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SHAKEN, NOT STIRRED: TEST PARTICLES IN BINARY-BLACK-HOLE MERGERS

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AIMS

To develop exploratory code to simulate the acceleration and spectral energy distribution of any resulting EM-radiation from charged particles accelerated only due to gravity in inspiraling binary black hole (BBH) merger environments.

Simplifying assumptions?

- Eccentricity of the BBH orbits are circular
- The BHs are non-rotating and uncharged
- The orbits of the BHs are determined using a 1^{st} order PN effective one-body problem
- Low particle density close to stellar mass BBH (accretion disk dynamics can be neglected)

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BUILDING THE MODEL
WHAT DO WE NEED?

- Some Newtonian-like potential to determine the dynamics of the particle
  - Paczynski-Wiita potential
  - Time dependent positions of BHs
  - BHs are in motion => Potentials evaluated at retarded position
- Momentum and Force equations with which to model the dynamics of the particles
- Numerical solver to calculate the Dynamics ODEs
- A way to calculate the SED from the Numerically calculated $\vec{a}$-components

BUILDING THE MODEL
PACZYNSKI-WIITA POTENTIAL

For a preliminary look at the possible accelerations of test particles in a BBH system, we use the pseudo-Newtonian Paczynski-Wiita potential:

$$\Phi_{PW} = -\frac{GM}{r - r_s}, \text{with } r_s = \frac{2GM}{c^2},$$

- Exactly reproduces the position of marginally bound and marginally stable circular orbit & form of the Keplerian angular momentum of a Schwarzschild BH
- Orbits are expected to be more chaotic than fully relativistic case for radii $< 6r_G$

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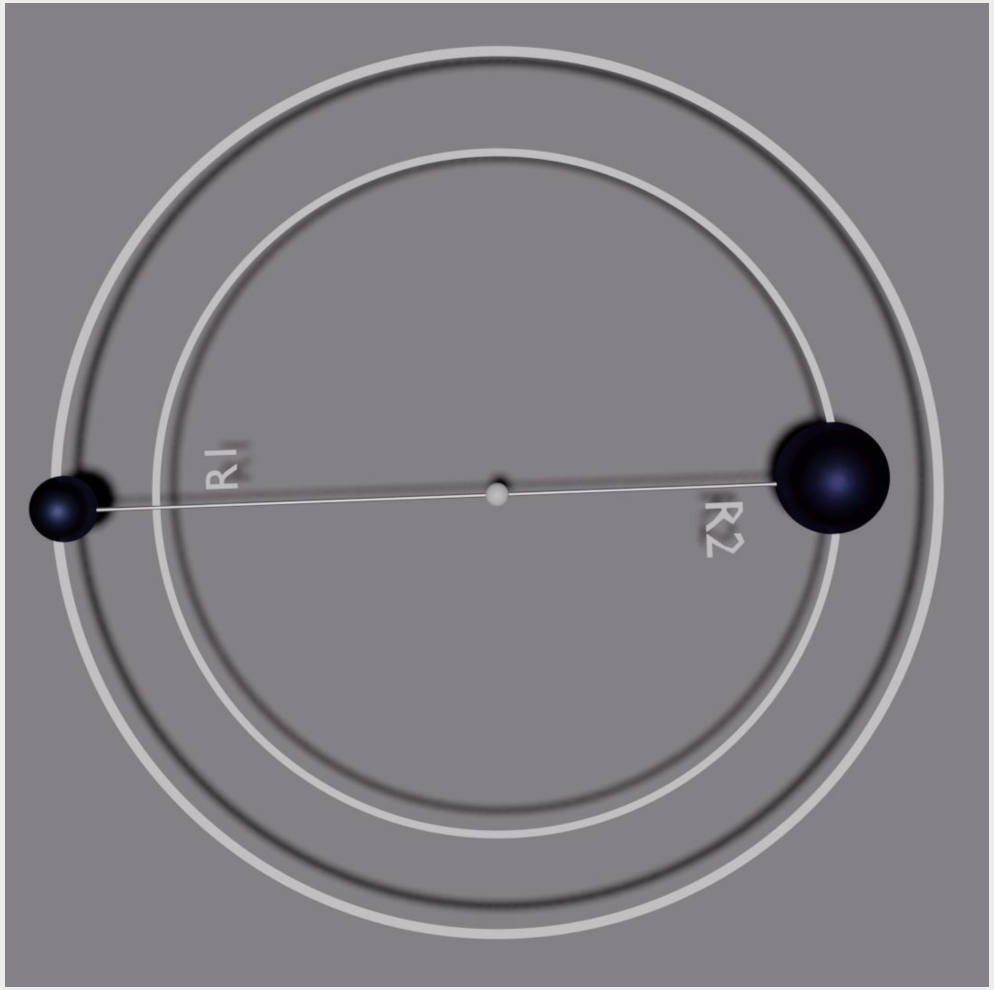
BUILDING THE MODEL
POSITION OF BBH

