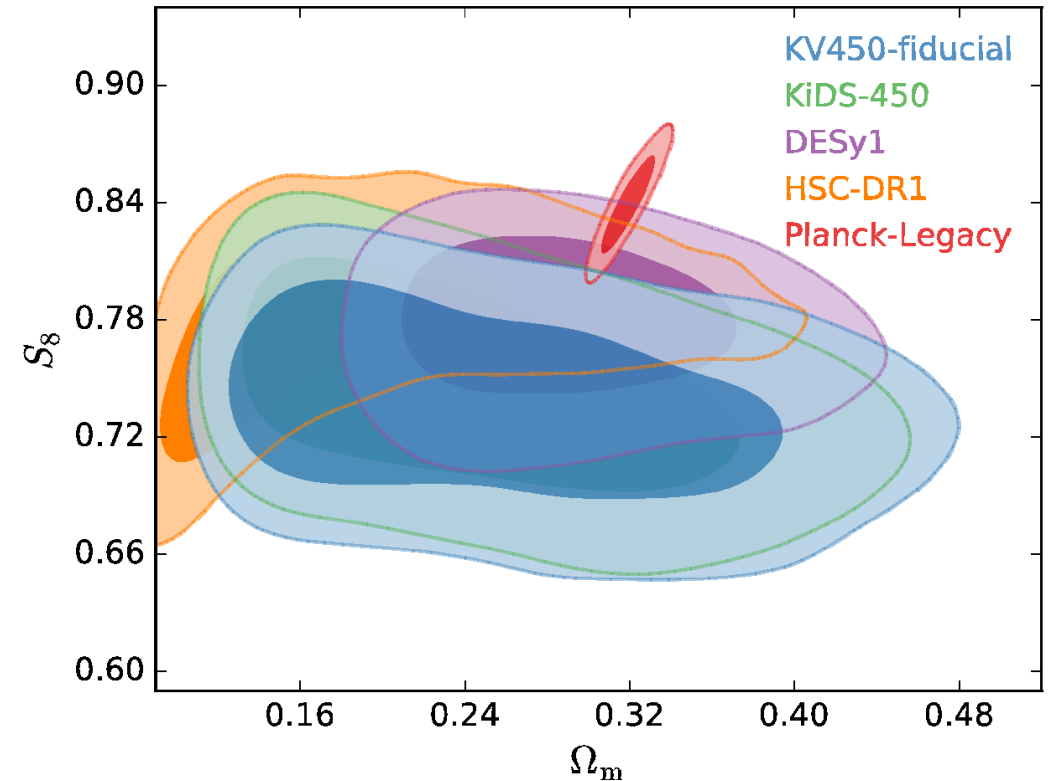
The amplitude of weak lensing is determined by the  $S_8$  parameters

$$S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$$

$\sigma_8$  : amplitude of fluctuations

The prediction from CMB is slightly larger than the values from WL surveys.

CMB constraint here assume LCDM



DES: <https://www.darkenergysurvey.org/>; HSC: <https://www.naoj.org/Projects/HSC/>;

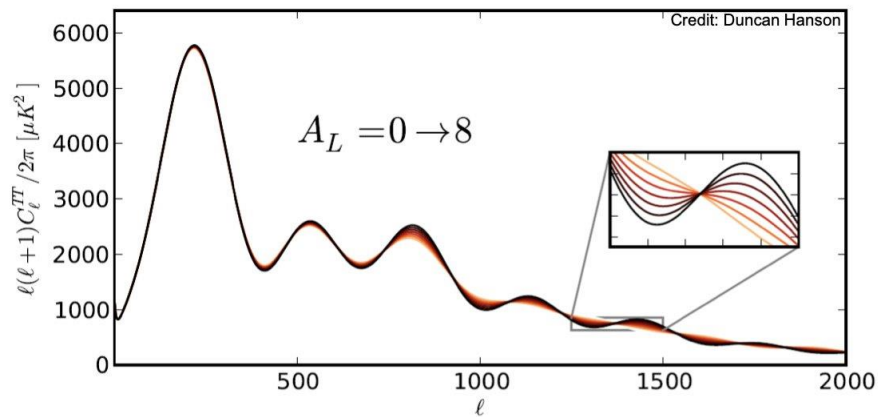
KiDS: <http://kids.strw.leidenuniv.nl/DR3/lensing.php>

