

Tenth International Fermi Symposium

9th-15th October 2022



Constraining the multipolar magnetic field of MSP J0030+0451 via *NICER* X-ray light curve fitting



Anu Kundu*, North-West University

TEAM:

Harding, Alice K. (LANL)

Kalapotharakos, Constantinos (NASA GSFC/UMCP/CRESST II)

Kazanas, Demosthenes (NASA GSFC)

Venter, Christo (CSR, NWU)

Wadiasingh, Zorawar (NASA GSFC/USRA)

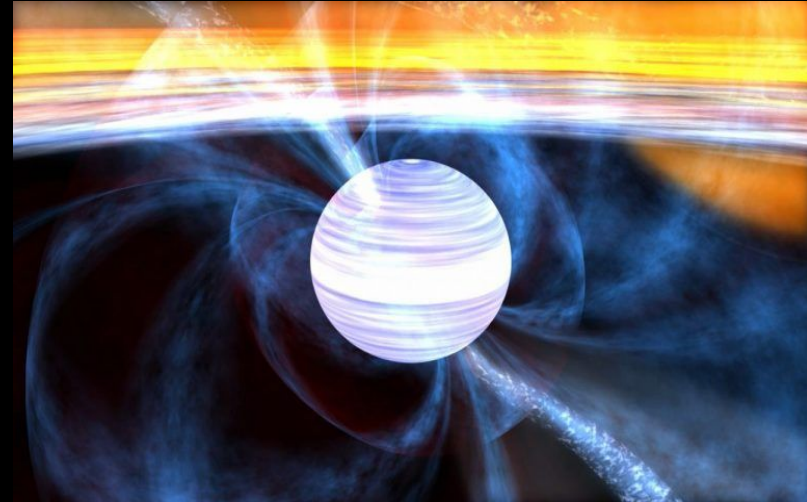
*anukunduo2@yahoo.com

Outline

- **Introduction**
- **Multipolar magnetic field**
- **Methods**
- **Markov chain Monte Carlo (MCMC) runs**
- **Summary**

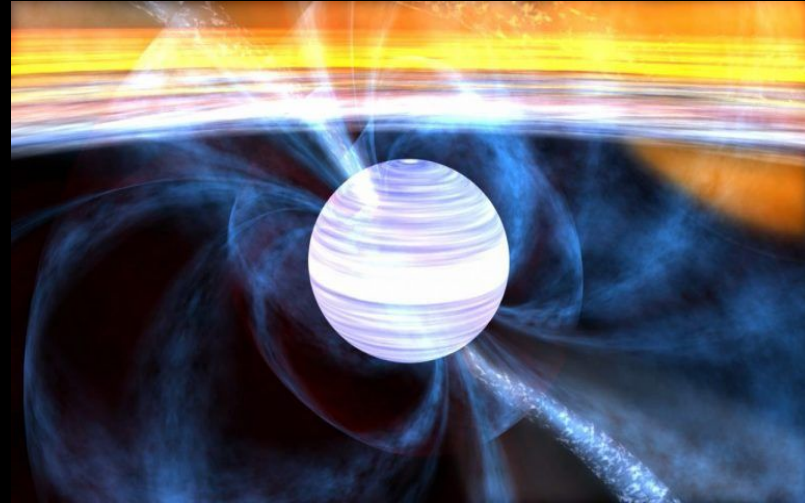
Introduction: MSP PSR J0030+0451

- Millisecond pulsars (MSPs)
 - Old pulsars
 - Period < 10 ms



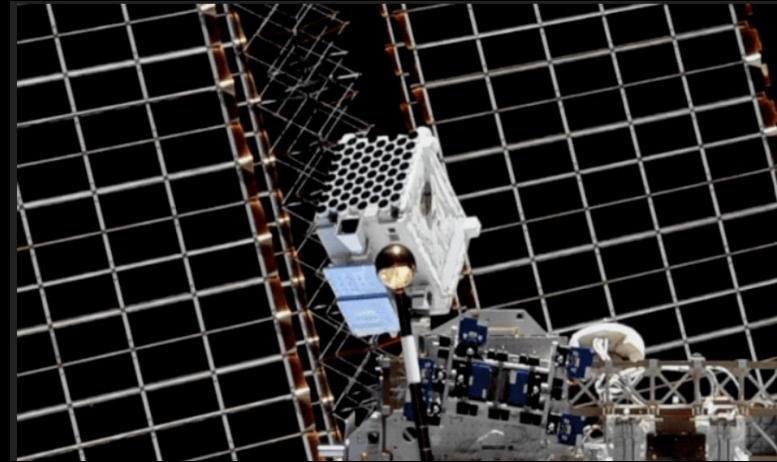
Introduction: MSP PSR J0030+0451

- Millisecond pulsars (MSPs)
 - Old pulsars
 - Period < 10 ms
- J0030: Isolated MSP, minority in MSP population
- Spin period 4.865 ms
- Multiwavelength emission:
 - discovered as a radio pulsar using *Arecibo*
 - identified as an X-ray pulsar with *ROSAT*
 - first gamma-ray MSP announced by *Fermi*



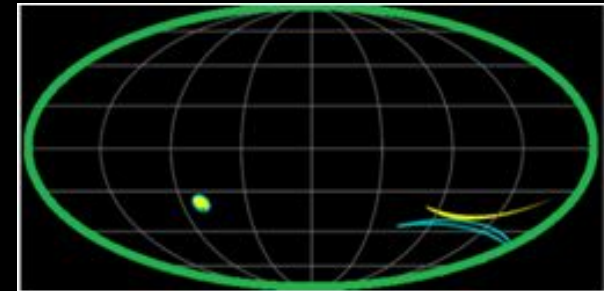
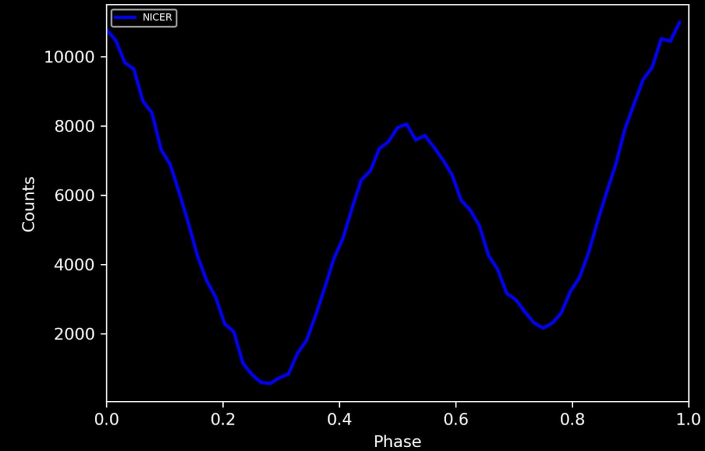
Introduction: NICER view of J0030

- Neutron star Interior Composition Explorer (*NICER*)
 - Soft X-ray band (0.2–12 keV)
 - Science goal: Constrain the equation of state



Introduction: NICER view of J0030

- Neutron star Interior Composition Explorer (*NICER*)
 - Soft X-ray band (0.2–12 keV)
 - Science goal: Constrain the equation of state
- Light curve (LC) of J0030: two peaks
- Possible origin: hotspots on surface
- Fit the light curve, model hotspots on the surface
 - Kalapotharakos et al. (2021): static offset dip+quad magnetic field
 - Updating magnetic field: retarded centred multipolar field, up to $l=3$; Pétri (2015)



Kalapotharakos et al. (2021)

Multipolar magnetic field equations

Multipolar magnetic field outside neutron star

$$B(r, \theta, \phi, t) = \sum_{l=1}^{\infty} \sum_{m=-l}^l [B_r(r, \theta, \phi, t) + B_\theta(r, \theta, \phi, t) + B_\phi(r, \theta, \phi, t)]$$

where,

$$B_r = -\frac{\sqrt{l(l+1)}}{r} f_{lm}^B Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\theta = -\frac{\partial_r(r f_{lm}^B)}{r \sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t} + \frac{i\mu_0 m \Omega f_{lm}^D}{\sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\phi = -\frac{\partial_r(r f_{lm}^B)}{r \sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t} - \frac{i\mu_0 m \Omega f_{lm}^D}{\sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t}$$

Multipolar magnetic field equations

Multipolar magnetic field outside neutron star

$l = 1$: Dipole

$l = 2$: Quadrupole

$l = 3$: Octopole

...

$m = -l$ to l : different orientations for the corresponding multipolar component

Terms with r dependence

$$B_r = -\frac{\sqrt{l(l+1)}}{r} f_{lm}^B Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\theta = -\frac{\partial_r(r f_{lm}^B)}{r \sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t} + \frac{i\mu_0 m \Omega f_{lm}^D}{\sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\phi = -\frac{\partial_r(r f_{lm}^B)}{r \sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t} - \frac{i\mu_0 m \Omega f_{lm}^D}{\sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t}$$

Multipolar magnetic field equations

Multipolar magnetic field outside neutron star

$l = 1$: Dipole

$l = 2$: Quadrupole

$l = 3$: Octopole

...