- Time dependent description of BBH motion
  - Classical effective one-body problem



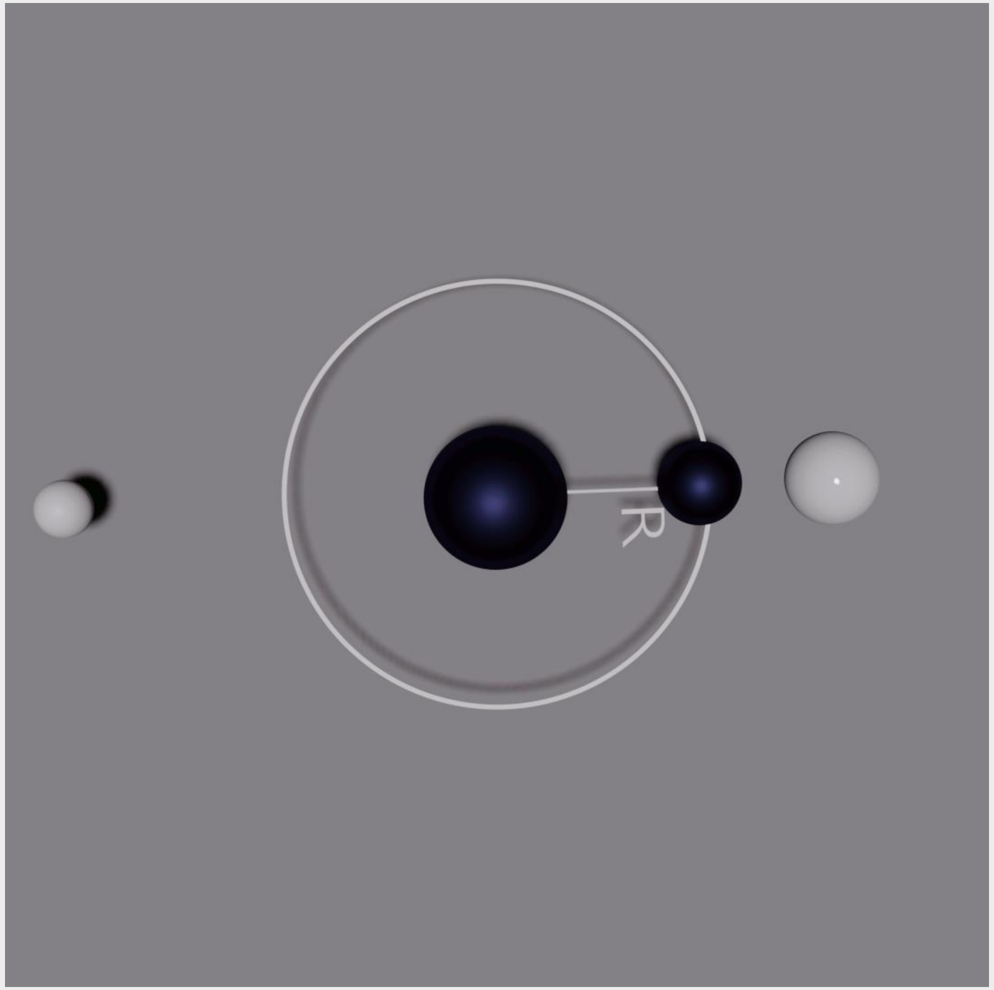
BUILDING THE MODEL

POSITION OF BBH (…THEN)

