



FCC-ee beam polarization and centre-of-mass energy calibration

Polarization and Centre-of-mass Energy Calibration at FCC-ee

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arXiv:1909.12245



Some references (not a complete set!):

B. Montague, Phys.Rept. 113 (1984) 1-96;

Polarization at LEP, CERN Yellow Report 88-02;

Beam Polarization in e+e-, AB, CERN-PPE-93-125 Adv.Ser.Direct.High Energy Phys. 14 (1995) 277-324;

L. Arnaudon et al., Accurate Determination of the LEP Beam Energy by resonant depolarization, Z. Phys. C 66, 45-62 (1995).

Spin Dynamics in LEP <http://dx.doi.org/10.1063/1.1384062>

Precision EW Measts on the Z Phys.Rept.427:257-454,2006 [arXiv:0509008v3](https://arxiv.org/abs/0509008v3)

D.P. Barber and G. Ripken ``Handbook of Accelerator Physics and Engineering'' World Scientific (2006), (2013)

D.P. Barber and G. Ripken, Radiative Polarization, Computer Algorithms and Spin Matching in Electron Storage Rings
[arXiv:physics/9907034](https://arxiv.org/abs/physics/9907034)

for FCC-ee:

First look at the physics case of TLEP [arXiv:1308.6176](https://arxiv.org/abs/1308.6176), **JHEP 1401 (2014) 164** DOI: [10.1007/JHEP01\(2014\)164](https://doi.org/10.1007/JHEP01(2014)164)

M. Koratzinos FCC-ee: Energy calibration IPAC'15 [arXiv:1506.00933](https://arxiv.org/abs/1506.00933)

E. Gianfelice-Wendt: Investigation of beam self-polarization in the FCC-ee [arXiv:1705.03003](https://arxiv.org/abs/1705.03003)

October 2017 EPOL workshop: <https://indico.cern.ch/event/669194/>

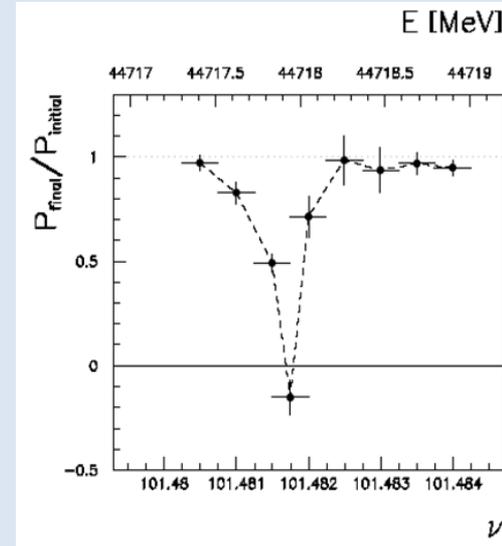
AB, P. Janot, J. Wenninger et al Polarization & Centre-of-mass Energy Calibration @ FCC-ee

[arXiv:1909.12245](https://arxiv.org/abs/1909.12245)

Beam Polarization can provide two main ingredients to Physics Measurements

1. Transverse beam polarization provides beam energy calibration by resonant depolarization

- low level of polarization is required ($\sim 10\%$ is sufficient)
- at Z & W pair threshold comes naturally $\sigma_E \propto E^2/\sqrt{\rho}$
- at Z use of asymmetric wigglers at beginning of fills
since polarization time is otherwise very long (250h \rightarrow ~ 1 h)
- should be used also at ee \rightarrow H(126)
- use 'single' non-colliding bunches and calibrate continuously during physics fills to avoid issues encountered at LEP
- Compton polarimeter for both e+ and e-
- should calibrate at energies corresponding to half-integer spin tune
- must be complemented by analysis of «average E_beam-to-E_CM» relationship



For beam energies higher than ~ 90 GeV can use $ee \rightarrow Z \gamma$ or $ee \rightarrow WW$ events to calibrate E_{CM} at $\pm 1-5$ MeV level: m_H (5 MeV) and m_{top} (20 MeV) measts

Beam Polarization can provide two main ingredients to Physics Measurements

2. Longitudinal beam polarization provides chiral e+e- system

- High level of polarization is required (>40%)
- Must compare with natural e+e- polarization due to chiral couplings of electrons (15%) or with final state polarization analysis for CC weak decays (polarized) (tau and top)
- **Physics case** for Z peak is very well studied and most measurements are polarized

$$A_{LR} = A_e, A_{FB}^{Pol}(f) \text{ etc... (CERN Y.R. 98)}$$

figure of merit is $L \cdot P^2$ --> must not lose factor ~ 10 in lumi.

self calibrating polarization measurement requires controlled e+ and e- polarization

at high statistics $A_{FB}^{Pol} = A_{FB}^{Pol}(f)$ of A_{LR} (Tenchini)

- enhance Higgs cross section ($\sim 30\%$)
- top quark cross section (state analysis does as well (Janot [arXiv:1503.01325](https://arxiv.org/abs/1503.01325)))
- enhance W production (extract/monitor backgrounds, for $ee \rightarrow WW$, $ee \rightarrow H$)
- require high polarization level and often both e- and e+ polarization

Warning: If loss of luminosity is too high

High level of polarization in high luminosity collisions is delicate in top-up mode

DECIDED to FOCUS ON TRANSVERSE POLARIZATION FOR ENERGY CALIBRATION

As far as we could check, there is no physics that can be done with longitudinal polarization that cannot be done without, given enough luminosity

Requirements from physics

1. Center-of-mass energy determination with precision of ± 100 keV around the Z peak
2. Center-of-mass energy determination with precision of ± 300 keV at W pair threshold
3. For the Z peak-cross-section and width, require energy spread uncertainty $\Delta\sigma_E/\sigma_E=0.2\%$

NB: at $2.3 \cdot 10^{36}/\text{cm}^2/\text{s}/\text{IP}$: **full LEP statistics** $10^6 \mu\mu$ $2 \cdot 10^7$ qq **in 6 minutes** in each expt

-- use resonant depolarization as main measuring method

-- use pilot bunches to calibrate during physics data taking: 100 calibrations per day each 10^{-6} rel.

-- long lifetime at Z requires the use of wigglers at beginning of fills

➔ take data at points where self polarization is expected

$$v_s = \frac{g-2}{2} \frac{E_b}{m_e} = \frac{E_b}{0.4406486(1)} \approx N + (0.5 \pm 0.1) \quad E_{\text{CM}} = (N + (0.5 \pm 0.1)) \times 0.8812972 \text{ GeV}$$

Given the Z and W widths of 2 GeV, this is easy to accommodate with little loss of statistics.

It might be more difficult for the Higgs 125.09 ± 0.2 corresponds to $v_s = 141.94 \pm 0.22$

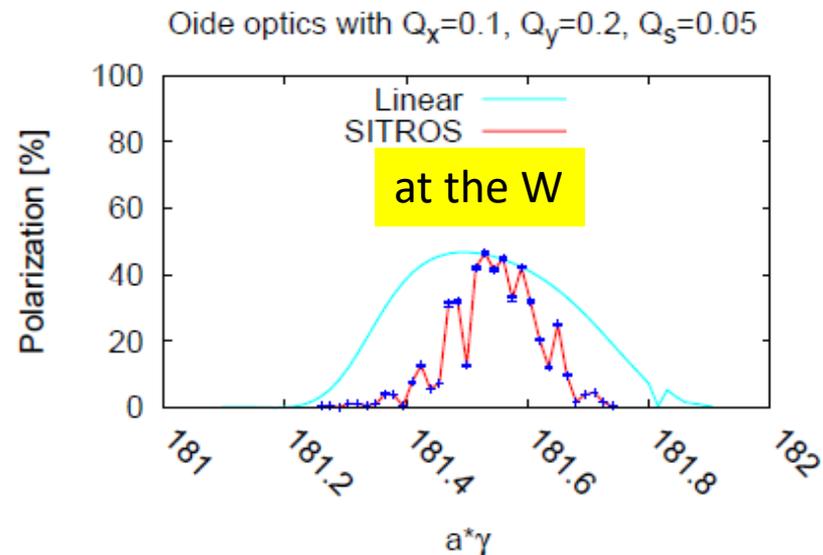
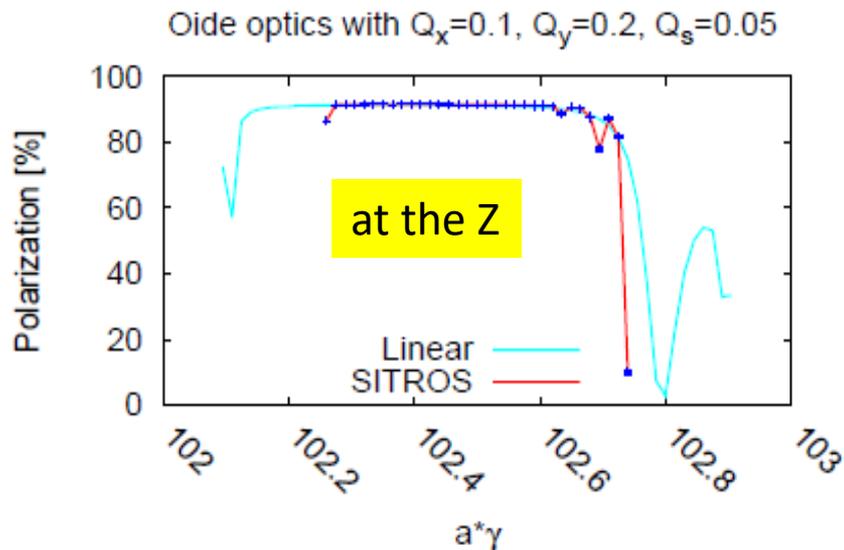
Simulations of self-polarization level with SITROS

Some results of coupling/dispersion correction

- $\delta y_{rms}^Q = 200 \mu\text{m}$ (including doublets)
- 250 μrad quadrupole roll angle (including doublets)
- 1086 BPMs w/o errors
- orbit corrected with 1086 CVs down to $y_{rms} = 0.05 \text{ mm}$
- coupling/dispersion correction with 289 skew quadrupoles

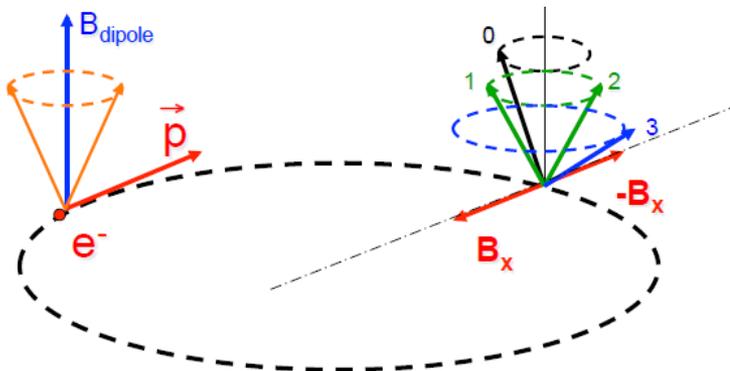
E. Gianfelice

1. orbit and emittance corrections needed for the FCC-ee luminosity seem sufficient to ensure useful levels of polarization
- 2. HOWEVER: same simulation does not produce luminosity and polarization, \rightarrow effect of simultaneous optimization could not be simulated**



Excellent level of polarization at the Z (even with wigglers) and sufficient at the W $\sigma_E \propto E^2/\rho$

RESONANT DEPOLARIZATION



Once the beams are polarized, an RF kicker at the spin precession frequency will provoke a spin flip and complete depolarization

Simulation of FCC-ee by I. Kopp:

spin precession (ν is the *spin tune*)

$$\delta\theta_{\text{spin}} = (g-2)/2 \cdot E/m \delta\theta_{\text{trajectory}}$$

$$= \nu \cdot \delta\theta_{\text{trajectory}}$$

$$\nu = E_{\text{beam}} / 0.4406486$$

$$= 103.5 \text{ at the Z peak}$$

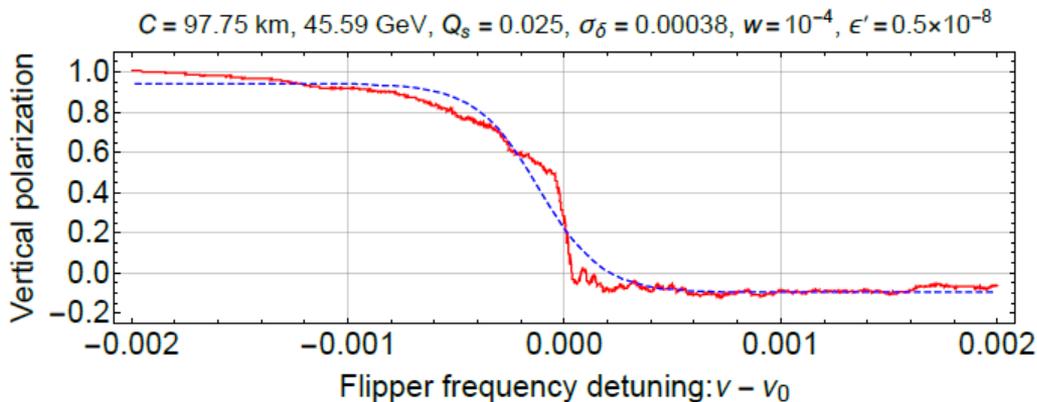
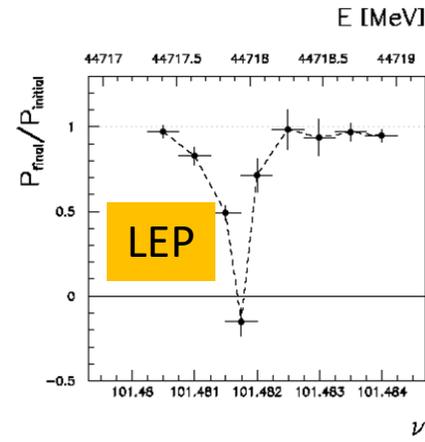
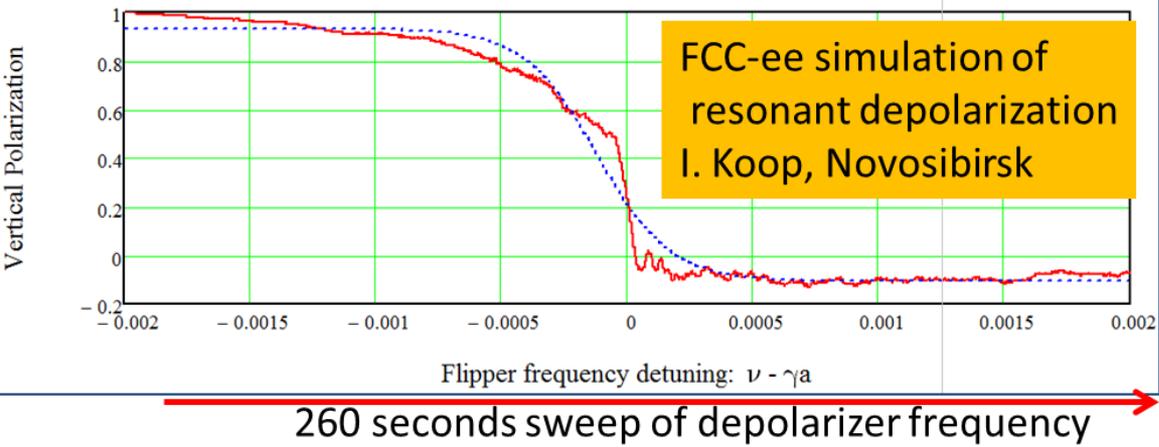
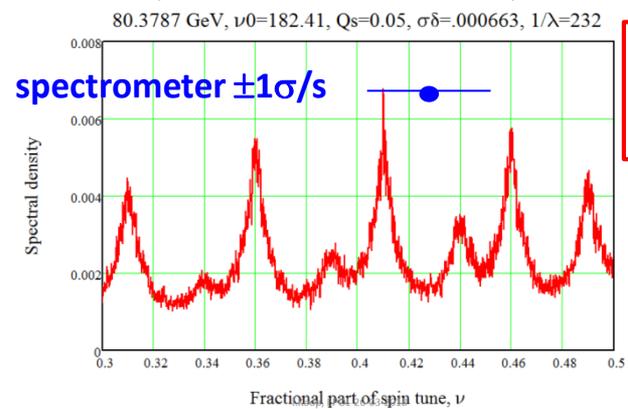


Figure 39. Simulation of a frequency sweep with the depolarizer on the Z pole showing a very sharp depolarization at the exact spin tune value.

