ACCELERATORS

with emphasis on hadron colliders

LECTURE 1

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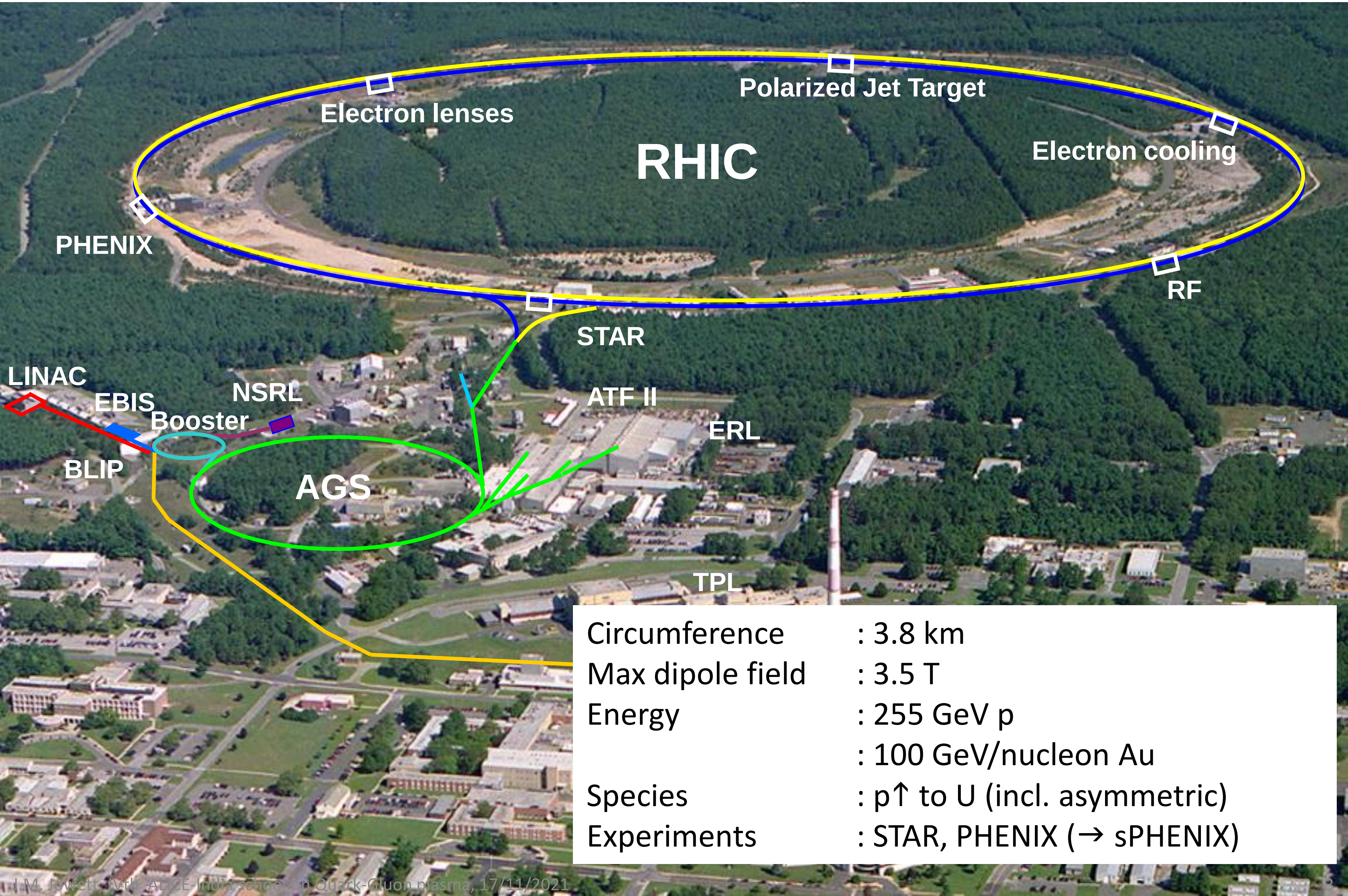
Introduction

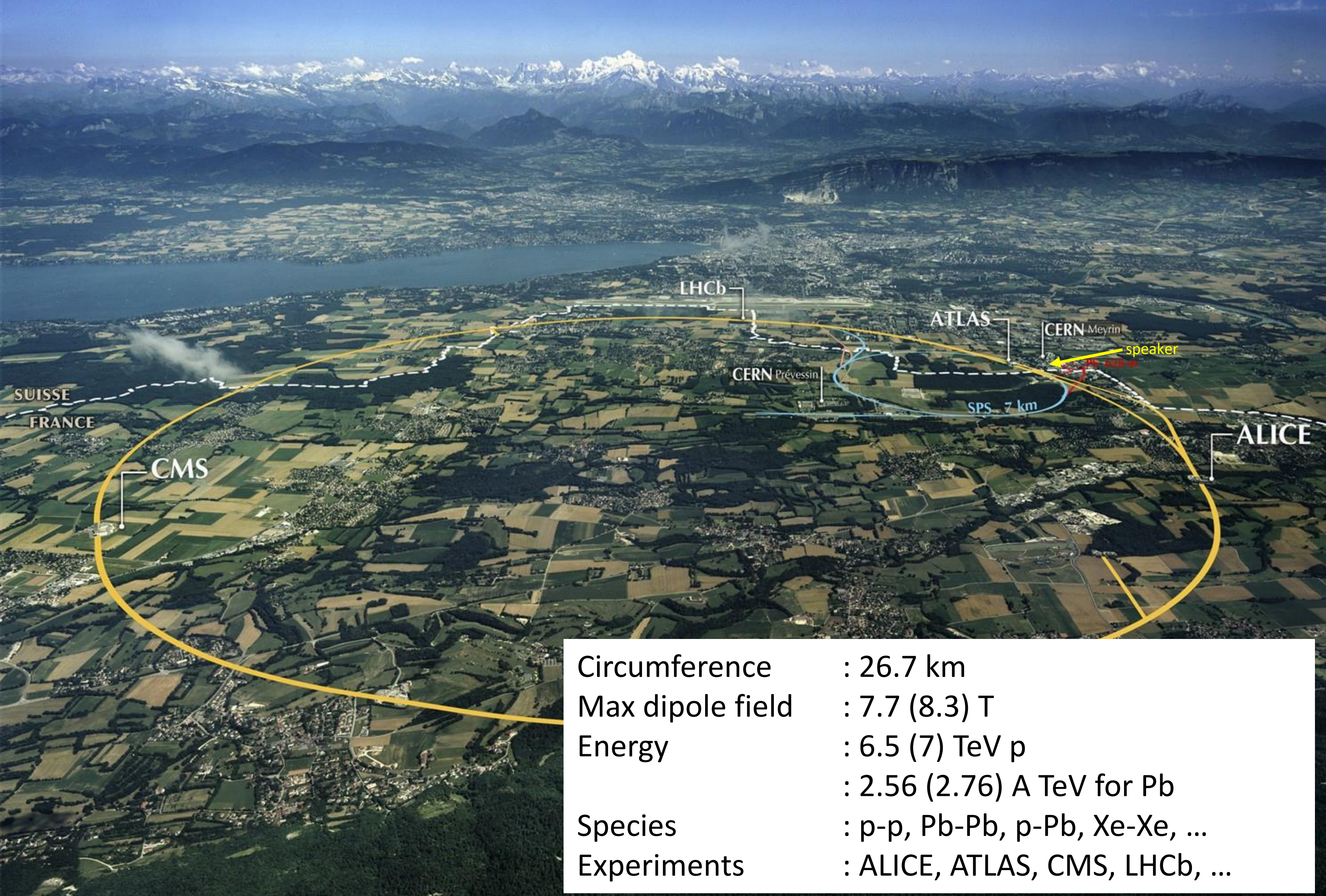
- Accelerator physics (and technology) is a field that has made spectacular progress for nearly a century
  - Growth in energy >> growth in size or cost
- It has often been said that this progress has reached its limits
- The times scales for frontier energy projects are certainly becoming longer
  - We look forward to ILC ? FCC-ee ? CepC ? FCC-hh? Muon collider? Plasma accelerators ?
- Nuclear physicists are lucky to be living in an exciting time with the LHC in mid-programme and RHIC still running but due to be reborn, quite soon, as the Electron-Ion Collider
- In these lectures, I will mainly use LHC examples.

Relativistic Heavy Ion Collider – main parameters



Circumference	: 3.8 km
Max dipole field	: 3.5 T
Energy	: 255 GeV p
	: 100 GeV/nucleon Au
Species	: p↑ to U (incl. asymmetric)
Experiments	: STAR, PHENIX (→ sPHENIX)

LHC at CERN, Geneva area



Circumference	: 26.7 km
Max dipole field	: 7.7 (8.3) T
Energy	: 6.5 (7) TeV p
	: 2.56 (2.76) A TeV for Pb
Species	: p-p, Pb-Pb, p-Pb, Xe-Xe, …
Experiments	: ALICE, ATLAS, CMS, LHCb, …

Accelerator physics, or the physics of particle beams, is a microcosm of the whole of physics

- Probably the most advanced applications of relativistic classical Hamiltonian dynamics of single particles (Floquet theory in an especially elegant formalism, non-linearities, long-term stability, …)
  - The formalism is a fundamental tool for design of circular accelerators
- Electromagnetism – obviously, in many ways
- Collective behaviour of many charged particles – intense beams interact electromagnetically with themselves (“space charge”), through their surroundings (impedances, collective instabilities)
- Beams collide and interact with each other (“beam-beam” effects)
- Statistical mechanics of beam distributions (eg, Bjorken-Mtingwa theory of intra-beam scattering)
- Quantum physics (radiating electron beams, …)
- Plasma physics (plasma wake-field accelerators)
- Superconductivity – magnets, RF accelerating systems
- Nuclear physics – performance limits with ultrarelativistic heavy ion beams

Accelerator physics: big field, many types, applications

- Our main tool to access the subatomic world
  - Concentrate energy in tiny volumes
- Applications of accelerators
  - Elementary particle physics (colliders, fixed-target)
  - Nuclear physics (ditto)
  - Synchrotron light sources (many applications in material science, medicine, archeology, lithography, chemistry, …)
  - Neutron sources for applied science
  - Medicine (diagnostics, cancer therapy, radioisotopes, …)
  - Food processing
- Technological spin-off from nuclear/particle physics to other fields

I will not discuss accelerator technology.

