The Standard Model of Particle Physics, Lecture 3

Sayantan Sharma

The Institute of Mathematical Sciences

10th November 2021

Interactions

- After having studied how to understand the hadron spectrum from the symmetry properties of Standard Model we now focus on the types of interactions.

$$\pi^- \to \mu^- + \bar{\nu}_\mu \quad , \quad \tau = 2.6 \times 10^{-8}\text{sec}$$
$$\pi^0 \to \gamma\gamma \quad , \quad \tau \sim 10^{-16}\text{sec}$$
$$\pi N \to \pi N \quad , \quad \tau \sim 10^{-23}\text{sec}$$

Interactions

- After having studied how to understand the hadron spectrum from the symmetry properties of Standard Model we now focus on the types of interactions.

$$\pi^- \rightarrow \mu^- + \bar{\nu_\mu} \quad , \quad \tau = 2.6 \times 10^{-8}\text{sec}$$
$$\pi^0 \rightarrow \gamma\gamma \quad , \quad \tau \sim 10^{-16}\text{sec}$$
$$\pi N \rightarrow \pi N \quad , \quad \tau \sim 10^{-23}\text{sec}$$

- We can categorize into weak, electromagnetic and strong interactions depending on the time scale of interactions.

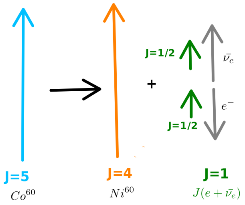
Interactions

- After having studied how to understand the hadron spectrum from the symmetry properties of Standard Model we now focus on the types of interactions.

$$\pi^- \to \mu^- + \bar{\nu}_\mu \quad , \quad \tau = 2.6 \times 10^{-8}\text{sec}$$
$$\pi^0 \to \gamma\gamma \quad , \quad \tau \sim 10^{-16}\text{sec}$$
$$\pi N \to \pi N \quad , \quad \tau \sim 10^{-23}\text{sec}$$

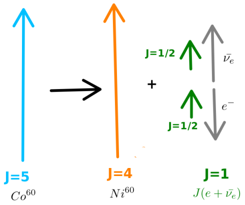
- We can categorize into weak, electromagnetic and strong interactions depending on the time scale of interactions.
- The beta decay $n \to p + e + \bar{\nu}_e$ is a weak decay. If there were no weak decays then neutron would be as stable as a proton (10^{30} yrs)!

Weak decays



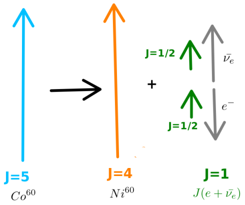
- It was observed by Madam Wu that weak decays violate parity. The electrons in the decay of $Co^{60} \rightarrow Ni^{60} + e + \bar{\nu_e}$ are emitted preferentially along a direction opposite to the Co-spin.

Weak decays



- It was observed by Madam Wu that weak decays violate parity. The electrons in the decay of $Co^{60} \rightarrow Ni^{60} + e + \bar{\nu_e}$ are emitted preferentially along a direction opposite to the Co-spin.
- Evidence collected from many experiments showed that only ν_L exist and there is no corresponding mirror image state ν_R.

Weak decays



- It was observed by Madam Wu that weak decays violate parity. The electrons in the decay of $Co^{60} \rightarrow Ni^{60} + e + \bar{\nu_e}$ are emitted preferentially along a direction opposite to the Co-spin.
- Evidence collected from many experiments showed that only ν_L exist and there is no corresponding mirror image state ν_R.
- In fact from experimental observations of weak decays reveal [Lee & Yang]

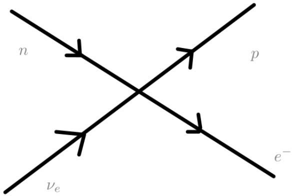
$$\Gamma\left(\pi^+ \rightarrow \mu^+ + \nu_{\mu,L}\right) \neq \Gamma\left(\pi^+ \rightarrow \mu^+ + \nu_{\mu,R}\right) \quad \text{Parity Violation}$$

$$\Gamma\left(\pi^+ \rightarrow \mu^+ + \nu_{\mu,L}\right) \neq \Gamma\left(\pi^- \rightarrow \mu^- + \nu_{\bar{\mu},L}\right) \quad \text{C Violation}$$

Weak decays: Maximal parity violation

- It is difficult to estimate the helicity of neutrinos emitted from beta decay experiments $n \rightarrow p + e + \bar{\nu_e}$. Experiments by Goldhaber, Grodzins and Sunyar measured the spin and momentum of the recoil nucleus and inferred the helicity of neutrinos.

