Instrumental Effects and Unfolding Method

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Introduction

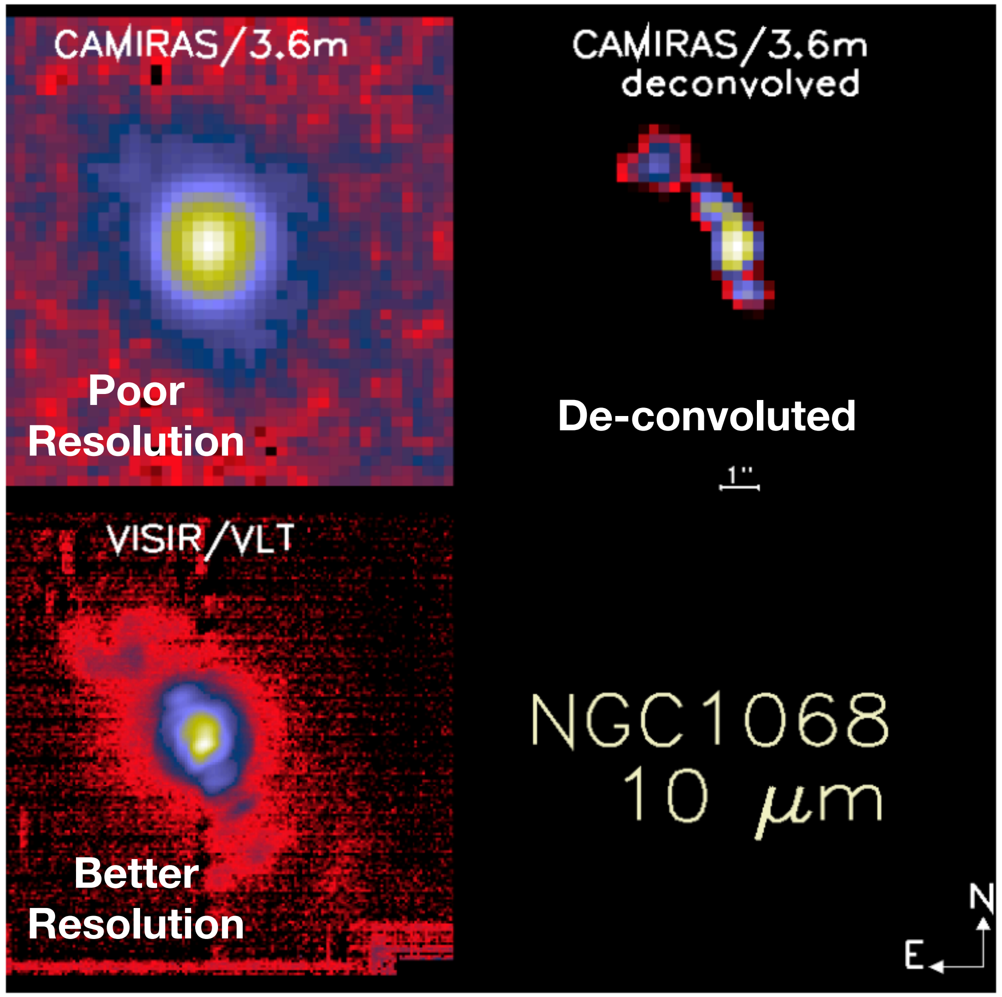
Image blurred due to motion

De-blurred image

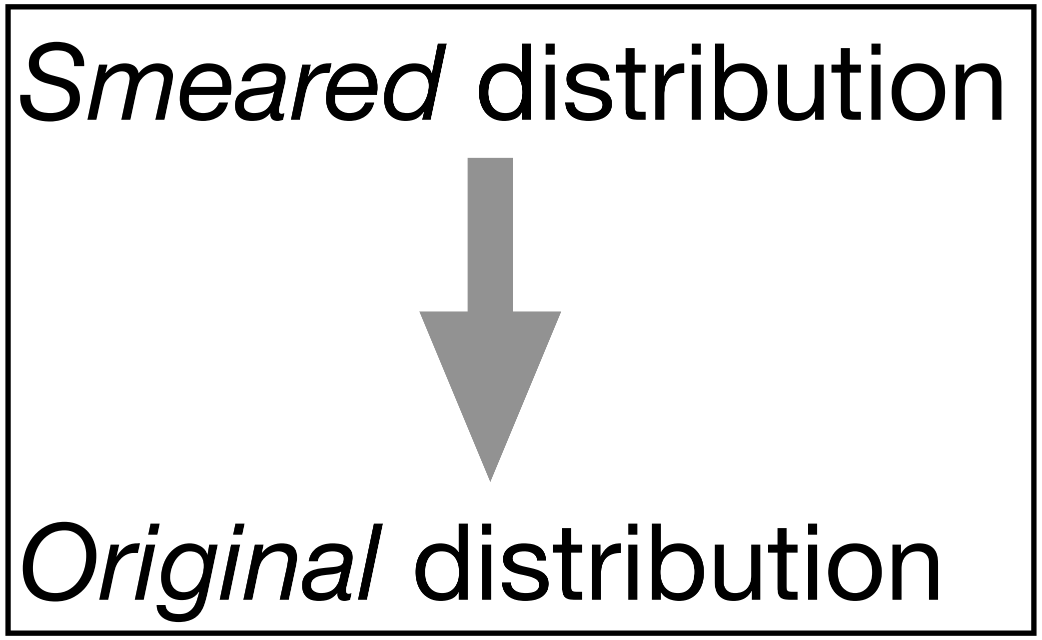
Figure 2: (a) First column: blurred images captured by a hand-held camera. (b)second column: corresponding outputs of our method.

Source:
DOI: 10.1145/1924559.1924595

Introduction



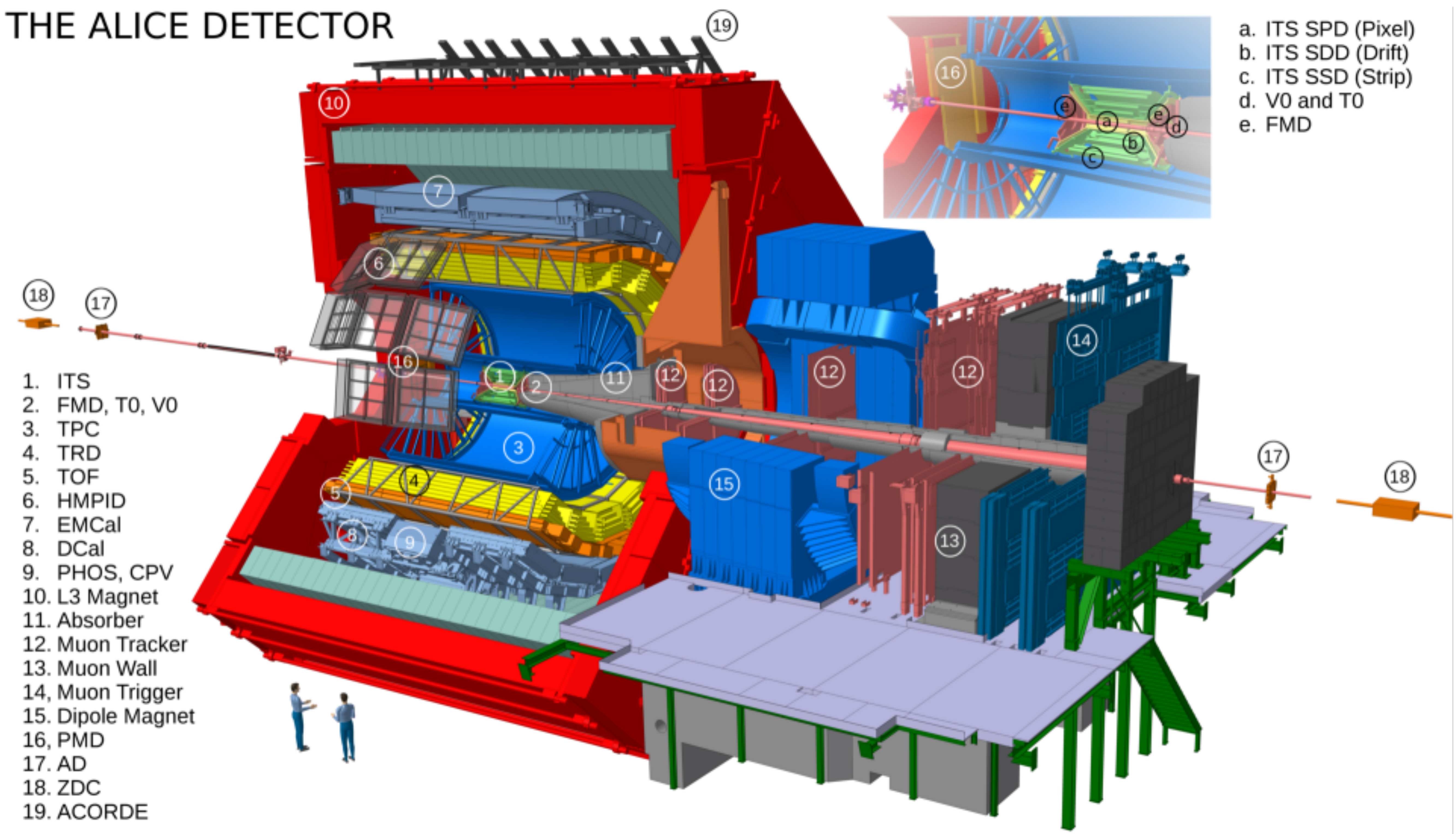
Finite detector resolution causes *smearing*