Outline

- Some keys to understanding discussions related to the LHC collider – fast track explanations
- Lecture 1:
  - Introduction
  - Transverse dynamics: mathematics of the strong-focusing principle, beam optics, emittance, illustrated with the LHC, particularly the ALICE interaction region
    - *Certainly the most important principle to understand*
- Lecture 2:
  - Nuclear beams, luminosity, energy
  - Limits to performance in heavy-ion collisions
  - Filling schemes
  - Evolution so far and outlook for LHC

Learning more - schools

- Joint Universities Accelerator School (Europe)
  - http://juas.eu
  - In depth courses at Archamps (near CERN), in normal times
    - Course 1: The science of particle accelerators (10 Jan-11 Feb)
    - Course 2: The technology and applications of particle accelerators (14 Feb-18 Mar)
  - Register now for online courses in early 2022 !!
    - (No need to travel …)
- CERN Accelerator School
  - https://cas.web.cern.ch/
    - Shorter courses, proceedings available for free
- US Particle Accelerator School
  - https://uspas.fnal.gov/

Textbooks, a few favourites

- D. Edwards, M. Syphers, An Introduction to the Physics of High Energy Accelerators | Wiley Online Books
  - Popular clear introduction
- S Y Lee, Accelerator Physics
  - Excellent advanced book , not so much about colliders (level comparable to Jackson’s Classical Electrodynamics)
- H Wiedemann, Particle Accelerator Physics
  - Very comprehensive text, not so much about colliders
- A Chao, M. Tigner (Eds), Handbook of Accelerator Physics and Engineering
  - Comprehensive but compact *vade mecum* (not a text), new edition in preparation
- There are now many others …

TRANSVERSE DYNAMICS OF SINGLE PARTICLES

The need for focusing

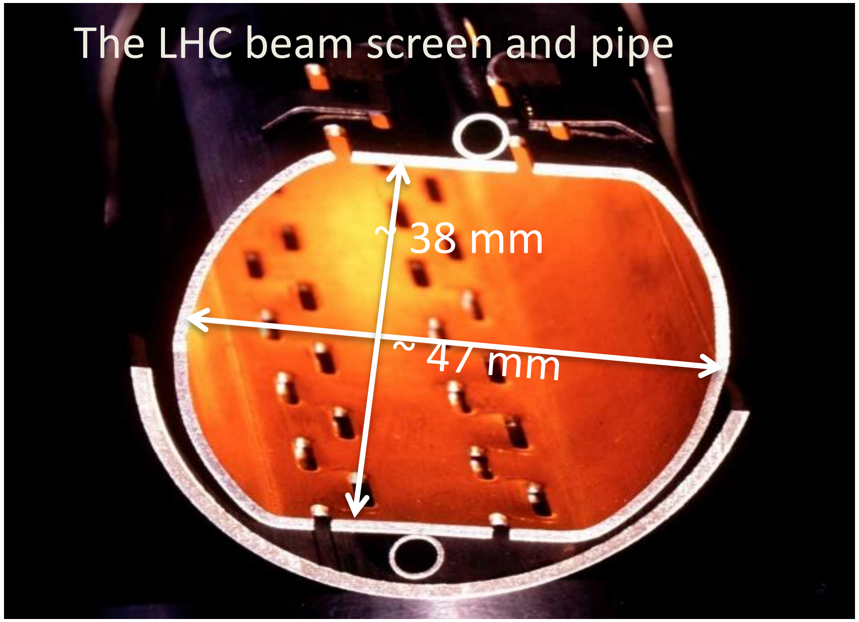
- Extremely naïve reasoning based on relation between bending field and momentum of a particle

$$p = eB\rho$$

$$\left[\frac{p}{\mathrm{GeV}/c}\right] = 0.2998\left[\frac{B}{\mathrm{T}}\right]\left[\frac{\rho}{\mathrm{m}}\right] \quad \Rightarrow \quad \frac{\Delta p}{p} = \frac{\Delta\rho}{\rho}$$

$$\sigma_\delta = \sqrt{\left\langle\left(\frac{p-p_0}{p_0}\right)^2\right\rangle} \approx 10^{-4} - 10^{-3}$$

$$\frac{\Delta\rho}{\rho} \approx 0.1\ \% \quad \text{for LHC} \quad \Delta\rho \approx 3\ \mathrm{m}$$



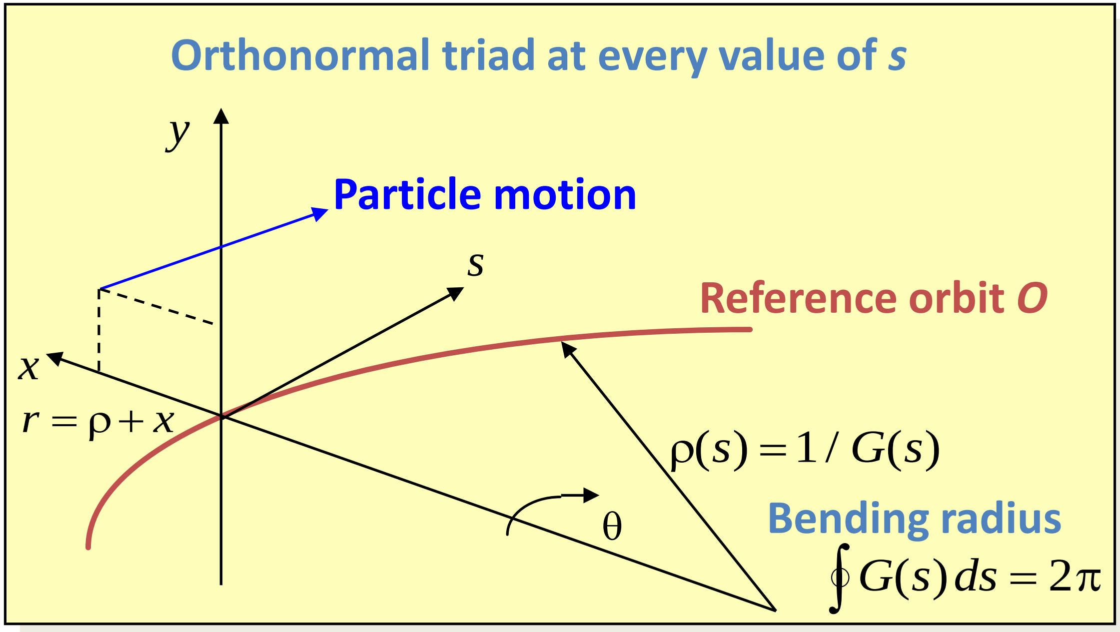
⇒ orbits accommodating the momentum spread in LHC would be spread over several metres of machine radius

⇒ Beam pipe/vacuum chamber bigger than tunnel!

⇒ Somebody must have had a better idea ....

Accelerator coordinates and dynamical variables

- Reference particle, momentum p_0, follows an ideal *reference orbit O*
- Azimuthal coordinate *s* (usually) plays role of time (independent variable) in accelerator dynamics.
  - Time *t* becomes the coordinate for the third degree of freedom (different particles pass *s* at different times). Usually use time-delay w.r.t. reference particle.
- Particles move in a neighbourhood of *O* (ideally a curve passing through centres of all magnets)

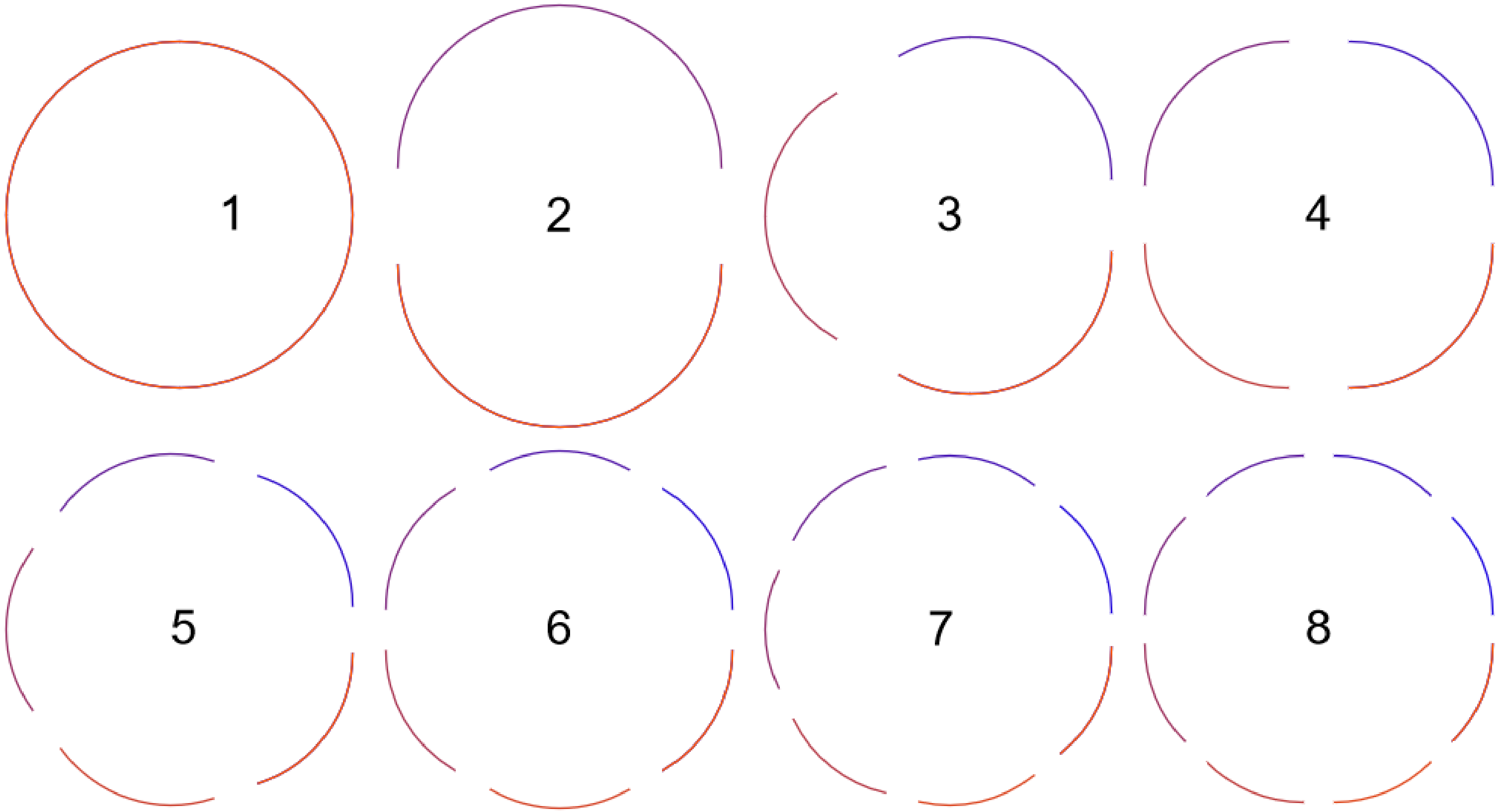


Each of the coordinates *(x,y,ct)* has a conjugate momentum variable

Usually measure in units of nominal momentum, p_0, so these momenta are dimensionless variables.