# “Tensions” with LCDM – CMB lensing

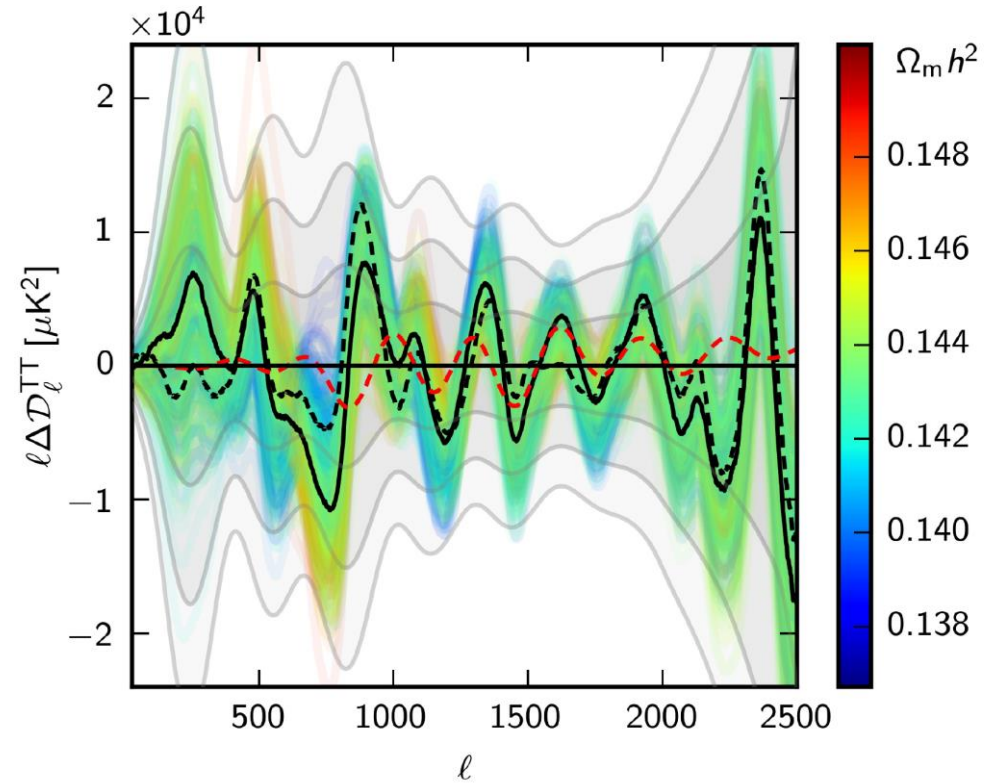
## ► CMB temperature power spectrum

The Planck CMB temperature power spectrum is well fitted by LCDM but at high  $\ell$ , there are residual oscillations

CMB peaks are smeared out by CMB lensing. These residuals are well fitted if CMB lensing amplitude is larger than that in LCDM



$$\tilde{C}_\ell^{\phi\phi} = A_L C_\ell^{\phi\phi}$$



$$A_L = 1.243 \pm 0.096$$

Planck 2018 arXiv:1807.06209

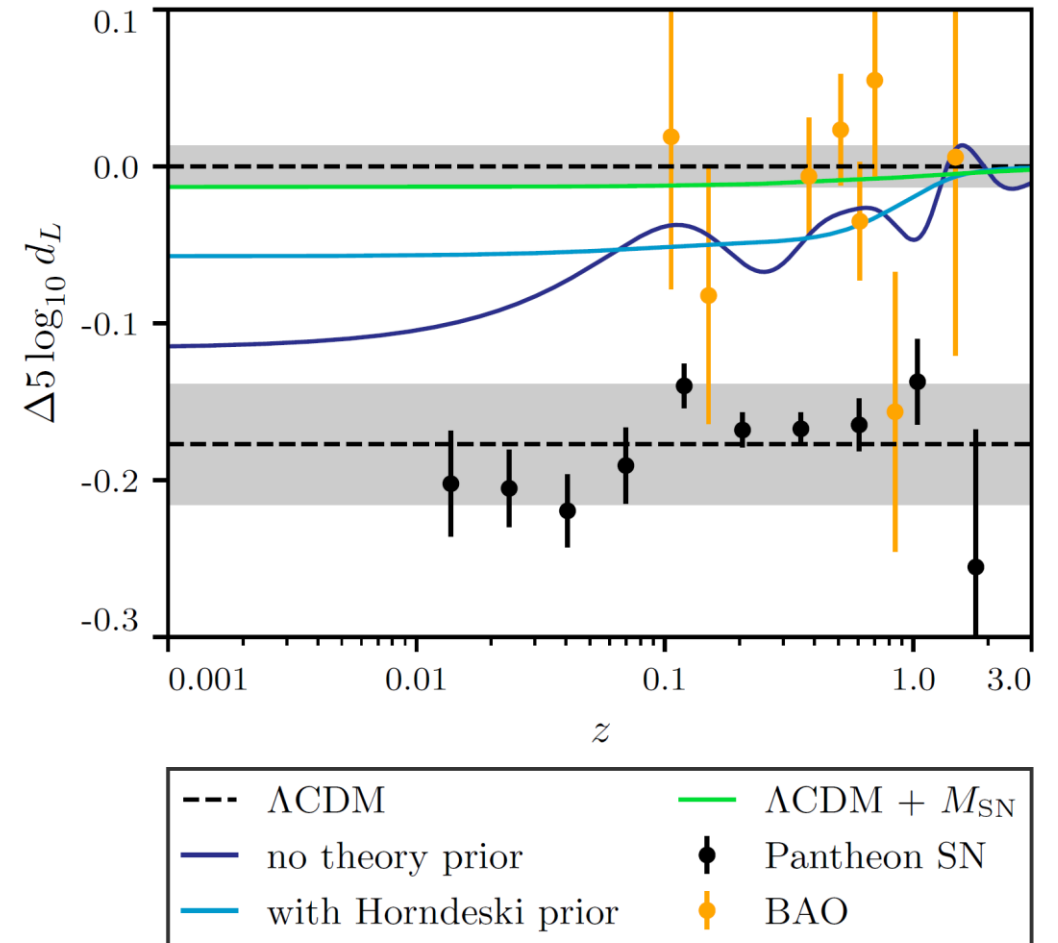
# Hubble constant tension in extended cosmologies