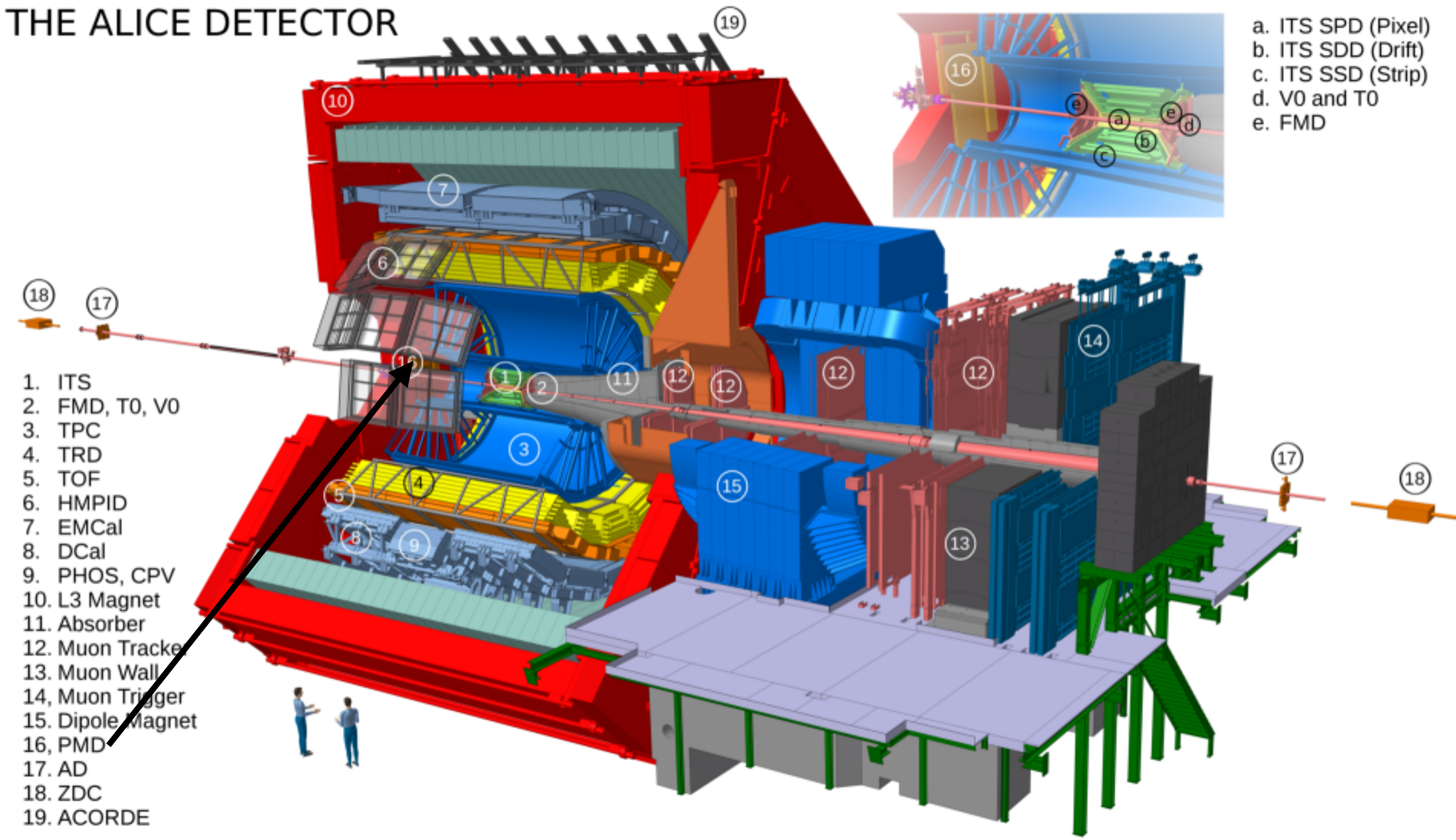
"De-convolution"
Or,
"Unfolding"

Source:
http://jstarck.free.fr/Blind07.pdf

Experimental Apparatus



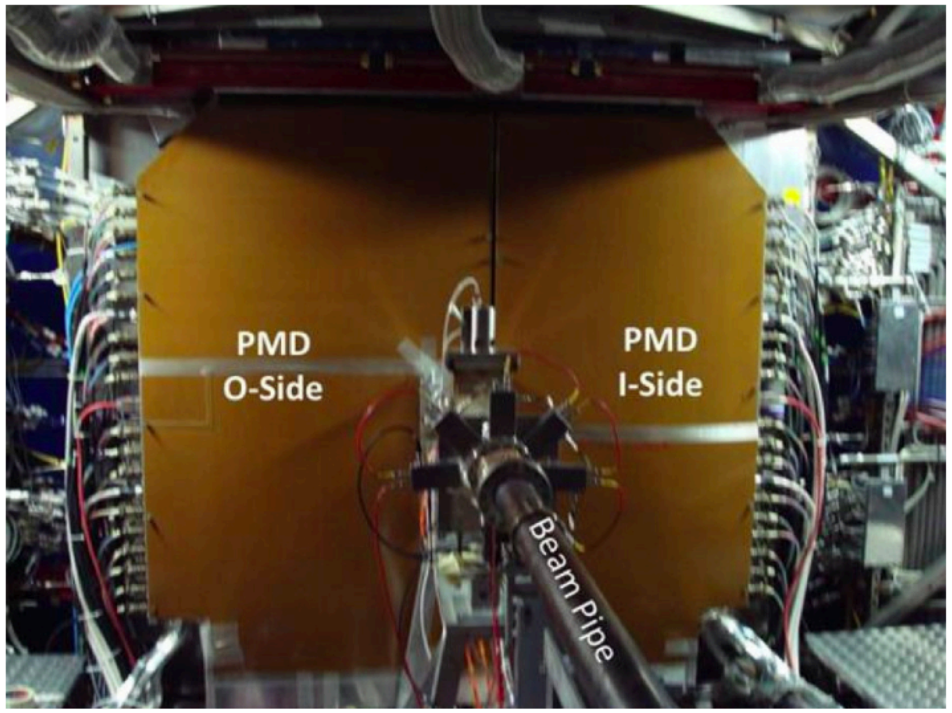
Experimental Apparatus

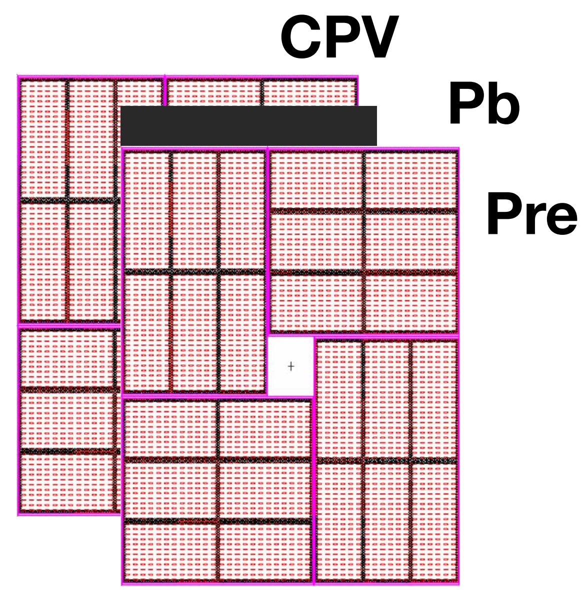


Experimental Apparatus

Detectors	Position (cm)	Acceptance (η)	Acceptance (ϕ)	technology	purpose
SPD (layer1, 2)	3.9, 7.6	± 2, ± 1.4	full	Si Pixel	tracking, vertex
SDD (layer 3, 4)	15.0, 23.9	± 0.9, ± 0.9	full	Si drift	tracking, PID
SSD (layer 5, 6)	38, 43	± 0.97, ± 0.1	full	Si strip	tracking, PID
TPC (IORC, OROC)	85, 247	± 0.9	full	Ne drift, MWPC	tracking, PID
TRD	290, 368	± 0.8	full	TR, Xe drift, MWPC	tracking, $e^{\pm}$ id
TOF	370, 399	± 0.9	full	MRPC	PID
PHOS	460, 478	± 0.12	220, 320	$PbWO_4$	photons
EMCAL	430, 455	± 0.7	80, 187	Pb, scint	photons, jets
HMPID	490	± 0.6	1, 59	$C_6F14, RICH, MWPC$	PID
ACORDE	850	± 1.3	30, 150	scint.	cosmics
FMD	320	$3.6 < \eta < 5.0$	full	Si strip	charged particle
	80	$1.7 < \eta < 3.7$	full	Si strip	
	-70	$-3.4 < \eta < -1.7$	full	Si strip	
PMD	367	$2.3 < \eta < 3.9$	full	Pb+PC	photons
ZDC	± 113 m	$\eta > 8.8$	full	W+quartz	forward neutrons
	± 113 m	$6.5 < \eta < 7.5$	$\phi < 10$	brass, quartz	forward protons
	7.3 m	$4.8 < \eta < 5.7$	$\phi < 32$	Pb, quartz	photons
V0	340	$2.8 < \eta < 5.1$	full	scint.	time, vettex
	-90	$-3.7 < \eta < -1.7$	full	scint.	
T0	370	$4.6 < \eta < 4.9$	full	quartz	time, vetex
	-70	$-3.3 < \eta < -3.0$	full	quartz	
MCH	-14.2, -5.4 m	$-4.0 < \eta < -2.5$	full	MWPC	muon tracking
MTR	-17.1, 16.1 m	$-4.0 < \eta < -2.5$	full	RPC	muon trigger

