The Standard Model of Particle Physics, Lecture 2

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9th November 2021

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  $Q = I_3 + \frac{Y}{2}$ [Gell-Mann & Nishijima].
- The hypercharge can be written in terms of baryon number B, strangeness S as
  $Y = S + B + ..$
  All of them are conserved in strong interactions!

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- One can show that there are 2 generators of $SU(3)$ which are diagonal and commute with each other

$$I_3 = F_3 = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}\ ,\ Y = \frac{F_8}{\sqrt{3}} = \frac{1}{3}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}\ .$$

$SU(2)$ subgroups

- Once can show that there are three different $SU(2)$ sub-algebra for $SU(3)$ Lie algebra. One can construct

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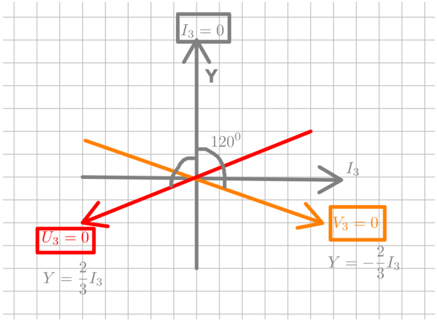
- Assignment: Show that $2U_3 = \frac{3}{2}Y - I_3$ and $2V_3 = \frac{3}{2}Y + I_3$
- These also satisfy the important algebra with respect to the commuting generators, for e.g.,

$$[Y, V_\pm] = \pm V_\pm \quad , \quad [I_3, V_\pm] = \pm\frac{1}{2}V_\pm \quad , \quad [Y, I_\pm] = 0 \ .$$

Visualizing U, V, I multiplets



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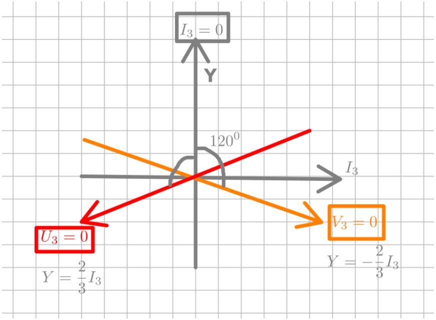

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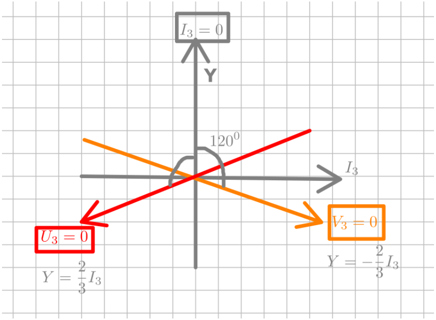

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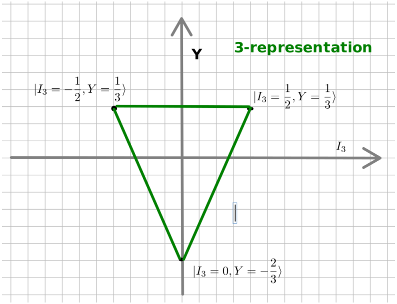
- The $U_3 = 0, \ V_3 = 0$ and the $I_3 = 0$ axes are at angles 120^0 to each other.
- Due to equivalence of these 3 sub-algebras the $SU(3)$ multiplets in the $I_3 - Y$ plane have to be regular hexagons or triangles.

Lowest irreducible representation of $SU(3)$



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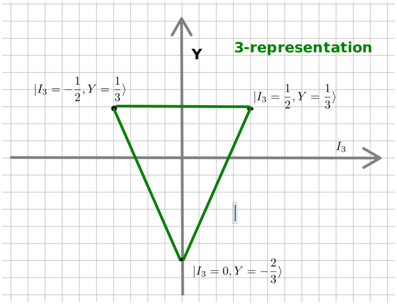

- The smallest representation must contain a U, V, I-spin doublet. The smallest isospin representation is for $I = \frac{1}{2}$.
- If the three states are denoted by $\psi_{1,2,3}$ they should satisfy

$$I_3\psi_{1,2} = \pm\psi_{1,2} \quad , \quad I_3\psi_3 = 0 \quad , \quad U_3\psi_1 = 0$$