## ► Hubble constant tension

The luminosity distance inferred from CMB and BAO does not agree with the one calibrated from SNe with the prior on the absolute magnitude from the local distance ladder

This makes it hard for late time modifications to fully resolve the tension even though the fit can be improved from LCDM

A prior on  $H_0$  can lead to an inconsistent result  
The proper way is to put a prior on the absolute magnitude [Efstathiou arXiv:2103.08723](https://arxiv.org/abs/2103.08723)





# Implications for tensions in extended cosmologies

## ▶ Extended cosmologies

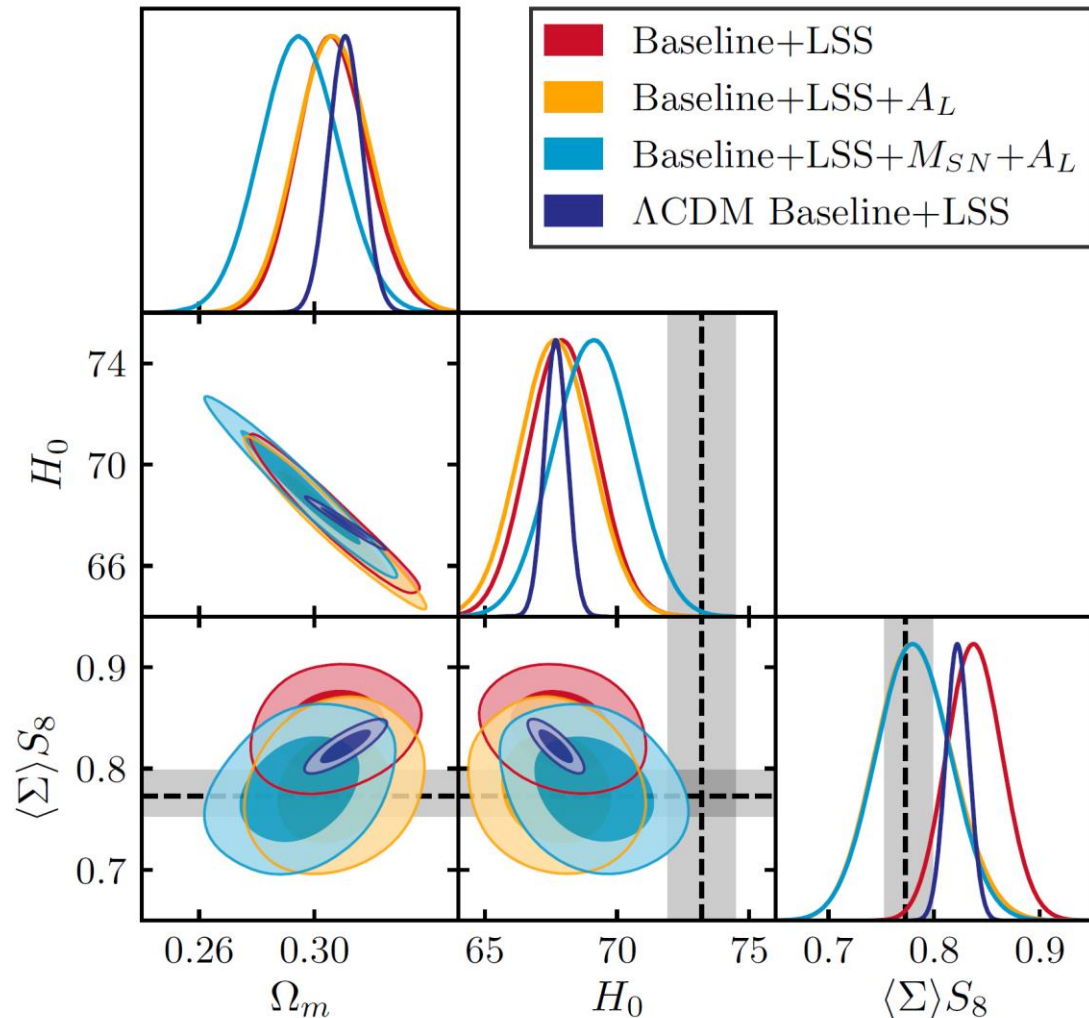
$$\Omega_{DE}(z), \mu(z), \Sigma(z)$$

## ▶ Hubble constant

- ▶ It is not possible to resolve the tension fully due to the inconsistency with BAO