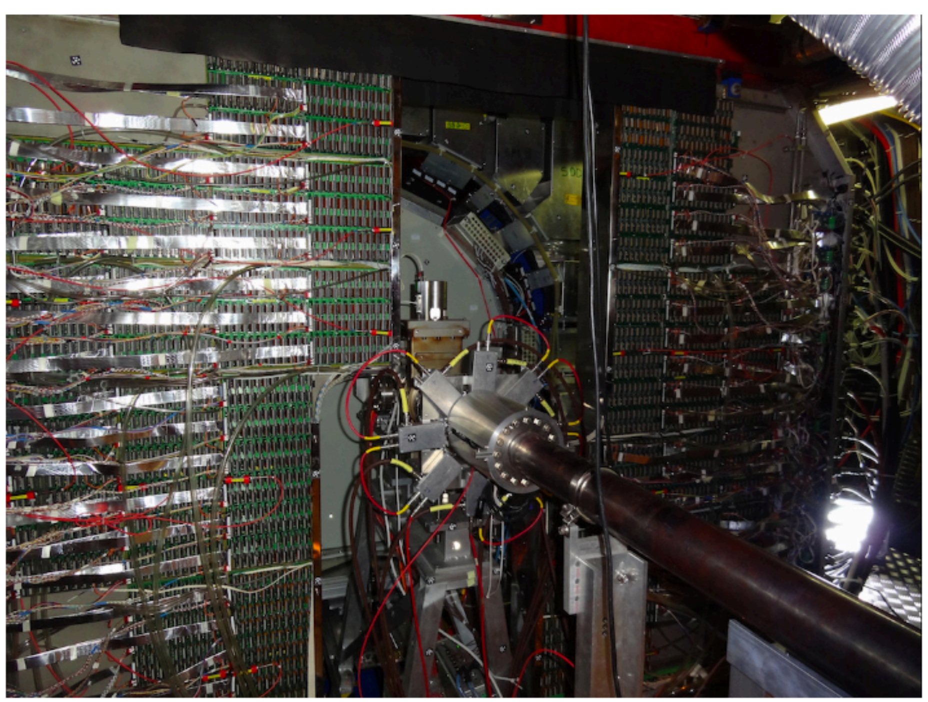
Photon Multiplicity Detector





Two planes
24 modules in each plane
4608 cells in each module

η: 2.3 to 3.9
Φ: 0 to 2π
Distance from IP: 367.5 cm
Cell size: 0.5 cm diameter



Observables in PMD:

Cluster X, Cluster Y, Cluster Z

Cluster η → $\eta = -\ln(\tan(\vartheta/2))$

$\vartheta = \tan^{-1}\left(\frac{\sqrt{clsX^2 + clsY^2}}{clsZ}\right)$

Cluster φ → $\phi = \tan^{-1}\left(\frac{clsY}{clsX}\right)$

Cluster ADC

Cluster Ncell

Instrumental Effects

Finite Acceptance

The acceptance of a measurement corresponds to the range in which an observable of interest can be measured.

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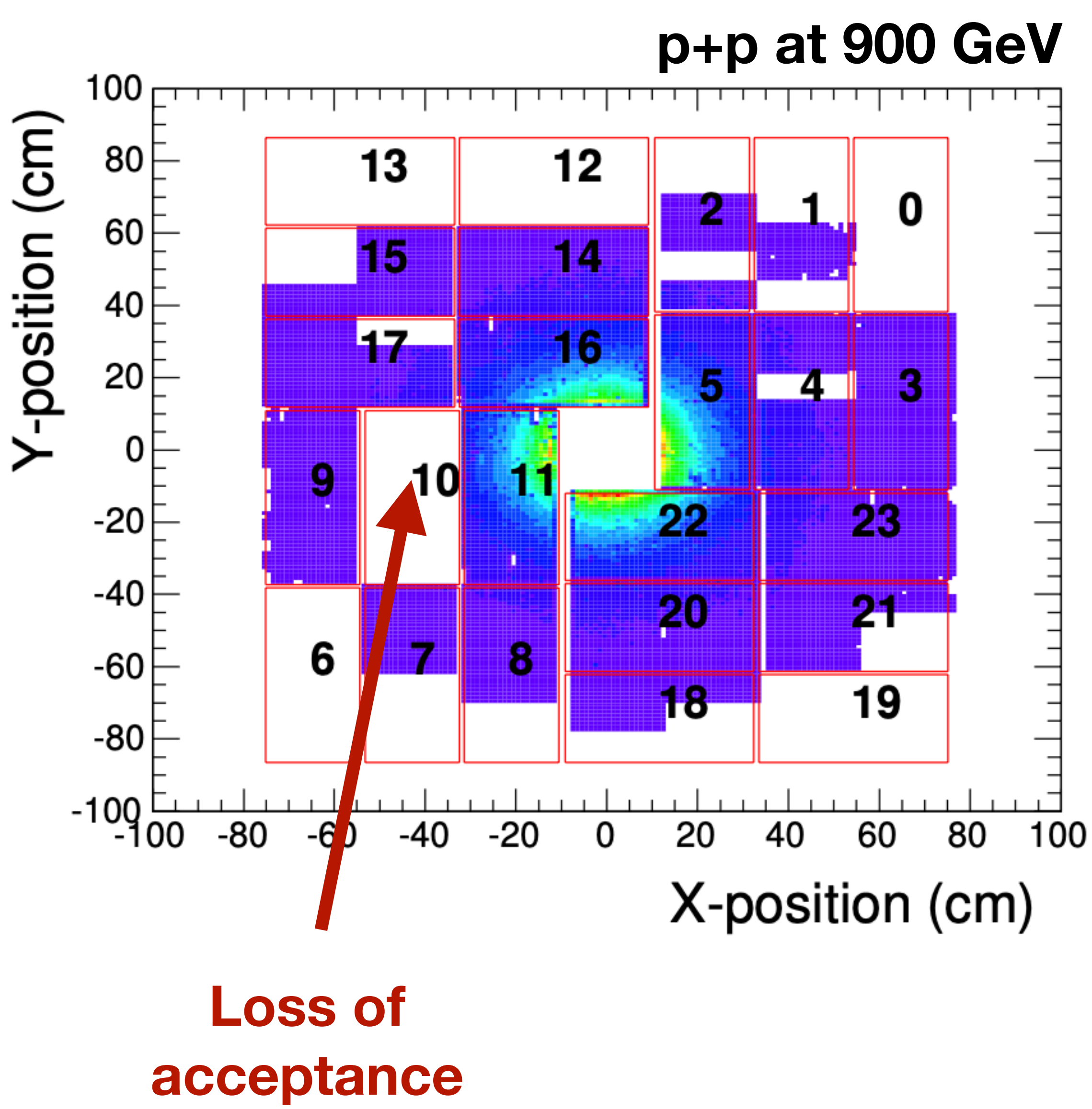
The <u>acceptance</u> of a measurement corresponds to the range in which an observable of interest can be measured.

Hits in pre-shower plane of PMD in ALICE



Finite Acceptance

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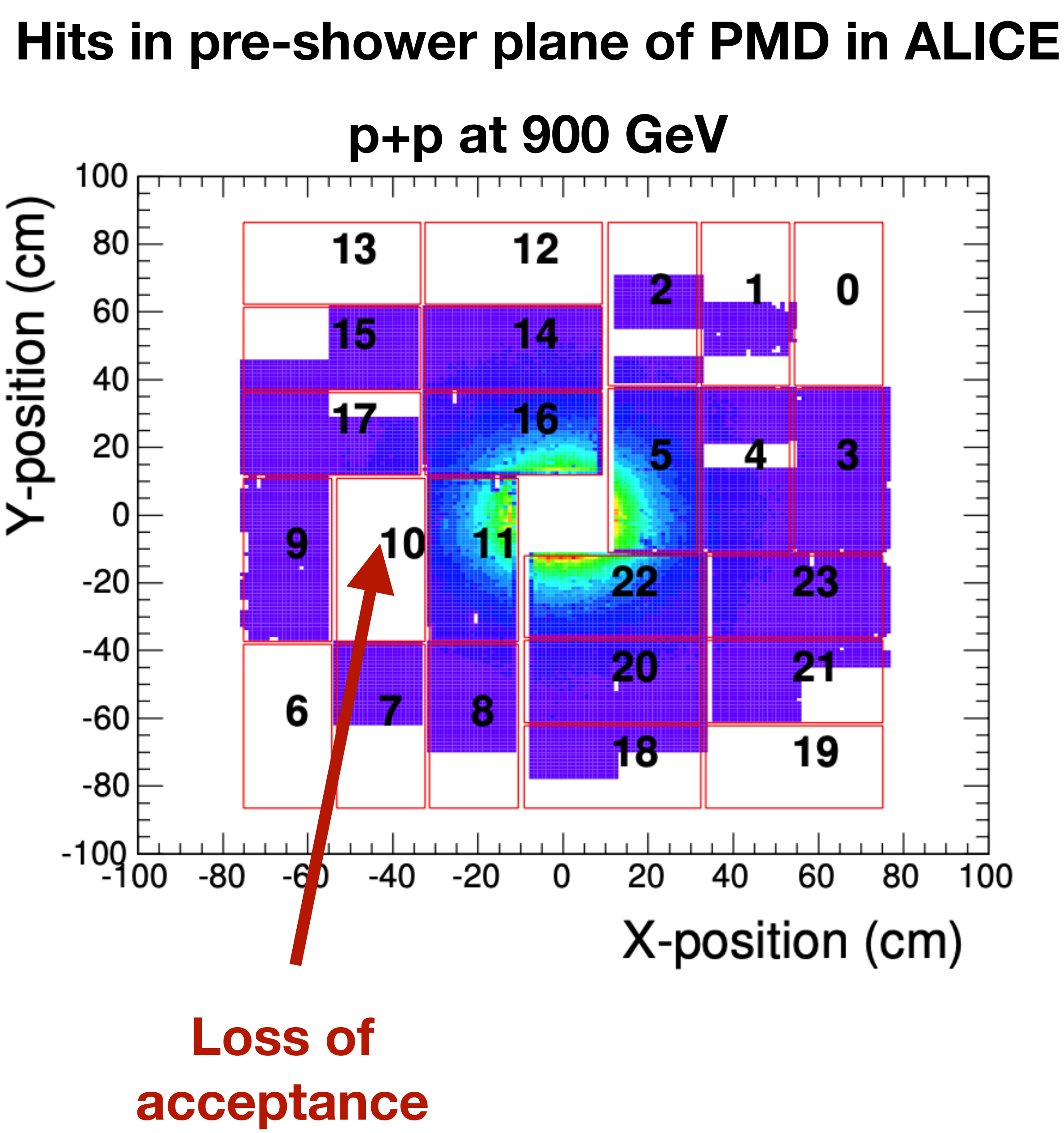
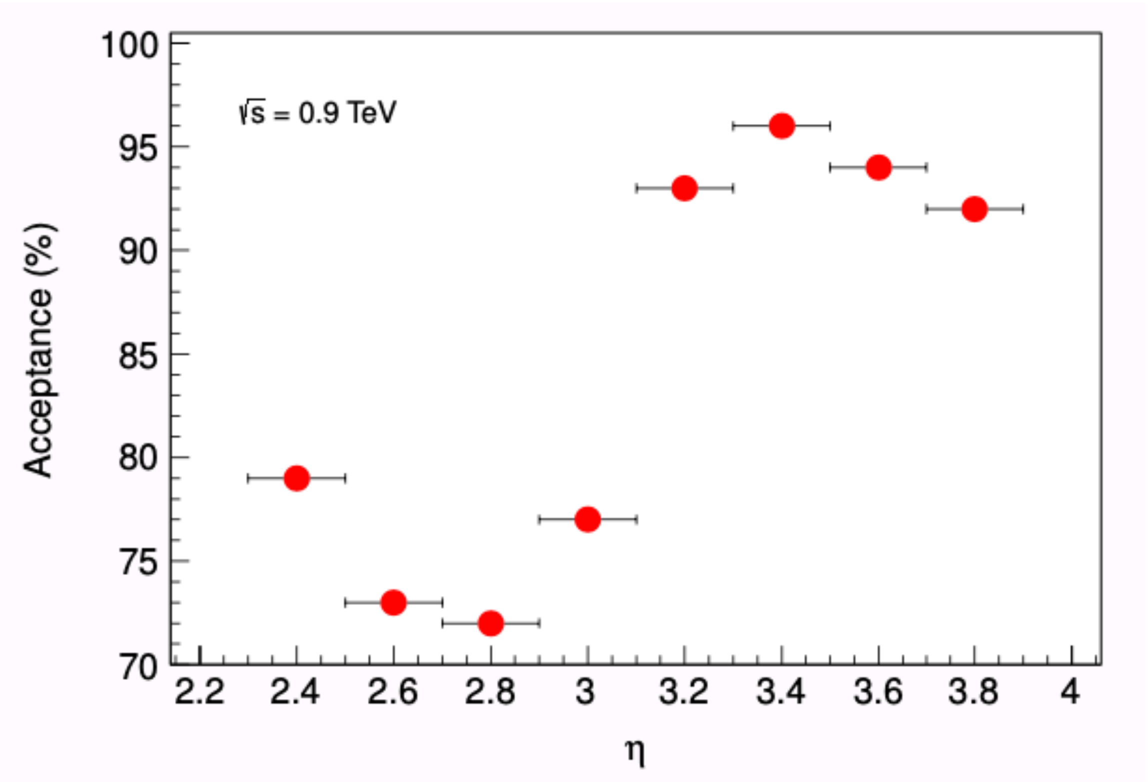


