The Standard Model of Particle Physics, Lecture 1

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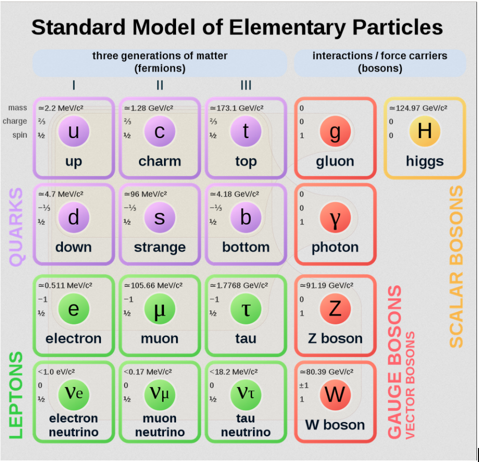
Overview of Standard Model

- The standard model of particle physics involves understanding and categorizing fundamental particles and their interactions.



[courtesy:www.wikipedia.org]

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- Discovery of neutrons by the seminal experiment by James Chadwik in 1932 is one of the milestones of particle physics research.
- It was observed that $M_N = 939.565$ MeV is strikingly close to the proton mass $M_P = 938.272$ MeV.
- This lead to a proposal by Heisenberg that these are two degenerate states of the same quantum number $\rightarrow$ Isospin !

Isospin

- The quantum number corresponding to isospin is denoted by I. One can denote the proton and neutron states by the component of the total isospin given by I_3. The wavefunctions can be denoted as,

$$|p\rangle \equiv |\mathbf{x}, t, s, I, I_3\rangle \ , \ |n\rangle \equiv |\mathbf{x}, t, s, I, -I_3\rangle \ .$$

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$$|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- If we take the states $|n, p\rangle$ eigenfunctions of $\hat{I}_3$, then denoting $I_3 = \frac{\sigma_3}{2}$ we get

$$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} |p, n\rangle = \pm\frac{1}{2}|p, n\rangle \ .$$

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- using which one can go from one component-state to the other

$$I_{+}|p>= 0 \quad , \quad I_{+}|n>= |p> \quad , \quad I_{-}|p>= |n> \quad , \quad I_{-}|n>= 0$$

More on Isospin

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$$|\pi^\pm >= |1, \pm 1> \quad , \quad |\pi^0 >= |1, 0> \ .$$

- It was observed that strong interactions conserve isospin! Interactions of two protons are of same strength as that between two neutrons. Mathematically $[H_{\text{strong}}, I] = 0$.

More on Isospin: Pion-nucleon scattering

- One can use this fact that strong interactions conserve isospin to calculate the ratio of cross sections of the decay process

$$\frac{\sigma(\pi^- p \to \pi^- p)}{\sigma(\pi^- p \to \pi^0 n)}$$

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- Step 1: One can write the states in terms of product states using addition of isospins $I = I^p + I^\pi$. The highest weight state, for example, is given as $|1, 1\rangle|\frac{1}{2}, \frac{1}{2}\rangle \equiv |\frac{3}{2}, \frac{3}{2} >$.

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- Thus acting $I_- = I_-^p + I_-^\pi$ on the highest state one gets the next state with $I = \frac{3}{2}$, $I_3 = \frac{1}{2}$.

[Note that $I_\pm|I, I_3> = \sqrt{I(I+1) - I_3(I_3 \pm 1)}|I, I_3>$].

$$I_-|\frac{3}{2},\frac{3}{2}> = |\frac{3}{2},\frac{1}{2}> = \sqrt{\frac{2}{3}}|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle \quad .$$

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- Step 2: Act I_- again on this state. We can show that both the states $\pi^- p$ and $\pi^0 n$ decay via $|\frac{3}{2},-\frac{1}{2}>$ with weights $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$ respectively.

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- Since cross-sections are probability densities of the isospin states hence,

$$\frac{\sigma(\pi^- p \rightarrow \pi^- p)}{\sigma(\pi^- p \rightarrow \pi^0 n)} = |\frac{\frac{1}{\sqrt{3}}}{\sqrt{\frac{2}{3}}}|^2 = \frac{1}{2} \ .$$

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- In fact if you look into the Particle Data Group this predicted ratio of cross-sections are quite close to experimental values! Small deviations due to the fact that $SU(2)$ isospin symmetry is near-exact.

Strangeness

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- Resolution of this puzzle: Gell-Mann and Nishijima introduced a new quantum number called strangeness.
- They postulated that strangeness has to be conserved in strong and electromagnetic interactions. Thus assigned $S = 0$ to Δ, $S = +1$ to K^+ and $S = -1$ to the baryons Σ, Λ. This also explains why strange particles are produced in pairs.

References

- F. Halzen and A. D. Martin, "Quarks and Leptons", John Wiley & Sons (1984).
- T-P Cheng, L-F Li, "Gauge Theory of Elementary Particle Physics", Oxford University Press (1984).