In these units, p_x, p_y are ~equal to the (small) *angles* of particle trajectory with respect to reference trajectory.

Reference orbit, *O,* need not be a circle



In a "circular" accelerator, *O* is designed to close on itself.

O need not lie in the (horizontal) plane (if the ring is not flat)

Insertions between curved *arcs* (bending magnets, same vertical field B_0) of *O* allow us to insert other things between the bending magnets, e.g.,

- Physics experiments(!), RF, collimation, beam dump, … in long straight sections
- Coordinate system at the interaction point (only) coincides with detector system
- Other kinds of magnets, instrumentation, vacuum pumps, … in many short straight sections (< ~1 m) in the arcs of the ring.

Transverse equations of motion, simplest case

Consider only magnetic fields for now.

Particle of momentum p_0, neighbourhood of *planar O*, passing *s*

$$r = \rho + x$$

$$\mathbf{r} = r\mathbf{e}_x + y\mathbf{e}_y, \quad \text{local direction of ref. orbit is } \mathbf{e}_s$$

$$\frac{d\mathbf{p}}{dt} = e(\mathbf{v}\times\mathbf{B})$$

$$= e\left\{-v_s B_y \mathbf{e}_x + v_s B_x \mathbf{e}_y + \left(v_x B_y - v_y B_x\right)\mathbf{e}_s\right\}$$

Energy constant $\Rightarrow \dfrac{d\mathbf{p}}{dt} = \dfrac{d}{dt}(m\gamma\dot{\mathbf{r}}) = m\gamma\ddot{\mathbf{r}}$, where $\gamma = \sqrt{1+(p/mc)^2}$

Need time derivatives of $\mathbf{r}$ in accelerator coordinates

$$\dot{\mathbf{r}} = \dot{r}\mathbf{e}_x + r\dot{\mathbf{e}}_x + \dot{y}\mathbf{e}_y, \quad \mathbf{e}_y \text{ constant}$$

$$= \dot{r}\mathbf{e}_x + r\dot{\theta}\mathbf{e}_s + \dot{y}\mathbf{e}_y, \qquad \dot{\mathbf{e}}_x = \dot{\theta}\mathbf{e}_s$$

$$\ddot{\mathbf{r}} = \ddot{r}\mathbf{e}_x + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\mathbf{e}_s + r\dot{\theta}\dot{\mathbf{e}}_s + \ddot{y}\mathbf{e}_y, \qquad \dot{\mathbf{e}}_s = -\dot{\theta}\mathbf{e}_x$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_x + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\mathbf{e}_s + \ddot{y}\mathbf{e}_y$$

Take x – component of equation of motion:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{e v_s}{m\gamma} B_y = -\frac{v_s^2}{(p/e)} B_y,$$

where $(p_0/e) = B_0\rho$ is magnetic rigidity

Transform to s as independent variable

$$\frac{ds}{dt} = v_s \frac{\rho}{r} \quad \Rightarrow \quad \frac{dt}{ds} = \frac{1}{v_s}\left(1 + \frac{x}{\rho}\right) = \frac{1}{v_s}(1 + Gx)$$

$$\frac{d^2x}{dt^2} = \frac{d^2x}{ds^2}\left(\frac{ds}{dt}\right)^2 + \frac{dx}{ds}\frac{d^2s}{dt^2}$$

Equation of motion in horizontal plane:

$$\frac{d^2x}{ds^2} - \frac{(\rho + x)}{\rho^2} = -\frac{B_y}{(p/e)}\left(1 + \frac{x}{\rho}\right)^2$$

Similarly for vertical motion

$$\frac{d^2y}{ds^2} = +\frac{B_x}{(p/e)}\left(1 + \frac{x}{\rho}\right)^2$$

N.B. The magnetic field components are themselves functions of *x,y,s* (field maps, multipole expansions)

Equations are non-linear in general.

Note sign difference on RHS of equations in *x* and *y*

In general, the change of independent variable from time, *t,* to the arc-length, *s,* is a special procedure in Hamiltonian mechanics in which the Hamiltonian function changes from the energy to *minus the longitudinal component of the momentum.*
See textbooks (e.g. Lee).

focusing fields

Transverse static magnetic field, the basis of the focussing structure in most particle accelerators. Expand fields around the reference orbit O:

$$\nabla \times \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

$$\mathbf{B}(x,y) = B_x(x,y)\mathbf{e}_x + B_y(x,y)\mathbf{e}_y$$

$$= \Bigg[\underbrace{B_x(0,0)}_{=0\ \text{(planar)}} + \underbrace{\frac{\partial B_x(0,0)}{\partial y}}_{\text{equal}} y + \frac{\partial B_x(0,0)}{\partial x} x + \cdots\Bigg]\mathbf{e}_x$$

$$+ \Bigg[\underbrace{B_y(0,0)}_{\text{dipole magnet}} + \overbrace{\underbrace{\frac{\partial B_y(0,0)}{\partial x}}_{\text{normal quadrupole}}}^{\downarrow} x + \underbrace{\frac{\partial B_y(0,0)}{\partial y}}_{\text{skew quadrupole}} y + \cdots\Bigg]\mathbf{e}_y$$

Skew quads $\Rightarrow$ x-y coupling, assume only normal quads.

Ignoring these and higher multipole fields for now but they are important!

First-order, uncoupled equations of motion :

$$\frac{d^2x}{ds^2} + \left[\frac{1}{\rho^2} + \frac{1}{(p/e)}\frac{\partial B_y(s)}{\partial x}\right]x = 0$$

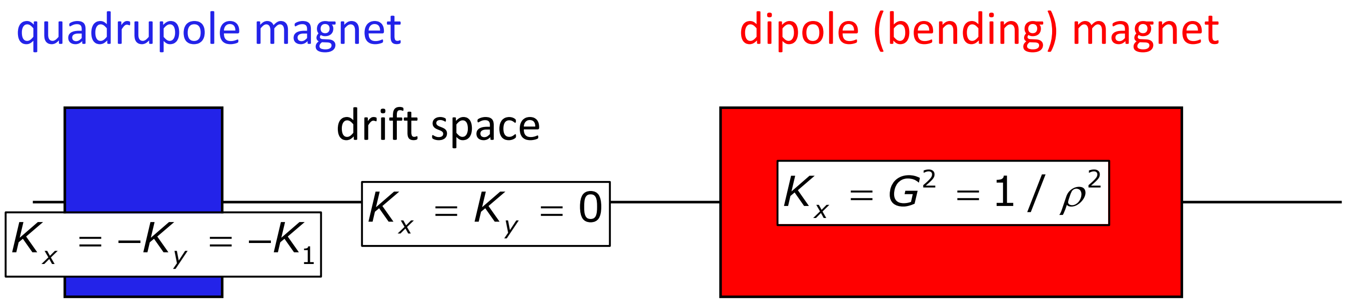
$$\frac{d^2y}{ds^2} - \left[\frac{1}{(p/e)}\frac{\partial B_y(s)}{\partial x}\right]y = 0$$

Hill equations with s-dependent, periodic force terms

$$x'' + K(s)x = 0, \quad K(s+C) = K(s), \text{ where } C \text{ is the circumference of } O.$$

Matrix solution, the linear part of an element map

In practice, the focusing functions $K(s)$ are often piecewise constant (neglecting edge effects)



Propagate vector $\begin{pmatrix} x \\ p_x \end{pmatrix} = \begin{pmatrix} x \\ x' \end{pmatrix}$ through each type of element, of length L, by matrix M (linear mapping).

Drift space:

$$\begin{pmatrix} x \\ p_x \end{pmatrix} \mapsto \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix}$$

F-quadrupole or dipole $(K > 0)$:

$$\begin{pmatrix} x \\ p_x \end{pmatrix} \mapsto \begin{pmatrix} \cos\left(\sqrt{|K|}L\right) & \frac{1}{\sqrt{K}} \sin\left(\sqrt{|K|}L\right) \\ \sqrt{K} \sin\left(\sqrt{|K|}L\right) & \cos\left(\sqrt{|K|}L\right) \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix}$$

D-quadrupole $(K < 0)$:

$$\begin{pmatrix} x \\ p_x \end{pmatrix} \mapsto \begin{pmatrix} \cosh\left(\sqrt{|K|}L\right) & \frac{1}{\sqrt{K}} \sinh\left(\sqrt{|K|}L\right) \\ \sqrt{K} \sinh\left(\sqrt{|K|}L\right) & \cosh\left(\sqrt{|K|}L\right) \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix}$$

Convention:
An F quadrupole focuses in *x* defocuses in *y*.
A D quadrupole defocuses in *x* and focuses in *y*.

How can we focus in both planes with the same magnets?

Extension to many elements

Build up the map for the lattice (sequence of elements) of a circular accelerator by taking composition of maps (product of matrices in linear approx.) for all elements:

$$M = M_n \cdots M_2 M_1$$

Note that each map M_i is area-preserving: $\det M_i = 1 \quad \Rightarrow \quad \det M = 1$

More generally, non-linear maps are symplectic because the system is Hamiltonian.

The determinant of M is related to its eigenvalues; in one degree of freedom (our x):

$$\det M = \lambda_1 \lambda_2 \quad \Rightarrow \quad \lambda_2 = \frac{1}{\lambda_1}$$

The 2×2 matrices for $\begin{pmatrix} x \\ p_x \end{pmatrix}$ generalise to 6×6 matrices for (x, p_x, y, p_y, z, p_z)

which have to be 2×2-block diagnolasied to find the eigenmodes of the motion.

We use specialised computer programs (e.g., MADX in the LHC community) to compute large and complex accelerator structures.
Matrix formalism for the linear part is fundamental in practical design work.
Extension to non-linear dynamics is done using Hamiltonian perturbation theory, Lie-algebraic operator formalisms and direct numerical tracking of particles.

Stability of transverse motion

Require that the motion of all particles remains bounded after many turns around the ring.