Acceptance



Detection Efficiency

$$\text{Efficiency}(\epsilon) = \frac{<\text{Measured events}>}{<\text{True events}>}$$

Take example of PMD

$$\text{Efficiency}(\epsilon) = \frac{<\text{N}_{\gamma-\text{det}}>}{<\text{N}_{\gamma-\text{inc}}>}$$

Detection Efficiency

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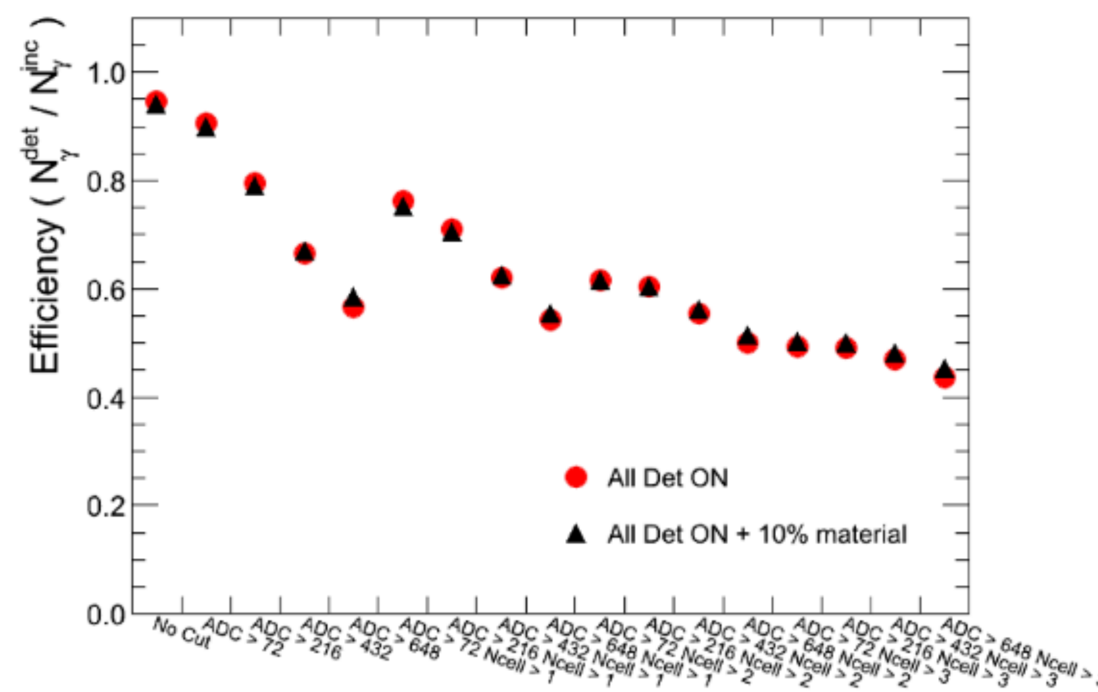
Efficiency



Loose cut

Strict cut

Reconstruction Efficiency

$$\text{Efficiency}(\epsilon) = \frac{<\text{Reconstructed tracks}>}{<\text{Input tracks}>}$$

e.g. p_T spectra analysis

Obtained by simulations:

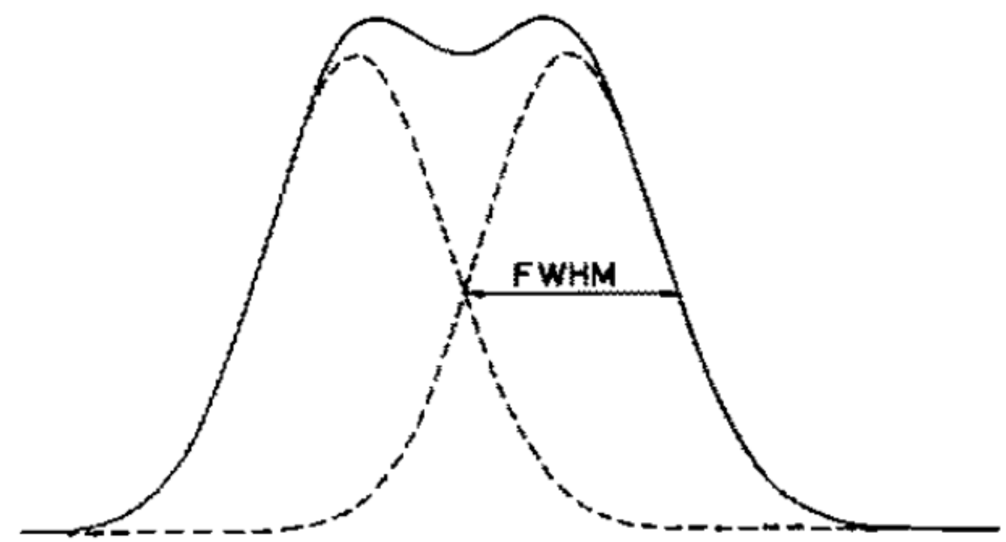
Input (PYTHIA/HIJING)	—>	**GEANT**	—>	Reconstructed

Resolution

Energy resolution: The ability to distinguish to close lying energies (separate two close by signals).