$m = -l$ to l : different orientations for the corresponding multipolar component

Terms with r dependence

Terms with (ϑ, φ) dependence

$$B_r = -\frac{\sqrt{l(l+1)}}{r} f_{lm}^B Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\theta = -\frac{\partial_r(r f_{lm}^B)}{r \sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t} + \frac{i\mu_0 m \Omega f_{lm}^D}{\sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\phi = -\frac{\partial_r(r f_{lm}^B)}{r \sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t} - \frac{i\mu_0 m \Omega f_{lm}^D}{\sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t}$$

Multipolar magnetic field equations

Multipolar magnetic field outside neutron star

$l = 1$: Dipole

$l = 2$: Quadrupole

$l = 3$: Octopole

...

$m = -l$ to l : different orientations for the corresponding multipolar component

Terms with r dependence

Terms with (ϑ, φ) dependence

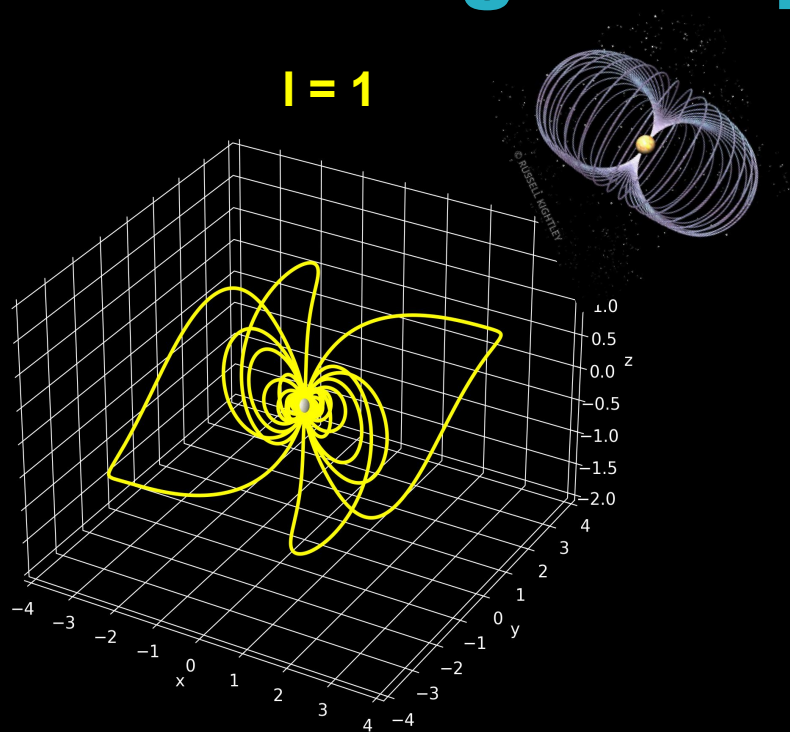
Terms with (l, m) dependence

$$B_r = - \frac{\sqrt{l(l+1)}}{r} f_{lm}^B Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

$$B_\theta = - \frac{\partial_r(r f_{lm}^B)}{r \sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t} + \frac{i\mu_0 m \Omega f_{lm}^D}{\sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t}$$

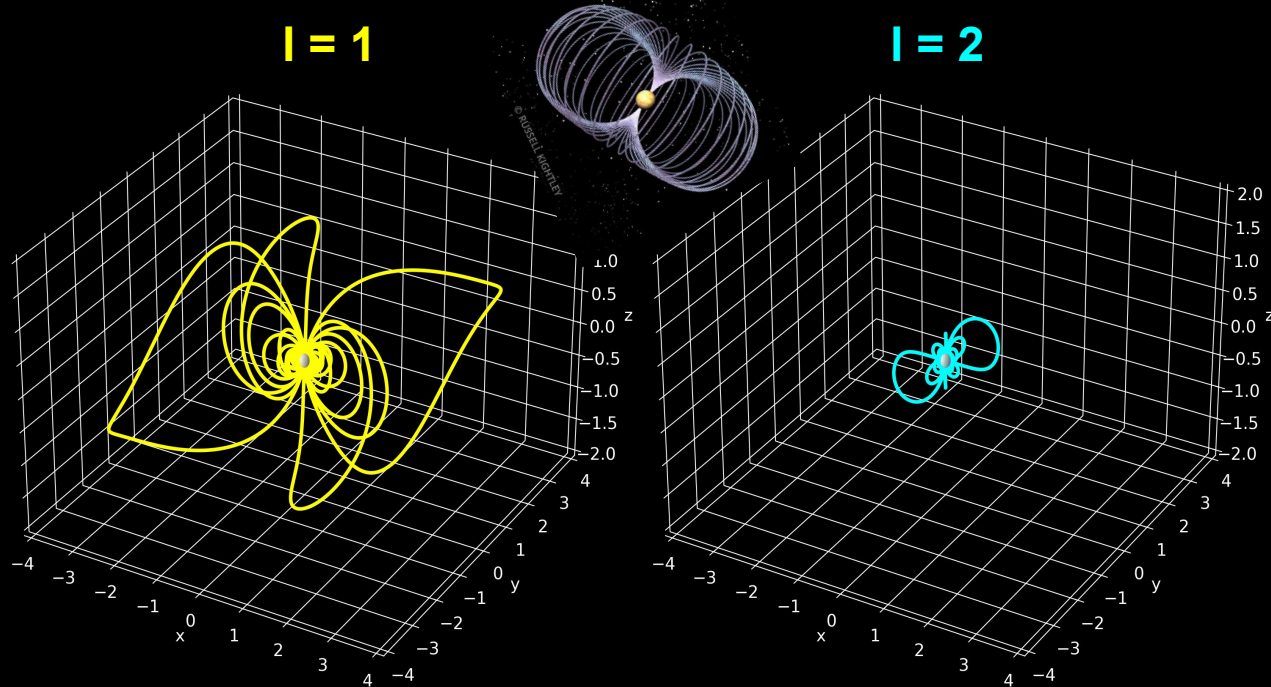
$$B_\phi = - \frac{\partial_r(r f_{lm}^B)}{r \sin \theta \sqrt{l(l+1)}} im Y_{lm}(\theta) e^{im\phi} e^{-im\Omega t} - \frac{i\mu_0 m \Omega f_{lm}^D}{\sqrt{l(l+1)}} \partial_\theta(Y_{lm}(\theta)) e^{im\phi} e^{-im\Omega t}$$