Any $\mathbf{x}=\begin{pmatrix} x \\ p_x \end{pmatrix}$ can be expressed in terms of eigenvectors

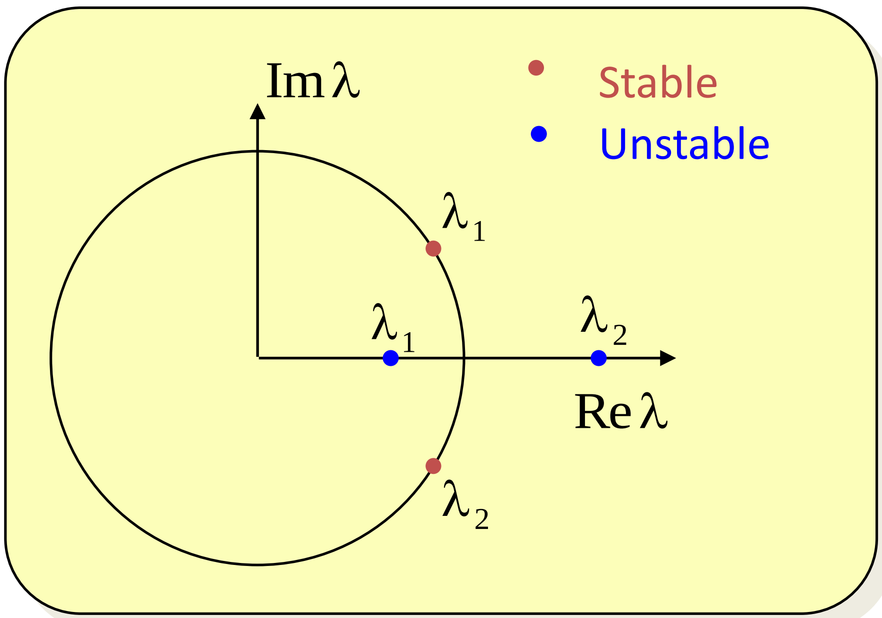
$\mathbf{x}=a_1\mathbf{v}_1 + a_2\mathbf{v}_2$

$$\left.\begin{aligned} M\mathbf{v}_1 &= \lambda_1\mathbf{v}_1 \\ M\mathbf{v}_2 &= \lambda_2\mathbf{v}_2 \end{aligned}\right\} \Rightarrow \quad M^n\mathbf{x} = a_1\lambda_1{}^n\mathbf{v}_1 + a_2\lambda_2{}^n\mathbf{v}_2$$

For stability the eigenvalues of the transfer matrix for a period must lie on the unit circle.

Clearly $|\mathbf{x}| \to \infty$ as $n \to \infty$, unless $|\lambda_1| = |\lambda_2| = 1$,

i.e., $\lambda_1 = e^{i\mu}$, $\lambda_2 = e^{-i\mu}$ for some real μ.



Exercise: Show that this condition is equivalent to

$$-1 \leq \cos\mu = \frac{1}{2}\mathrm{Tr}\,M \leq 1$$

An accelerator lattice may be built up of many identical *periods*. The stability criterion may be applied to each of them or to the whole ring. Since the trace is invariant under cyclic permutation of matrices, the stability criterion is independent of the starting point in the ring.

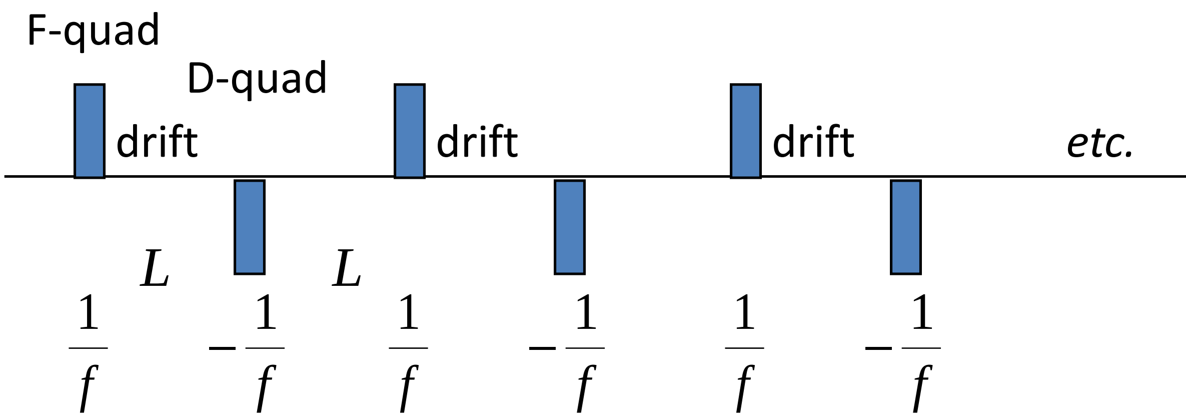
Thin lens approximation and Alternating Gradient Principle

Limit of elements with constant strength KL as $L \to 0$. In this limit $KL \to 1/f$, where f is the focal length of a quadrupole lens.

Exercise: show that in the thin lens approximation the matrices for a focusing and defocusing quadrupole are:

$$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Consider a periodic FODO lattice (with no bends) and alternating focusing and defocusing quads



The matrix for one period is

$$\begin{pmatrix} 1 - \frac{L}{f} - \left(\frac{L}{f}\right)^2 & 2L + \frac{L^2}{f} \\ \frac{-L}{f^2} & 1 + \frac{L}{f} \end{pmatrix}$$

Exercise: For this lattice, show that stability requires

$$f > \frac{L}{2}$$

This is the basic principle of *Alternating Gradient or Strong Focusing,* the foundation of all circular accelerators since the late 1950s.

Optical functions (sometimes called "Twiss" functions)

General solution of Hill equation (see also Floquet's Theorem, etc.)

$$x'' + K(s)x = 0, \quad \text{with } K(s+C) = K(s)$$

Try quasi-harmonic oscillator form:

$$x(s) = \sqrt{2J\beta(s)}\cos\left(\psi(s) + \psi_0\right)$$

Substitution gives

$$x'' + Kx = \sqrt{2J}\left[-\sqrt{\beta}\psi'' - \frac{\beta'\psi'}{\sqrt{\beta}}\right]\sin\left(\psi + \psi_0\right)$$

$$+\sqrt{2J}\left[-\sqrt{\beta}\psi'^2 - \frac{\beta'^2}{4\beta^{3/2}} + \frac{\beta''}{2\sqrt{\beta}} + \sqrt{\beta}K\right]\cos\left(\psi + \psi_0\right)$$

Equate sin and cos terms to zero:

$$\beta\psi'' + \beta'\psi' = \left(\beta\psi'\right)' = 0 \Rightarrow \psi' = \frac{1}{\beta} \quad \Rightarrow \quad \psi(s) = \psi_0 + \int_0^s \frac{d\sigma}{\beta(\sigma)}$$

$$\frac{\beta\beta''}{2} - \frac{\beta'^2}{4} + \beta^2 K = 0 \quad \Leftrightarrow \quad \alpha' = K\beta - \gamma, \ \text{ where } \alpha = -\frac{\beta'}{2}, \ \gamma = \frac{1+\alpha^2}{\beta}$$

Exercise: Check all the steps!
Derive expression for $x'(s)=p_x$

This is the β-function that you will often hear about.

Periodic solutions and the "tune"

Description of *betatron motion* is now formally analogous to harmonic oscillator.
Transfer matrix from point 1 to point 2 can be written as

$$M_{21} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\psi_{21} + \alpha_1 \sin\psi_{21}) & \sqrt{\beta_1\beta_2}\sin\psi_{21} \\ -\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\psi_{21} + \frac{\alpha_1-\alpha_2}{\sqrt{\beta_1\beta_2}}\cos\psi_{21} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\psi_{21} - \alpha_2\sin\psi_{21}) \end{pmatrix}, \quad \psi_{21} = \psi(s_2) - \psi(s_1)$$

Look at physical meaning of factors of β.
Generally, the equations for β can be solved for arbitrary β > 0.

Unique periodic solutions $\beta(s+C) = \beta(s)$ can be found and are of special interest. The one-turn transfer matrix is

$$M = \begin{pmatrix} \cos\psi + \alpha_1\sin\psi & \beta\sin\psi \\ -\gamma\sin\psi & \cos\psi - \alpha\sin\psi \end{pmatrix}$$

Comparison with our earlier expression shows that we can identify the parameter μ with the phase advance around the ring ψ.

The tune is the number of betatron oscillations around the ring

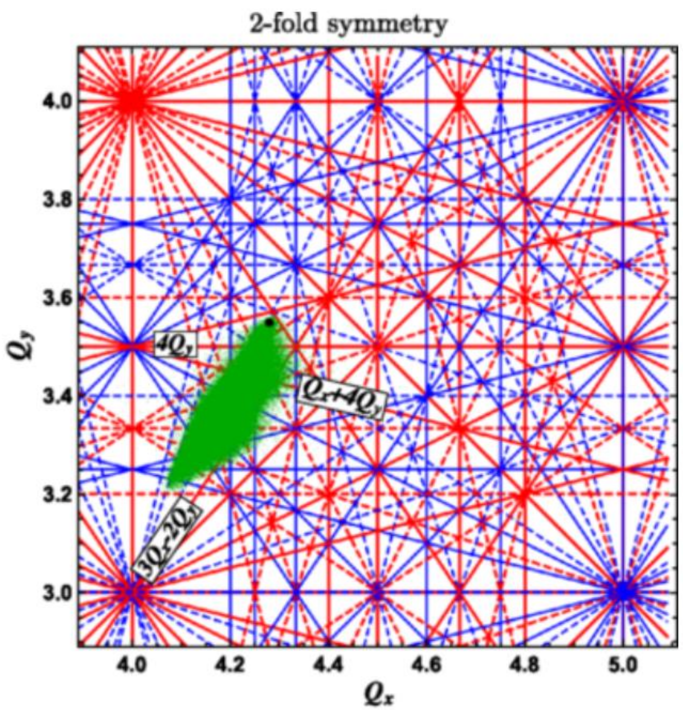
$$Q = \frac{\psi}{2\pi} = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$$