Usually, represented in terms of *full width at half maximum* of the peak

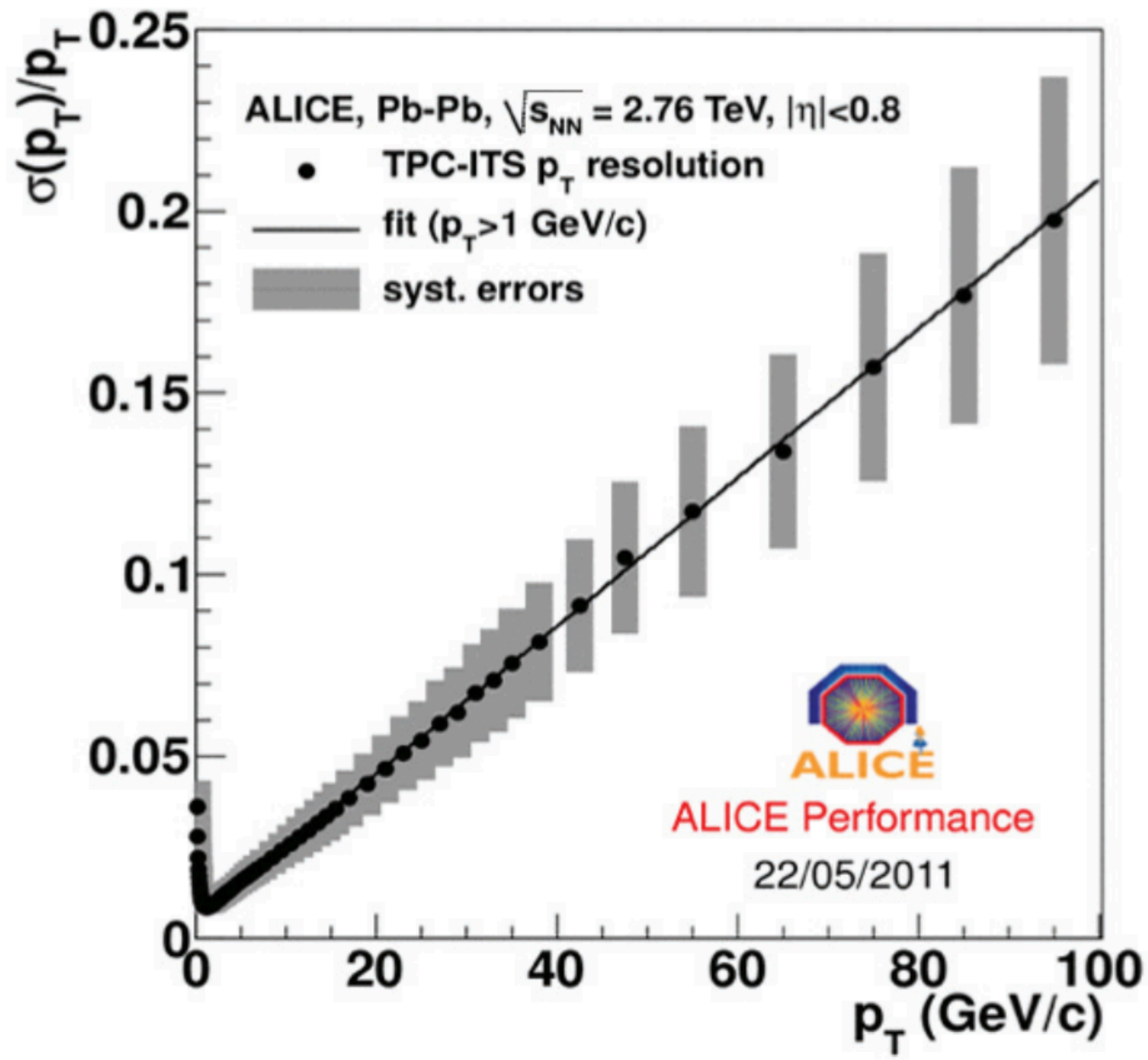
$$\frac{\sigma_E}{E} \sim \frac{a}{\sqrt{E}}(\%)$$



Resolution

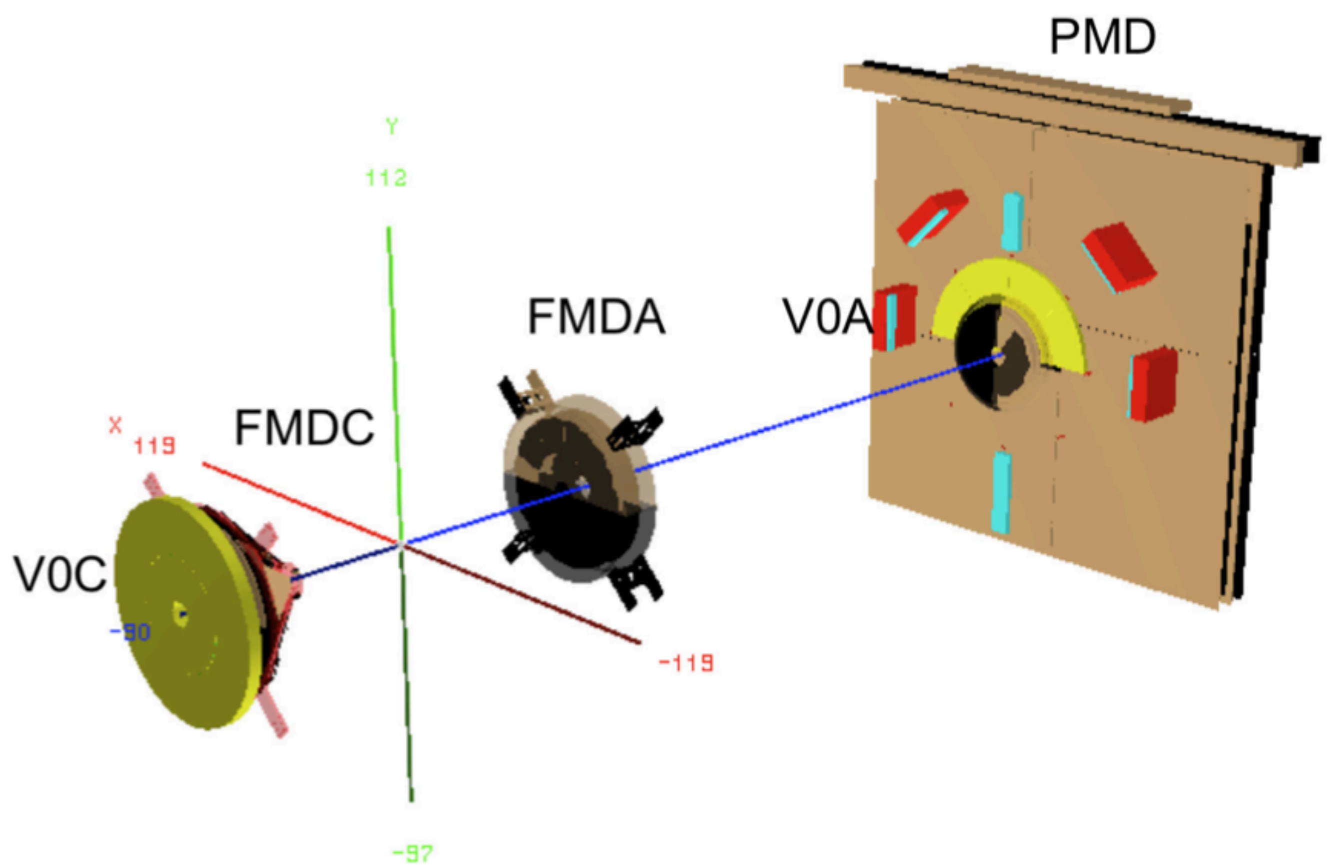
Momentum resolution:

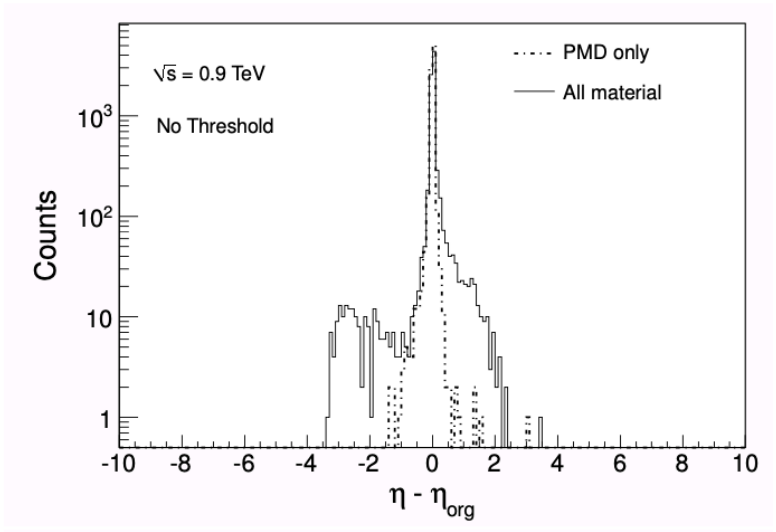
$$\frac{\sigma(p_T)}{p_T} \sim \sigma_s \frac{p_T}{BL^2\sqrt{N_{\text{pad-rows}}}}$$

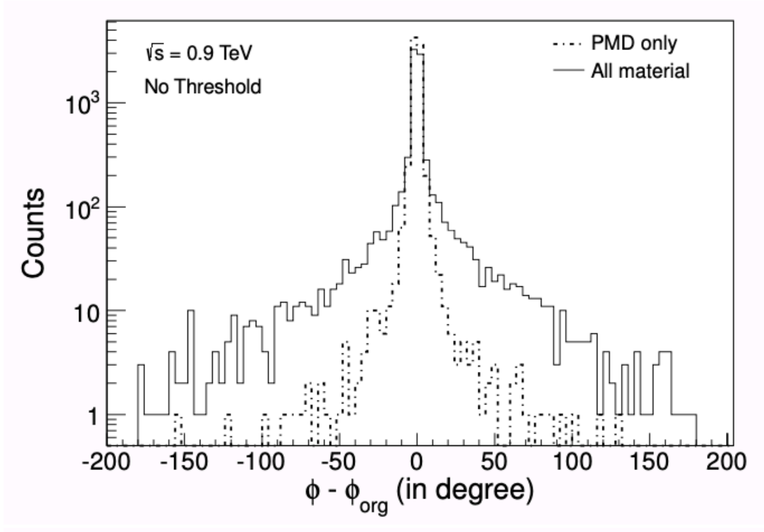


Backgrounds

Materials in front of PMD







Experimental Measurements

Any experimental measurements are subjected to effects from:

1. Finite Acceptance
2. Detection Efficiency
3. Detection Resolution
4. Backgrounds
5. …..

Measurements need to be corrected for these effects

Unfolding Method

Unfolding

True distribution: T(x)

Measured distribution: M (y)

M (y) = **R(y,x)** T(x)

R (y,x) is called the Response Matrix,

R_{ji} is the conditional probability that a collision with true multiplicity *i* is measured as an event with multiplicity *j*

Unfolding

Unfolding: Estimating probability distribution from data that are smeared by random fluctuations

$T(x) = \mathbf{R}^{-1} M(y)$

Matrix Inversion can cause large fluctuation

Inversion of Response Matrix

Unfolding is an *ILL-Posed* problem

M (y) = **R(y, x)** T(x)