## ▶ Lensing anomalies

- ▶ CMB lensing anomaly can be resolved either by  $\Sigma > 1$  or  $A_L > 1$ .
- ▶ Fit to DES cannot be improved even if  $S_8$  is lower if  $\Sigma > 1$  as  $\Sigma \times S_8$  stays the same.
- ▶ We need  $A_L > 1$  to improve fit to DES



# Going beyond linear scales

---

- ▶ **Ample information on non-linear scales**

Parametrisation is valid only for linear perturbations

Conservative cut-offs are required to remove data on non-linear scale, which significantly degrade the constraining power

- ▶ **Extraction of linear information**

For redshift distortions, non-linear modelling is required to extract the linear growth rate, which is done normally within LCDM

- ▶ **New information on non-linear scales**

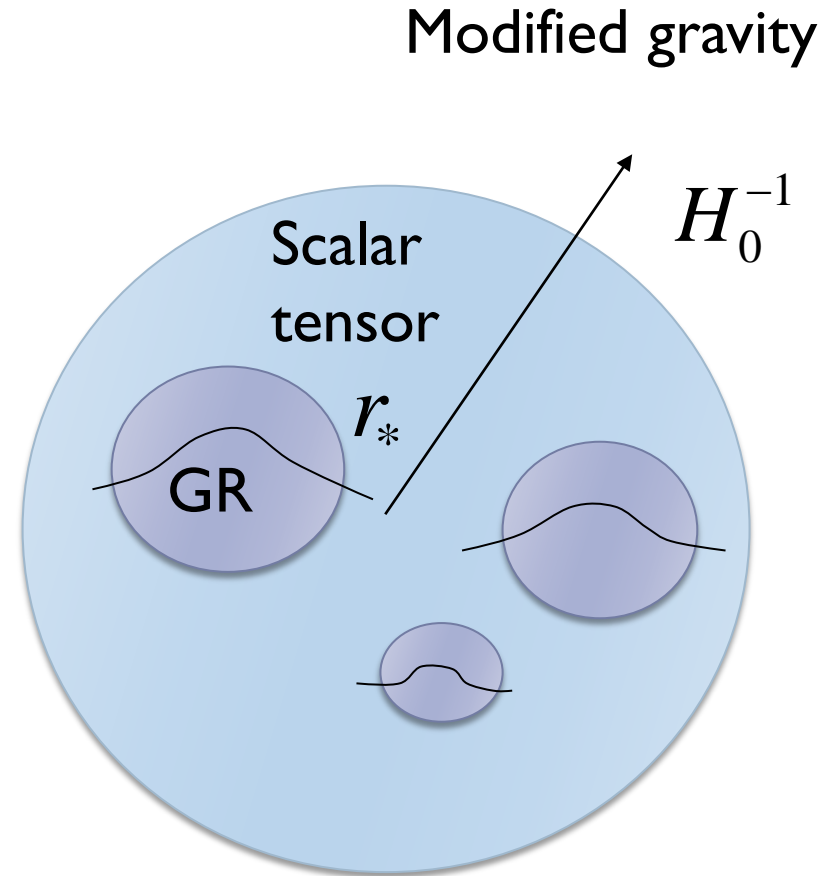
On non-linear scales, **screening mechanisms** can be important leaving interesting signatures



# General picture

---

- ▶ **Largest scales**  
gravity is modified so that the universe accelerates without dark energy
- ▶ **Large scale structure scales**  
gravity is still modified by a fifth force from scalar graviton
- ▶ **Small scales (solar system)**  
GR is recovered by “screening mechanism”