C=97.75 km, 45.59 GeV, $Q_s=0.025$, $\sigma_\delta=0.00038$, $w=1*10^{-4}$, $\epsilon'=0.5*10^{-8}$

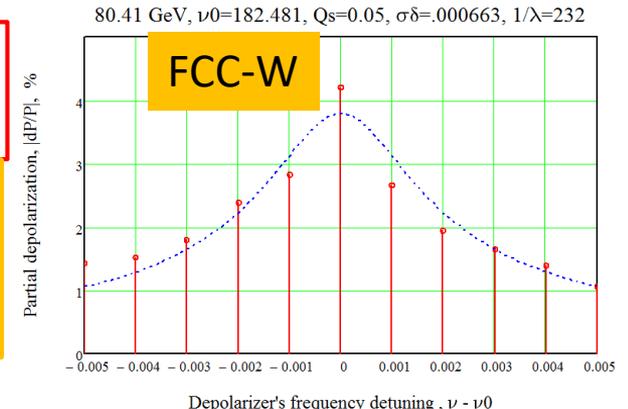


long sweep works well at the Z. Several depolarizations needed: eliminate Q_s side band and 0.5 ambiguity
Less well at the W: the Q_s side bands are much more excited because of energy spread, need iterations with smaller and smaller sweeps – work in progress. see *I. Koop* presentations at FCC weeks.



← Fourier analysis shows the side band situation at W.

First attempt at 'LEP' multiple sweep technique →



scan points for m_Z and m_W

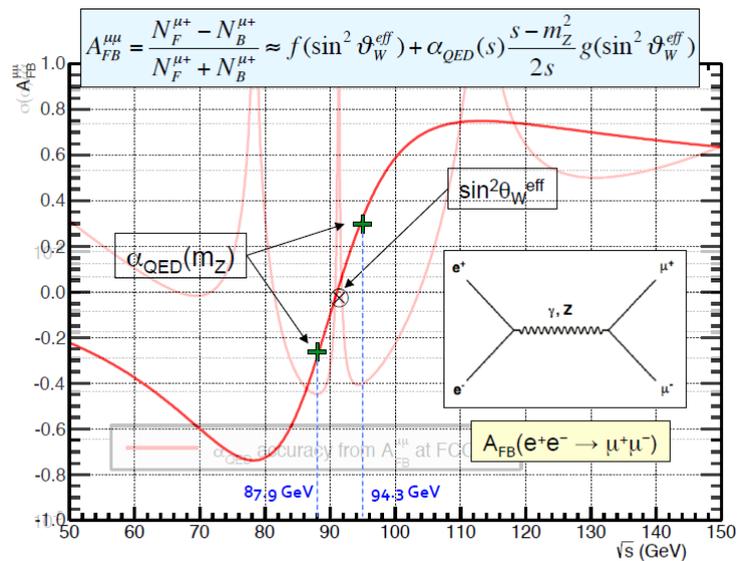
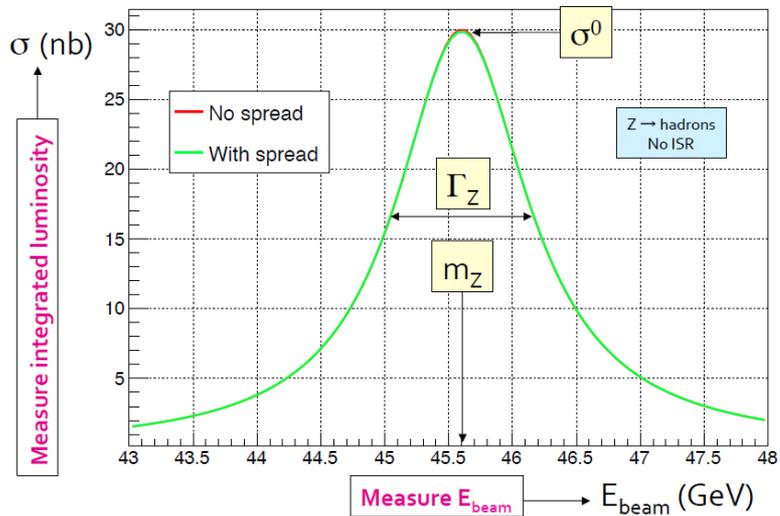
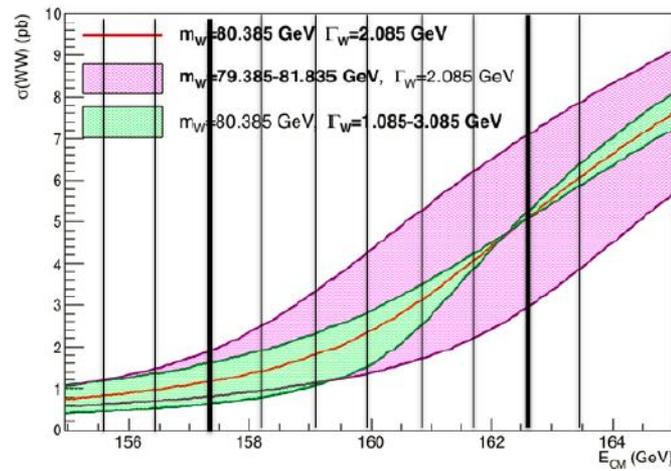


Table 3: Center-of-mass energies for the proposed Z scan. The points noted A and B are half integer spin tune points with energies closest to the requested energies.

Scan point	Centre-of-mass Energy	Beam Energy	Spin tune
E_{CM}^- A	87.69	43.85	99.5
E_{CM}^- Request	87.9	43.95	99.7
E_{CM}^- B	88.57	44.28	100.5
E_{CM}^0	91.21	45.61	103.5
E_{CM}^+ A	93.86	46.93	106.5
E_{CM}^+ Request	94.3	47.15	107.0
E_{CM}^+ B	94.74	47.37	107.5



statistical precision at the Z

centre-of-mass energy errors:

$$\begin{aligned}
 \frac{\Delta m_Z}{m_Z} &= \left\{ \frac{\Delta \sqrt{s}}{\sqrt{s}} \right\}_{\text{abs}} \oplus \left\{ \frac{\Delta(\sqrt{s_+} + \sqrt{s_-})}{\sqrt{s_+} + \sqrt{s_-}} \right\}_{\text{ptp-syst}} \oplus_i \left\{ \frac{\Delta \sqrt{s_{\pm}^i}}{\sqrt{s_{\pm}^i N_{\pm}^i}} \right\}_{\text{sampling}}, \\
 \frac{\Delta \Gamma_Z}{\Gamma_Z} &= \left\{ \frac{\Delta \sqrt{s}}{\sqrt{s}} \right\}_{\text{abs}} \oplus \left\{ \frac{\Delta(\sqrt{s_+} - \sqrt{s_-})}{\sqrt{s_+} - \sqrt{s_-}} \right\}_{\text{ptp-syst}} \oplus_i \left\{ \frac{\Delta \sqrt{s_{\pm}^i}}{\sqrt{s_{\pm}^i N_{\pm}^i}} \right\}_{\text{sampling}}, \\
 \Delta A_{\text{FB}}^{\mu\mu}(\text{pole}) &= \frac{\partial A_{\text{FB}}^{\mu\mu}}{\partial \sqrt{s}} \left\{ \Delta(\sqrt{s_0} - 0.5(\sqrt{s_+} + \sqrt{s_-})) \right\}_{\text{ptp-syst}} \oplus_i \frac{\partial A_{\text{FB}}^{\mu\mu}}{\partial \sqrt{s}} \left\{ \frac{\Delta \sqrt{s_{0,\pm}^i}}{\sqrt{N_{0,\pm}^i}} \right\}_{\text{sampling}}, \\
 \frac{\Delta \alpha_{\text{QED}}(m_Z^2)}{\alpha_{\text{QED}}(m_Z^2)} &= \left\{ \frac{\Delta \sqrt{s}}{\sqrt{s}} \right\}_{\text{abs}} \oplus \left\{ \frac{\Delta(\sqrt{s_+} - \sqrt{s_-})}{\sqrt{s_+} - \sqrt{s_-}} \right\}_{\text{ptp-syst}} \oplus_i \left\{ \frac{\Delta \sqrt{s_{\pm}^i}}{\sqrt{s_{\pm}^i N_{\pm}^i}} \right\}_{\text{sampling}},
 \end{aligned} \tag{3.1}$$

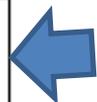
with $\frac{\partial A_{\text{FB}}^{\mu\mu}}{\partial \sqrt{s}} \simeq 0.09/\text{GeV}$.

Three categories:

- **Absolute** dominate for Z and W mass
- **ptp** Point-to-point dominate for Γ_Z & $A_{\text{FB}}^{\mu\mu}$ (peak and off-peak)
- Due to **sampling** – turns out to be negligible for 1meast / (15 min = 1000s) $\rightarrow 10^4$ measts

Table 4. Calculated uncertainties on the quantities most affected by the centre-of-mass energy uncertainties, under the initial systematic assumptions.

Observable	statistics	$\Delta\sqrt{s}_{\text{abs}}$ 100 keV	$\Delta\sqrt{s}_{\text{syst-ptp}}$ 100 keV	calib. stats. 200 keV/ $\sqrt{N^i}$	$\sigma_{\sqrt{s}}$ 85 \pm 0.5 MeV
m_Z (keV)	4	100	70	1	–
Γ_Z (keV)	4	2.5	55	1	100
$\sin^2 \theta_W^{\text{eff}} \times 10^6$ from $A_{\text{FB}}^{\mu\mu}$	2	–	6	0.1	–
$\frac{\Delta\alpha_{\text{QED}}(m_Z^2)}{\alpha_{\text{QED}}(m_Z^2)} \times 10^5$	3	0.1	2.2	–	1



From beam energy to E_{CM}

$$\sqrt{s} = 2\sqrt{E_b^+ E_b^-} \cos \alpha/2, \approx E_b^+ + E_b^-$$

Energy gain (RF) = losses in the storage ring

Synchrotron radiation (SR)

beamstrahlung (BS)

$$\Delta_{RF} = 2\Delta_{SRi} + 2\Delta_{SRe} + 2\Delta_{BS}$$

at the Z (O of mag.):

$$\Delta_{SR} = 2\Delta_{SRi} + 2\Delta_{SRe} = 36 \text{ MeV} \quad \Delta_{RF}$$

$$\Delta_{SRe} - \Delta_{SRi} \approx \alpha/2\pi \Delta_{SR} = 0.17 \text{ MeV}$$

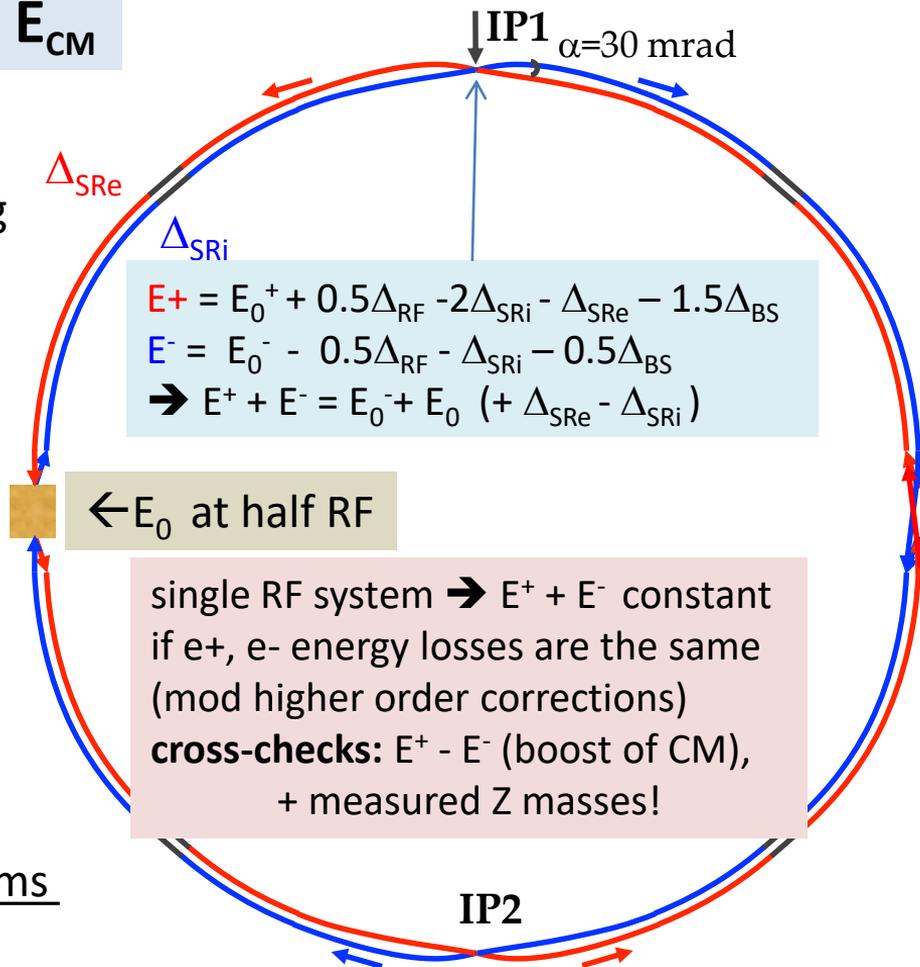
$$\Delta_{BS} = 0 \text{ up to } 0.62 \text{ MeV}$$