Let's take an example

$$\begin{pmatrix} y1 \\ y2 \end{pmatrix} = \begin{pmatrix} 0.9 & 0.0 \\ 0.0 & 0.8 \end{pmatrix} \times \begin{pmatrix} x1 \\ x2 \end{pmatrix}$$

R With
Only diagonal terms

Inversion of Response Matrix

Unfolding is an *ILL-Posed* problem

M (y) = **R** T(x)

$$\begin{pmatrix} y1 \\ y2 \end{pmatrix} = \begin{pmatrix} 0.9 & 0.6 \\ 1.0 & 0.8 \end{pmatrix} \times \begin{pmatrix} x1 \\ x2 \end{pmatrix}$$

R With
off-diagonal terms

Matrix inversion cause oscillations
with *large variances*

Regularization

Unfolding via matrix inversion cause oscillations with large *variances* due to the resolution of the measurement

↓ Apply ***“Regularization”***

Reduce reduce large fluctuations and *variance* of unfolded results

Unfolding by χ^2-minimization

Unfolding by χ²-minimization

$$\chi^2(U) = \sum_i \left(\frac{M_i - \sum_j R_{ij} U_j}{e_i} \right)$$

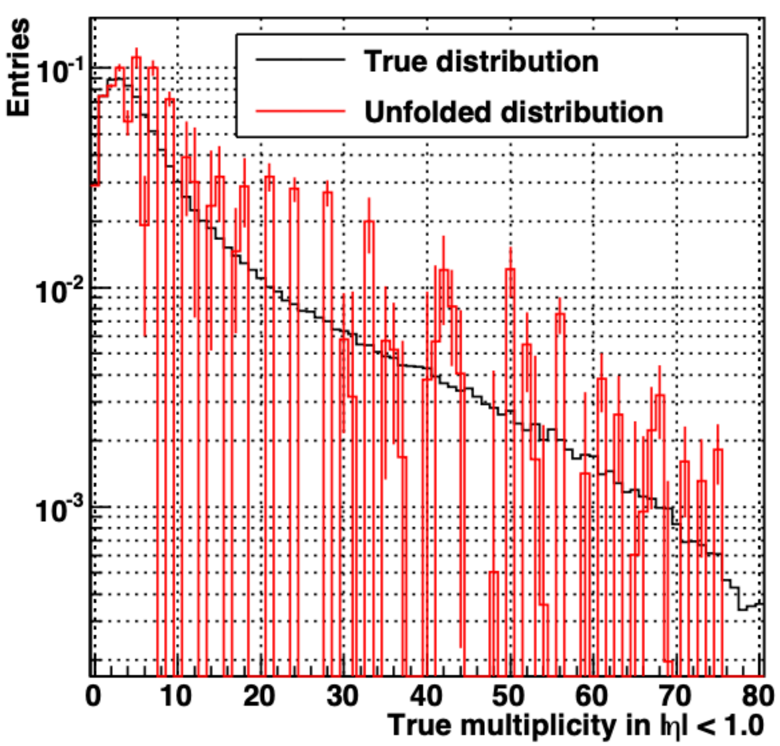
U_j is the guessed distribution

Can cause large fluctuation in unfolded distribution

Unfolding by χ²-minimization

$$\chi^2(U) = \sum_i \left(\frac{M_i - \sum_j R_{ij} U_j}{e_i} \right)$$

U_j is the guessed distribution



Large fluctuation in unfolded distribution

Regularization in χ^2-minimization

$$\chi^2(U) = \sum_i \left(\frac{M_i - \sum_j R_{ij} U_j}{e_i} \right)$$

To minimize oscillation, we add a ***regularization*** term with $P(U)$ a weight factor β

$$\chi^2(U) = \hat{\chi}^2(U) + \beta P(U)$$

Regularization add a constraint that favors a certain shape of unfolded distributions

What is optimum β and $P(U)$?

Regularization in χ^2-minimization

Different form of regularizations $P(U)$

ad hoc information

Favored shape

$$P(U) = \sum_t \left(\frac{U_t'}{U_t}\right)^2 = \sum_t \left(\frac{U_t - U_{t-1}}{U_t}\right)^2,$$

Constant function

$$P(U) = \sum_t \left(\frac{U_t''}{U_t}\right)^2 = \sum_t \left(\frac{U_{t-1} - 2U_t + U_{t+1}}{U_t}\right)^2,$$

Linear function

$$P(U) = P(\hat{U} := \ln U) = \sum_t \left(\frac{\hat{U}_t''}{\hat{U}_t}\right)^2$$

$$= \sum_t \left(\frac{\ln U_{t-1} - 2\ln U_t + \ln U_{t+1}}{\ln U_t}\right)^2,$$

Exponential function

What happens,

when β is very small … ?

or, when β is very large … ?

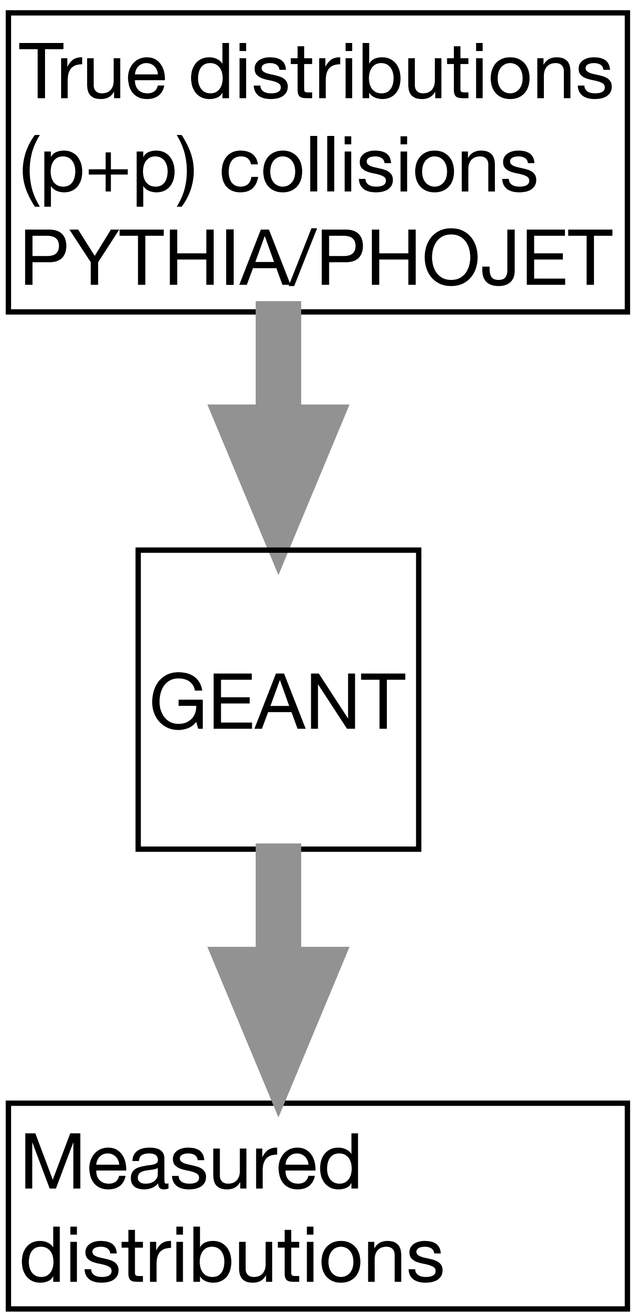
Unfolding by χ^2-minimization

Let's take an example: Photon Multiplicity Detector

Measured multiplicity of photons: $P(N_{\gamma\text{-meas}})$
True multiplicity of photons: $P(N_{\gamma\text{-true}})$