- Time dependent description of BBH motion
  - Classical effective one-body problem
  - Replace M_1 & M_2 with $\mu = \frac{M_1 M_2}{M_1 + M_2}$ & $M = M_1 + M_2$
  - Solve using Lagrangian mechanics

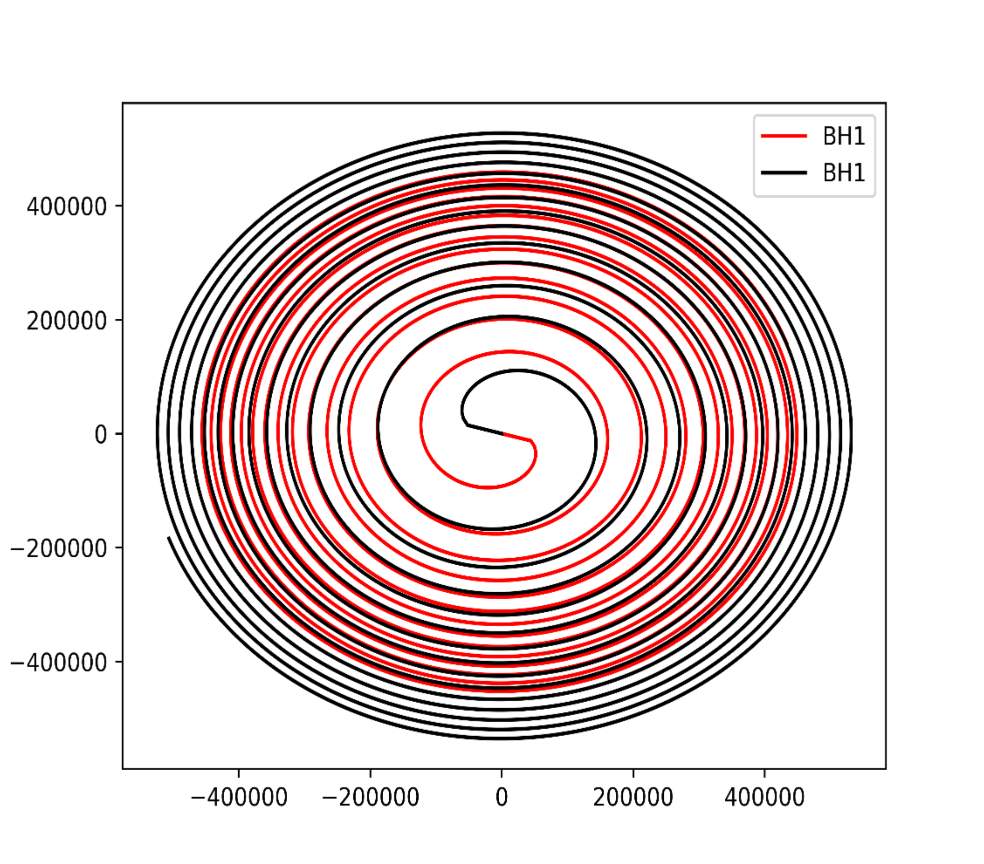
▸ $$\vec{r}_1 = \begin{pmatrix} \frac{M_2 P_\phi^2}{\mu G M^2} \cos\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \\ \frac{M_2 P_\phi^2}{\mu G M^2} \sin\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \end{pmatrix}$$

▸ $$\vec{r}_2 = -\begin{pmatrix} \frac{M_1 P_\phi^2}{\mu G M^2} \cos\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \\ \frac{M_1 P_\phi^2}{\mu G M^2} \sin\left(\frac{\mu G^2 M^2}{P_\phi^3} t\right) \end{pmatrix}$$



BUILDING THE MODEL
POSITION OF BBH (...NOW)