Visualizing multipolar field lines



Retarded vs static field line geometry

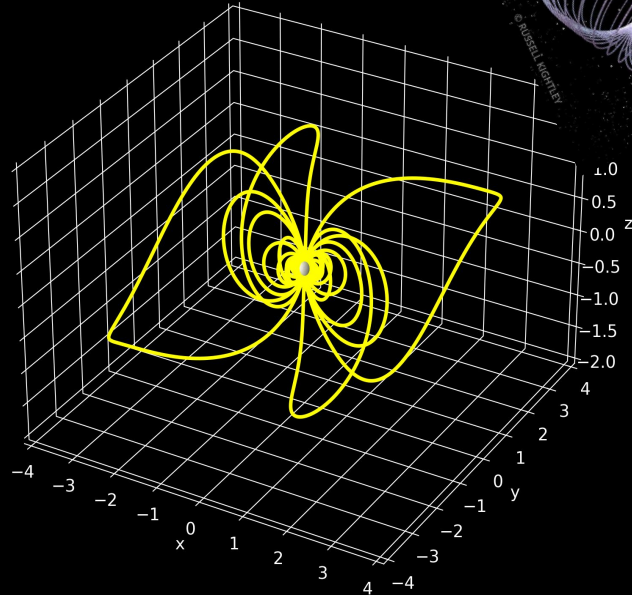
Visualizing multipolar field lines



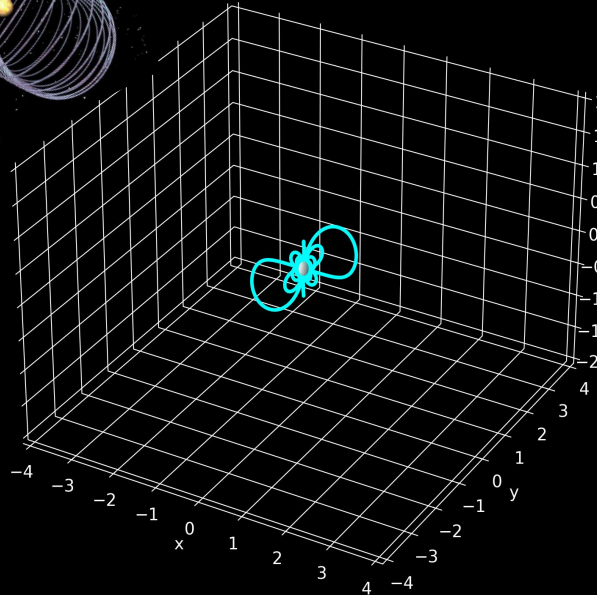
Retarded vs static field line geometry

Visualizing multipolar field lines

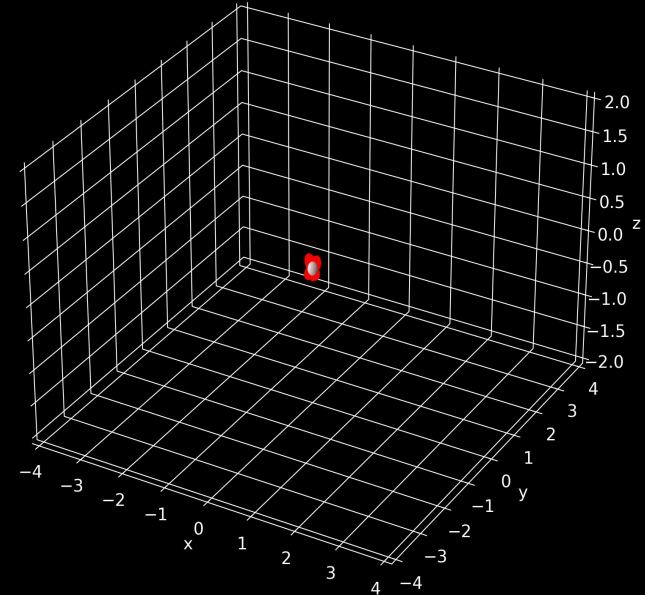
$l = 1$



$l = 2$



$l = 3$



Retarded vs static field line geometry

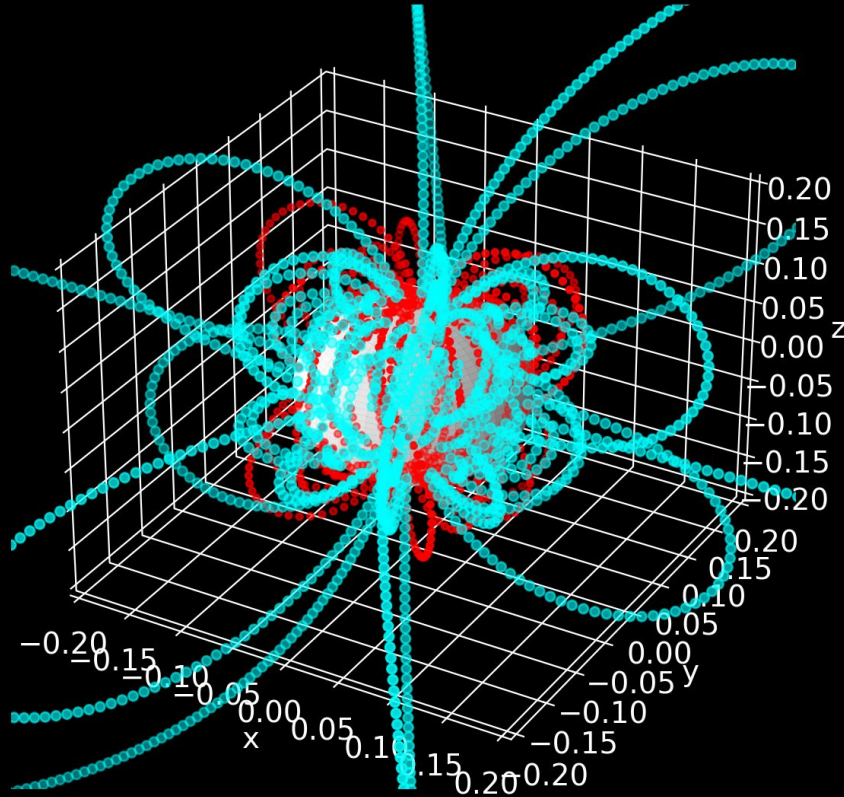
$\frac{1}{r^{l+1}}$ dependence

Visualizing multipolar field lines

$l = 2, 3$ overlap

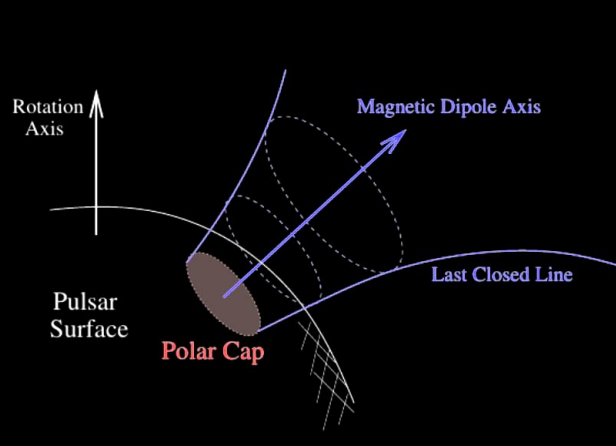
l_2 in cyan

l_3 in red



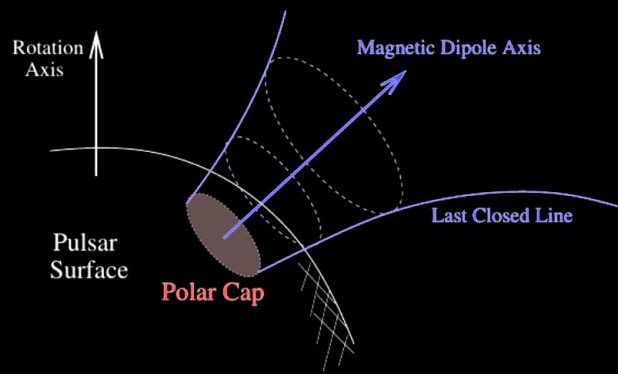
Close to the surface, higher components contribute

Testing multipolar field: Polar caps



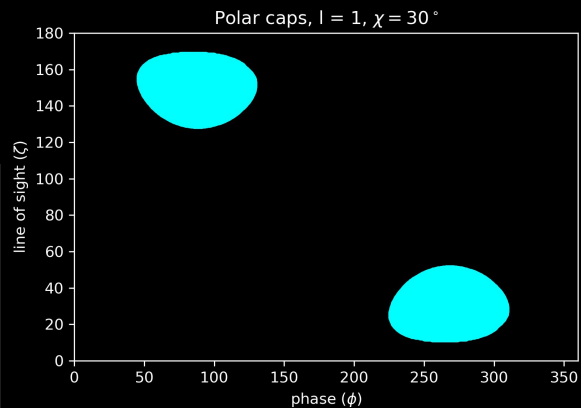
Saito (2011)

Testing multipolar field: Polar caps

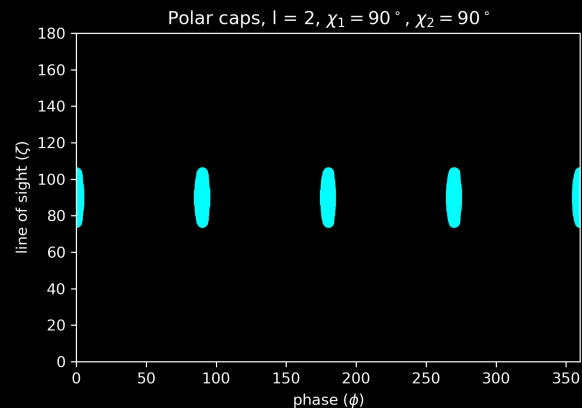
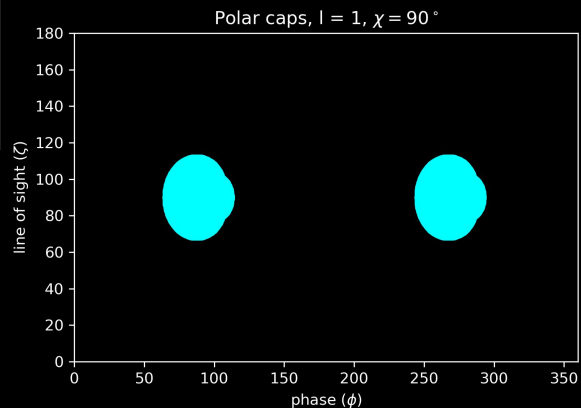
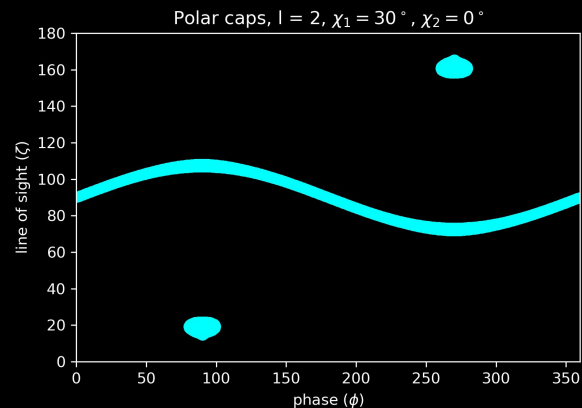