# Where to test GR

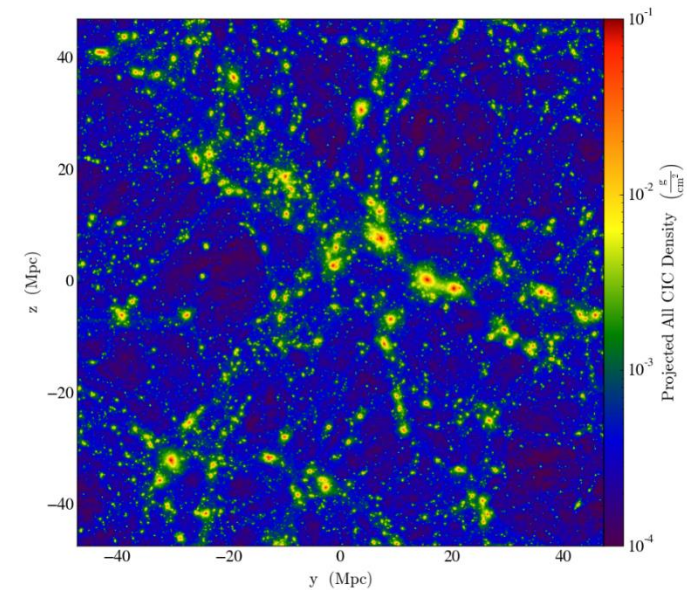
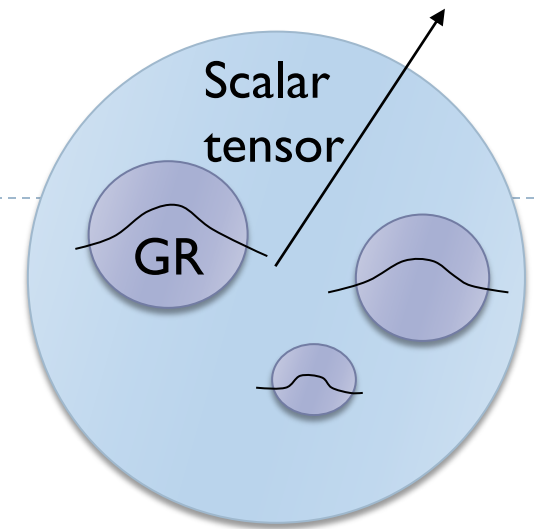
- ▶ GR is recovered in “high dense regions”  
Details depend on the screening mechanism

## Chameleon mechanism (environmental dependent mass)

- ▶ Screening of dark matter halos depends on mass and environment
- ▶ Strongest constraints come from objects with a shallow potential in low density environment

## Vainshtein mechanism (derivative interaction)

- ▶ Screening of dark matter halos does not depend on mass and environment
- ▶ Strongest constraints come from linear scales

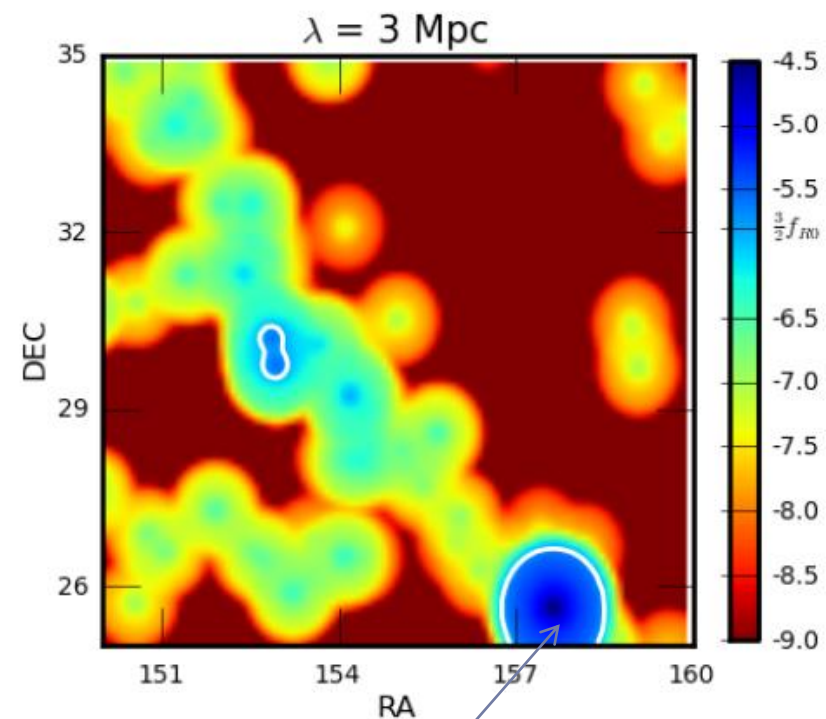
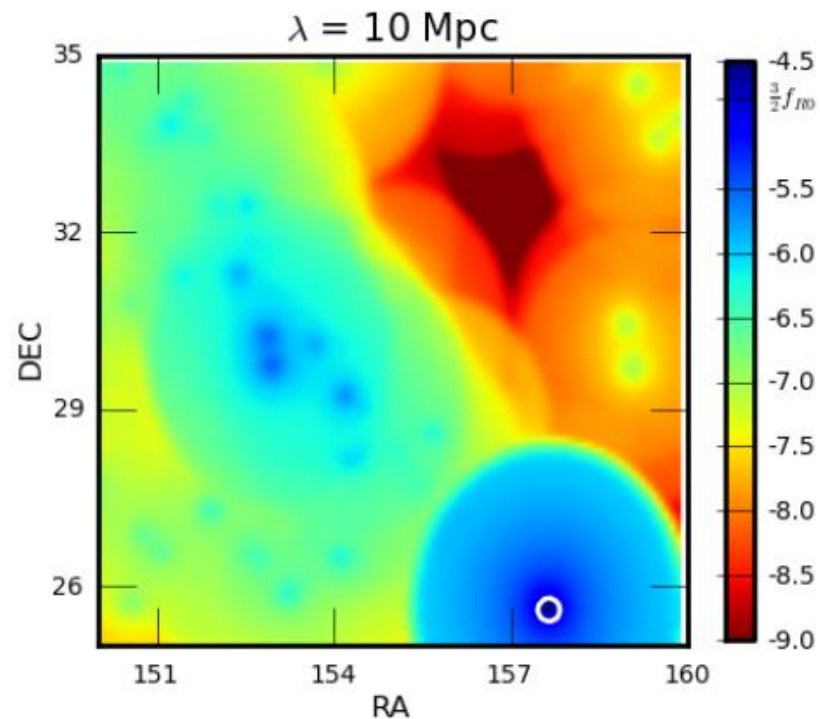