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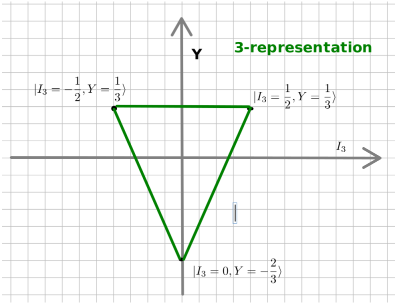

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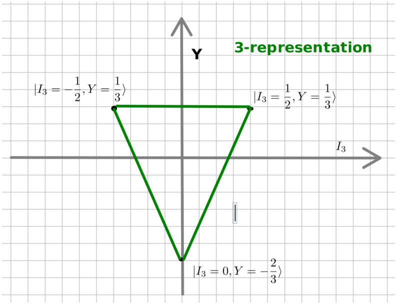
- Using the fact that $2U_3 = \frac{3}{2}Y - I_3$, one can show that $Y\psi_1 = \frac{1}{3}\psi_1$.

Lowest irreducible representation of $SU(3)$



- One can also calculate the electric charge of these states using Gell-Mann and Nishijima formula $Q = I_3 + \frac{Y}{2}$.

$$Q\psi_1 = \frac{2}{3}\psi_1 \quad , \quad Q\psi_2 = -\frac{1}{3}\psi_2 \quad , \quad Q\psi_3 = -\frac{1}{3}\psi_3 \ .$$

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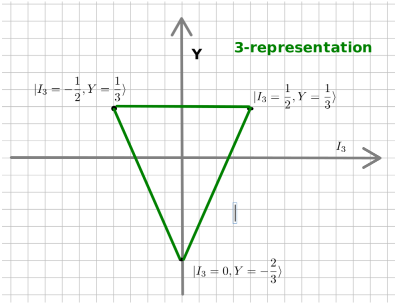

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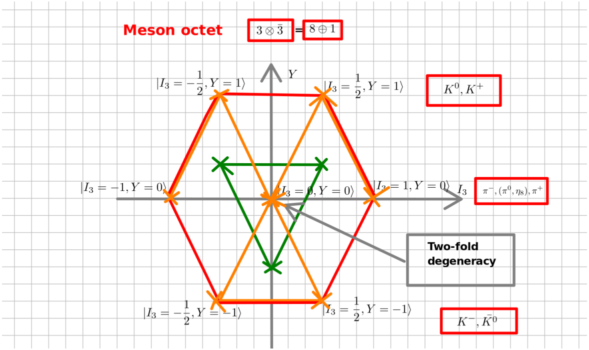
- Since these states carry fractional charges Gell-Mann named then quarks and Zweig called them aces. There is also another distinct $\bar{3}$-representation for the anti-quarks.

Constructing mesons out of the fundamental representations



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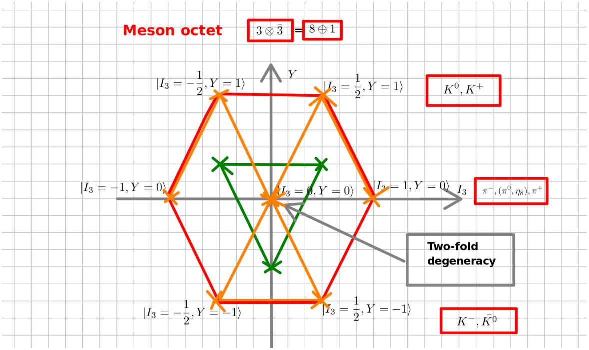
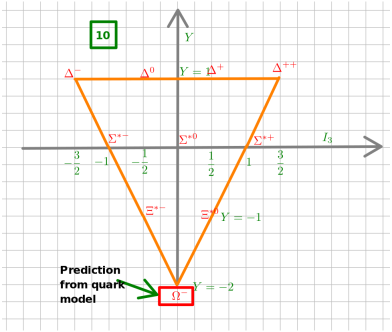


- Due to advent of cyclotron a large number of baryons and mesons were discovered $\rightarrow$ particle zoo!. Is there a way to classify them according to their symmetries.
- For mesons it was observed that particles and anti-particles are members of same $SU(3)$ multiplet and carry same spin. Hence unlike baryons it has $B = 0$ and consists of quark, anti-quark. These can be constructed using $[3] \otimes [\bar{3}] = [8] \oplus [1]$.