the average energies E_0 around the ring are determined by the magnetic fields

→ same for colliding or non-colliding beams

-- measured by resonant depolarization

-- can be different for e^+ and e^-



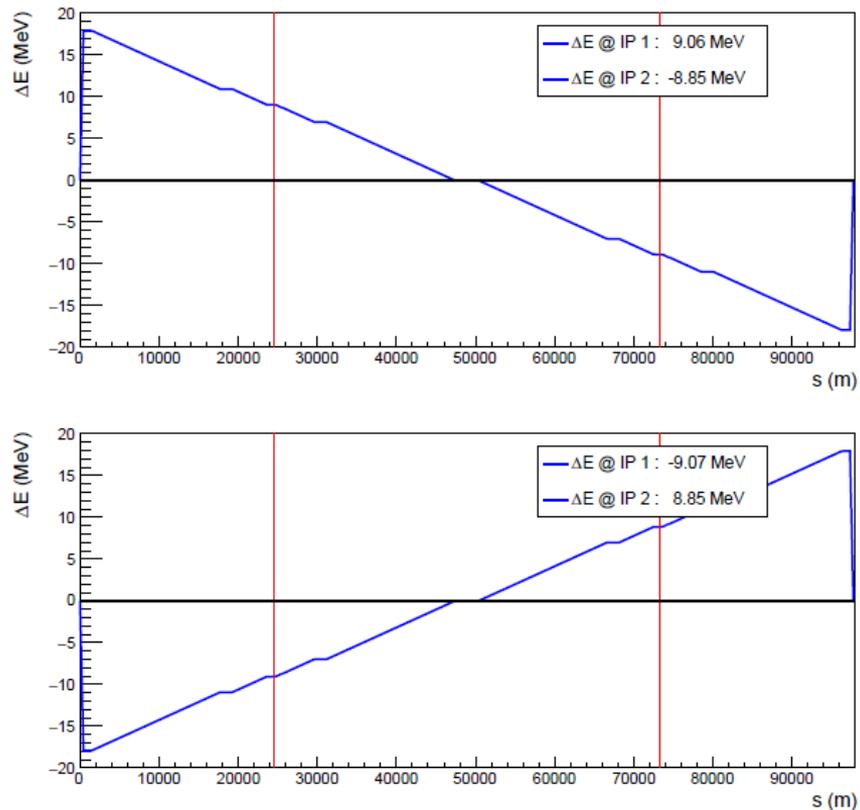


Figure 43. Energy sawtooth at the Z pole for the two beams with a single RF station per beam in the same location (top: beam direction left to right, bottom: beam direction right to left), the vertical axis corresponds to the relative energy offset and the horizontal axis to the longitudinal coordinate. The two IPs are indicated by the red vertical lines.

3. From spin tune measurement to center-of-mass determination $v_s = \frac{g-2}{2} \frac{E_b}{m_e} = \frac{E_b}{0.4406486(1)}$

3.1 Synchrotron Radiation energy loss (9 MeV @Z in 4 'arcs') calculable to < permil accuracy

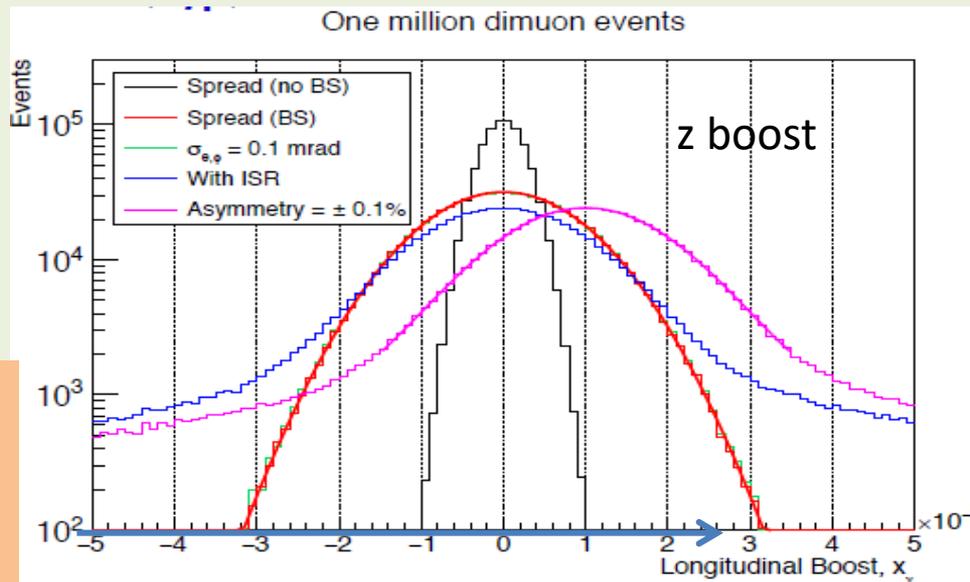
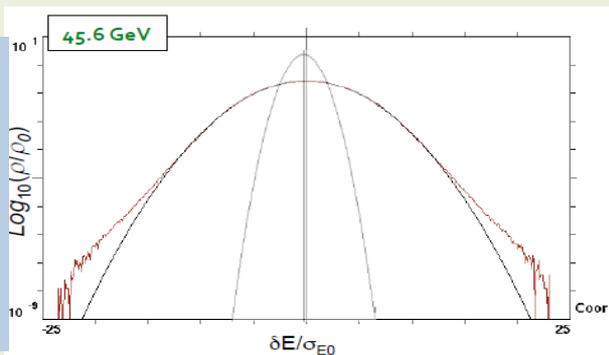
3.3 Beamstrahlung energy loss (0.62 MeV per beam at Z pole), compensated by RF (Shatilov)

3.4 layout of accelerator with IPs between two arcs well separated from single RF section

3.5 E_b^+ vs E_b^- asymmetries and energy spread can be measured/monitored in expt:

e+e- \rightarrow $\mu^+ \mu^-$ longitudinal momentum shift and spread (Janot)

D. Shatilov:
beam energy
spectrum
without/with
beamstrahlung



P. Janot: 5 min/exp @Z \rightarrow $10^6 \mu^+ \mu^-$ /expt \rightarrow
 \rightarrow 50 keV meast both on σ_{ECM} and $E^+ - E^-$
 \rightarrow and beam crossing angle α (error negl.)
 \rightarrow also monitor relative ECM (p-t-p!)

Given the long polarization time at Z, wigglers will be necessary.
 An agreement was reached on a set of **8 wiggler units per beam**

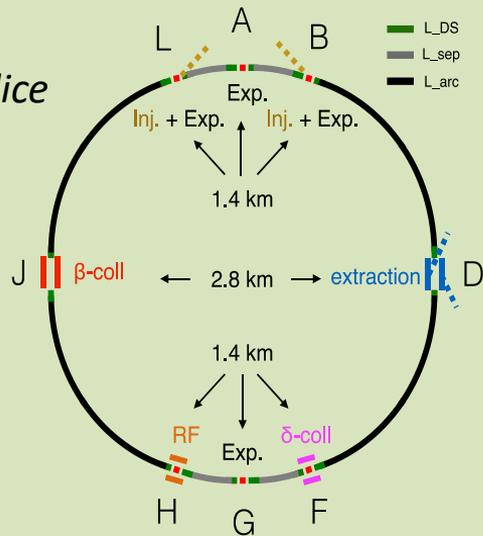
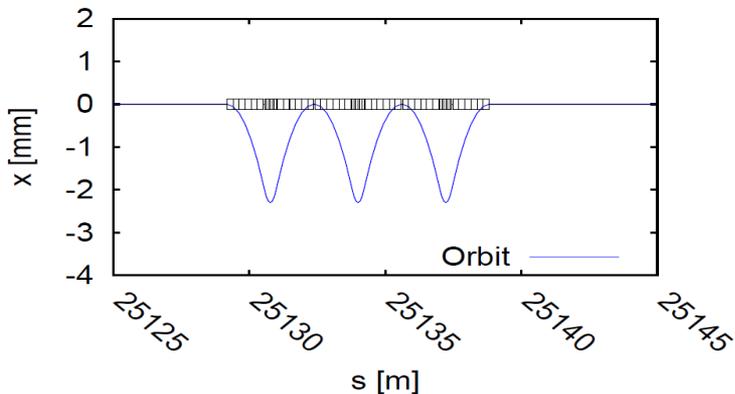
Polarization wigglers

8 units per beam, as specified by *Eliana Gianfelice*

$B^+ = 0.7\text{ T}$ $L^+ = 43\text{ cm}$ $L^-/L^+ = B^+/B^- = 6$

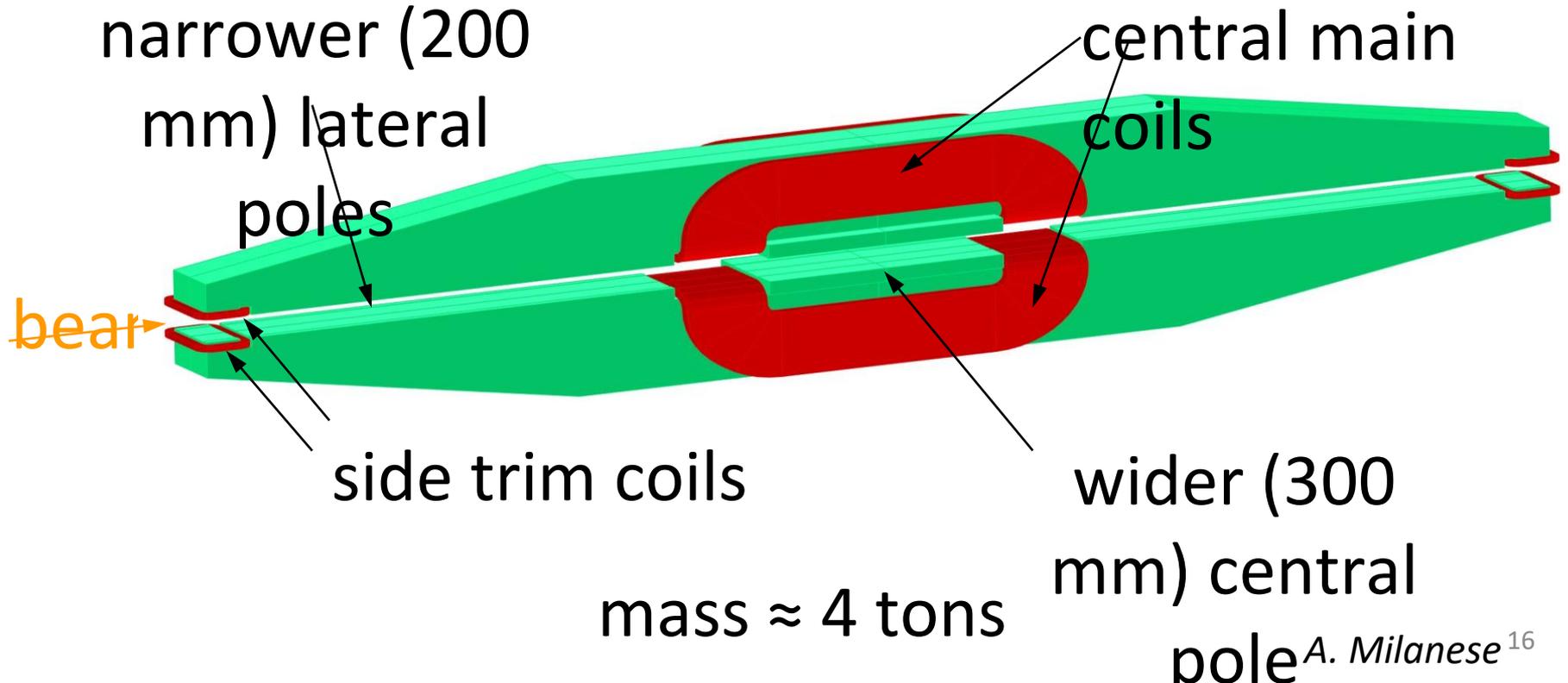
at $E_b = 45.6\text{ GeV}$ and $B^+ = 0.67\text{ T}$

$\Rightarrow P = 10\%$ in 1.8 H $\sigma_{E_b} = 60\text{ MeV}$ $E_{\text{crit}} = 902\text{ keV}$



placed e.g. in dispersion-free straight section H and/or F

First single pole magnetic concept, keeps some of the ideas of the LEP design, in particular the “floating” poles





polarimeter-spectrometer situated 100m from end of dipole.

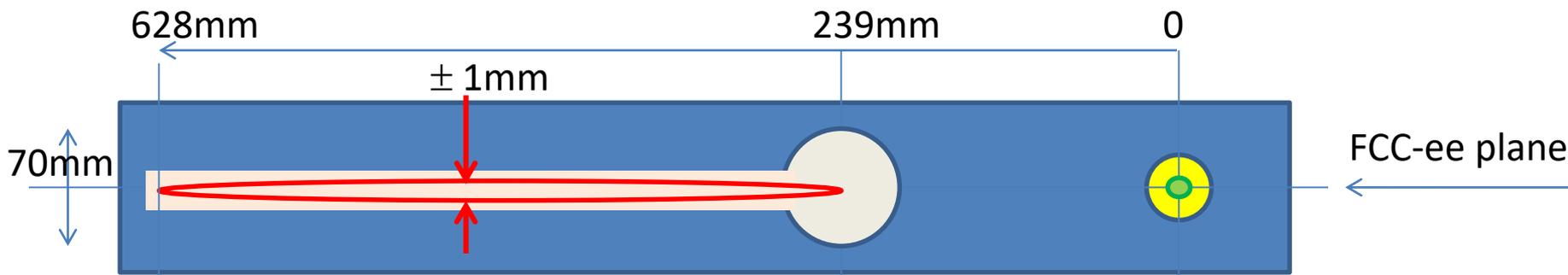
Using the dispersion suppressor dipole with a lever-arm of **100m** from the end of the dipole, one finds

-- minimum compton scattering energy at 45.6 GeV is 17.354 GeV

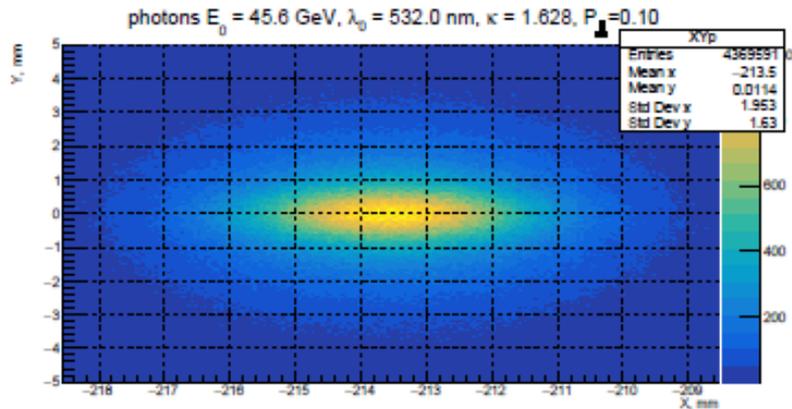
-- distance from photon recoil to Emin electron is 0.628m

	laser (eV)	beam (GeV)	mc2(MeV)	B field	R	LM	theta	L	true beam
	2.33	45.6	0.511	0.013451	11300	24.119	0.002134	100	45.60005
nominal kappa = 4. E_laser.Ebeam_nom/mc2	1.627567296								
true kappa = 4. E_laser.Ebeam_true/mc2	1.627568924								
nominal Emin	17.35445561								
true Emin	17.35446221								
position of photons	0								
nominal position of beam (m)	0.239182573								
true position of beam (m)	0.239182334	2.39182E-07							
nominal position of min (m)	0.628468308								
true position of min (m)	0.628468069	2.39182E-07							