**Modified Gravity Simulations comparison**  
**Winther et.al. arXiv: 1506.06384**



# Creating a screening map (chameleon mechanism)

- ▶ It is essential to find places where GR is not recovered
  - ▶ Small galaxies in underdense regions [Cabre, Vikram, Zhao, Jain, KK JCAP 1207 \(2012\) 034](#)
  - ▶ SDSS galaxies within 200 Mpc



GR is recovered

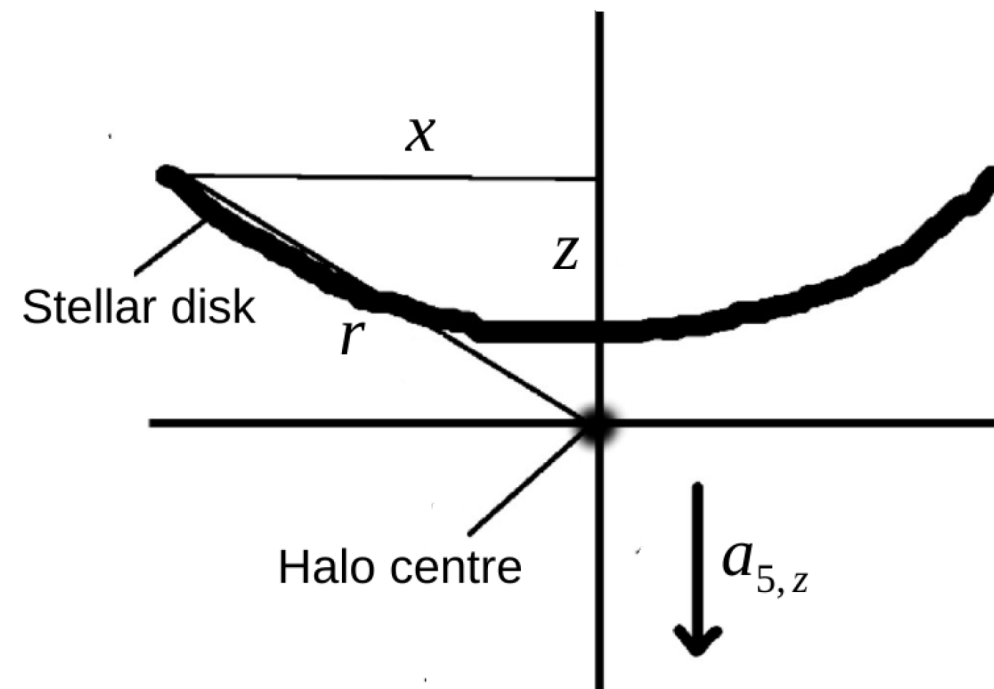
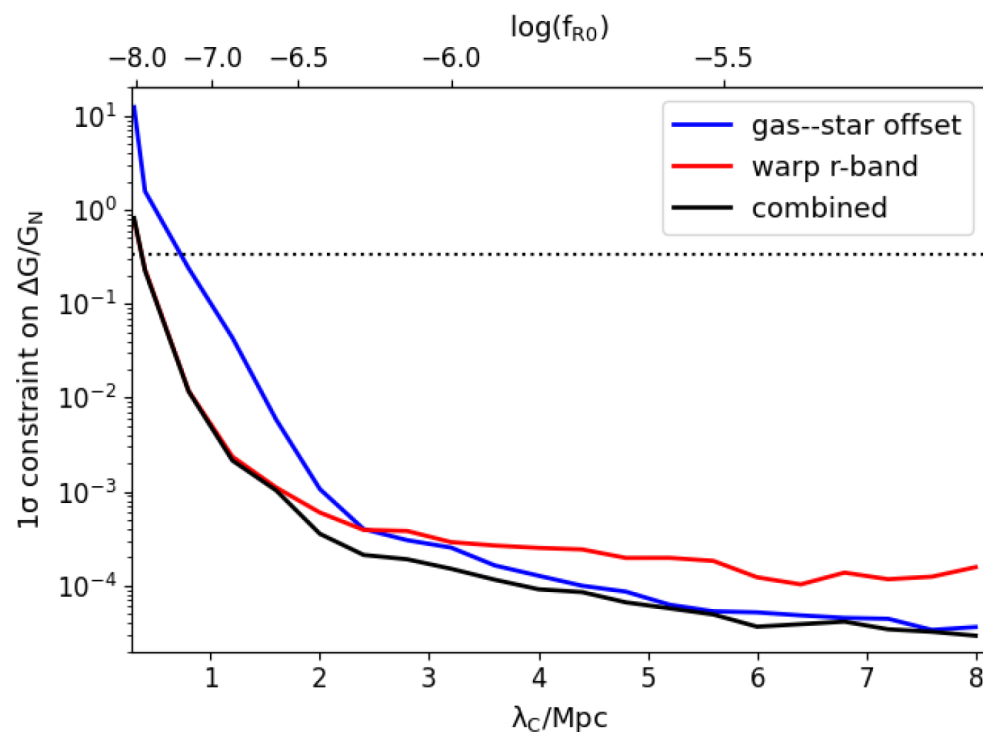
# Tests of chameleon gravity