$n \rightarrow p + e + \bar{\nu}_e$



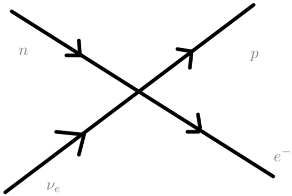
amplitude~ (hadron current)$^\mu$ $\bar{e}_L \gamma_\mu \nu_L$

$$\bar{e_L}\gamma_\mu \nu_L = \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu$$

Weak decays: Maximal parity violation

- It is difficult to estimate the helicity of neutrinos emitted from beta decay experiments $n \to p + e + \bar{\nu_e}$. Experiments by Goldhaber, Grodzins and Sunyar measured the spin and momentum of the recoil nucleus and inferred the helicity of neutrinos.
- Experiments revealed that neutrinos are always left handed! $\to$ maximum parity violation!

$n \to p + e + \bar{\nu}_e$



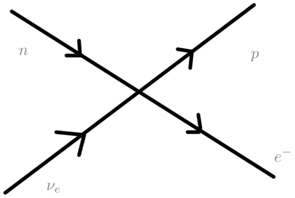
amplitude$\sim$ $(\text{hadron current})^\mu\ \bar{e}_L \gamma_\mu \nu_L$

$$\bar{e_L}\gamma_\mu \nu_L = \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu$$

Weak decays: Maximal parity violation

- It is difficult to estimate the helicity of neutrinos emitted from beta decay experiments $n \rightarrow p + e + \bar{\nu_e}$. Experiments by Goldhaber, Grodzins and Sunyar measured the spin and momentum of the recoil nucleus and inferred the helicity of neutrinos.
- Experiments revealed that neutrinos are always left handed! $\rightarrow$ maximum parity violation!
- $j^\mu = \bar{e}\left[C_V\gamma^\mu - C_A\gamma^\mu\gamma^5\right]\nu_e \quad , \quad C_V = C_A = \frac{1}{2}$.

$n \rightarrow p + e + \bar{\nu}_e$



amplitude~ $(\text{hadron current})^\mu \; \bar{e}_L\gamma_\mu\nu_L$

$$\bar{e_L}\gamma_\mu\nu_L = \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu$$

Weak decays

- It was also observed that weak decay $\mu^- \to e^- + \nu_\mu + \bar{\nu_e}$ is allowed whereas $\mu \to e\gamma$ is forbidden!

Weak decays

- It was also observed that weak decay $\mu^- \to e^- + \nu_\mu + \bar{\nu_e}$ is allowed whereas $\mu \to e\gamma$ is forbidden!
- This tells us that there is an additive conserved quantum number $\to$ Lepton number

$$L_e = +1 \text{ for } e^-, \nu_e \quad , \quad L_e = 0 \text{ for } \mu, \tau, \nu_\mu, \nu_\tau \ .$$

Weak decays

- It was also observed that weak decay $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu_e}$ is allowed whereas $\mu \rightarrow e\gamma$ is forbidden!
- This tells us that there is an additive conserved quantum number $\rightarrow$ Lepton number

$$L_e = +1 \text{ for } e^-, \nu_e \quad , \quad L_e = 0 \text{ for } \mu, \tau, \nu_\mu, \nu_\tau \ .$$

- One can therefore construct three families of leptons which forms doublets

$$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \ , \ \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \ , \ \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix} \ , \ e_R \ , \ \mu_R \ , \ \tau_R \ .$$

Hadronic weak decays

- Hadrons also undergo weak decays to leptons.

$$\pi^- \rightarrow \mu^- + \bar{\nu_\mu} \quad , \quad \Sigma^- \rightarrow n + e^- + \bar{\nu_e}$$

Hadronic weak decays

- Hadrons also undergo weak decays to leptons.

$$\pi^- \to \mu^- + \bar{\nu_\mu} \quad , \quad \Sigma^- \to n + e^- + \bar{\nu_e}$$

- Furthermore it has been observed that strangeness-changing process are suppressed which involves no change in electric charge $\rightarrow$ no mixing between d and s weak states [Glashow-Iliopoulos-Maiani].

Hadronic weak decays

- Hadrons also undergo weak decays to leptons.

$$\pi^- \to \mu^- + \bar{\nu_\mu} \quad , \quad \Sigma^- \to n + e^- + \bar{\nu_e}$$

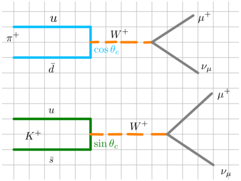
- Furthermore it has been observed that strangeness-changing process are suppressed which involves no change in electric charge $\to$ no mixing between d and s weak states [Glashow-Iliopoulos-Maiani].
- One can analogously write the weak eigenstates of quark families as

$$\begin{pmatrix} u_L \\ d_L^{'} \end{pmatrix} , \begin{pmatrix} c_L \\ s_L^{'} \end{pmatrix} , \begin{pmatrix} t_L \\ b_L^{'} \end{pmatrix} \quad , \quad u_R, d_R, s_R, c_R, b_R, t_R \ .$$

Cabibbo angles

- It was observed that decays with $\Delta S = \pm 1$ are always suppressed.