In the LHC collision optics,
$Q_x = 62.30, Q_x = 60.32$

- The instability conditions mentioned above correspond to the tune becoming equal to an integer or half-integer.
- More generally, instabilities or beating of oscillation amplitudes can occur when the tunes satisfy non-linear resonance conditions, driven by higher multipole fields

$$nQ_x + mQ_y = p, \quad n, m, p \in \text{integers}$$

- Spread in tunes in a beam can make this difficult …



A quadrupole perturbation

- Real machines are never perfect as designed.
  - We must consider imperfections as perturbations.
- Consider the insertion of a single thin-lens quadrupole in a ring whose Twiss functions and matrix M_0 *(from the same point) we already know. The new transfer matrix is

$$M = M_0 \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

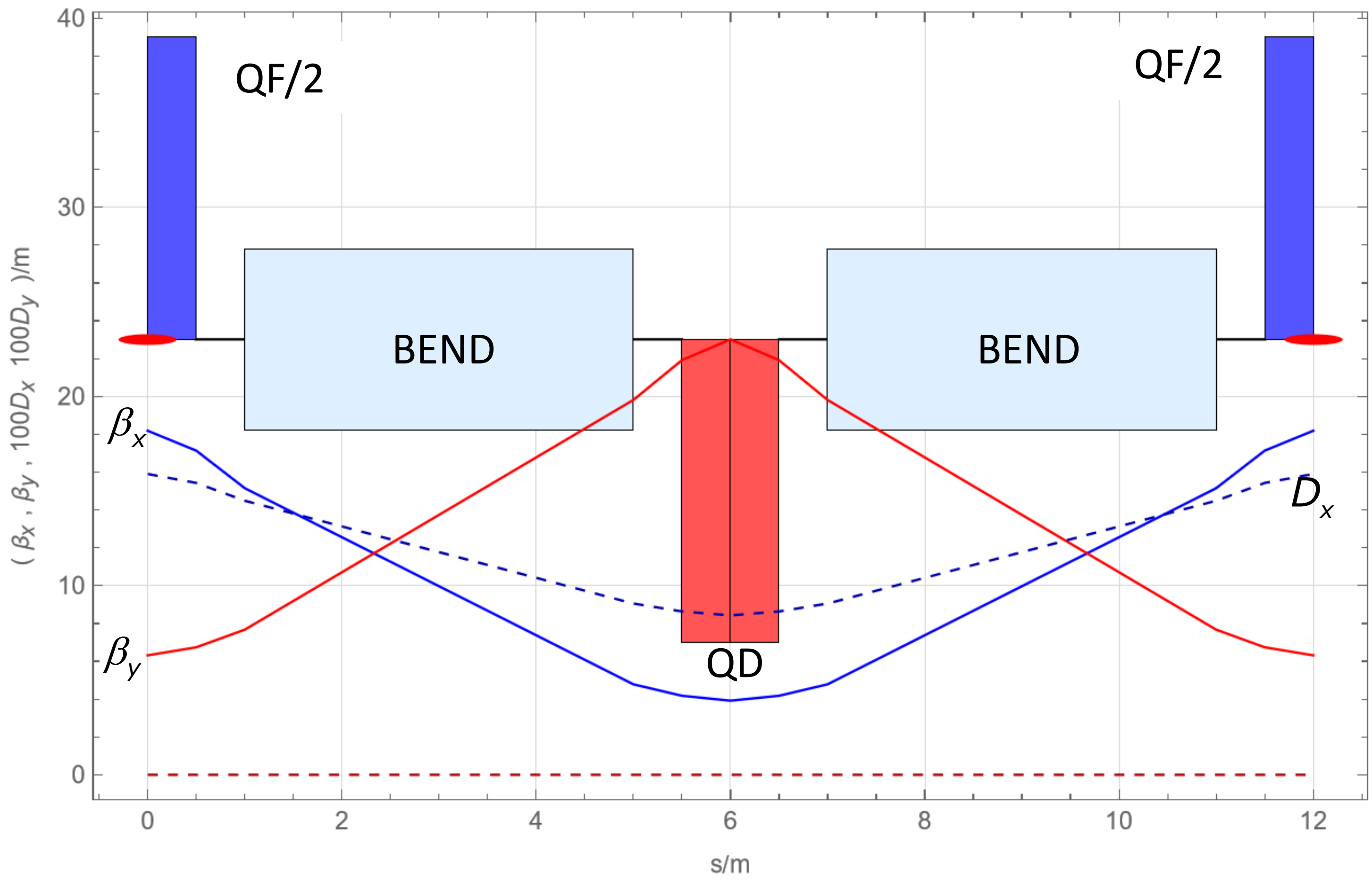
$$\Rightarrow \cos 2\pi Q = \cos 2\pi Q_0 - \frac{\beta_0}{2f} \sin 2\pi Q_0$$

Writing $Q = Q_0 + \Delta Q,$ we find, by expanding that

$$\Delta Q = \frac{\beta_0}{4\pi f}$$

- The β-function measures the sensitivity of the optics of the ring to generic perturbations. We already saw that it provides a measure of the amplitude of oscillations, ie, relative size of the beam in different places.
- Design of accelerator ring optics is usually concerned with providing a suitable form of the β-functions in both planes to achieve design goals.
  - Analytic understanding of special cases is a useful guide.

A FODO cell - basic unit of collider arcs



Not LHC!

Emittance of a beam

- Measure of beam quality in hadron machines
  - Beams have a transverse size and divergence (spread in transverse momenta or angle)

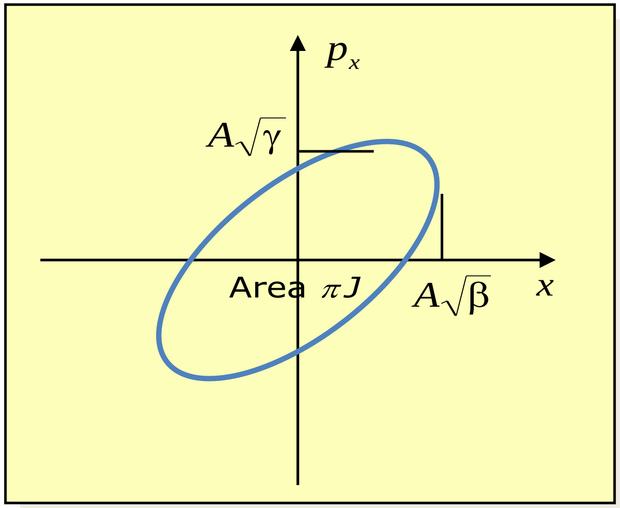
Horizontal emittance defined by

$$\varepsilon_x = \langle J_x \rangle = \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2}$$

$$\approx \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle} \quad \text{(upright beam)}$$

p_x measured in units of the longitudinal momentum p_0

$\langle \cdots \rangle$ denotes average over all particles in beam



  - Alternative definitions in terms of phase space area often include conventional factor π related to area of phase-space ellipse.
  - Analogous definitions for vertical and longitudinal motion
  - Property of a *beam* (many particles), not of a particle (despite what is often written).
  - In a (matched) *linear* focusing system with no coupling to other degrees of freedom, each emittance is preserved as a consequence of Liouville's theorem.

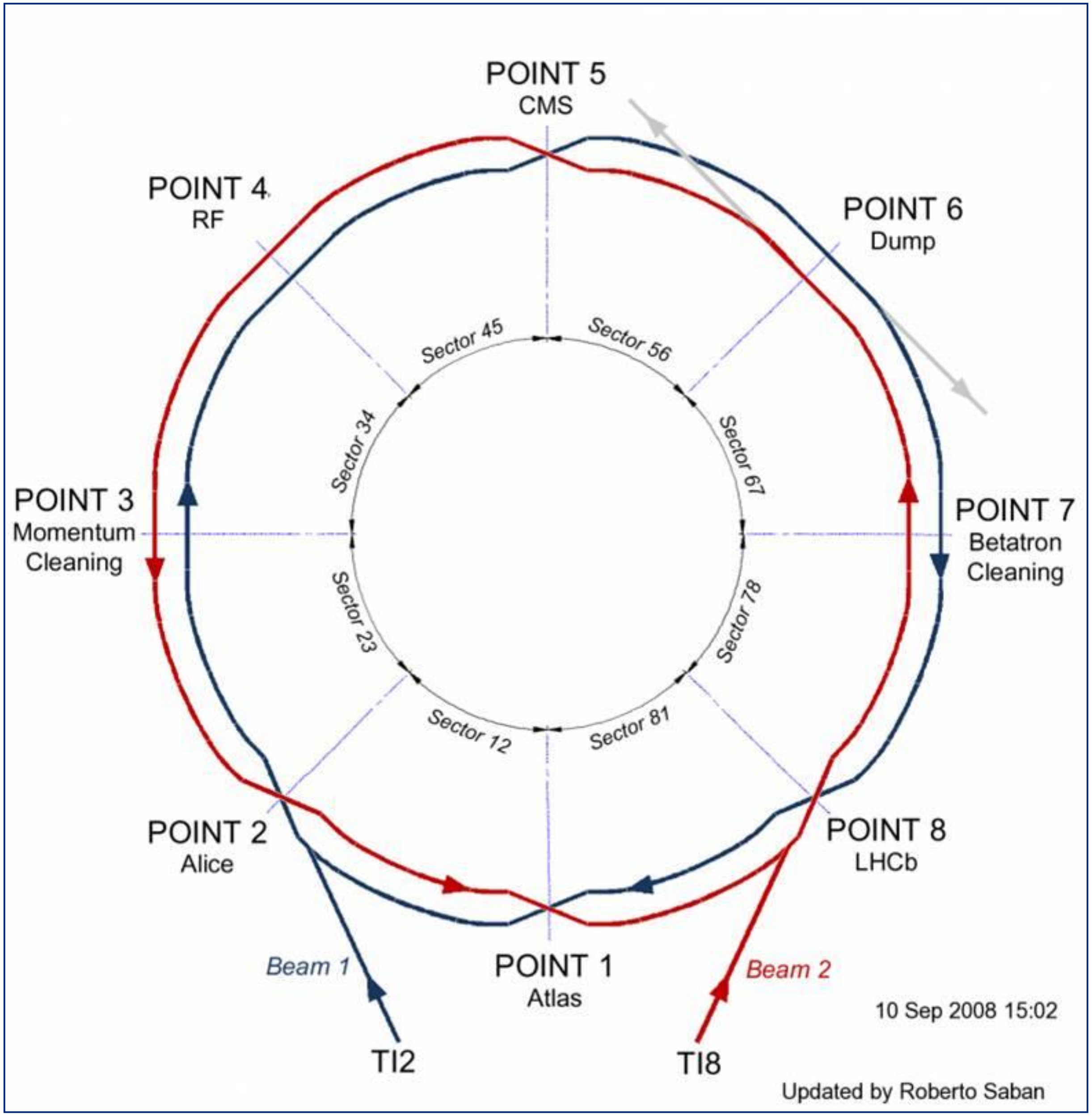
Longitudinal motion - intuitive, without derivations

- What about the other degree of freedom?
- Particles may have another momentum $p = p_0(1+\delta)$
- They are bent less by the magnets and travel on longer paths towards the outside of the ring
- But these paths are modified by the focusing, this effect is described by the *dispersion function* $D_x(s)$

$$x(s) = \sqrt{2J\beta(s)}\cos\left(\psi(s)+\psi_0\right) + D_x(s)\delta$$

- For ultrarelativistic particles, this lengthens the revolution period
- This has to be counteracted by longitudinal focusing: electric fields from RF cavities accelerate or decelerate creating analogous oscillations of energy and lag time around the reference “synchronous” particle.
- These oscillations are much slower, the synchrotron tune Q_s=0.002
- This bunches the beam, in time, within one RF wavelength
- 35640 RF wavelengths (400 MHz) in LHC circumference are “buckets” which can contain particle bunches