Saito (2011)

$l = 1$

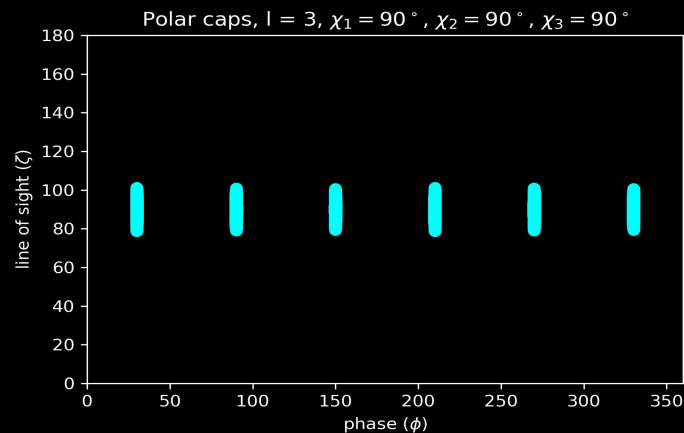
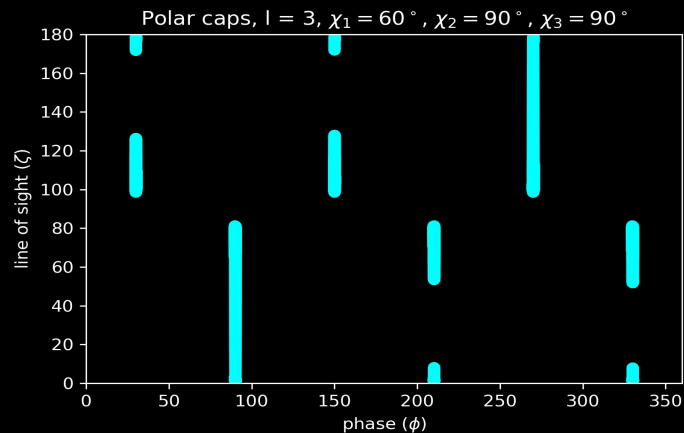
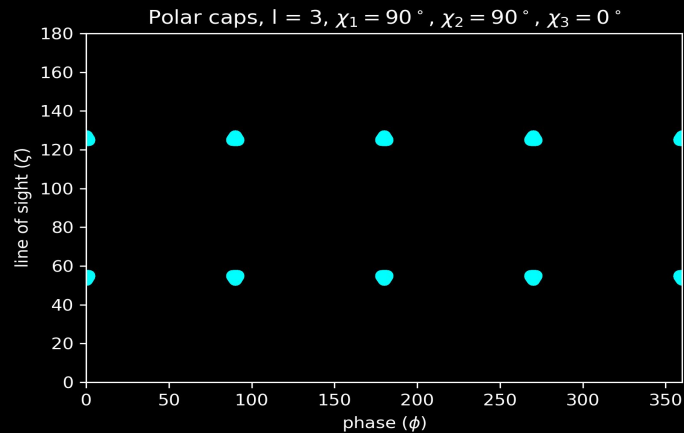
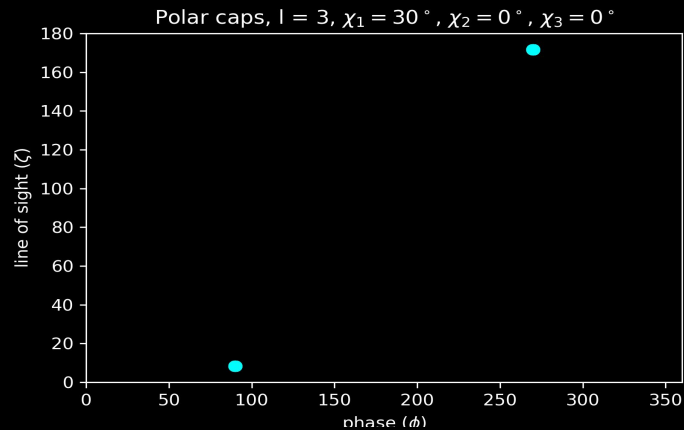


$l = 2$



Testing multipolar field: Polar caps

$l = 3$



Methods

Retarded multipolar magnetic field

Geodesic Integration in Kerr Spacetime (GKS) ray-tracing:

Find hotspots on the surface -> Produce corresponding light curve

MCMC: updated code from Kalapotharakos et al. (2021)

Kalapotharakos et al. (2021)	Current work
$I_1 + I_2$	$I_1 + I_2 + I_3$
Static	Retarded
Offset	Centred (offset later)
11 parameters	11 parameters (6 up to I_2)

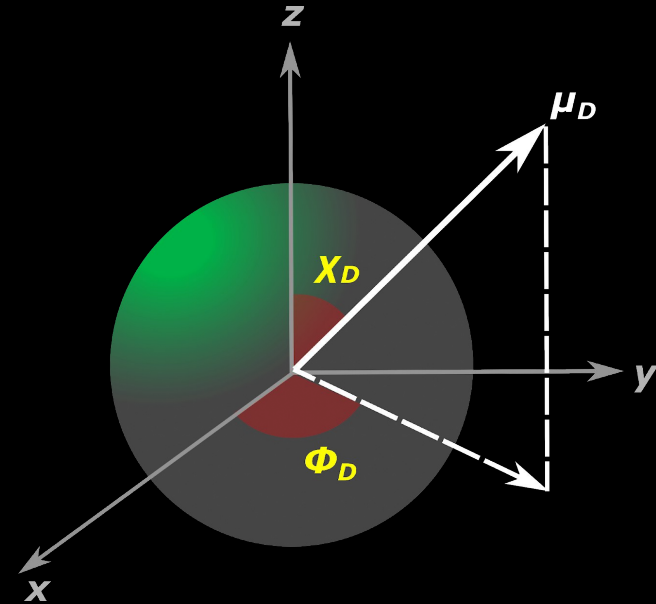
Methods: MCMC parameter space

Phase - 3 parameters: $\varphi_D, \varphi_Q, \varphi_O$

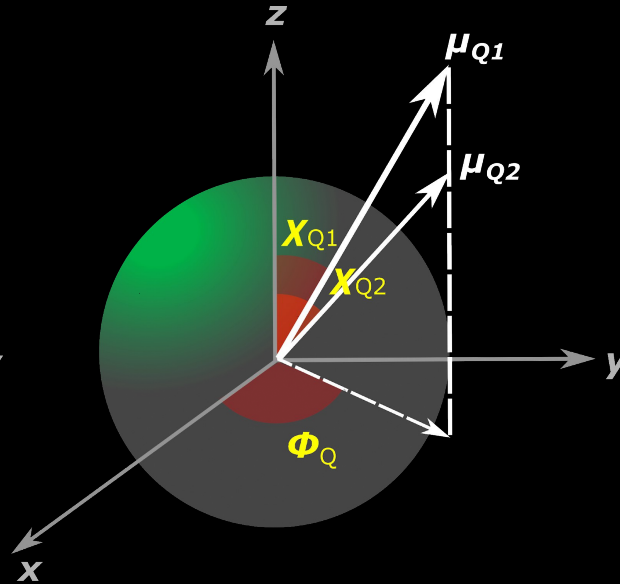
Angular - 6 parameters: $\chi_D, \chi_{Q1}, \chi_{Q2}, \chi_{O1}, \chi_{O2}, \chi_{O3}$