$$\frac{m_K^3 f_\pi^2}{m_\pi^3 f_K^2}\left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 \frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \sim 5 \times 10^{-2} .$$



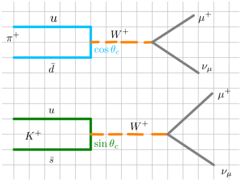
Cabibbo angles

- It was observed that decays with $\Delta S = \pm 1$ are always suppressed.

$$\frac{m_K^3 f_\pi^2}{m_\pi^3 f_K^2}\left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 \frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \sim 5 \times 10^{-2} \ .$$

- This can happen if the u quark couples with both d and s quarks. Hence the weak eigenstates are [Cabibbo]

$$d' = d\cos\theta_c + s\sin\theta_c \quad , \quad s' = -d\sin\theta_c + s\cos\theta_c \ .$$



Cabibbo angles

- It was observed that decays with $\Delta S = \pm 1$ are always suppressed.

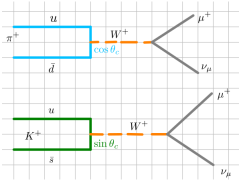
$$\frac{m_K^3 f_\pi^2}{m_\pi^3 f_K^2}\left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 \frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \sim 5 \times 10^{-2} \ .$$

- This can happen if the u quark couples with both d and s quarks. Hence the weak eigenstates are [Cabibbo]

$$d^{'} = d\cos\theta_c + s\sin\theta_c \quad , \quad s^{'} = -d\sin\theta_c + s\cos\theta_c \ .$$

- One can readily show that

$$\frac{m_K^3 f_\pi^2}{m_\pi^3 f_K^2}\left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 \frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \sim \tan^2\theta_c \quad , \quad \theta_c \simeq 13^0 .$$



Quark flavor mixing

- In fact for three families of quarks one can show that the weak eigenstates $d^{'}, s^{'}, b^{'}$ are related to mass eigenstates through CKM mixing matrix [Cabibbo-Kobayashi-Maskawa]

$$\begin{pmatrix} d^{'} \\ s^{'} \\ c^{'} \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ c \end{pmatrix}$$

Quark flavor mixing

- In fact for three families of quarks one can show that the weak eigenstates $d^{'}, s^{'}, b^{'}$ are related to mass eigenstates through CKM mixing matrix [Cabibbo-Kobayashi-Maskawa]

$$\begin{pmatrix} d^{'} \\ s^{'} \\ c^{'} \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ c \end{pmatrix}$$

- The CKM matrix is a unitary matrix. There are 9 independent parameters. We can change the phase of the quark mass and weak eigenstates and absorb 6 parameters, but one is a global one. Therefore the CKM matrix contains $9 - 5 = 4$ independent parameters.

Quark flavor mixing

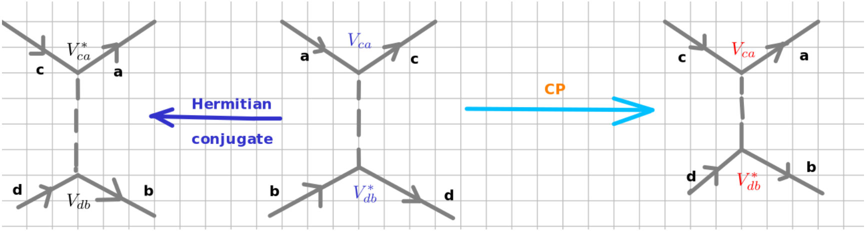
- In fact for three families of quarks one can show that the weak eigenstates $d^{'}, s^{'}, b^{'}$ are related to mass eigenstates through CKM mixing matrix [Cabibbo-Kobayashi-Maskawa]

$$\begin{pmatrix} d^{'} \\ s^{'} \\ c^{'} \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ c \end{pmatrix}$$

- The CKM matrix is a unitary matrix. There are 9 independent parameters. We can change the phase of the quark mass and weak eigenstates and absorb 6 parameters, but one is a global one. Therefore the CKM matrix contains $9 - 5 = 4$ independent parameters.
- Any orthogonal matrix has only $N(N-1)/2$ real parameters, so for $N = 3$ there are 3 angles and one residual phase for three families of quarks. This complex phase leads to CP violation in weak interactions.

CKM phase and CP violation

- There is a complex phase in V_{CKM} since there are 3 families of quarks.
- If there were only 2 families of quarks V_{CKM} is a 2×2 real matrix which is parametrized solely by the Cabibbo angle θ_c.
- If we look at a process $a + b \to c + d$ then the amplitude of its Hermitian conjugate process is not identically equal to its CP conjugate $\bar{a} + \bar{b} \to \bar{c} + \bar{d}$ because of the complex phase factor in V_{CKM}. This is the origin of CP violation.



References

- F. Halzen and A. D. Martin, "Quarks and Leptons", John Wiley & Sons (1984).
- T-P Cheng, L-F Li, "Gauge Theory of Elementary Particle Physics", Oxford University Press (1984).