- Time dependent description of BBH motion
  - Classical effective one-body problem
  - Replace M_1& M_2 with $\mu = \frac{M_1 M_2}{M_1+M_2}$
    & $M = M_1+M_2$
  - Take into account the energy loss due to GW to obtain 1PN accurate equations for the BHs



$$R(\tau) = R_0 \left(\frac{\tau}{\tau_0}\right)^{1/4} = R_0 \left(\frac{t_{\rm coal} - t}{t_{\rm coal} - t_0}\right)^{1/4}$$

$$\Phi(\tau) = -2 \left(\frac{5GM_c}{c^3}\right)^{-5/8} \tau^{5/8}$$

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BUILDING THE MODEL
RETARDED TIME

To account for the motion of the BH, the implicit definition of retarded time is used

$$t_r = t - \frac{1}{c}\left|\vec{r} - \vec{r}_{BH}\left(t_r\right)\right|$$

Using the position equations for each BH as a function from the EOB problem, we can solve this numerically using Newton's method

$$t_{r,n+1} = t_{r,n} - \frac{f\left(t_{r,n}\right)}{f'\left(t_{r,n}\right)}$$

$$f\left(t_{r,n}\right) = t - \frac{1}{c}\left|\vec{r} - \vec{r}_{BH}\left(t_{r,n}\right)\right| - t_{r,n}$$

BUILDING THE MODEL

DETERMINING THE POSITION AND ACCELERATION EVOLUTION OF A PARTICLE THROUGH THE SYSTEM

- The evolution of the position of a particle orbiting in the system is found by simultaneously solving the following set of coupled ODEs using the RKF7(8) adaptive time-step method, with upper and lower limits on the time step size:

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = -\vec{\nabla}\left[\Phi_{PW,1}\left(t_{r,1},\vec{r}\right) + \Phi_{PW,2}\left(t_{r,2},\vec{r}\right)\right]$$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\vec{p}}{m_e}\sqrt{1 - \frac{p^2}{m_e^2}\left(1 + \frac{p^2}{m_e^2}\right)^{-1}}$$

where we stop the solver once the particles plummets into a BH or the BHs get too close to one another.

- The acceleration of the particle as a function of time is found using

$$\vec{a}_i = \frac{1}{m_e \gamma_i}\left[\vec{F}\left(t_{t,i}\right) - \frac{\left(\vec{v}_i \cdot \vec{F}\left(t_{r,i}\right)\right)}{c^2}\vec{v}_i\right]$$

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BUILDING THE MODEL
CALCULATING THE SED

- SED of each particle in the system is calculated by

$$I(\nu) = \frac{\mu_0 q^2}{3\pi c}\left[\left|\gamma^2 \vec{a}_{\parallel}(\nu)\right|^2 + \left|\gamma^3 \vec{a}_{\perp}(\nu)\right|^2\right]$$

- A non-uniform Fourier transform is applied to the components of the acceleration parallel and perpendicular to the direction of motion using the fiNUFFT library.
- Total SED from the system is calculated from: $I_{Total}(\nu) = \sum_{particles} I_n(\nu)$
- Due to variable lengths of the generated data, it is necessary to use cubic spline methods to calculate the total SED.