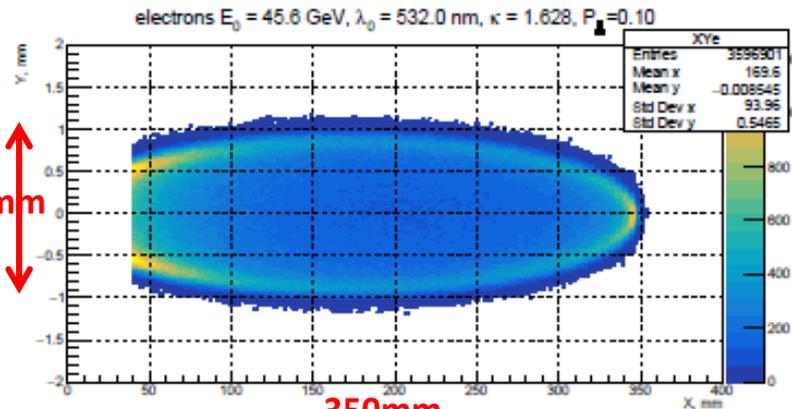
mouvement of beam and end point are the same:
 0.24microns for $\delta E_b/E_b=10^{-6}$ ($\delta E_b=45\text{keV}$)



end point elliptic distribution of scattered electrons beam spot and BPM recoil photon spot *A. Blondel*

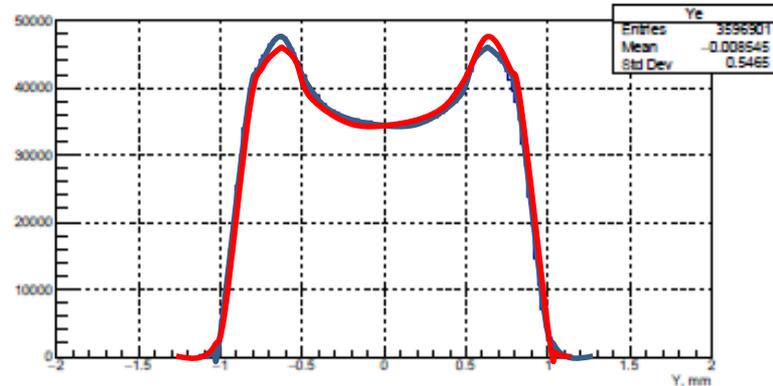
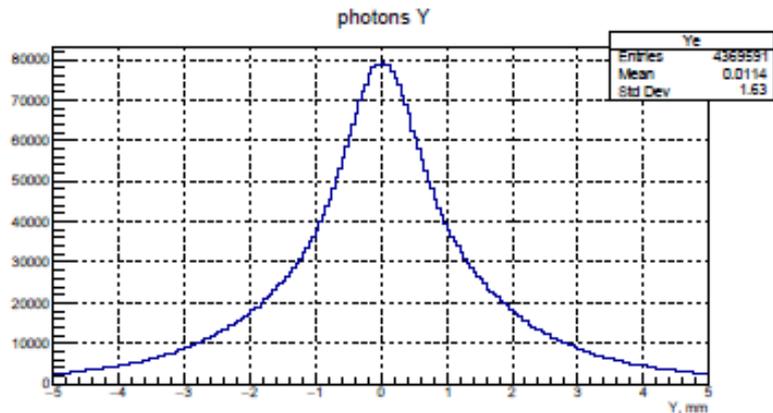


± 1 mm



350mm

electrons Y



- Laser wavelength $\lambda = 532$ nm.
 - Waist size $\sigma_0 = 0.250$ mm. Rayleigh length $z_R = 148$ cm.
 - Far field divergence $\theta = 0.169$ mrad
 - Interaction angle $\alpha = 1.000$ mrad
 - Compton cross section correction 0.5
 - Pulse energy: $E_L = 1$ [mJ]; $\tau_L = 5$ [ns] (sigma)
 - Pulse power: $P_L = 80$ [kW]
 - Ratio of angles $R_a = 5.905249$
 - Ratio of lengths $R_l = 0.984208$
 - $P_L/P_c = 1.1 \cdot 10^{-6}$
 - “efficiency” = 0.13
 - Scattering probability $W \simeq 7 \cdot 10^{-8}$
-
- With 10^{10} electrons and 3 kHz rep. rate: $\dot{N}_\gamma \simeq 2 \cdot 10^6$

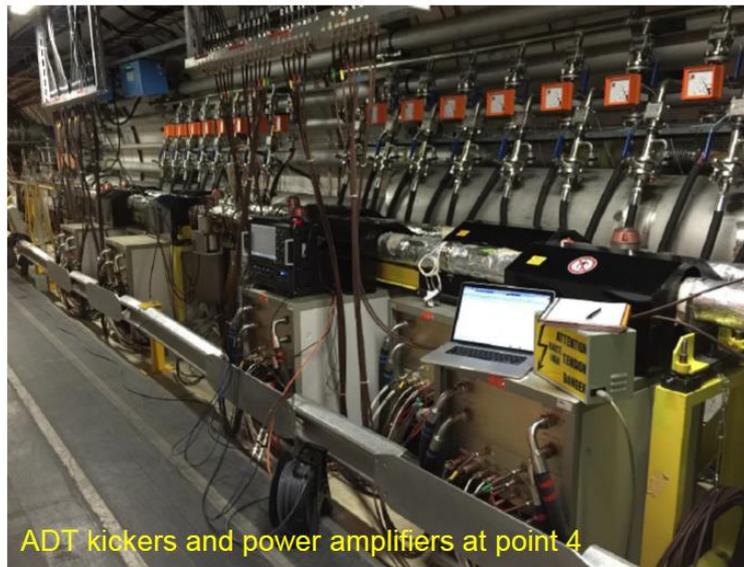


This is not-so trivial in FCC-ee!
16700 bunches circulate
time-between-bunches = 19ns,
depolarize one-and-only-one
of them.

Kicker must have fast (<9ns) rise.

The LHC TF system works essentially on
a bunch by bunch basis for 25ns.
They would provide a transverse kick of
up to ~ 20 mrad at the Z peak with ~ 10
MHz bandwidth. This is 10x more than
what we may need-
→ a priori OK !

- Four kickers per beam, per plane, located in RF zone (UX451) at point 4
 - **Electrostatic kicker**, length 1.5 m.
 - Providing a **kick of $\sim 2 \mu\text{rad}$ @ 450 GeV** (all 4 units combined).
 - Useful bandwidth ~ 1 kHz – 20 MHz.



From resonant depolarization to Center-of-mass energy -- 1. from spin tune to beam energy--

The spin tune may not be an exact measurement of the average of the beam energy along the magnetic trajectory of particles. Additional spin rotations may bias the issue. *Anton Bogomyagkov* and *Eliaana Gianfelice* have made many estimates.

synchrotron oscillations	$\Delta E/E$	$-2 \cdot 10^{-14}$
Energy dependent momentum compaction	$\Delta E/E$	10^{-7}
Solenoid compensation		$2 \cdot 10^{-11}$
Horizontal betatron oscillations	$\Delta E/E$	$2.5 \cdot 10^{-7}$
Horizontal correctors*)	$\Delta E/E$	$2.5 \cdot 10^{-7}$
Vertical betatron oscillations **)	$\Delta E/E$	$2.5 \cdot 10^{-7}$
Uncertainty in chromaticity correction $O(10^{-6})$	$\Delta E/E$	$5 \cdot 10^{-8}$
invariant mass shift due to beam potential		$4 \cdot 10^{-10}$

*) $2.5 \cdot 10^{-6}$ if horizontal orbit change by $>0.8\text{mm}$ between calibration is unnoticed or if quadrupole stability worse than 5 microns over that time. **consider that 0.2 mm orbit will be noticed**

***) $2.5 \cdot 10^{-6}$ for vertical excursion of 1mm. Consider orbit can be corrected better than 0.3 mm.



There was an issue here

-- Anton calculated the possible change in particle energies due to

electric potential effects

-- this is a small effect (few 10's of keV) but it is not clear that it should be included for relativistic particles

-- we agreed (with help from E. Levichev) to ask a theorist (A. Milstein) to have a look and ...

"I just had a conversation with A. Milstein about invariant mass correction due to the fields of opposite and own bunches.

*He convinced me that the matter is more complicated than what I wrote in the CDR paragraph (the bound state). **The effect seems to be small, however complicated for calculation. I propose to drop this paragraph, while A. Milstein will continue efforts for correct calculations"***

Anton



this section of the paper will be edited with inclusion of the following:

"an order-of-magnitude non-relativistic calculation indicates that intensity-dependent effects due to the potentials induced by the beam charges may affect the center-of-mass energy prediction by up to about 100 keV"

It is also indicated in section 7.4 that "intensity-dependent effects at the level of a up to a few 100 keV can be verified experimentally".

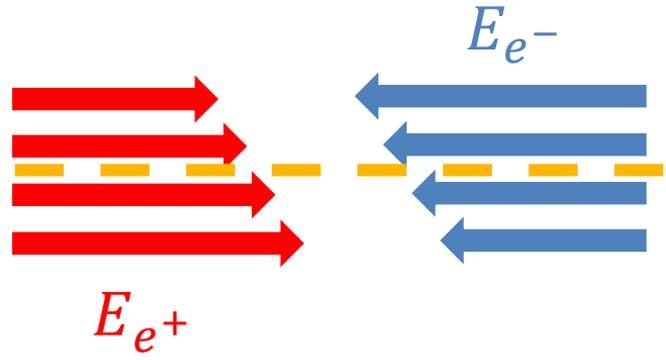
Then I would propose to submit the paper to PRSTAB

From resonant depolarization to Center-of-mass energy

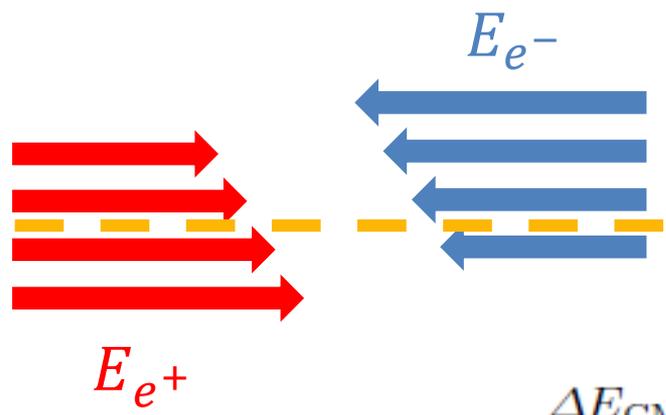
opposite sign dispersion

2. from beam energy to E_{CM}

Experience from LEP – Vernier scans



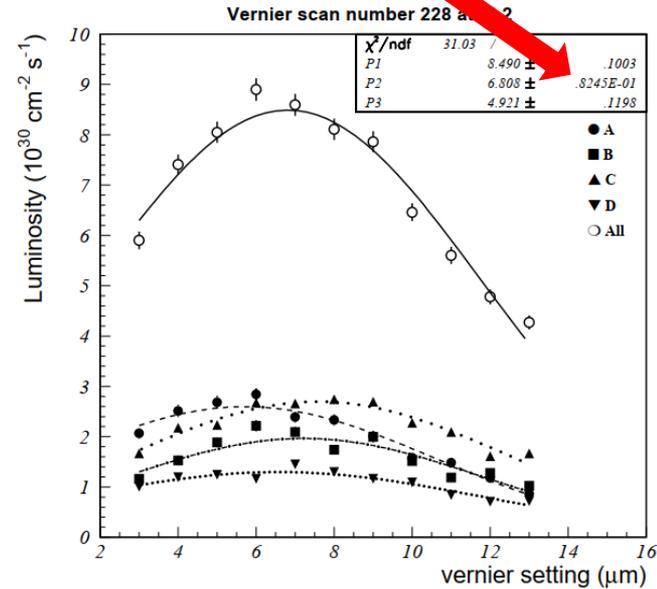
No effect.
 $ECM = (E_{e+} + E_{e-})$
 NB energy spread is reduced.



ECM lower than
 $(E_{e+} + E_{e-})$

$$\Delta E_{CM} = -\frac{1}{2} \cdot \frac{\delta y}{\sigma_y^2} \cdot \frac{\sigma E_b^2}{E_b} \cdot \Delta D_y^*$$

Van Der Meer today
 Relative position of beams measured to 80 nanometers from one scan



7.2 Dispersion at the IP

For beams colliding with an offset at the IP, the CM energy spread and shift are affected by the local dispersion at the IP. For a total IP separation of the beams of $2u_0$ the expressions for the CM energy shift and spread are [72]

$$\Delta\sqrt{s} = -2u_0 \frac{\sigma_E^2 (D_{u1} - D_{u2})}{E_0 (\sigma_{B1}^2 + \sigma_{B2}^2)} \quad (90)$$

$$\sigma_{\sqrt{s}}^2 = \sigma_E^2 \left[\frac{\sigma_\epsilon^2 (D_{u1} + D_{u2})^2 + 4\sigma_u^2}{\sigma_{B1}^2 + \sigma_{B2}^2} \right] \quad (91)$$

D_{u1} and D_{u2} represent the dispersion at the IP for the two beams labelled by 1 and 2. σ_E is the beam energy spread assumed here to be equal for both beams and $\sigma_\epsilon = \sigma_E/E$ is the relative energy spread. σ_{Bi} is the total transverse size of beam (i) at the IP,

$$\sigma_{Bi}^2 = \sigma_u^2 + (D_{ui}\sigma_\epsilon)^2 \quad (92)$$

with σ_u the betatronic component of the beam size.

If the beam sizes at the IP are dominated by the betatronic component which is rather likely, the energy shift simplifies to

$$\Delta\sqrt{s} = -u_0 \frac{\sigma_E^2 \Delta D^*}{E_0 \sigma_u^2} \quad (93)$$

where $\Delta D^* = D_{u1} - D_{u2}$ is the difference in dispersion at the IP between the two beams. This effect applies to both planes ($u = x, y$). In general due to the very flat beam shapes the most critical effect arises in the vertical plane.