Hui, Nicolis & Stubbs Phys. Rev. D80 (2009) 104002

## ▶ dwarf galaxies in voids

strong modified gravity effects on dark matter (but not on stars)

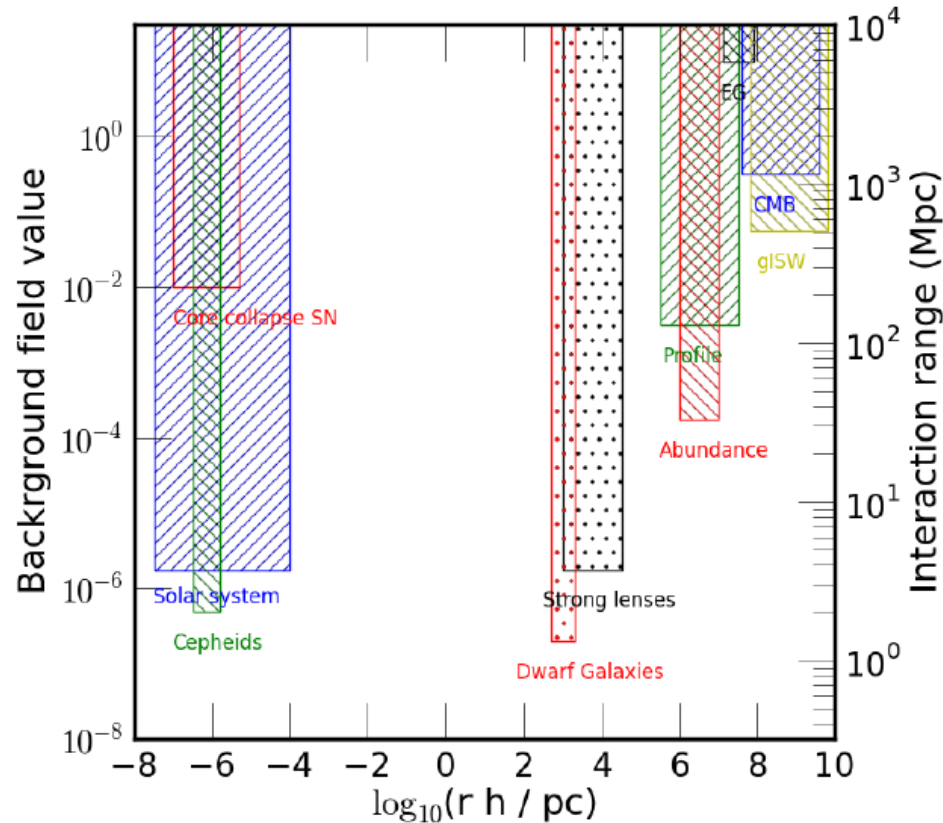
- ▶ Warping of stellar disks
- ▶ Gas-star offset



Desmond & Ferreira arXiv:2009.08743



# Constraints on chameleon gravity



Jain et.al. 1309.5389

Interaction range of the extra force that modifies GR in the cosmological background

GR

- ▶ Non-linear regime is powerful for constraining chameleon gravity
- ▶ Astrophysical tests could give better constraints than the solar system tests

# Summary

- ▶ In the next decade, we will be able to test the nature of dark energy and may be able to detect the failure of GR on cosmological scales