Field strength - 2 parameters: $B_Q/B_D, B_O/B_D$

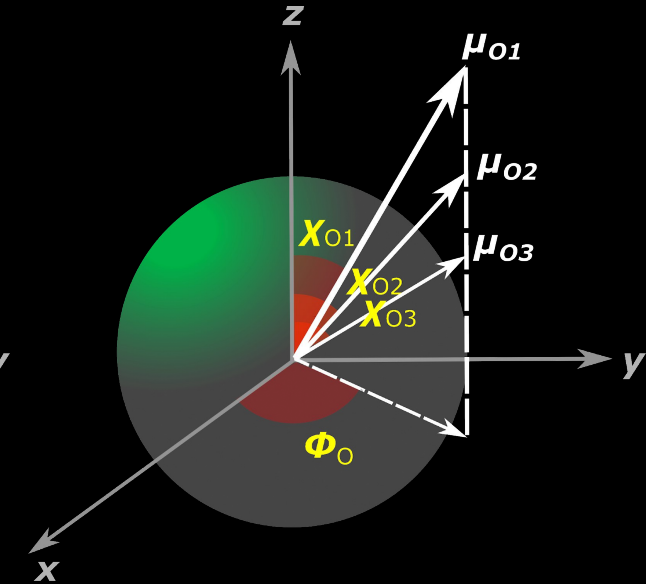
$l = 1$



$l = 2$

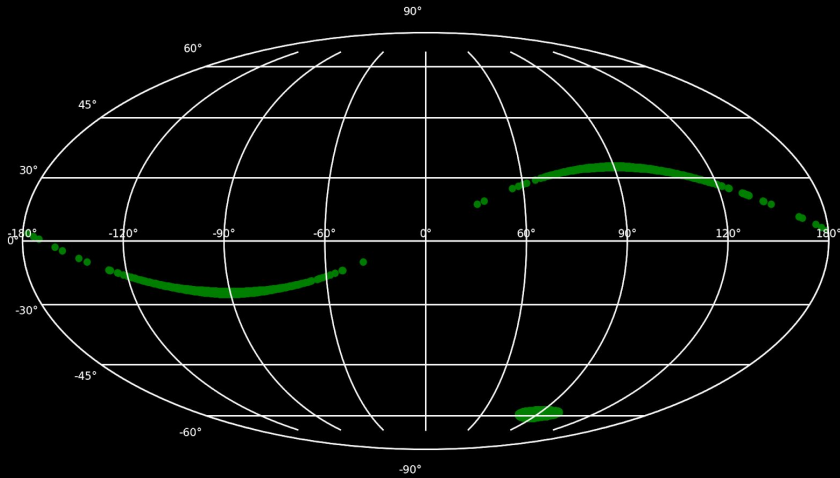


$l = 3$



MCMC runs: $l = 1+2$

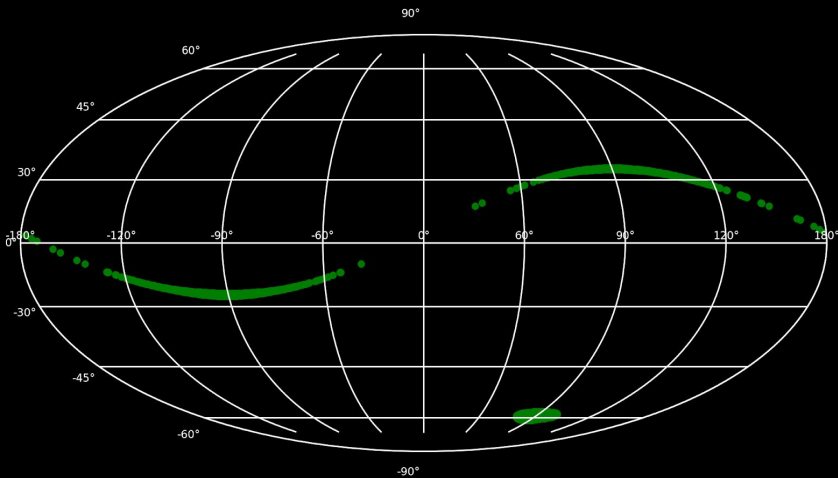
Mollweide projection showing hotspots



Low resolution

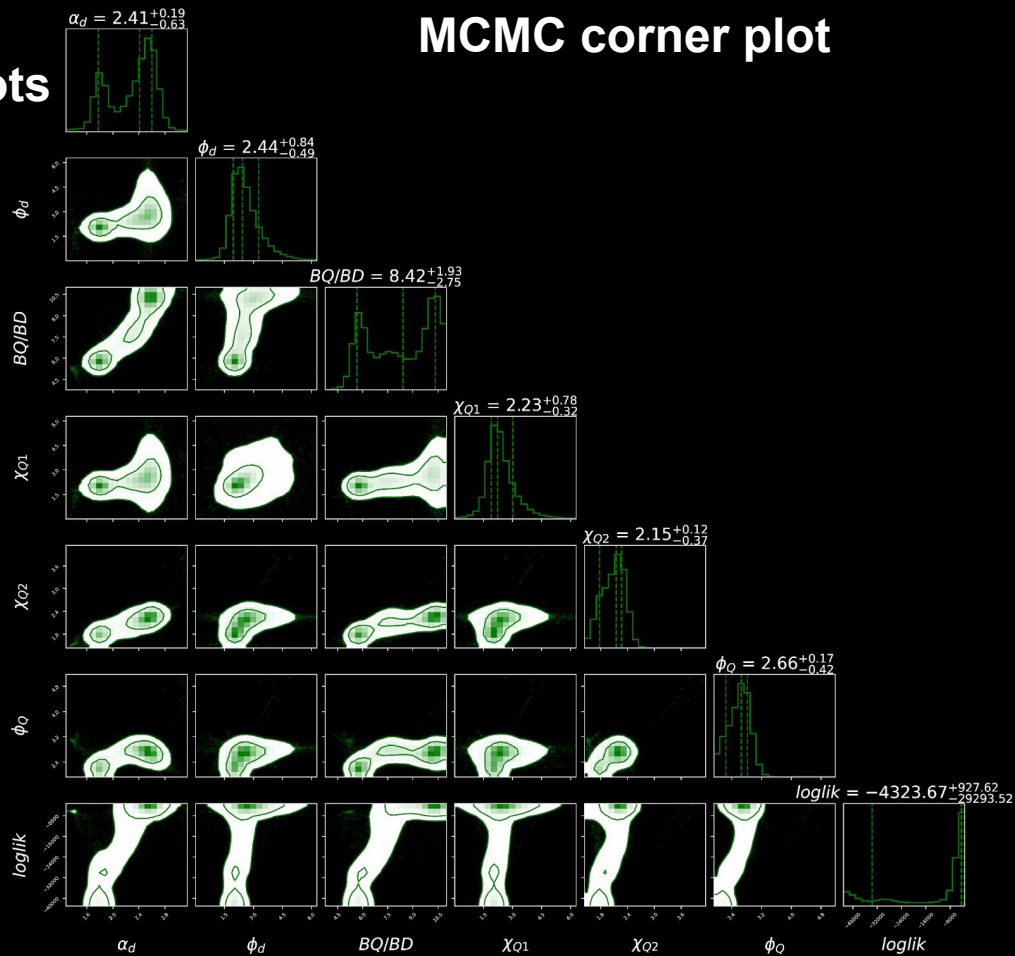
MCMC runs: I = 1+2

Mollweide projection showing hotspots



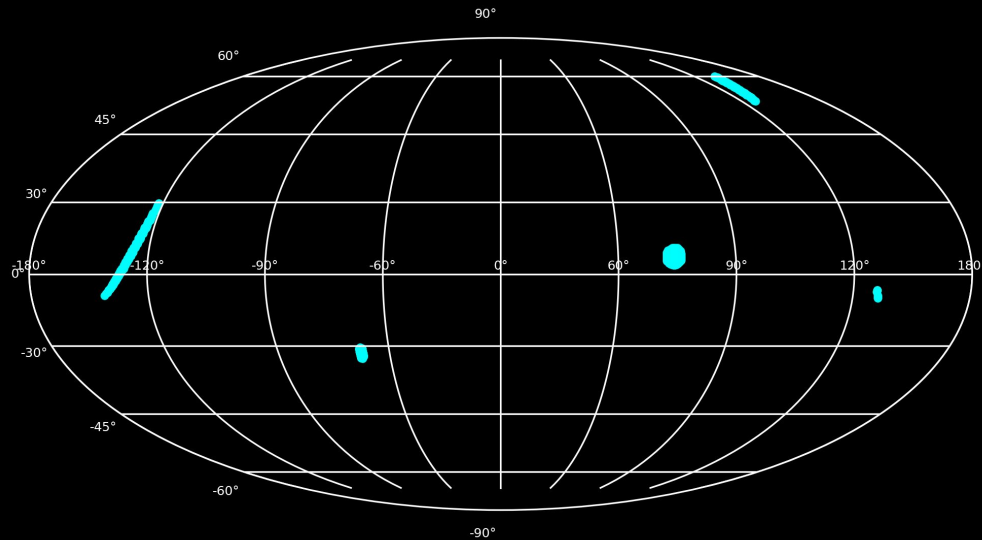
Low resolution
Bimodal!