For FCC-ee at the Z we have in vertical direction:

- Parasitic dispersion of e+ and e- beams at IP **10um** the difference is $\Delta D_y^* = 14\mu m$.
- Sigma_y is 28nm
- Sigma_E is 0.132%*45000MeV=60MeV
- **Delta_ECM is therefore 1.4MeV for a 1nm offset**
- Note that we cannot perform Vernier scans like at LEP, we can only displace the two beams by $\sim 10\% \sigma_y$
- Assume each Vernier scan is accurate to 1% σ_y , we get a precision of 400 keV.
- **the process should be simulated**
- we need 100 beams scans to get an E_{CM} accuracy of 40keV – suggestion: vernier scan every hour or more.
- It is likely that Vernier scans will be performed regularly at least once per hour or more. ($\rightarrow 100$ per week) we end up with an uncertainty of ~ 10 keV over the whole running period.
- **The dispersion must be measured as well; this can be done by using the vernier scans with offset RF frequency**

critical effect is in the vertical plane, but horizontal plane should be investigated as well

At full luminosity, a vernier scan is a tricky operation and beam beam blow up effects might affect the result

Therefore a beamstrahlung or radiative bhabha monitor seems highly worthwhile as it gives information on the direction of the interacting particles.

it detects

the hard photons emitted in either $e^+e^- \rightarrow e^+e^- \gamma$

or

the hard beamstrahlung photons

Photons are not affected by the IR magnetic fields.

The beam-beam offset leads to a shift in the beamstrahlung photon beam which is **proportional** to the offset (and to the charge of the opposite beam) for small offsets.

the measurement is passive

the zero position can be operationally established by colliding beams at lower intensity where large vernier scan amplitude is possible.

An angular kick of up to 0.18 mrad is expected in the horizontal plane due to EM attraction.

Table 15: Calculated uncertainties on the quantities most affected by the center-of-mass energy uncertainties, under the final systematic assumptions.

Quantity	statistics	ΔE_{CMabs} 100 keV	$\Delta E_{CMSyst-ptp}$ 40 keV	calib. stats. 200 keV/ $\sqrt{(N^i)}$	σE_{CM} (84) \pm 0.05 MeV
m_Z (keV)	4	100	28	1	–
Γ_Z (keV)	7	2.5	22	1	10
$\sin^2\theta_W^{eff} \times 10^6$ from $A_{FB}^{\mu\mu}$	2	–	2.4	0.1	–
$\frac{\Delta\alpha_{QED}(M_Z)}{\alpha_{QED}(M_Z)} \times 10^5$	3	0.1	0.9	–	0.05

- the point-to-point uncertainty estimate is O(10 keV) (M.K.) It can be controlled in two ways
1. compare the momentum as measured with the polarimeter spectrometer between different energies (monitored constantly at each energy)
 - Magnet must be very precisely monitored (<10⁻⁶) and dedicated monitoring of the main beam after the collision and magnet should be discussed.
 - this requires dedicated design of polarimeter
 2. use the e⁺e⁻ → μ⁺μ⁻ events in the detectors to measure ECM for each of the energies.
 - monitor experimental magnet to (<10⁻⁶) precision + QED issues etc..

Conclusions

We had a first look at the determination of centre of mass energy and energy spread in FCC-ee
Results are promising of extraordinary, historical measurements.

This must be improved and secured further towards the TDR

To do list next page



Energy calibration and polarization to-do list (I)

1. The tools used for simulation of the orbit correction process, and of the simultaneous optimization of luminosity and polarization in realistic machines should be integrated.

This is essential to confirm the feasibility and operability of the proposed data taking scheme.

This tool should allow for simulation of the resonant depolarization process and of the calculation of the collision energy.

2. A critical point to address is the design of the diagnostics allowing control of the beam-beam offsets and the measurement of residual dispersion and the interaction point. This will allow the centre-of-mass energy shifts to be reduced and monitored, but should also benefit the optimization of luminosity.

IP collision monitoring and necessary measurements:

- vernier scans, fast lumi monitor,
- beam-beam deflection monitoring,
- beamstrahlung monitor etc...

devise means to make it all automatic (RD, Verniers, Dispersion, etc...)

3. The resonant depolarization process and its sensitivity to the energy spread and synchrotron tune should be further studied to optimize the procedures and the machine settings.

The precision obtainable at the W threshold energy should be clarified and, if possible, improved.



Energy calibration and polarization to-do list(II).

4. A detailed design and implementation of wigglers & management of the radiation is required.
4'. The depolarization kicker should be finalized.

5. Further reduction of the point-to-point uncertainties would still be welcome. This should involve development of an energy model, a thorough design of the monitoring devices, and of the data recording strategy.

5'. This includes a careful design of the polarimeter implementation and of its associated spectrometer, as well as a study of devices capable of providing a relative ECM measurement in (at least one of) the experiments.

6. Given that the Z run is scheduled at the beginning of the life of FCC-ee, all procedures, instrumentation, data processing and analysis should be ready well before the commissioning of the machine. (NB This will hugely pay off in overall performance)



Finally:

all this work will be essential for the effort towards the observation of the s-channel Higgs production and the measurement of the electron Yukawa coupling

-- monochromatization is new and running scheme remains to be developed

-- but most of the rest

control of ECM,

monitoring of the energy spread and $E_{e^+} - E_{e^-}$ (across the bunch x-width!),

beam offsets etc..

will have been developed for the Z and W scans.

(still 'optional' and incredibly difficult, but **unique**)



Various complimentary spares

Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties

The LEP Energy Working Group

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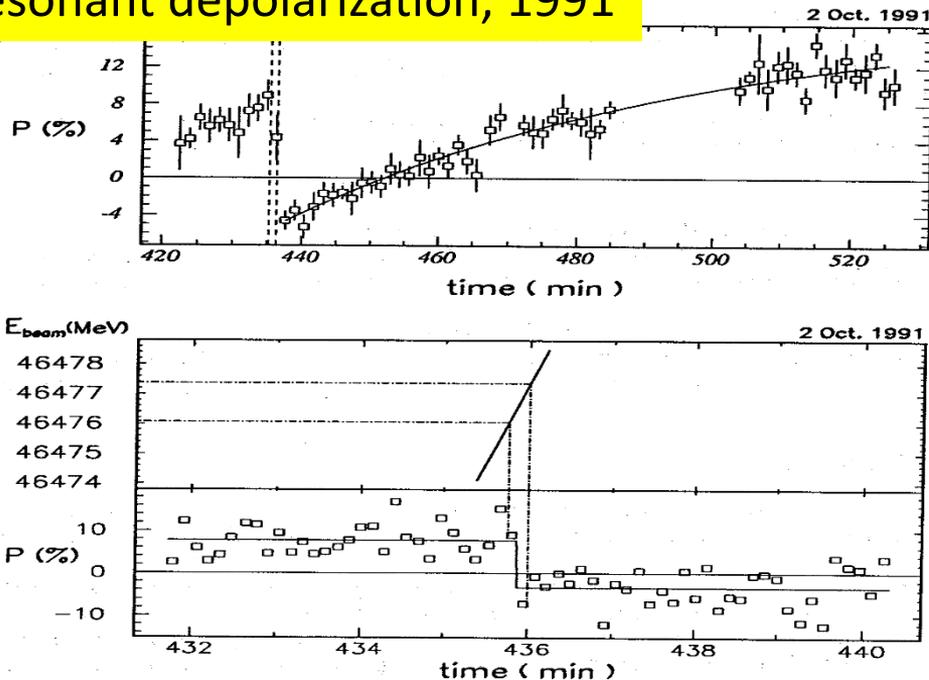
Abstract

The determination of the centre-of-mass energies from the LEP1 data for 1993, 1994 and 1995 is presented. Accurate knowledge of these energies is crucial in the measurement of the Z resonance parameters. The improved understanding of the LEP energy behaviour accumulated during the 1995 energy scan is detailed, while the 1993 and 1994 measurements are revised. For 1993 these supersede the previously published values. Additional instrumentation has allowed the detection of an unexpectedly large energy rise during physics fills. This new effect is accommodated in the modelling of the beam-energy in 1995 and propagated to the 1993 and 1994 energies. New results are reported on the magnet temperature behaviour which constitutes one of the major corrections to the average LEP energy.

The 1995 energy scan took place in conditions very different from the previous years. In particular the interaction-point specific corrections to the centre-of-mass energy in 1995 are more complicated than previously: these arise from the modified radiofrequency-system configuration and from opposite-sign vertical dispersion induced by the bunch-train mode of LEP operation.

Finally an improved evaluation of the LEP centre-of-mass energy spread is presented. This significantly improves the precision on the Z width.

LEP Resonant depolarization, 1991

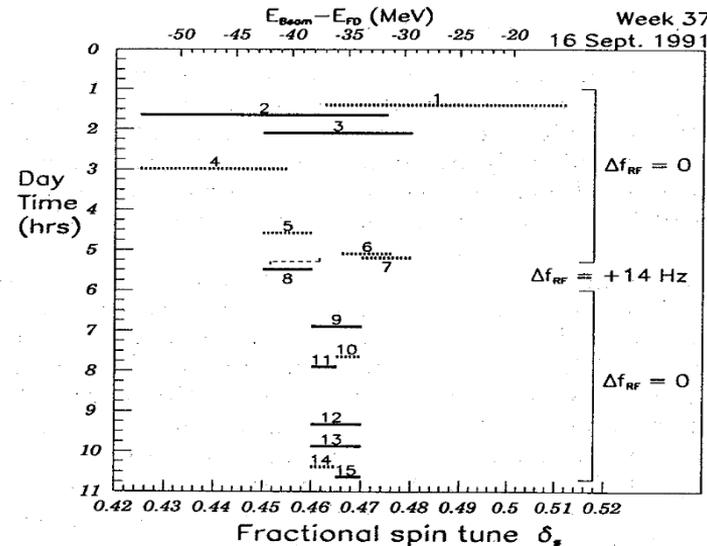


$$E_{beam} = 46,466.6 \pm 0.6 \text{ MeV, e.g. precise to } \pm 1.5 \cdot 10^{-5}.$$

Figure 20: Polarization signal on 2 October 1991, showing the localization of the depolarizing frequency within the sweep.

Top: display of data points, with the frequency sweep indicated with vertical dashed lines. The full line represents the result of a fit with starting polarization $(-4.9 \pm 1.1)\%$, polarization rise-time (60 ± 13) minutes, asymptotic polarization $(18.4 \pm 4.1)\%$.

Bottom: expanded view of the sweep period, with the individual data sets displayed (there are 10 sets per point); The frequency sweep lasted 7 data sets. The corresponding beam energy is shown in the upper box. Spin flip occurred between the two vertical dash-dotted lines.



variation of RF frequency to eliminate half integer ambiguity

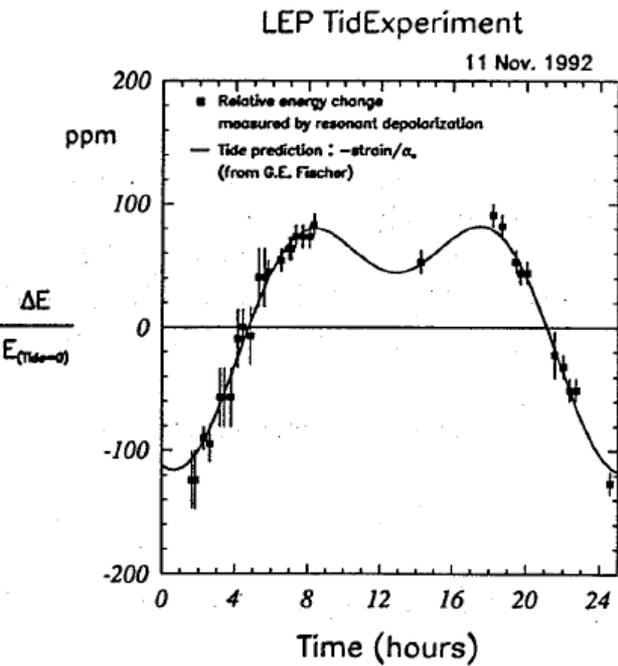
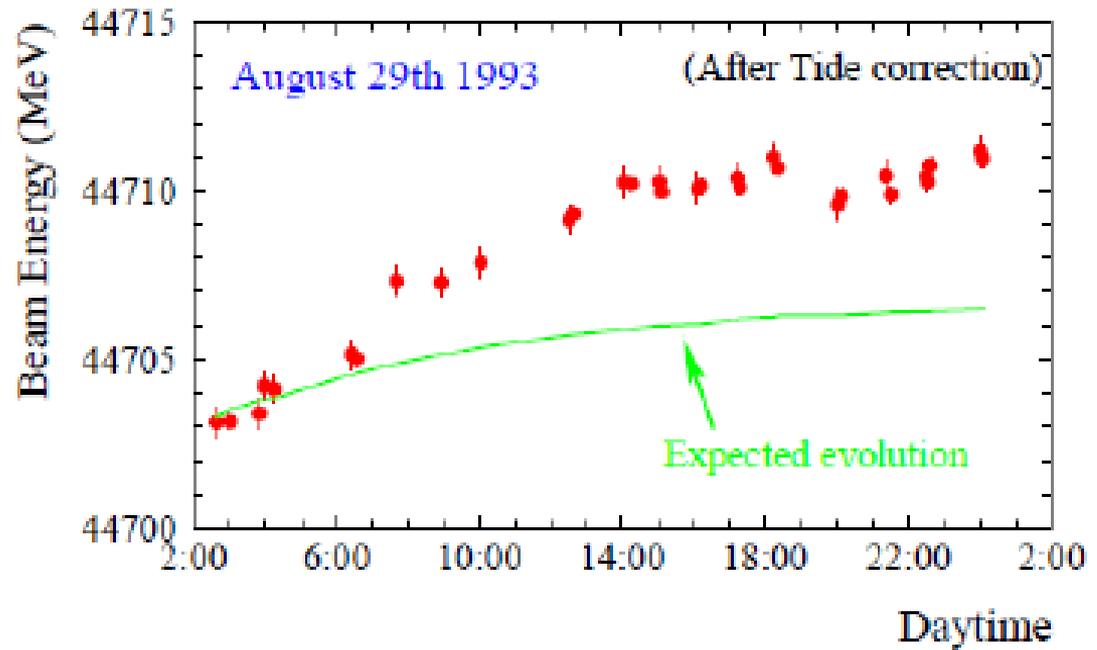


Figure 23: Beam energy variations measured over 24 hours compared to the expectation from the tidal LEP deformation.