Success of Gell-Mann's quark model



- Baryons carry half-integral spin so can be constructed out of three quarks. You can show that $[3] \otimes [3] \otimes [3] = [10] \oplus [8] \oplus [8] \oplus [1]$.

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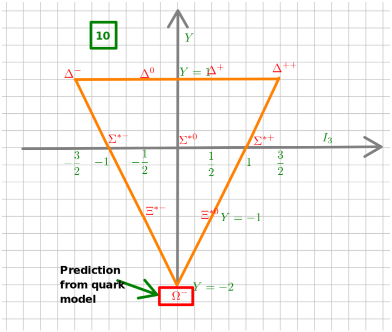

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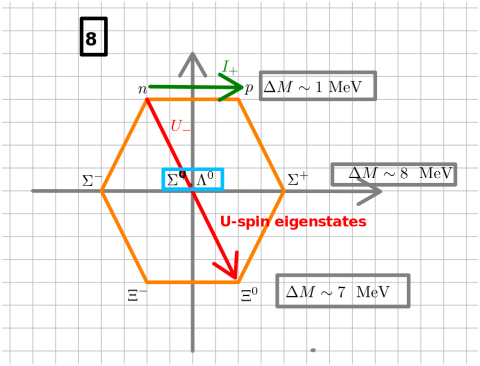
- The $\Omega^-(sss)$ was predicted and later confirmed experimentally $\rightarrow$ one of the most successful predictions of quark model! It was predicted that one needs color quantum number to explain existence of such a state [Oscar Greenberg].

Predictions from quark model



- Since $[Q, U_3] = 0$, the U-spin states carry same charge. Furthermore only states with multiplicity one could be eigenstate of U-spin. Hence for $I_3 = 0$ one has $\chi = \frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda$ as its eigenstate.

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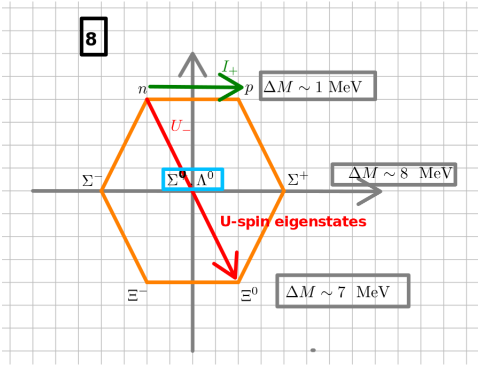

- Mass difference along $U_3 = 0$ is $\mathcal{O}(100)$ MeV! However since charge is same for U-spin states the electromagnetic corrections to the mass are the same which leads to Coleman-Glashow relation,

$$M_n - M_P + M_{\Xi^-} - M_{\Xi^0} = M_{\Sigma^-} - M_{\Sigma^+} \ .$$

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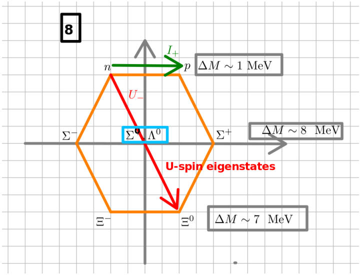

- If there were only strong interactions and u, d, s quark masses were degenerate then states in these multiplets should be degenerate. Since the amount of isospin breaking is small one can account for the mass differences using the mass formula of Gell-Mann and Okubo

$$M = a + bY + c[I(I+1) - \frac{Y^2}{4}] \Rightarrow M_n + M_{\Xi^0} \simeq \frac{3}{2} M_{\Lambda^0} + \frac{1}{2} M_{\Sigma^0} \ .$$

References

- F. Halzen and A. D. Martin, "Quarks and Leptons", John Wiley & Sons (1984).
- T-P Cheng, L-F Li, "Gauge Theory of Elementary Particle Physics", Oxford University Press (1984).