MCMC corner plot



MCMC runs: I = 1+2+3

Mollweide projection showing hotspots

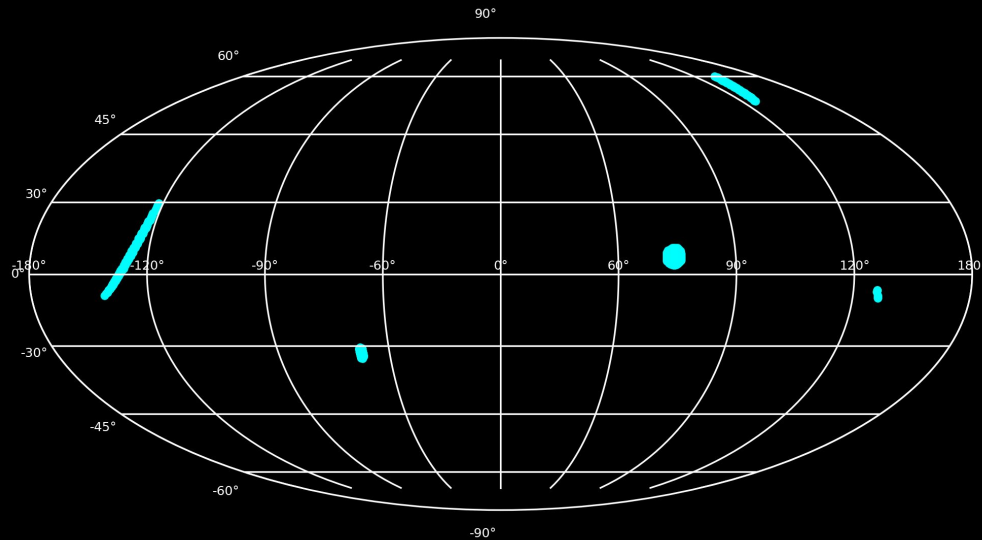


Work in progress...

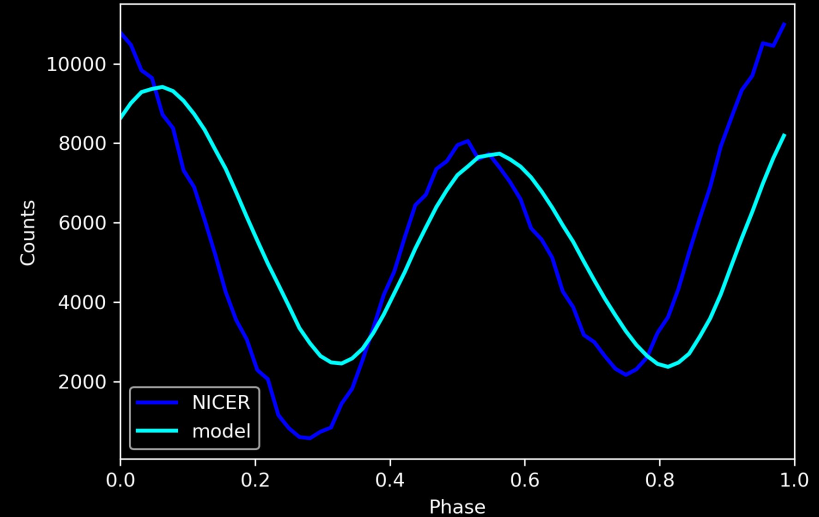
20 chains: 100 or 200 iterations
Grid size: 1000x1000 grid or 800x800

MCMC runs: $I = 1+2+3$

Mollweide projection showing hotspots



Light curve, model in red

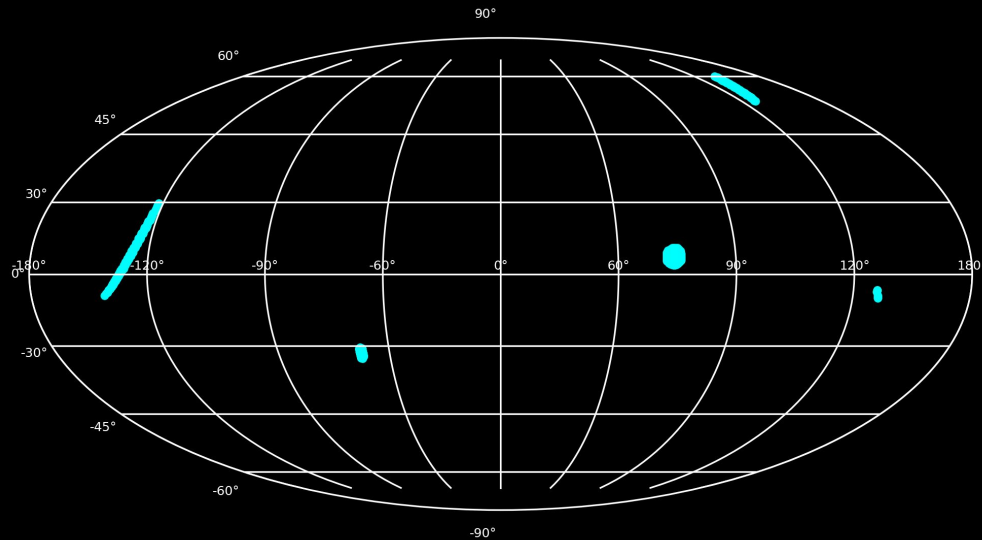


Work in progress...

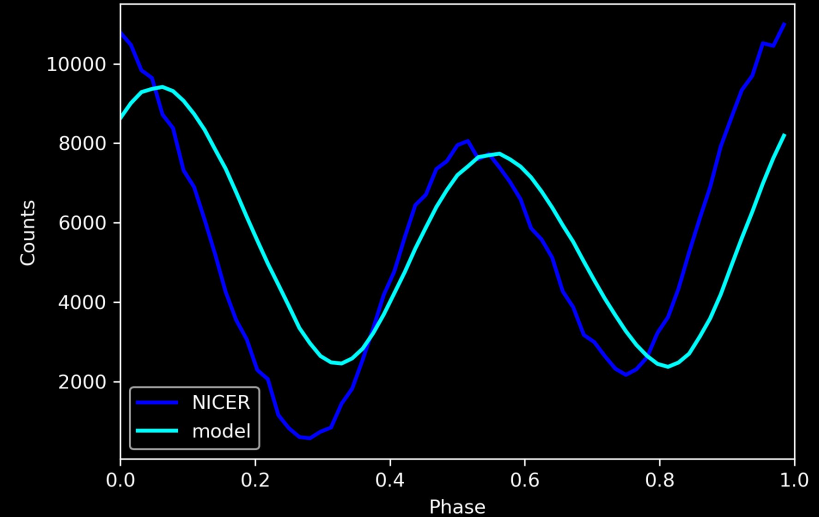
20 chains: 100 or 200 iterations
Grid size: 1000x1000 grid or 800x800

MCMC runs: I = 1+2+3

Mollweide projection showing hotspots



Light curve, model in red



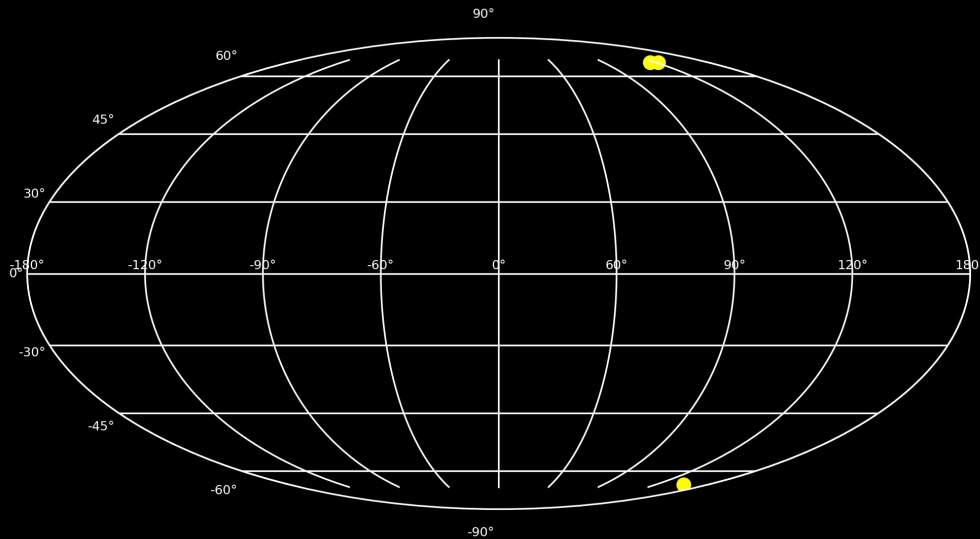
Incorrect phase transformation...

Work in progress...