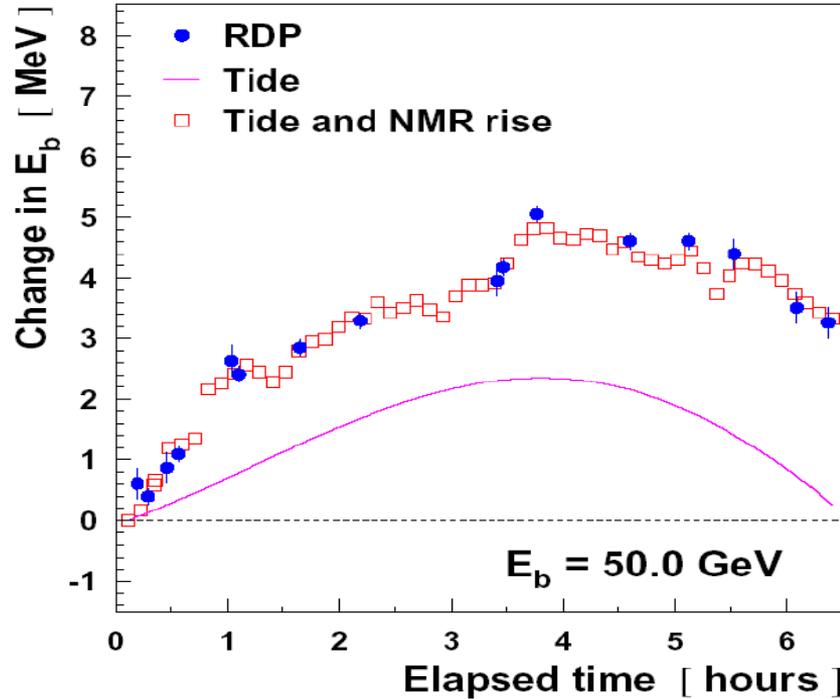
Many effects spoil the calibration if it is performed outside physics time

- tides and other ground motion
- RF cavity phases

hysteresis effects and environmental effects (trains etc)



Modelling of energy rise by (selected) NMR sampling of B-field is excellent !



(Experiment from 1999)

by 1999 we had an excellent model of the energy variations...
but we were not measuring the Z mass and width anymore
– we were hunting for the Higgs boson!

EXPERIMENTS ON BEAM-BEAM DEPOLARIZATION AT LEP

R. Assmann*, A. Blondel*, B. Dehning, A. Drees^o, P. Grosse-Wiesmann, H. Grote, M. Placidi, R. Schmidt, F. Tecker[†], J. Wenninger

PAC 1995

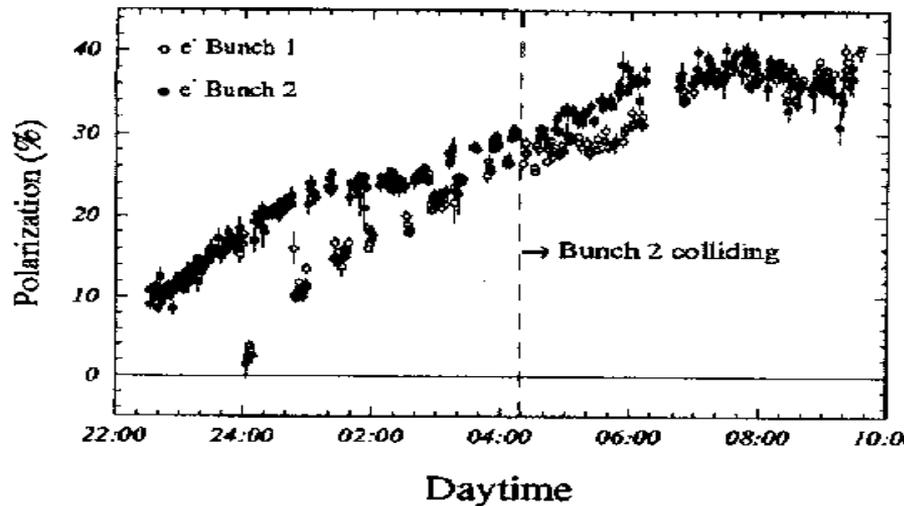


Figure. 3. Polarization level during third experiment

- With the beam colliding at one point, a polarization level of 40 % was achieved. The polarization level was about the same for one colliding and one non colliding bunch.
- It was observed that the polarization level depends critically on the synchrotron tune : when Q_s was changed by 0.005, the polarization strongly decreased.

experiment performed at an energy of 44.71 GeV the polarization level was 40 % with a linear beam-beam tune shift of about 0.04/IP. This indicates, that the beam-beam depolarization does not scale with the linear beam-beam tune shift at one crossing point. Other parameters as spin tune and synchrotron tune are also of importance.

LEP:

This was only tried 3 times!

Best result: $P = 40\%$, $\xi_y^* = 0.04$, one IP

FCC-ee

Assuming 2 IP and $\xi_y^* = 0.01 \rightarrow$

reduce luminosity, $10^{10} Z @ P \sim 30\%$

Longitudinal polarization at FCC-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of $\sin^2\theta_w^{lept} = e^2/g^2 (m_z)$
(-- not to be confused with -- $\sin^2\theta_w = 1 - m_w^2/m_z^2$)

Useful references from the past:

«polarization at LEP» CERN Yellow Report 88-02

Precision Electroweak Measurements on the Z Resonance

Phys.Rept.427:257-454,2006 <http://arxiv.org/abs/hep-ex/0509008v3>

GigaZ @ ILC by K. Moenig

Longitudinal polarization: reduction of polarization due to continuous injection

The colliding bunches will lose intensity continuously due to collisions.

In FCC-ee with 4 IPs, $L = 28 \cdot 10^{34}/\text{cm}^2/\text{s}$ beam lifetime is 213 minutes

In FCC-ee with 2 IPs, $L = 1.4 \cdot 10^{36}/\text{cm}^2/\text{s}$ beam life time is 55minutes

Luminosity scales inversely to beam life time.

The injected e^+ and e^- are not polarized \rightarrow asymptotic polarization is reduced.

Assume here that machine has been well corrected and beams (no collisions, no injection) can be polarized to nearly maximum.

(Eliaana Gianfelice in Rome talk)

- 45 GeV
 - limit $\Delta E = 50$ MeV (extrapolating from LEP)
 - 4 wigglers with $B^+ = 0.7$ T
 - 10% polarization in 2.9 h for energy calibration

(polarization time is 26h)



We have simulated the simultaneous effect of

-- natural polarization

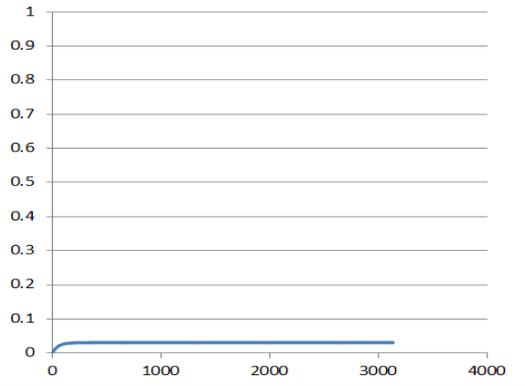
-- beam consumption by e+e- interactions

-- replenishment with unpolarized beams

assuming *optimistically* a maximal 90% asymptotic polarization

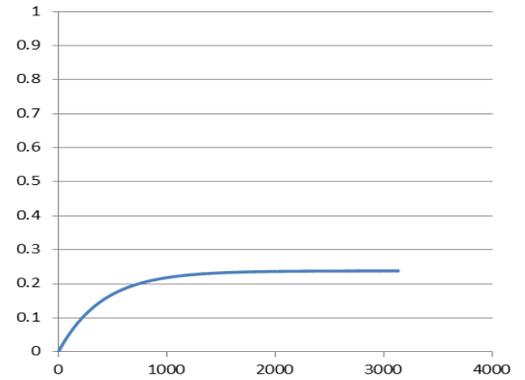
Running at full luminosity
 $P_{\text{max}}=0.03!$ $P_{\text{eff}}=0.03$

polariz



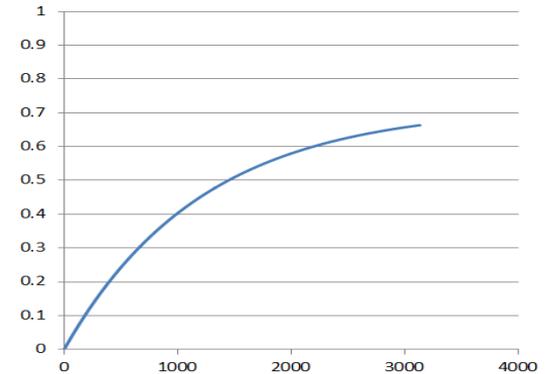
Running at 10% Lumi
 $P_{\text{max}}=0.24$, $P_{\text{eff}}=0.21$

polariz



Running at 1% Lumi
 $P_{\text{max}}=0.66$, $P_{\text{eff}}=0.5$

polariz



ΔA_{LR} scales as $1/\sqrt{(P^2L)}$



Lumi loss factor	L.10^34	Figure of merit: sum(P^2L)	Peff	Pmax
1	220	0.195	0.03	0.03
2	110	0.367	0.059	0.06
4	55	0.627	0.1078	0.11
6	37	0.805	0.149	0.16
8	27	0.924	0.184	0.2
10	22	1.003	0.214	0.24
12	18	1.053	0.24	0.27
15	15	1.09	0.27	0.32
18	12	1.101	0.3	0.35
22	10	1.088	0.33	0.4
26	8	1.059	0.354	0.43
30	7	1.023	0.37	0.46
40	5	0.92	0.41	0.52

Optimum around a reduction of luminosity by a factor 18.

This is still a luminosity of $\sim 10^{35}$ per IP... and the effective polarization is 30%.
 This is equivalent to a 100% polarization expt with luminosity reduced by 180.

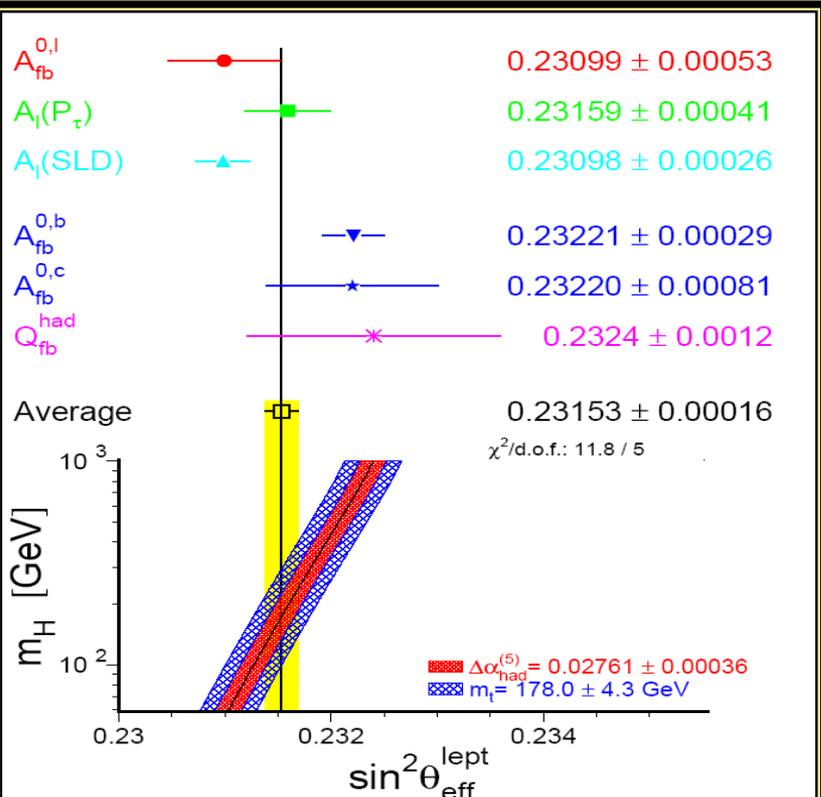


Measuring $\sin^2\theta_W^{\text{eff}} (m_Z)$

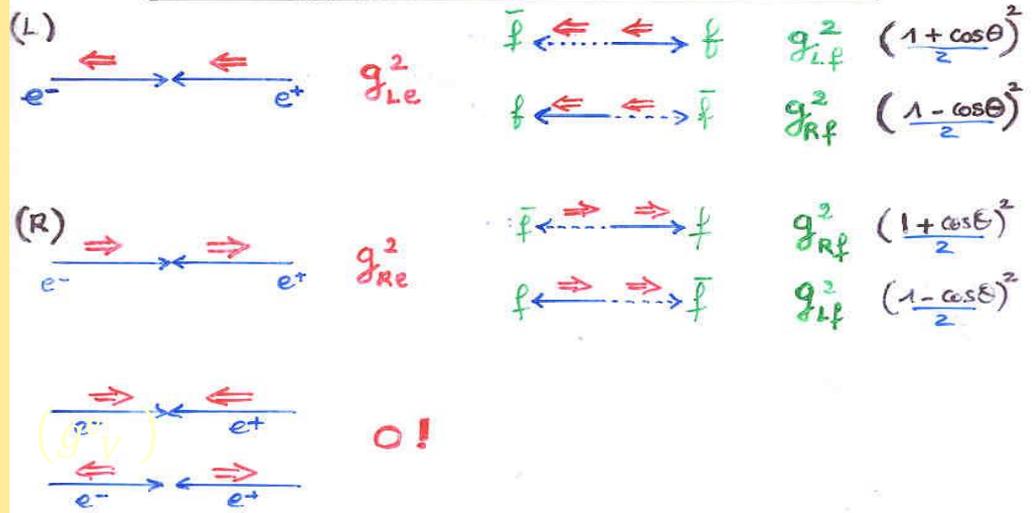
$$\sin^2\theta_W^{\text{eff}} \equiv \frac{1}{4} (1 - g_V/g_A)$$