LHC orientation - schematic



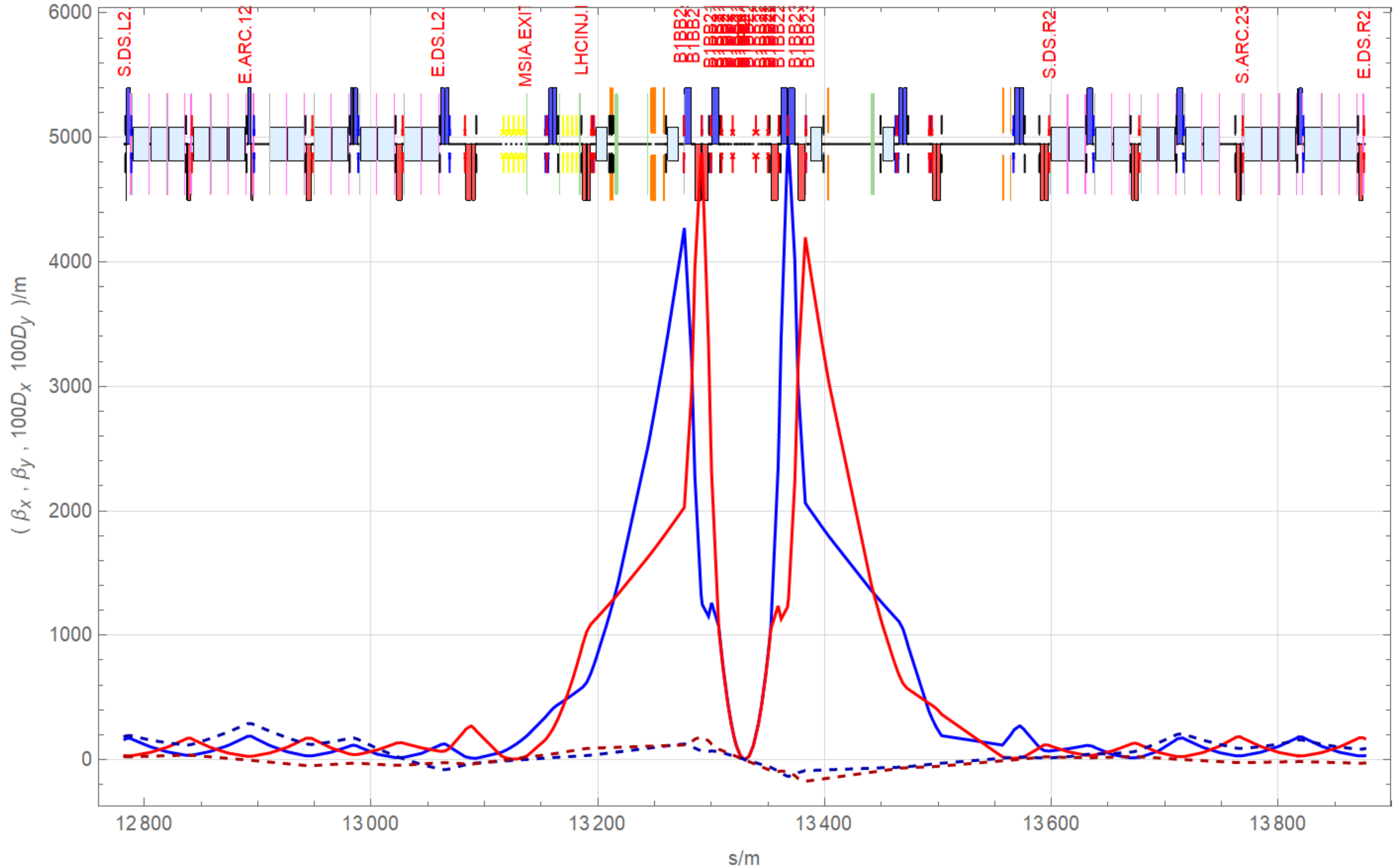
Four large and highly capable heavy-ion physics experiments: ALICE, CMS, ATLAS
LHCb since 2012

Each beam has its own reference orbit in the twin-aperture magnets of the arcs, common in interaction regions.

$s_1(\mathrm{IP1}) = s_2(\mathrm{IP1}) = 0$ (ATLAS) for both beams by convention

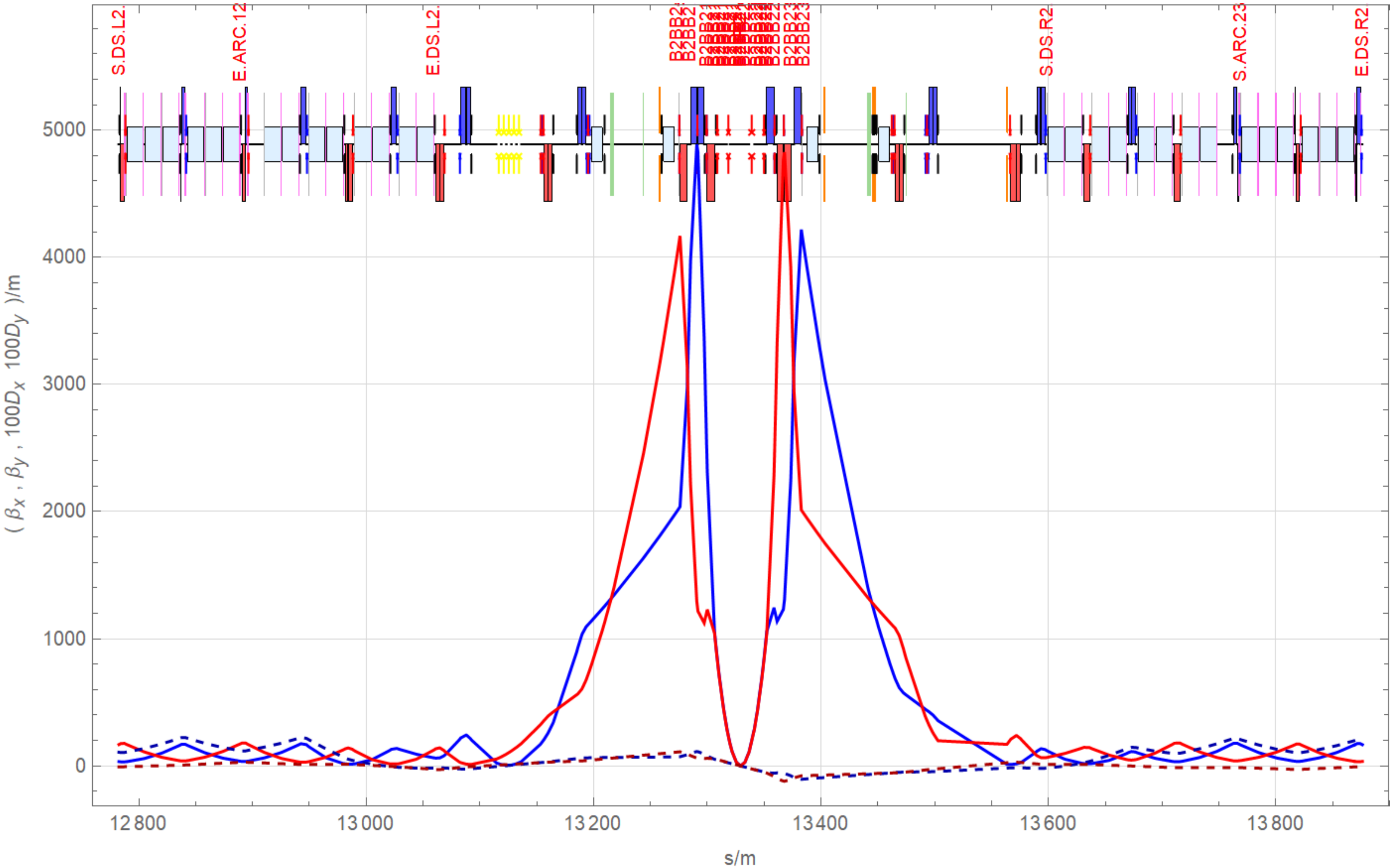
Inner and outer arc lengths are slightly different so
$s_1(\mathrm{IP2}) = 3332.436$ m
$s_2(\mathrm{IP2}) = 3332.284$ m

Optical functions for Beam 1 in LHC IR2, 2018



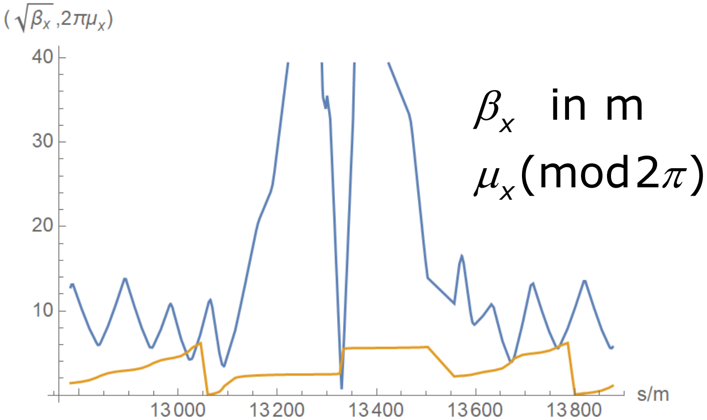
Optics solutions must be found for each beam with these constraints.

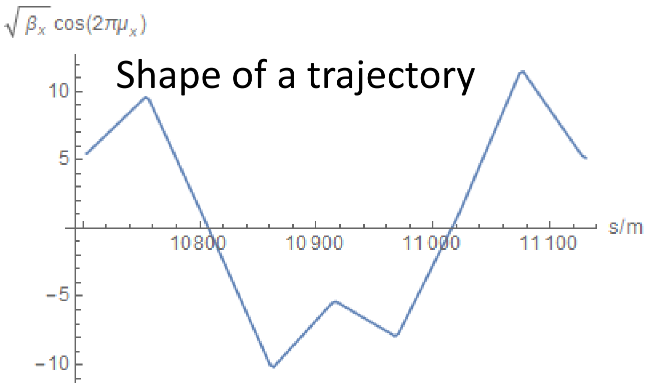
Optical functions or Beam 2 in LHC IR2, 2018