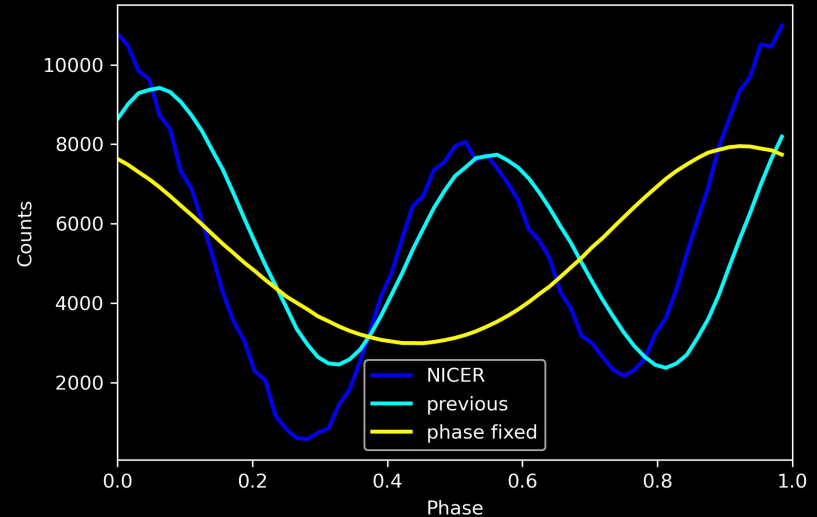
20 chains: 100 or 200 iterations
Grid size: 1000x1000 grid or 800x800

MCMC runs: I = 1+2+3

Mollweide projection showing hotspots



Light curve, model in cyan, yellow



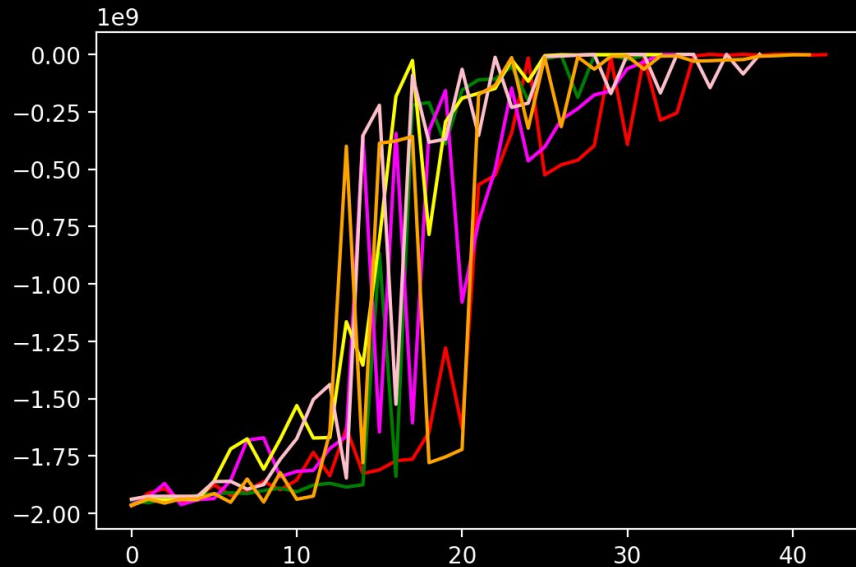
Incorrect phase transformation...
After the fixes...

Work in progress...

Grid size: 600x600, 50x50

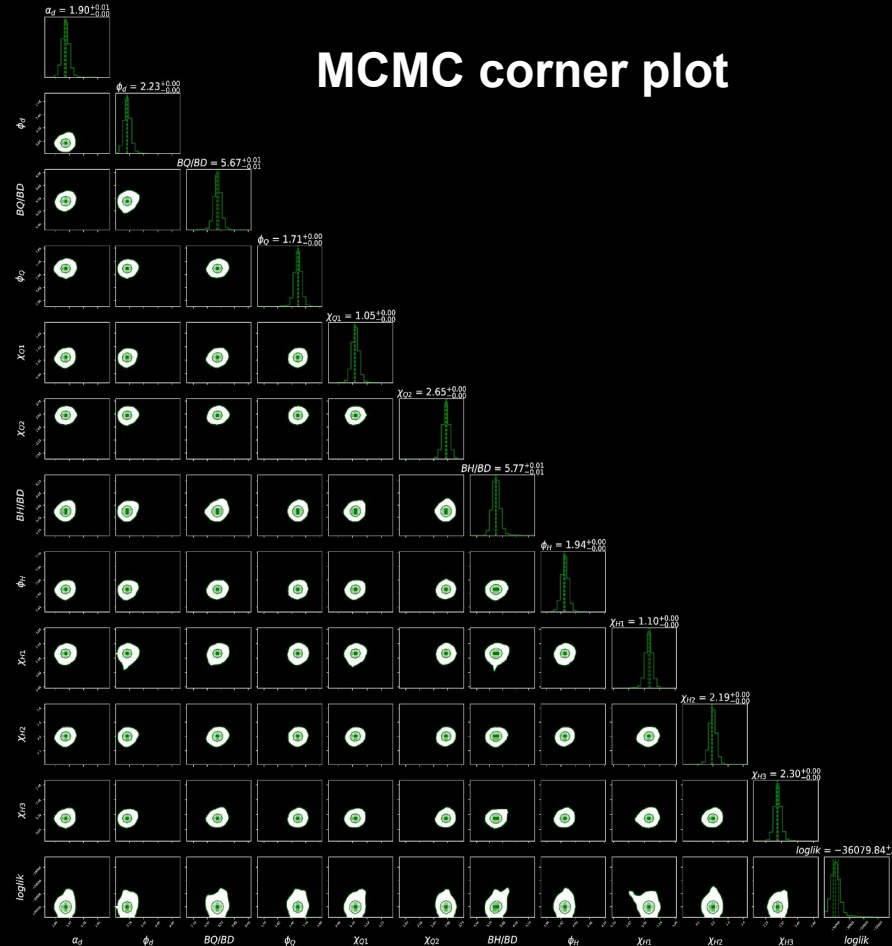
MCMC runs: I = 1+2+3

Trace plot for log-likelihood



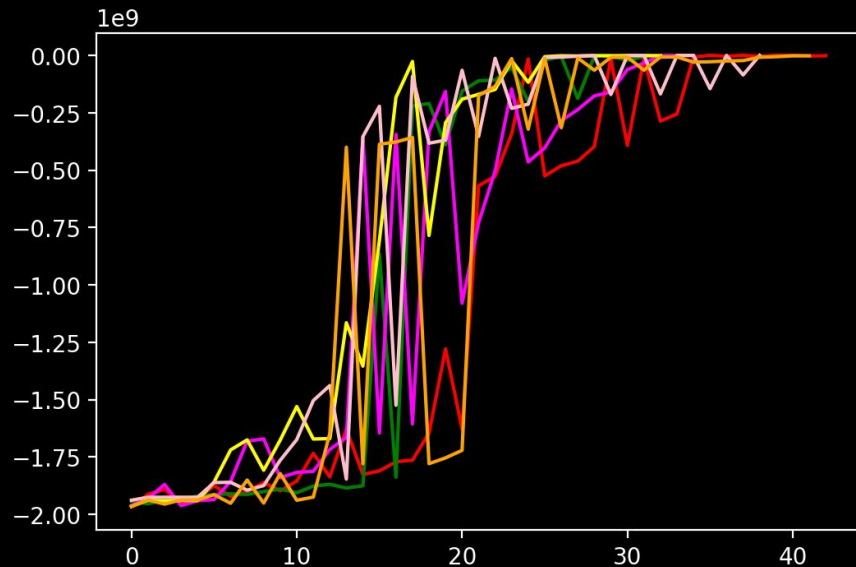
Maximizing negative log-likelihood :
For 50 steps, from order of billion to 1000

MCMC corner plot



MCMC runs: $I = 1+2+3$

Trace plot for log-likelihood

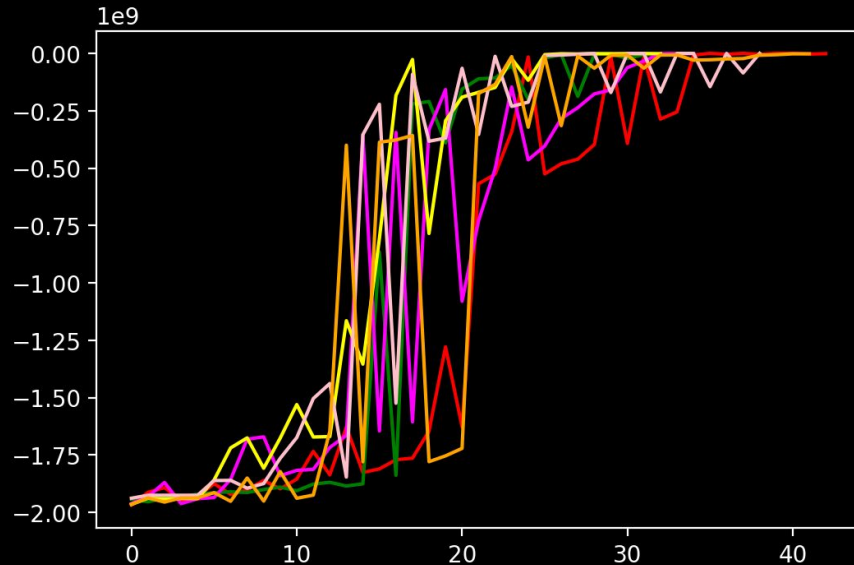


Current focus:

- Number of iterations, runtime
- Decent set of initial conditions

MCMC runs: $I = 1+2+3$

Trace plot for log-likelihood



Current focus:

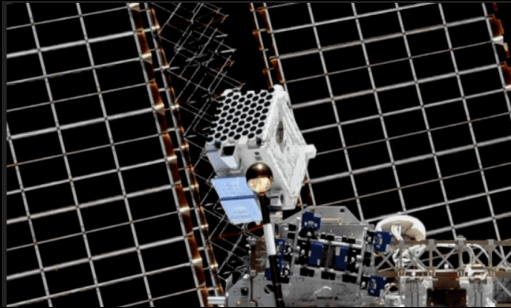
- Number of iterations, runtime
- Decent set of initial conditions

Current limitations:

- Cluster resources, limits
- Loadshedding, resulting in longer queues

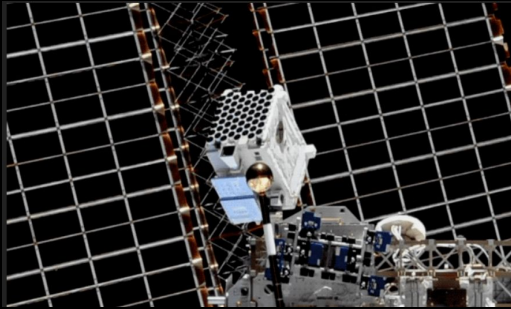
The full story...