$$g_V = g_L + g_R$$

arXiv:0509008



Helicity effects in $e^+e^- \rightarrow f\bar{f}$

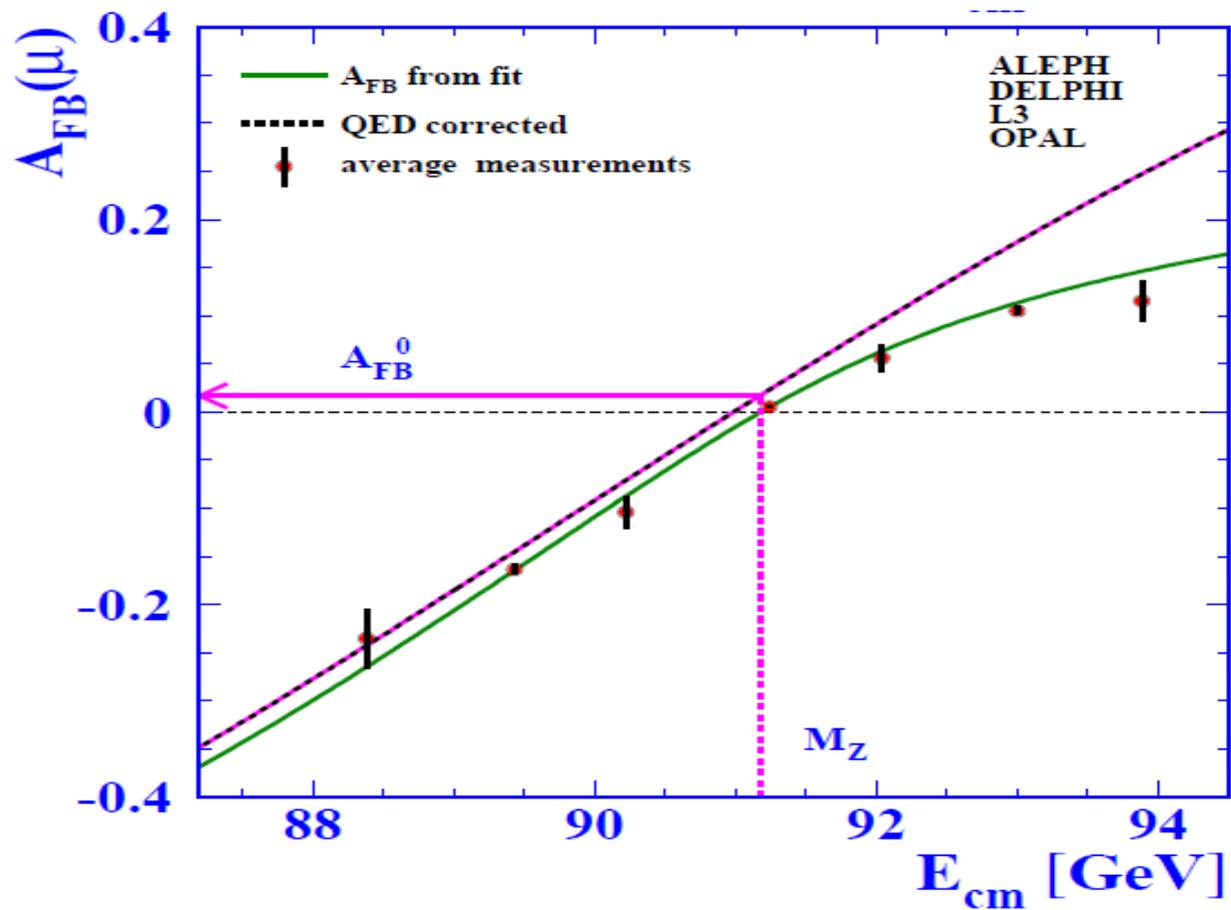


Red BEAM \Rightarrow $A_{LR} = \frac{\sigma_L^{\text{tot}} - \sigma_R^{\text{tot}}}{\sigma_L^{\text{tot}} + \sigma_R^{\text{tot}}} = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} \equiv \mathcal{A}_e = \frac{2g_V g_A}{g_V^2 + g_A^2}$

no Pol available: $A_{FB}^{\text{Pol}f} = \frac{\sigma_{L^+f^-} - \sigma_{L^-f^+} - (\sigma_{R^+f^-} - \sigma_{R^-f^+})}{\sigma_{L^+f^-} + \sigma_{L^-f^+} + \sigma_{R^+f^-} + \sigma_{R^-f^+}} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$

Pol analysis: $\langle P_f \rangle = \frac{\sigma_{e^+R^-} - \sigma_{e^+L^-}}{\sigma_{e^+R^-} + \sigma_{e^+L^-}} = -\mathcal{A}_f$

τ $A_{FB}^{\text{Pol}e} = \frac{\sigma_{e^+R^-} - \sigma_{e^+L^-} - (\sigma_{e^-R^+} - \sigma_{e^-L^+})}{\sigma_{e^+R^-} + \sigma_{e^+L^-} + \sigma_{e^-R^+} + \sigma_{e^-L^+}} = -\frac{3}{4} \mathcal{A}_e$



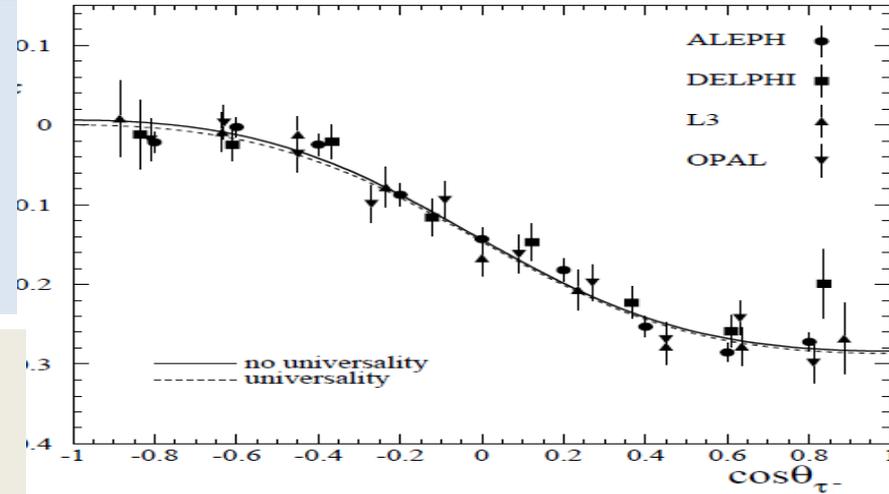
	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^9	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3 \cdot 10^{-6}$	ΔA_{LR} (stat)	$4.2 \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$
ΔE_{cm} (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}(E_{CM})$	$9.2 \cdot 10^{-6}$	$\Delta A_{LR}(E_{CM})$	$4.1 \cdot 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0 \cdot 10^{-5}$	ΔA_{LR}	$5.9 \cdot 10^{-5}$	
$\Delta \sin^2\theta_{lept}^w$	$5.9 \cdot 10^{-6}$		$7.5 \cdot 10^{-6}$	$6 \cdot 10^{-6} + ?$

All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ ($3 \cdot 10^{-5}$) or the comparison with m_W (500keV)

But this precision on $\Delta \sin^2\theta_{lept}^w$ can only be exploited at FCC-ee!



Measured P_τ vs $\cos\theta_{\tau^-}$.



4.7: The values of P_τ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP experiments. Only the statistical errors are shown. The values are not corrected for radiation, interference or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of \mathcal{A}_τ and \mathcal{A}_e . The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of \mathcal{A}_e .

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on \mathcal{A}_τ and \mathcal{A}_e by category for each of the LEP experiments.



The forward backward tau polarization asymmetry is very clean.
 Dependence on E_{CM} same as A_{LR} negl.
 At FCC-ee
 ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

Already syst. level of $6 \cdot 10^{-5}$ on $\sin^2\theta_W^{\text{eff}}$
 much improvement possible
 by using dedicated selection
 e.g. $\tau \rightarrow \pi \nu$ to avoid had. model

Concluding remarks

1. There are very strong arguments for precision energy calibration with transverse polarization at the Z peak and W threshold.
2. Given the likely loss in luminosity, and the intrinsic uncertainties in the extraction of the weak couplings, the case for longitudinal polarization is limited

→ **We have concluded that first priority is to achieve transverse polarization** in a way that allows continuous beam calibration by resonant depolarization

- this is all possible with a very high precision, both at the Z and the W. calibration at higher energies can be made from the data themselves at sufficient level.
- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy with the aim of achieving a precision of $O(100 \text{ keV})$ on E_{CM}

$\Delta\rho$
 $\equiv \epsilon_1$

$$\Gamma_e = (1 + \Delta\rho) \frac{G_F m_Z^3}{24\pi\sqrt{2}} \left(1 + \left(\frac{g_{Ve}}{g_{Ae}} \right)^2 \right) \left(1 + \frac{3}{4} \frac{\alpha}{\pi} \right)$$

ϵ_3

$$\sin^2\theta_w^{\text{eff}} \cos^2\theta_w^{\text{eff}} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2} G_F m_Z^2} \frac{1}{1 + \Delta\rho} \frac{1}{1 - \frac{\epsilon_3}{\cos^2\theta_w}}$$

δ_{vb}

$$\Gamma_b = (1 + \delta_{vb}) \Gamma_d \left(1 - \frac{\text{mass corrections}}{\alpha m_b^2/M_Z^2} \right)$$

ϵ_2

$$M_W^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2} G_F \sin^2\theta_w^{\text{eff}}} \cdot \frac{1}{(1 - \epsilon_3 + \epsilon_2)}$$

$\sin^2\theta_w^{\text{eff}}$ is defined from

$$\sin^2\theta_w^{\text{eff}} = \frac{1}{4} \left(1 - \frac{g_{Ve}}{g_{Ae}} \right) = \sin^2\theta_w^{\text{opt}}$$

obtained from asymmetries at the Z.

also

$\Delta\alpha$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2} G_F} \cdot \frac{1}{\left(1 - \frac{M_W^2}{M_Z^2} \right)} \frac{1}{(1 - \Delta\alpha)}$$

$$\Delta\alpha = \Delta\alpha - \frac{\cos^2\theta_w}{\sin^2\theta_w} \Delta\rho + 2 \frac{G^2\theta_w}{\sin^2\theta_w} \epsilon_3 + \frac{C^2 - S^2}{S^2} \epsilon_2$$

EWRCs

relations to the well measured

$G_F m_Z \propto_{\text{QED}}$
at first order:

$$\Delta\rho = \alpha/\pi (m_{\text{top}}/m_Z)^2$$

$$- \alpha/4\pi \log(m_h/m_Z)^2$$

$$\epsilon_3 = \cos^2\theta_w \alpha/9\pi \log(m_h/m_Z)^2$$

$$\delta_{vb} = 20/13 \alpha/\pi (m_{\text{top}}/m_Z)^2$$

complete formulae at 2d order
including strong corrections
are available in fitting codes

e.g. ZFITTER, GFITTER



Extracting physics from $\sin^2\theta_W^{lept}$

1. Direct comparison with m_Z

$$\sin^2\theta_W^{eff} \cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2} GF m_Z^2} \frac{1}{1+\Delta\rho} \frac{1}{1-\frac{\epsilon_3}{\cos^2\theta_W}}$$

Uncertainties in m_{top} , $\Delta\alpha(m_Z)$, m_H , etc....

$\Delta\sin^2\theta_W^{lept} \sim \Delta\alpha(m_Z) / 3 = 10^{-5}$ if we can reduce $\Delta\alpha(m_Z)$ (see P. Janot)

2. Comparison with m_W/m_Z

Compare above formula with similar one:

$$\sin^2\theta_W \cos^2\theta_W = \frac{\pi\alpha(M_Z^2)}{\sqrt{2} GF m_Z^2} \frac{1}{1 - \left(-\frac{\cos^2\theta_W}{\sin^2\theta_W} \Delta\rho + 2\frac{G^2\theta_W}{\sin^2\theta_W} \epsilon_3 + \frac{C^2 - S^2}{S^2} \epsilon_2 \right)}$$

Where it can be seen that $\Delta\alpha(m_Z)$ cancels in the relation.

The limiting error is the error on m_W .

For $\Delta m_W = 0.5$ MeV this corresponds to $\Delta\sin^2\theta_W^{lept} = 10^{-5}$

Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare

ILC measurement of A_{LR} with $10^9 Z$ and $P_{e^-} = 80\%$, $P_{e^+} = 30\%$

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{Pol}(\tau)$ with $2 \cdot 10^{12} Z$

Comparing $A_{LR}(P)$ and $A_{FB}(\mu\mu)$

Both measure the weak mixing angle as **defined** by the relation $A_\ell = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2}$

with $(g_L^e) = \frac{1}{2} - \sin^2\theta_{W}^{lept}$ and $(g_R^e) = -\sin^2\theta_{W}^{lept}$ $A_\ell \approx 8(1/4 - \sin^2\theta_{W}^{lept})$

$$A_{LR} = A_e$$

$$A_{FB}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_\ell^2$$

- $A_{FB}^{\mu\mu}$ is measured using muon pairs (5% of visible Z decays) and unpolarized beams
- A_{LR} is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization both with very small experimental systematics

-- **parametric sensitivity** $\frac{dA_{FB}^{\mu\mu}}{d\sin^2\theta_{W}^{lept}} = 1.73$ vs $\frac{dA_{LR}}{d\sin^2\theta_{W}^{lept}} = 7.9$

-- **sensitivity to center-of-mass energy** (w.r.t. m_Z) is larger for $A_{FB}^{\mu\mu}$

$$\frac{\partial A_{FB}^{\mu\mu}}{\partial\sqrt{s}} = 0.09/\text{GeV} \text{ vs } \frac{\partial A_{LR}}{\partial\sqrt{s}} = 0.019/\text{GeV}$$

“an 80 MeV uncertainty in E_{cm} corresponds to a 1% error on A_{LR} ” (relative error)

But of course $A_{FB}^{\mu\mu}$ benefits from much larger statistics and E_{cm} precision of circular collider

Measurement of A_{LR}

electron bunches	1 \Leftarrow	2	3	4 \Leftarrow
positron bunches	1	2 \Rightarrow	3	4 \Rightarrow
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

$$\sigma_1 = \sigma_u (1 - P_e^- \Lambda_{LR})$$

$$\sigma_2 = \sigma_u (1 + P_e^+ \Lambda_{LR})$$

$$\sigma_3 = \sigma_u$$

$$\sigma_4 = \sigma_u [1 - P_e^+ P_e^- + (P_e^+ - P_e^-) \Lambda_{LR}]$$

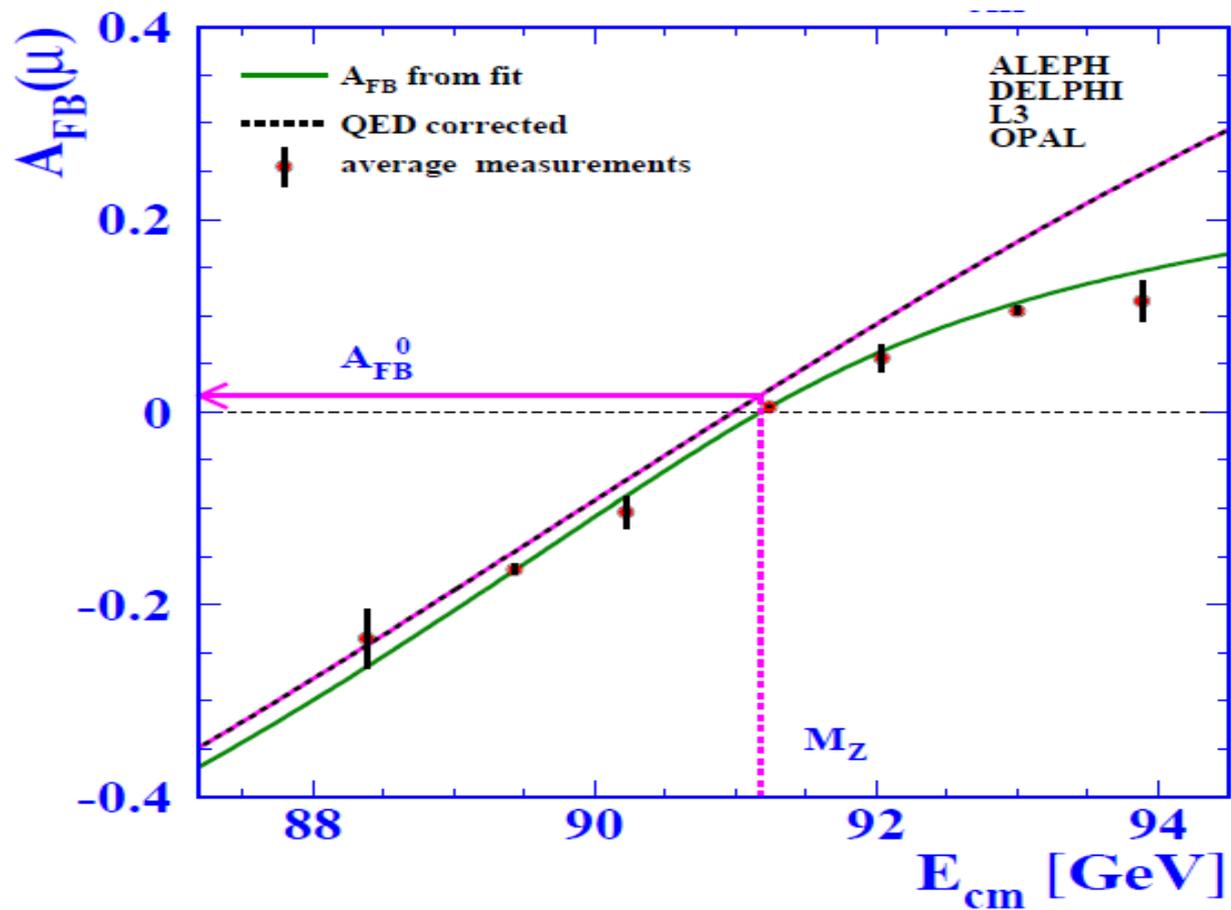
Verifies polarimeter with experimentally measured cross-section ratios