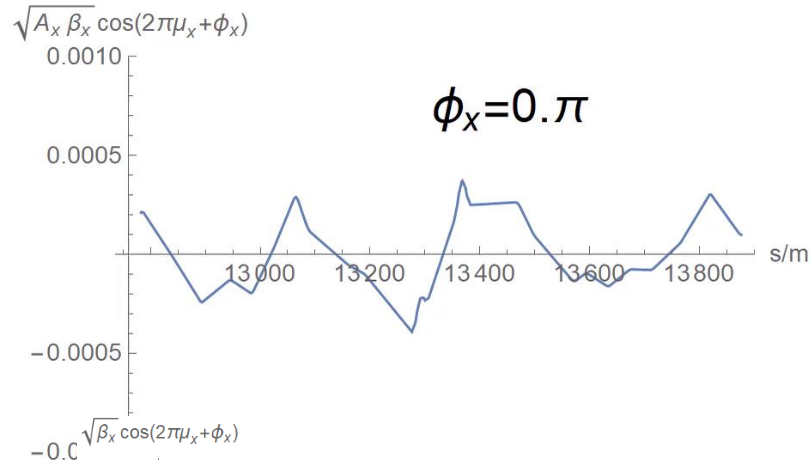


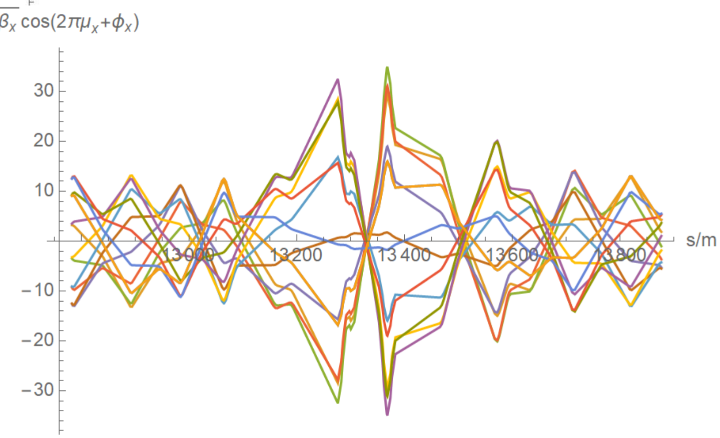
Approximate symmetry with Beam 1 under Left <->Right and x <-> y

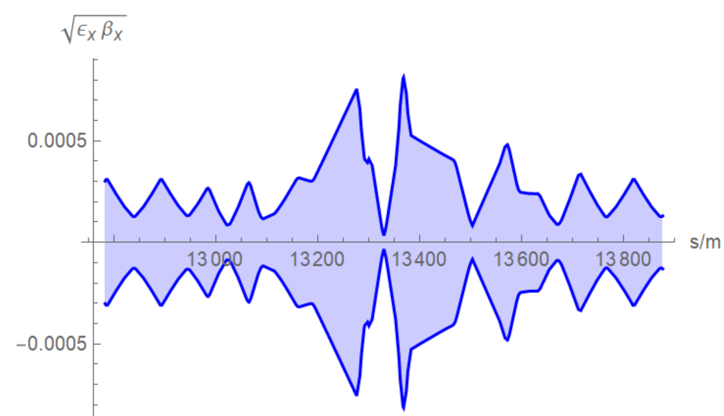
Optical functions and beam envelope in IR2



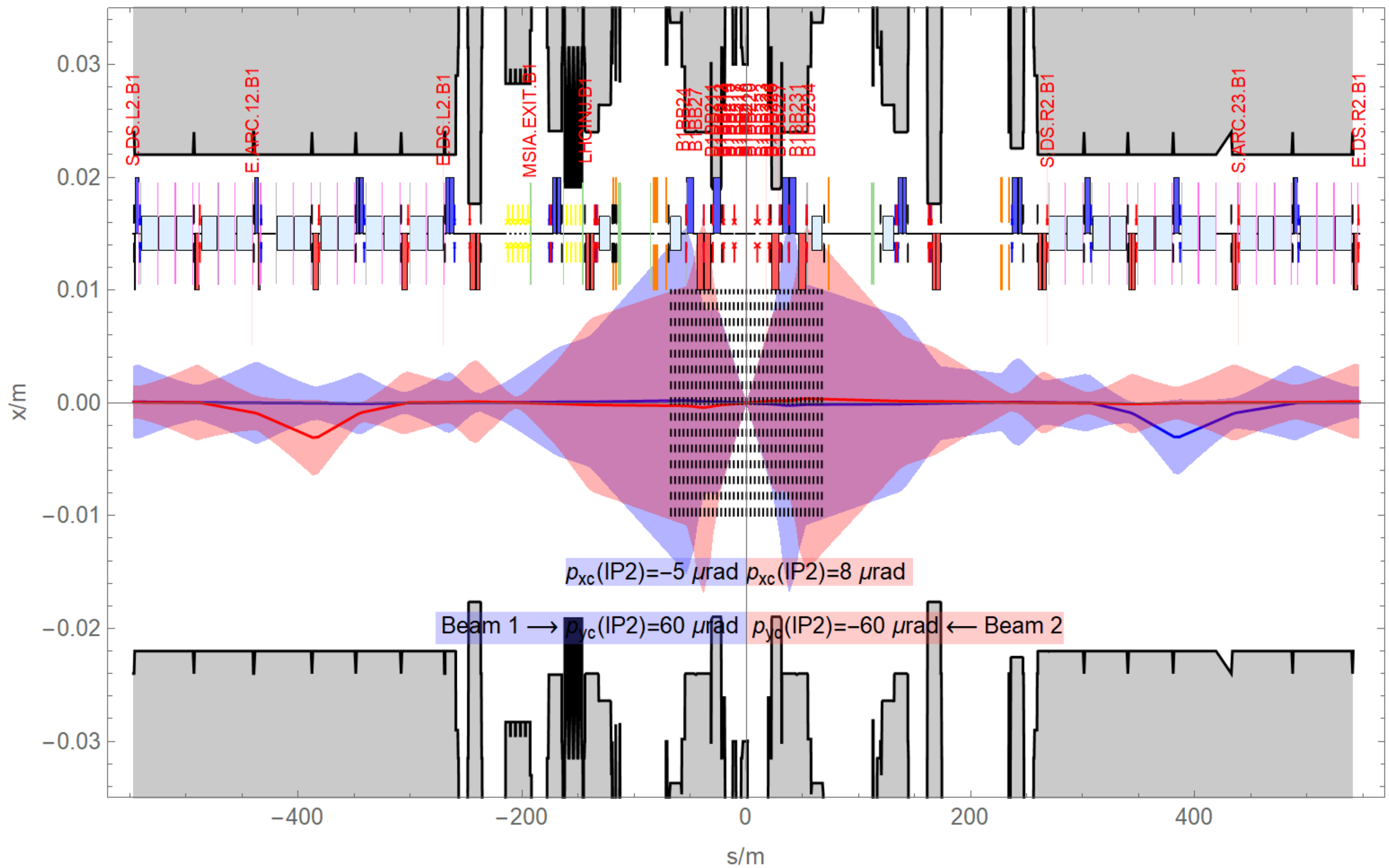






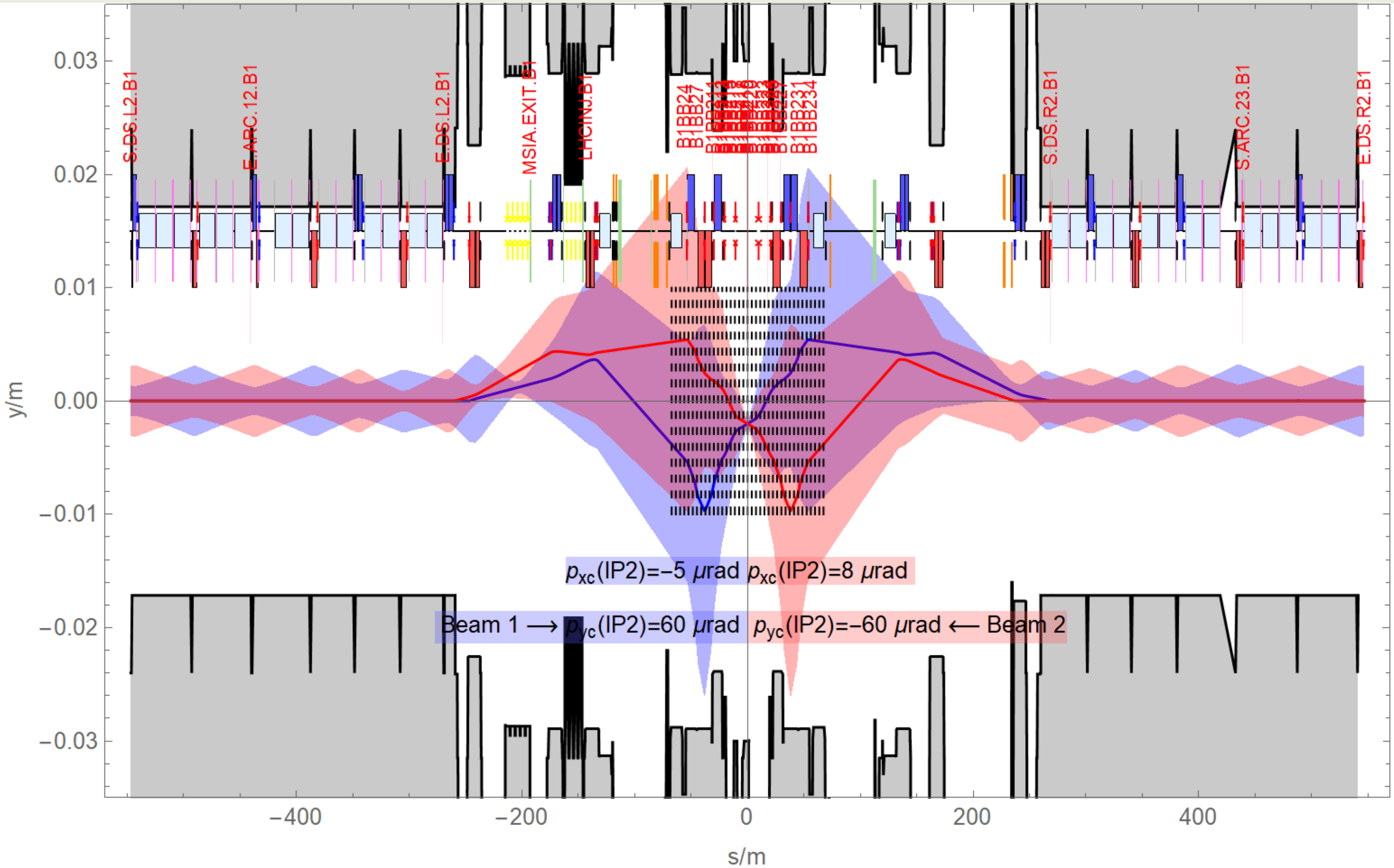


Collision conditions in LHC, IR2 horizontal plane 2018



Aim for small β-functions at IP (called β* by convention).
Gives small beams, higher luminosity and collision rate.
Keep beam envelopes sufficiently well within beam pipe (aperture, shown in grey).