Constraining the multipolar magnetic field of MSP
J0030+0451 via *NICER* X-ray light curve fitting



The full story...

Constraining the multipolar magnetic field of MSP J0030+0451 via *NICER* X-ray light curve fitting



Constraining the multipolar magnetic field of MSP J0030+0451 via combined *NICER* X-ray and *Fermi* gamma-ray light curve fitting

<https://heasarc.gsfc.nasa.gov/docs/nicer/>

<https://www.nasa.gov/sites/default/files/thumbnails/image/glap0588.jpg>

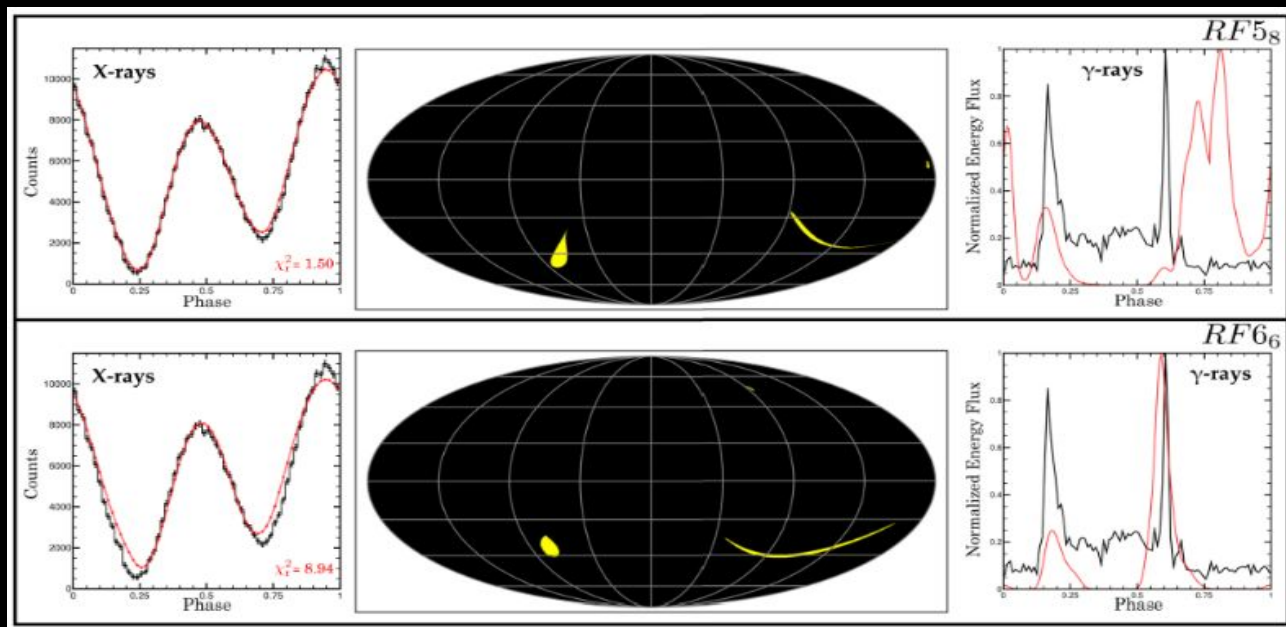
The full story...

Yellow:
Hotspots

LCs:
Black: NICER/Fermi data
Red: Model

X-ray LCs:
Field degeneracies

Gamma-ray LCs:
Lifts the degeneracies



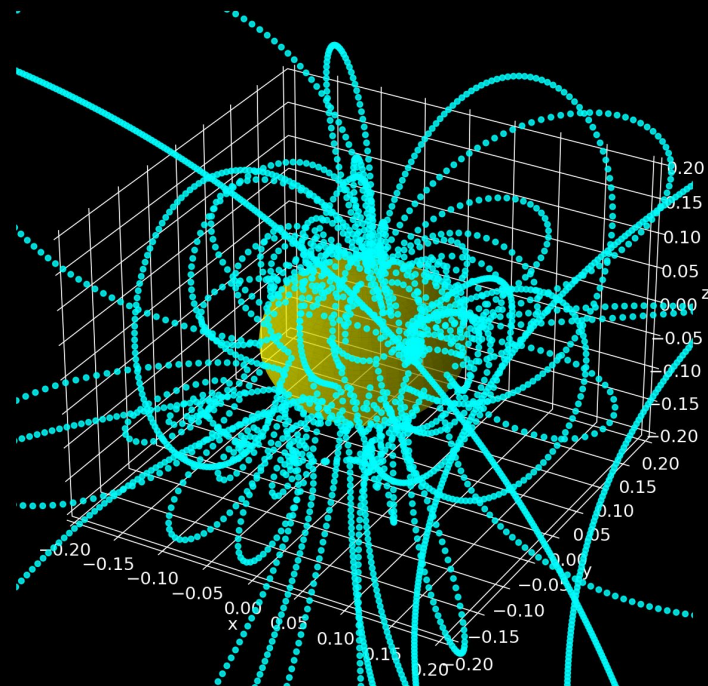
Summary and outlook

Now: Multipolar field + MCMC runs:

- Runtime optimization: Ideas?
- Constrain by simultaneous fitting with *Fermi* gamma-ray data: Degeneracies?
- Offset multipolar field

Next: Constraining equation of state:

- Self-consistent mass and radius determination
- Apply to other *NICER* MSPs



Tenth International Fermi Symposium

9th-15th October 2022



Thank you for your attention!



Anu Kundu*, North-West University

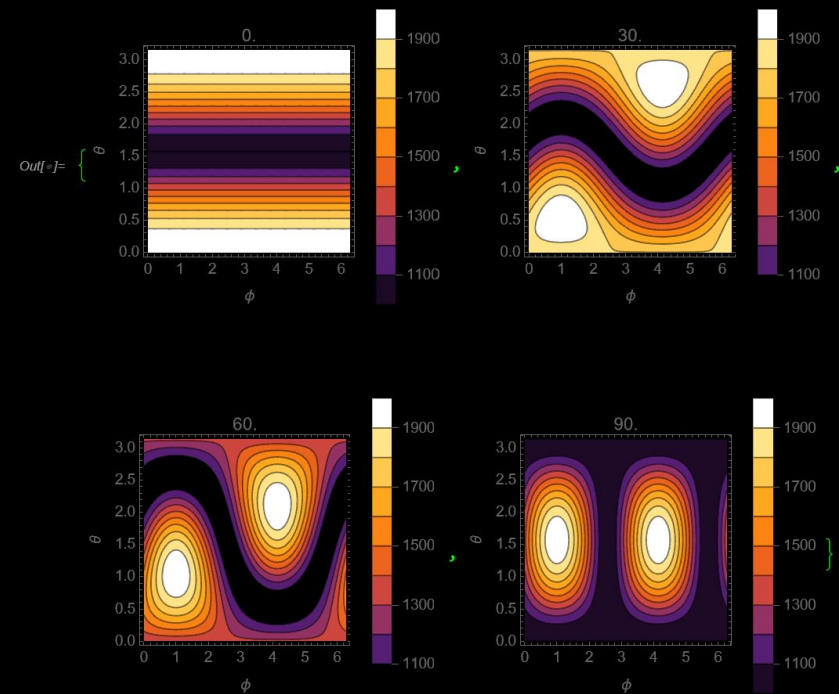
Can't think of any questions? Maybe you are curious about:

1. *“Normalization of the field strength of the different components?”*
2. *“Difference from what NICER team is doing?”*
3. *“Are you crazy enough to activate I4 and higher components?”*
4. *“What is this... loadshedding?”*
5. *(If you attended HEASA 2022:) “Updates on the lost peak...?”*

***anukunduo2@yahoo.com**

Extra slides

Surface magnetic field for $l = 1$



Surface magnetic field for $l = 2$