statistics

$$\Delta A_{LR} = 0.0025 \text{ with about } 10^6 \text{ } Z^0 \text{ events,}$$

$$\Delta A_{LR} = 0.000045 \text{ with } 5 \cdot 10^{10} \text{ } Z \text{ and 30\% polarization in collisions.}$$

$$\Delta \sin^2 \theta_W^{\text{eff}} (\text{stat}) = O(2 \cdot 10^{-6})$$



	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^9	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3 \cdot 10^{-6}$	ΔA_{LR} (stat)	$4.2 \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$
ΔE_{cm} (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}(E_{CM})$	$9.2 \cdot 10^{-6}$	$\Delta A_{LR}(E_{CM})$	$4.1 \cdot 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0 \cdot 10^{-5}$	ΔA_{LR}	$5.9 \cdot 10^{-5}$	
$\Delta \sin^2\theta_{lept}^w$	$5.9 \cdot 10^{-6}$		$7.5 \cdot 10^{-6}$	$6 \cdot 10^{-6} + ?$

All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ ($3 \cdot 10^{-5}$) or the comparison with m_W (500keV)

But this precision on $\Delta \sin^2\theta_{lept}^w$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.

Dependence on E_{CM} same as A_{LR} negl.

At FCC-ee

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

Already syst. level of $6 \cdot 10^{-5}$ on $\sin^2\theta_W^{\text{eff}}$

much improvement possible

by using dedicated selection
e.g. $\tau \rightarrow \pi \nu$ to avoid had. model

Measured P_τ vs $\cos\theta_{\tau^-}$.

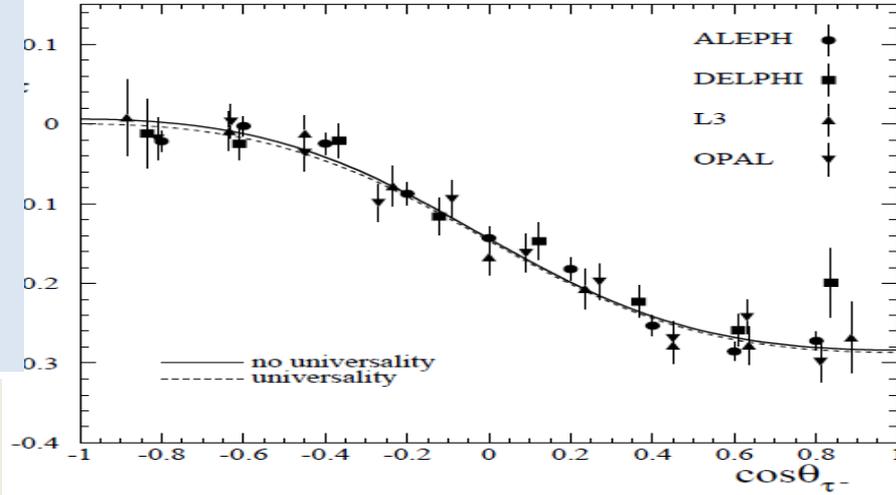
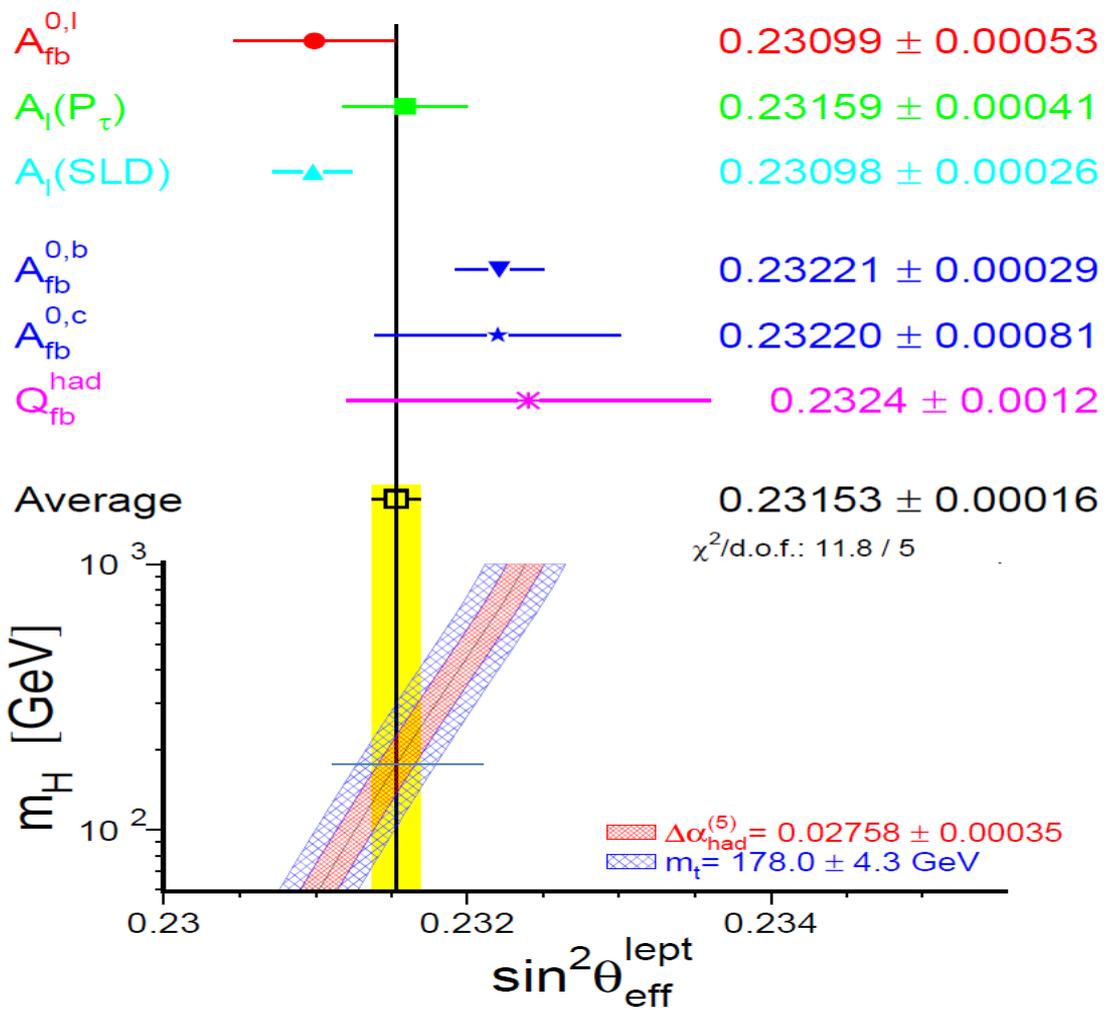
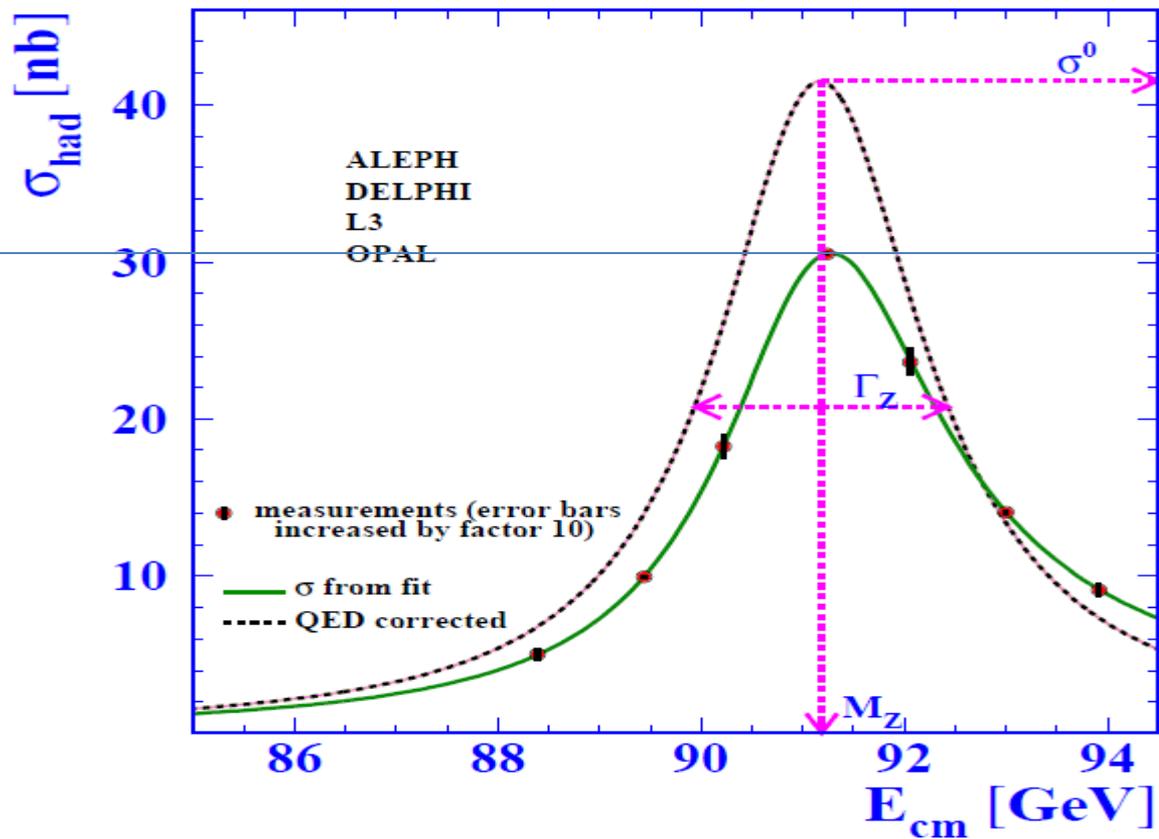


Figure 4.7: The values of P_τ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP experiments. Only the statistical errors are shown. The values are not corrected for radiation, interference or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP value of \mathcal{A}_τ and \mathcal{A}_e . The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of \mathcal{A}_e .

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on \mathcal{A}_τ and \mathcal{A}_e by category for each of the LEP experiments.





Going through the observables

the weak mixing angle as **defined** by the relation

$$A_\ell = \frac{2g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2}$$

with $(g_L^e) = \frac{1}{2} - \sin^2\theta_W^{\text{lept}}$ and $(g_R^e) = -\sin^2\theta_W^{\text{lept}}$

$A_\ell \approx 8(1/4 - \sin^2\theta_W^{\text{lept}})$ very sensitive to $\sin^2\theta_W^{\text{lept}}$!

Or

$A_{LR} = A_e$ measured from $(\sigma_{\text{vis,L}} - \sigma_{\text{vis,R}}) / (\sigma_{\text{vis,L}} + \sigma_{\text{vis,R}})$

(total visible cross-section had + $\mu\mu$ + $\tau\tau$ (35 nb) for 100% Left Polarization

$$g_{Vf} = \sqrt{R_f} (T_3^f - 2Q_f^f \sin^2\theta_W)$$

$$A_{FB}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_e^2$$

$$\begin{aligned} A_{FB}^{0,f} &= \frac{3}{4} A_e A_f \\ A_{LR}^0 &= A_e \\ A_{LRFB}^0 &= \frac{3}{4} A_f \\ \langle P_\tau^0 \rangle &= -A_\tau \\ A_{FB}^{\text{pol},0} &= -\frac{3}{4} A_e \end{aligned}$$

$$\begin{aligned} A_{FB} &= \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ A_{LR} &= \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{\langle |P_e| \rangle} \\ A_{LRFB} &= \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \frac{1}{\langle |P_e| \rangle} \end{aligned}$$

Beam polarization and E-calibration

Precise meas of E_{beam} by resonant depolarization

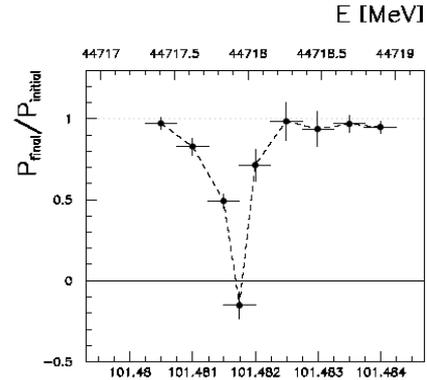
~100 keV each time the meas is made

LEP →

At LEP transverse polarization was achieved routinely at Z peak.

but measurement only performed at the end of fills (and only for e-!)

lots of effects (tides, trains, lake level, rain (...and tears) in data vs calib!



Polarization in collisions was observed (*40% at BBTS = 0.04*)

At LEP a beam energy spread $\sigma_E > 55$ MeV destroyed polarization above 61 GeV

$\sigma_E \propto E^2/\sqrt{\rho} \rightarrow$ *At FCC-ee transverse polarization up to > 81 GeV (WW threshold)*

FCC-ee: use 'single' pilot bunches to measure the beam energy continuously

no interpolation errors due to tides, ground motion or trains etc...

<< 100 keV beam energy calibration around Z peak and W pair threshold.

$\Delta m_Z \sim 0.1$ MeV, $\Delta \Gamma_Z < 0.1$ MeV, $\Delta m_W \sim 0.5$ MeV