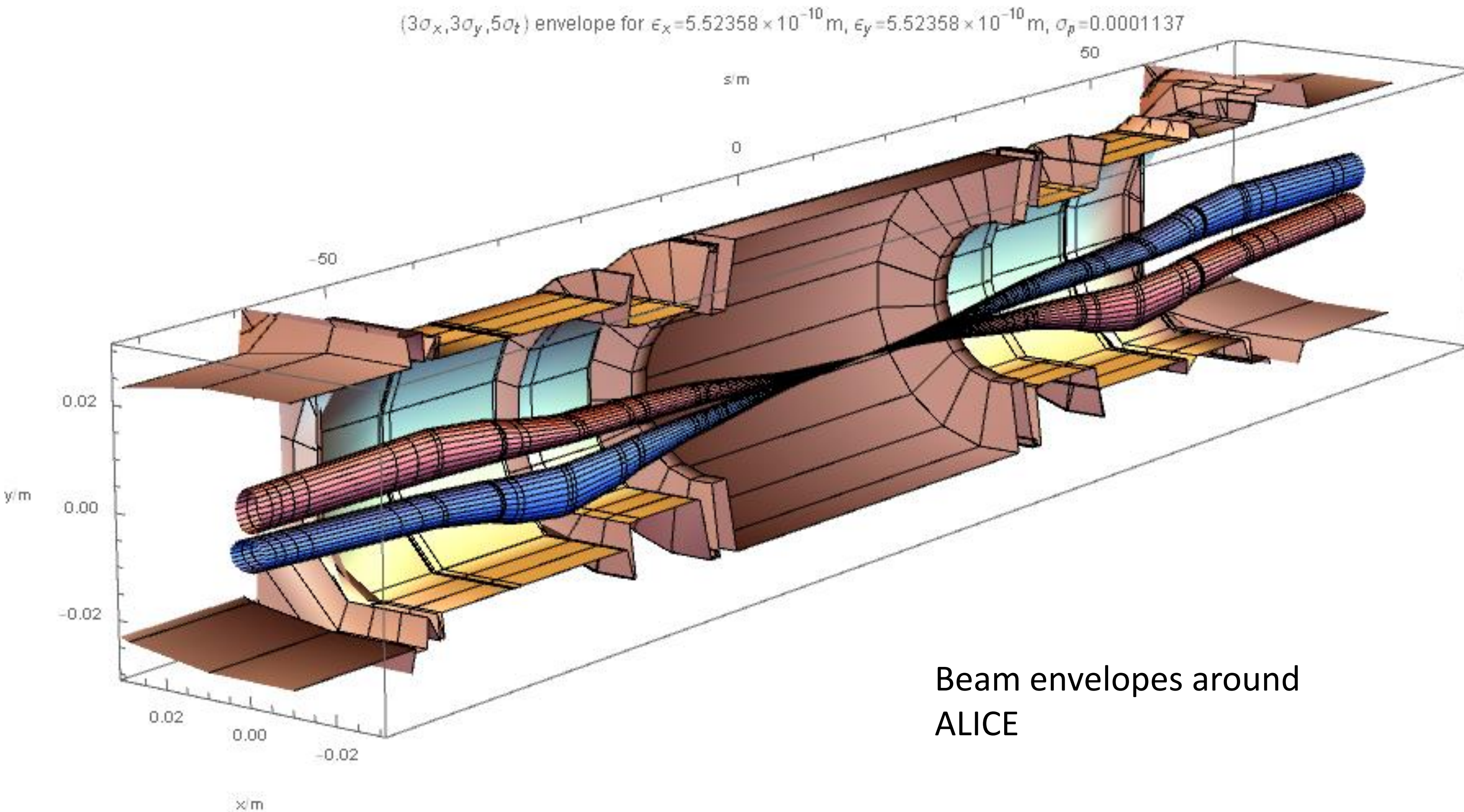
Collision conditions in LHC, IR2 vertical plane 2016



Combination of three orbit bumps (displacement from reference orbit by small dipole magnets called correctors):

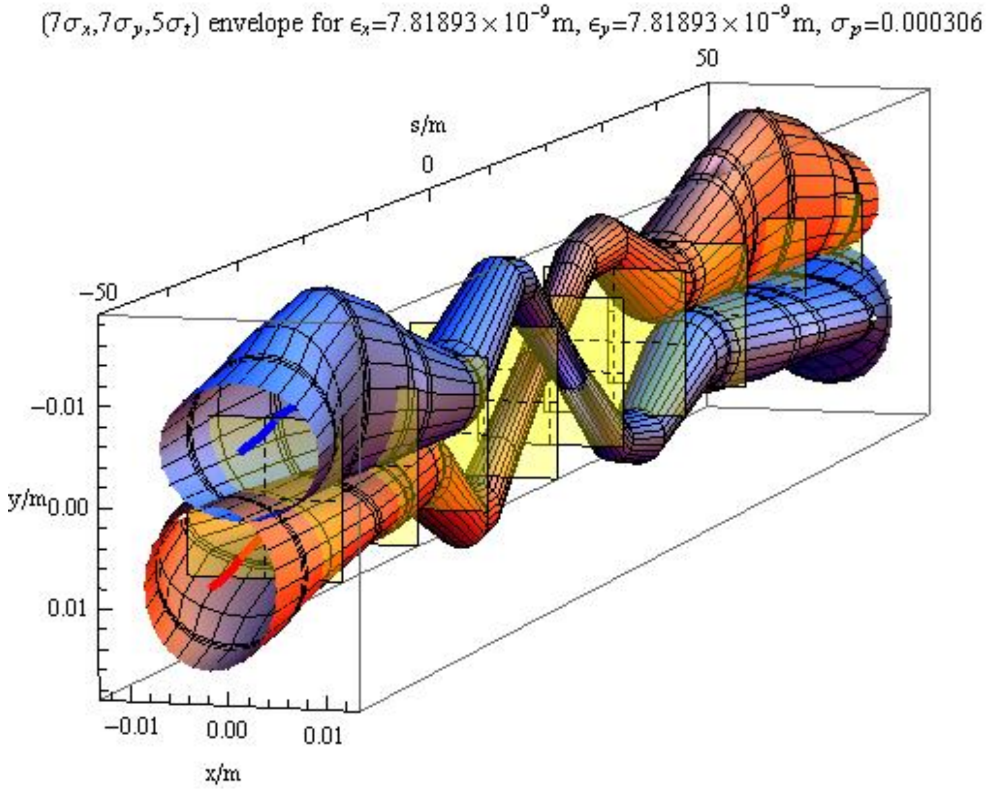
1. Compensate magnetic field of ALICE experiment spectrometer magnet
2. Arrange for vertical crossing angle of beams (avoid unwanted encounters)
3. Lower collision point by 2 mm (the experiment sank …)

Optics for Pb-Pb collisions in ALICE



Beam envelopes around ALICE

Beam envelopes around ALICE at injection

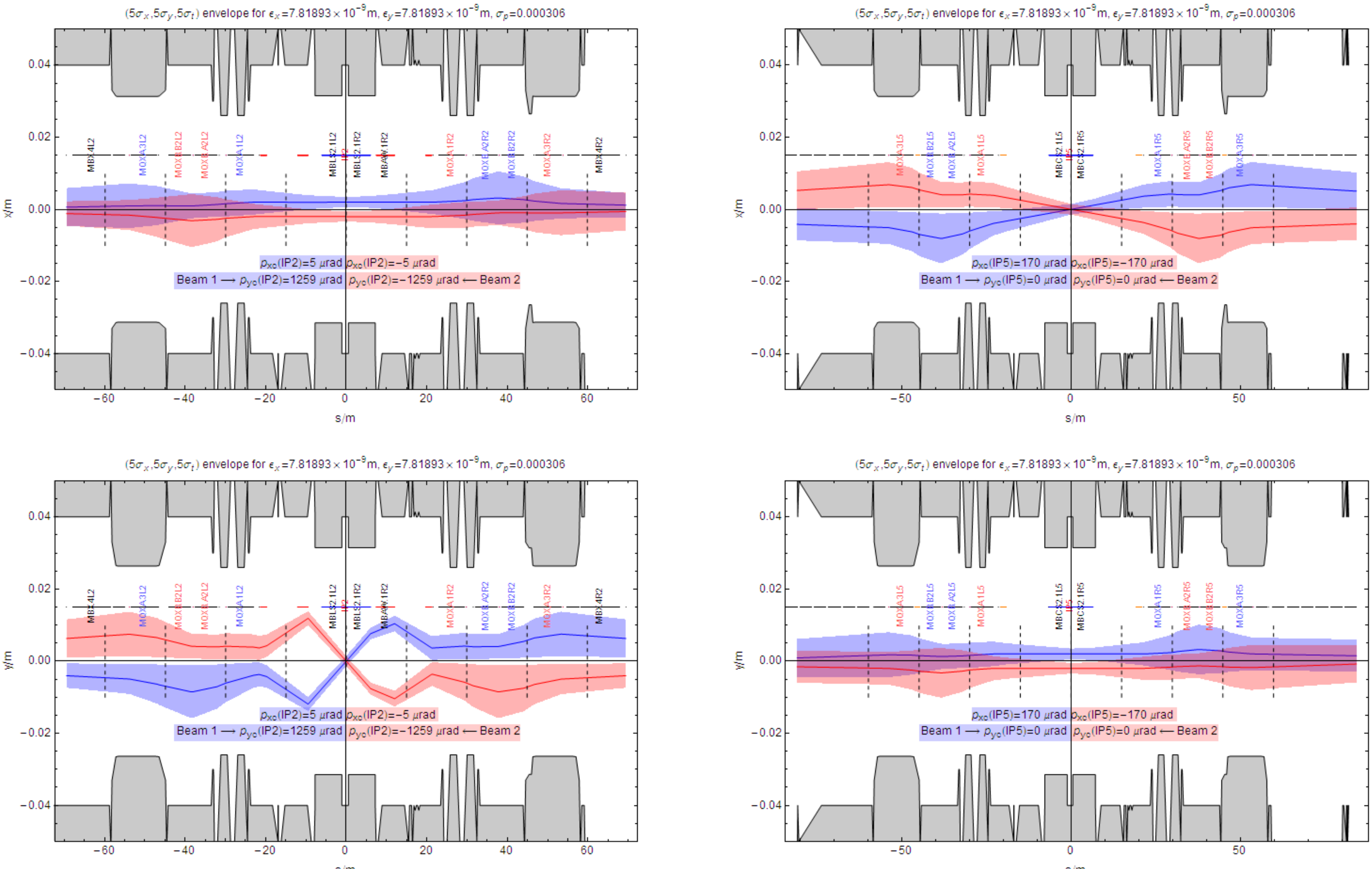


Crossing angle from spectrometer and external bump separates beams vertically everywhere except at IP (also in physics).

Parallel separation also separates beams horizontally to avoid unwanted beam-beam interactions at the IP during injection, ramp, squeeze.

Other experiments have different separation schemes …

ALICE – Separation at injection - CMS



Summary of Lecture 1

- Using the simplest possible, but still meaningful, approximations, we introduced the basic concepts
  - Coordinate system
  - Single particle motion and the optical functions
  - Conditions for stability of motion
  - Perturbations
- Quick intuitive description of longitudinal motion
  - Not harder, just to save time
  - Off-momentum orbit described by dispersion function
- Illustration of the concepts in a real collider, the LHC
  - Example of the ALICE interaction region
  - Beam envelopes