- ▶ **Linear scales**

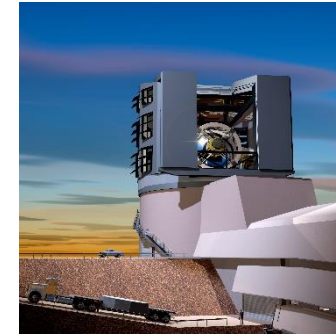
model independent tests of gravity



DARK ENERGY  
SURVEY



HSC

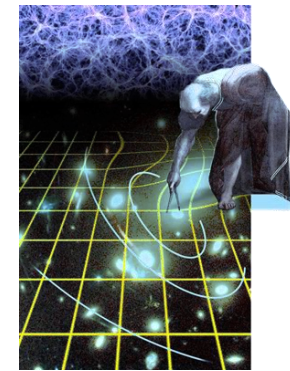


Vera C. Rubin  
Observatory



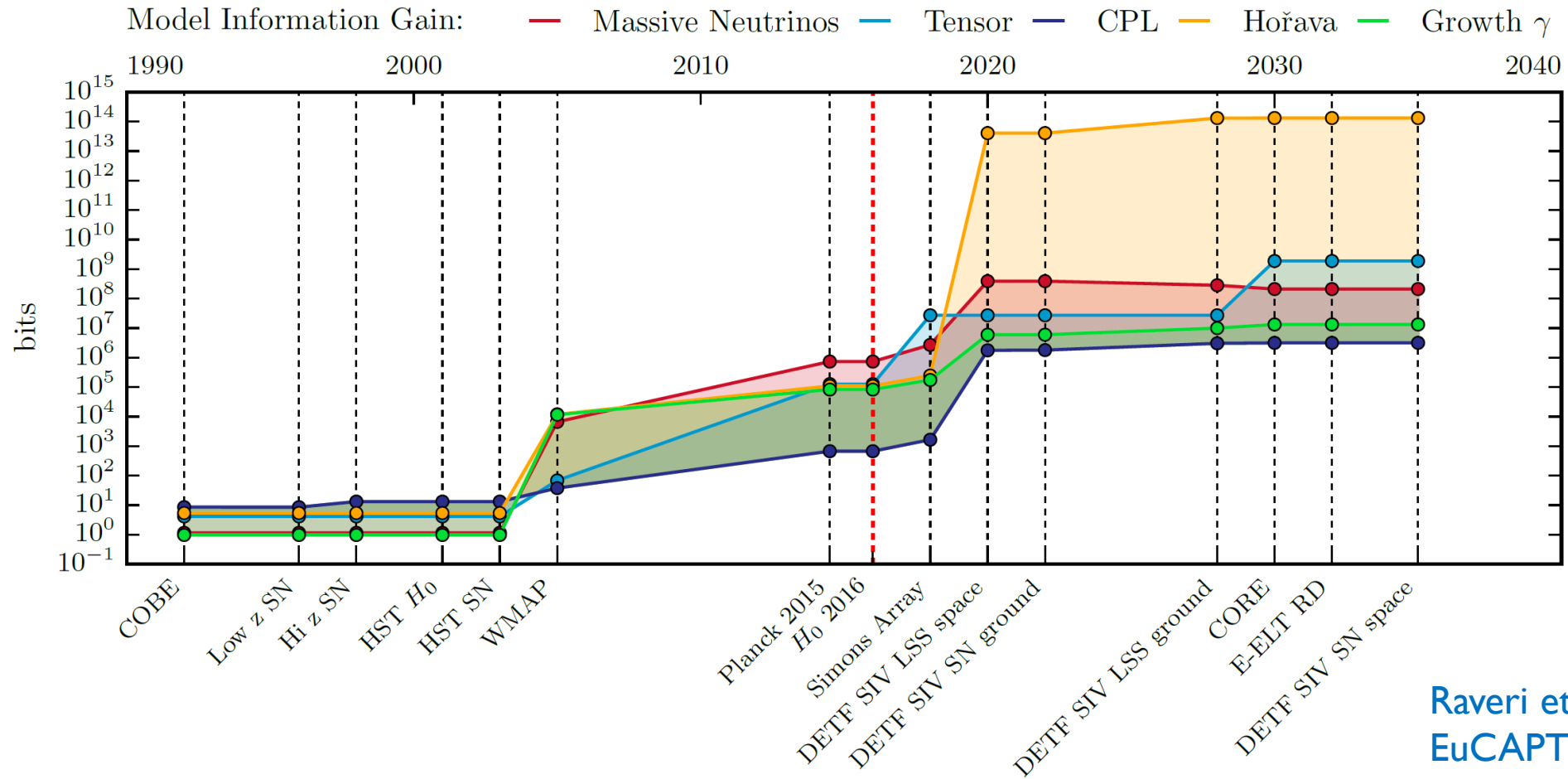
- ▶ **Non-linear scales**

novel astrophysical tests of gravity  
(in a model dependent way)



EUCLID





- ▶ *It is required to develop more theoretical models as theoretical inputs are crucial for testing dark energy and modified gravity*