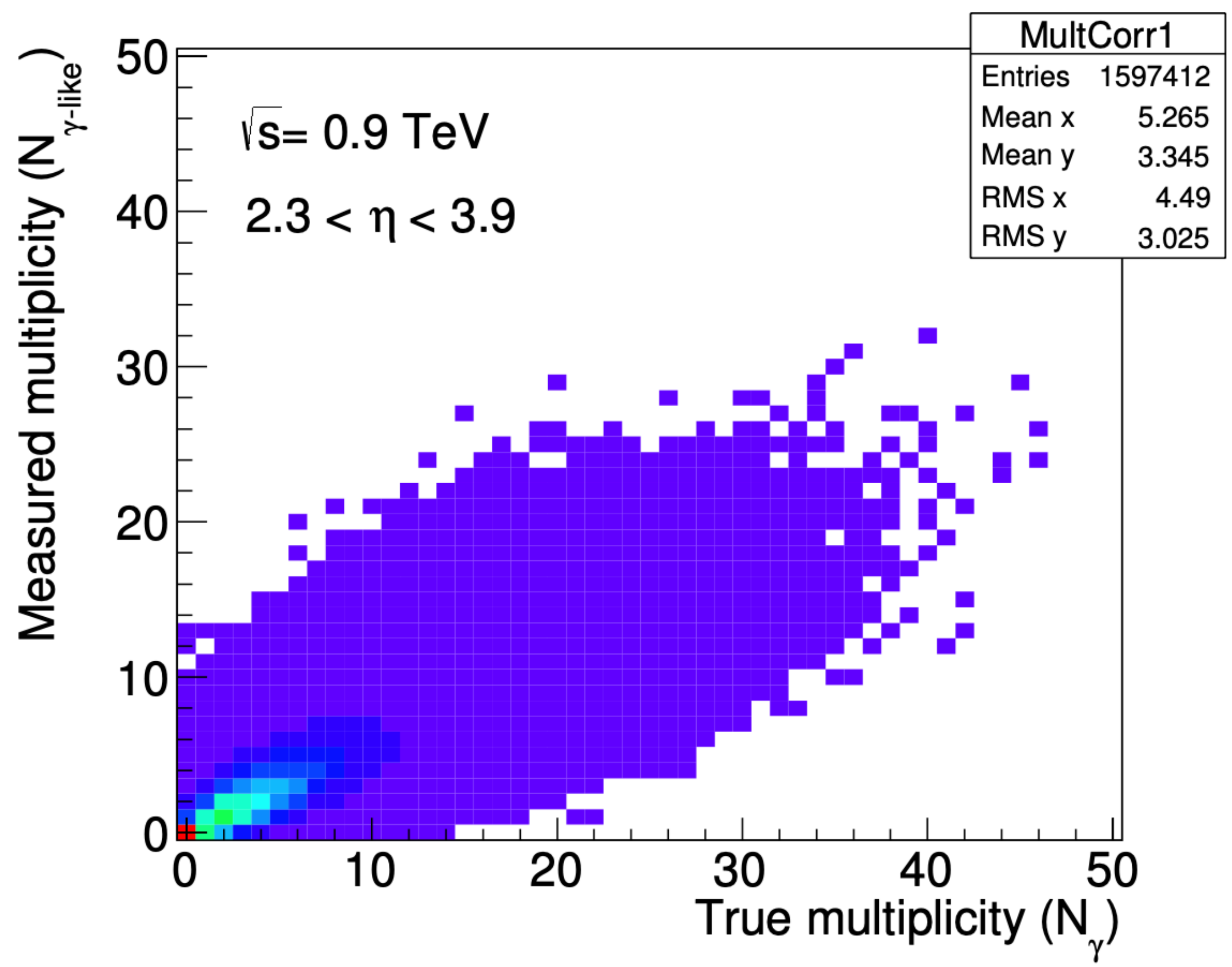
Unfolding by χ^2-minimization

Simulation



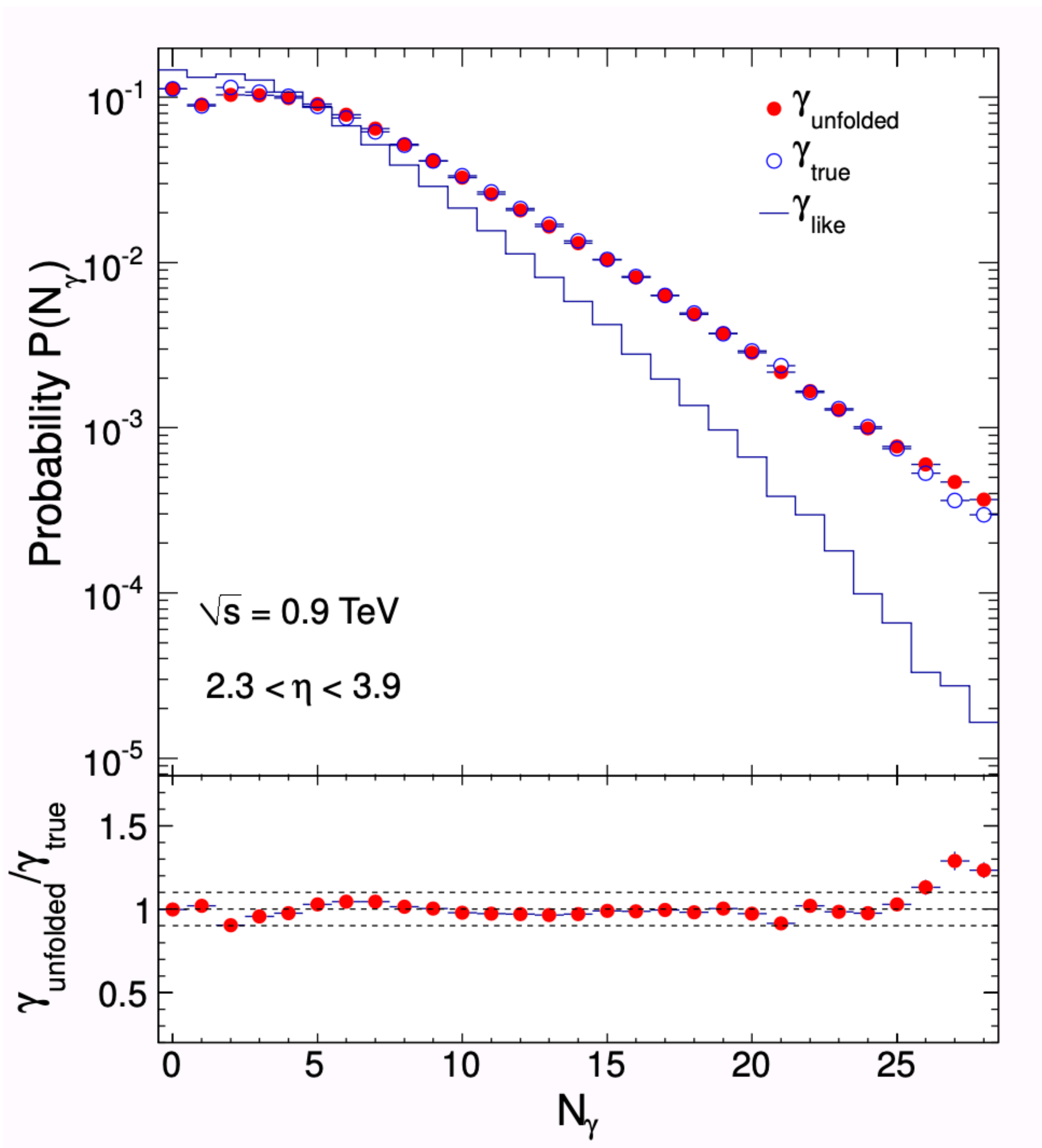
Response Matrix

$P(N_{\gamma\text{-meas}})$ versus $P(N_{\gamma\text{-true}})$



Unfolding by χ²-minimization

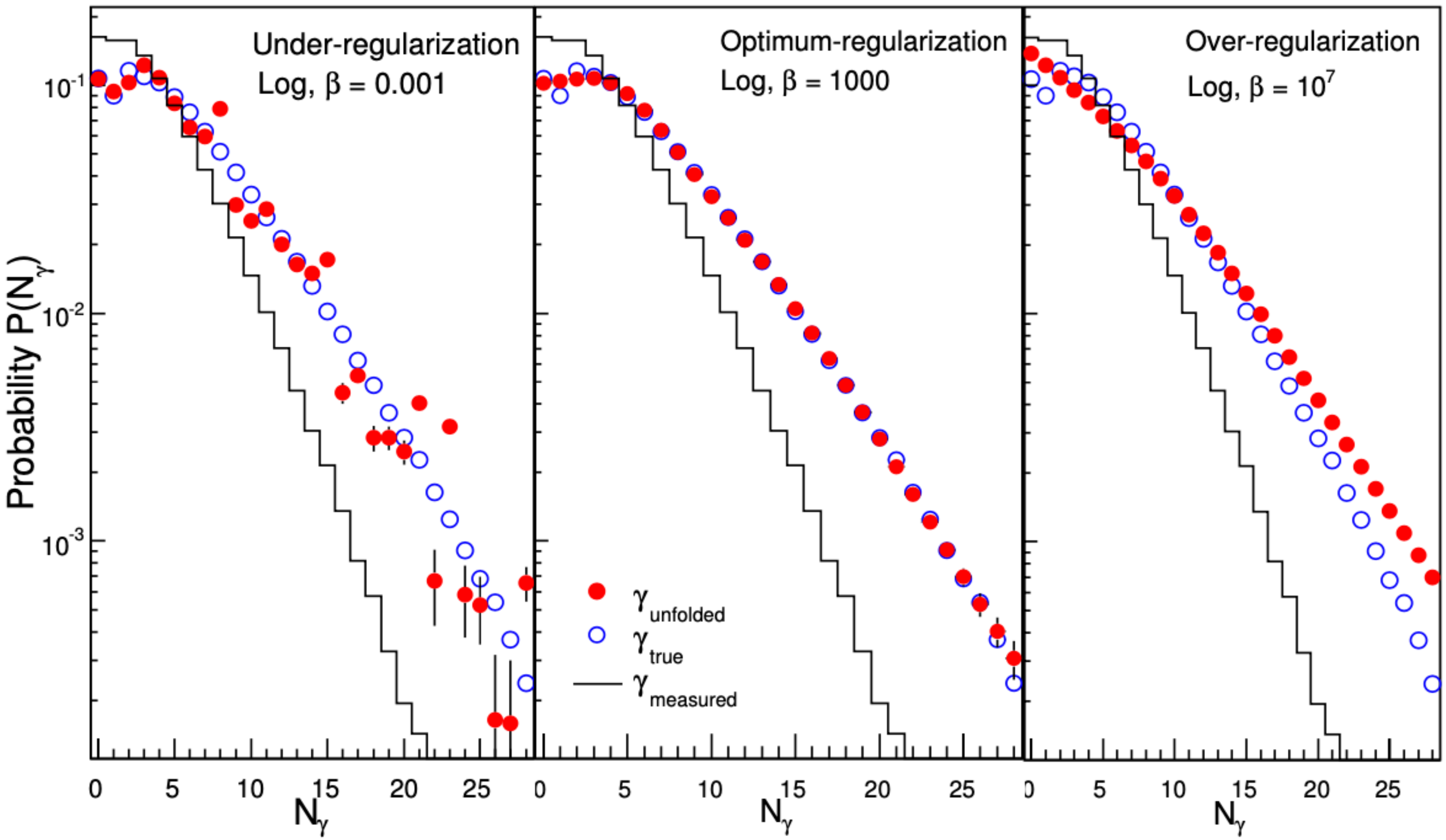
Closure test of unfolding or training



Perform test using simulations whether or, not we can recover the true distribution

Regularization by χ^2-minimization

Optimize parameters β and $P(U)$ using simulation



Apply parameters β and $P(U)$ on real data to get unfolded (true) distribution

Unfolding by Bayesian Method

Unfolding by Bayesian Method

Bayes Theorem

$$P(A\,|\,B) = \frac{P(B\,|\,A)P(A)}{P(B)}$$

P(A) : Prob of event A
P(B) : Prob of event B
P(A|B) : Prob of event A when B is true
P(B|A) : Prob of event B when A is true

$$\hat{R}_{ij} = \frac{R_{ij}T_i}{\sum_k R_{ik}T_k}$$

T_k : *Prior distribution* for the true distribution T_i

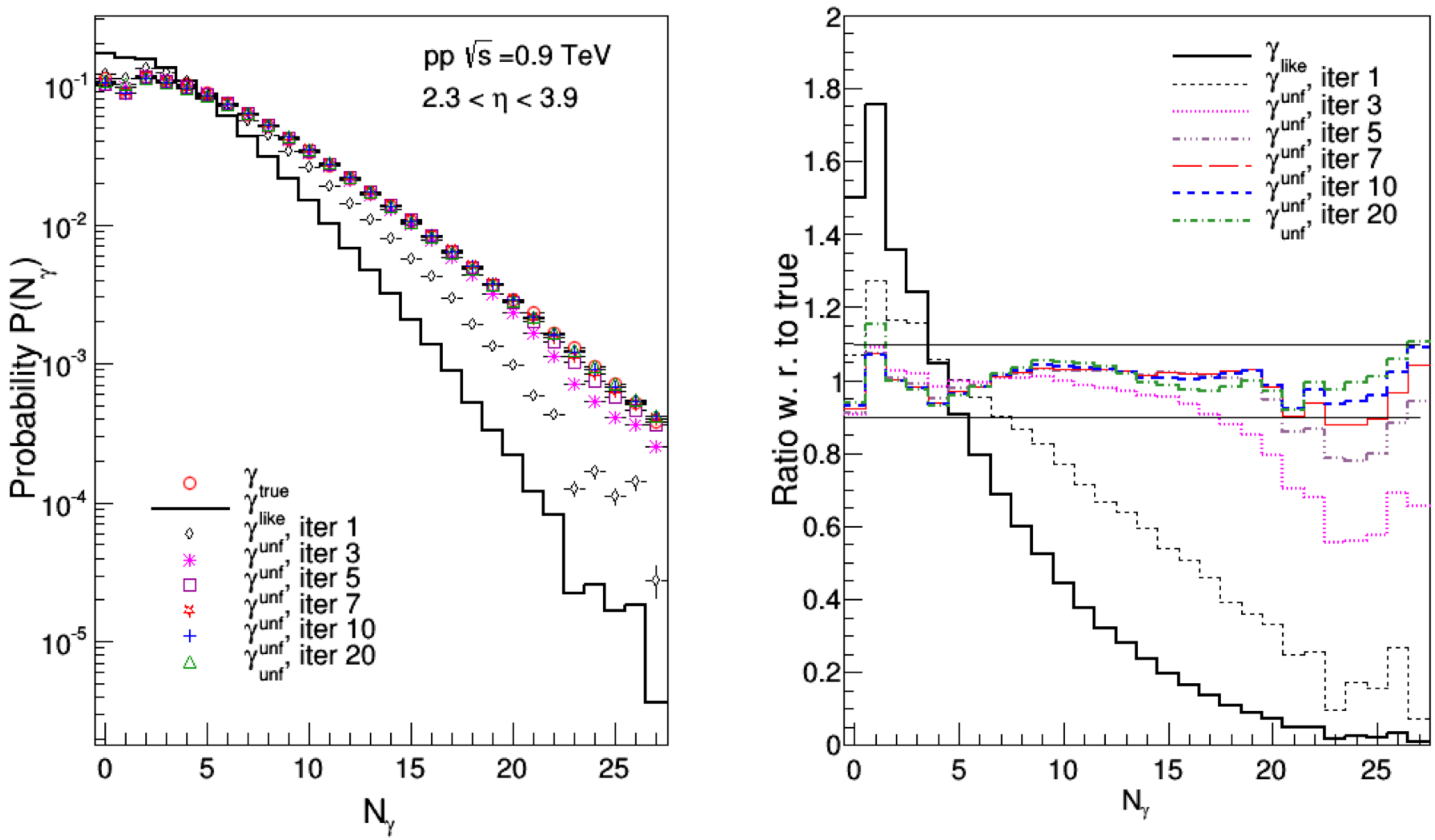
$$U_i = \sum_j \hat{R}_{ij}M_j$$

Smoothening parameter, Iteration method ….

Unfolding by Bayesian Method

<u>Example</u>

Measured distribution taken as *a priory* distribution



Applications of unfolding

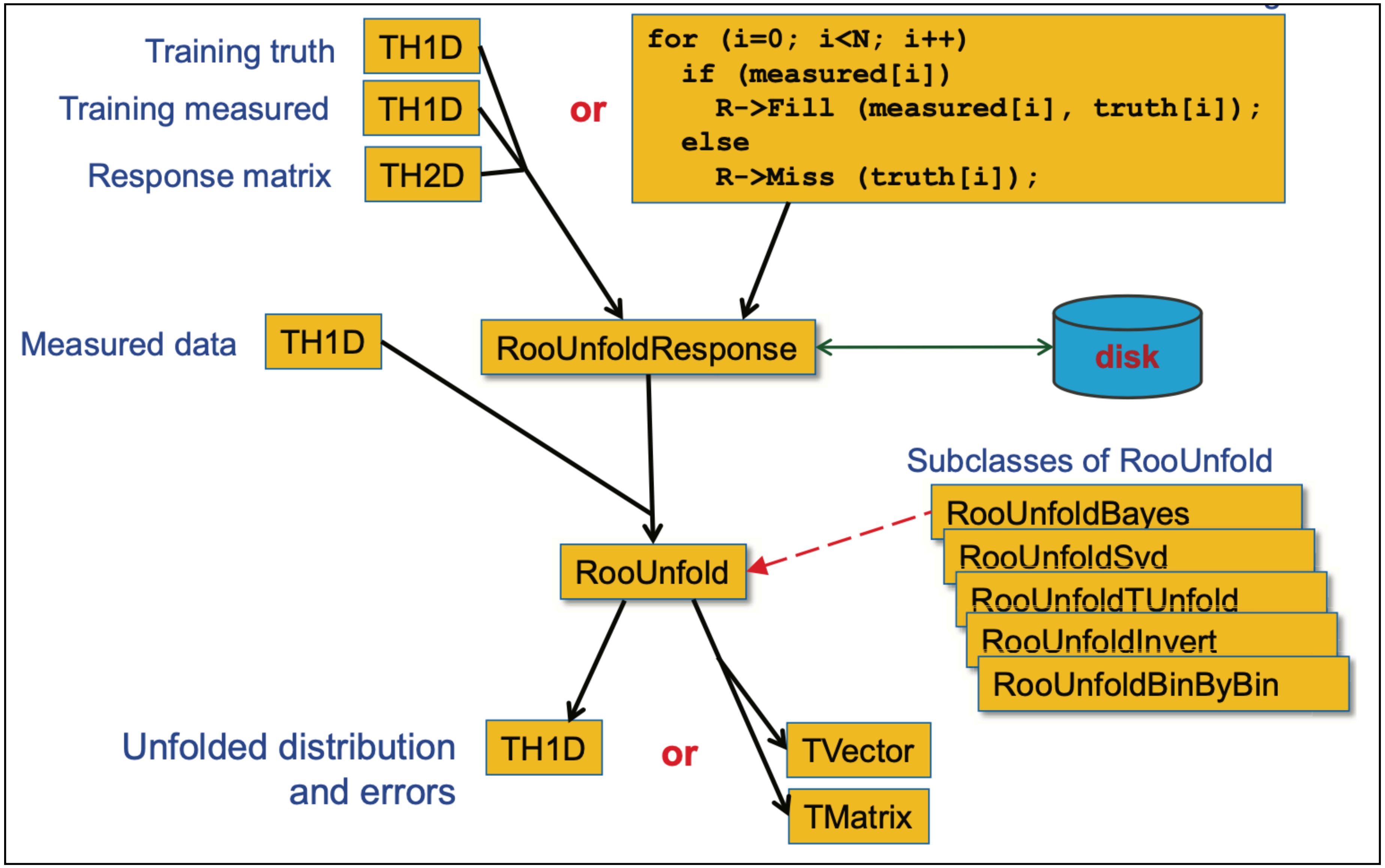
Unfolding techniques are widely used in many analysis:

1. Multiplicity distribution
2. Collective flow analysis (flow fluctuation)
3. Jet analysis (Jet spectra)
4. Net-charge fluctuation
5. …..

— https://arxiv.org/pdf/1004.3514.pdf
— https://arxiv.org/pdf/1711.05594.pdf
— https://journals.aps.org/prc/pdf/10.1103/PhysRevC.101.034911
— https://arxiv.org/abs/1211.2074

RooUnfold

RooUnfold package



Download

git clone https://gitlab.cern.ch/RooUnfold/RooUnfold.git

cd RooUnfold

Make

gSystem->Load("RooUnfold/libRooUnfold")

Let's look at: RooUnfold/examples/RooUnfoldExample.cxx

Thank you for your attention!

Some references and further reading:

1. V. Blobel (https://arxiv.org/abs/hep-ex/0208022)
2. G. Cowan (https://www.ippp.dur.ac.uk/Workshops/02/statistics/proceedings/cowan.pdf)
3. A. N. Tikhonov, (Sov. Math. 5 (1963) 1035): On method of Regularization
4. G. D'Agostini NIM A 362 (1995), 487: On Bayesian unfolding

6. RooUnfold by Tim Adye: https://hepunx.rl.ac.uk/~adye/software/unfold/RooUnfold.html

   https://gitlab.cern.ch/RooUnfold/RooUnfold
7. Book: Data analysis Techniques for Physical Scientists, by C. Prenau
8. PhD Thesis:

   https://www.hep.lu.se/staff/gustafsson/alice/thesis/janfietethesis.pdf

   http://www.hbni.ac.in/phdthesis/phys/PHYS07200904008.pdf
5. …….