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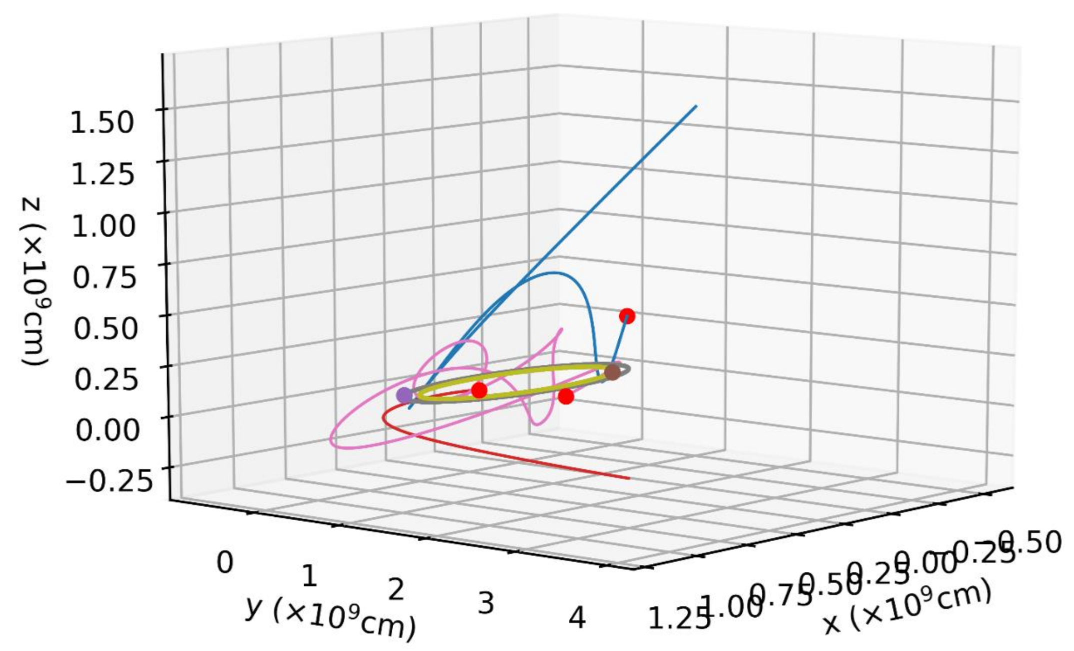
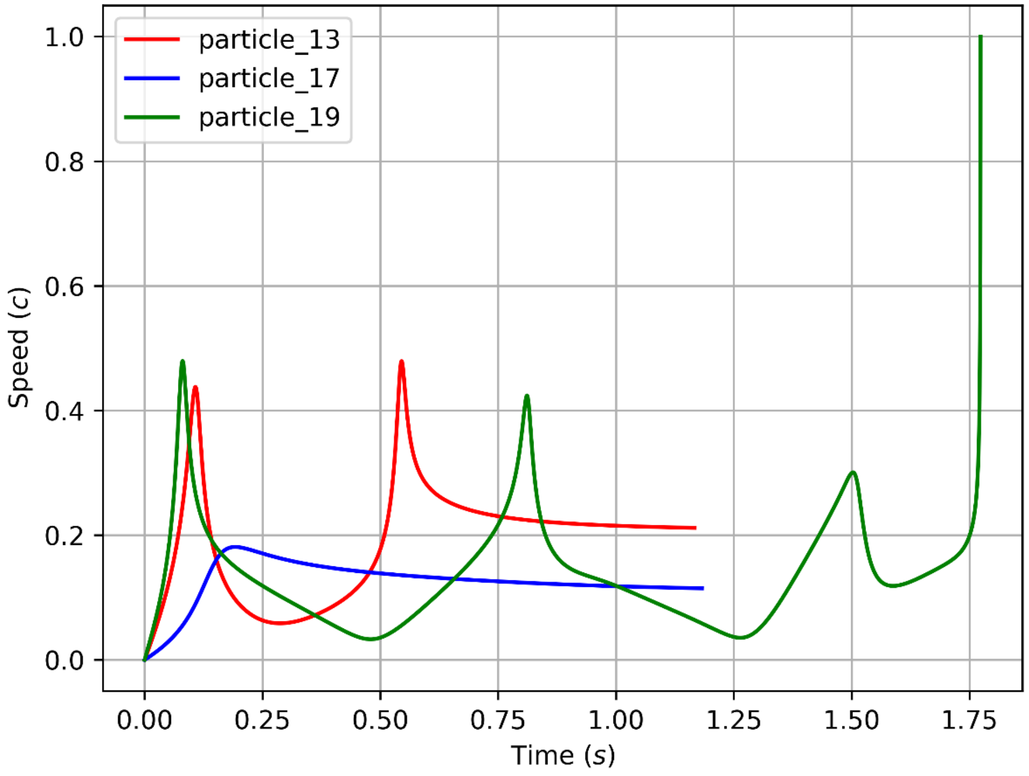
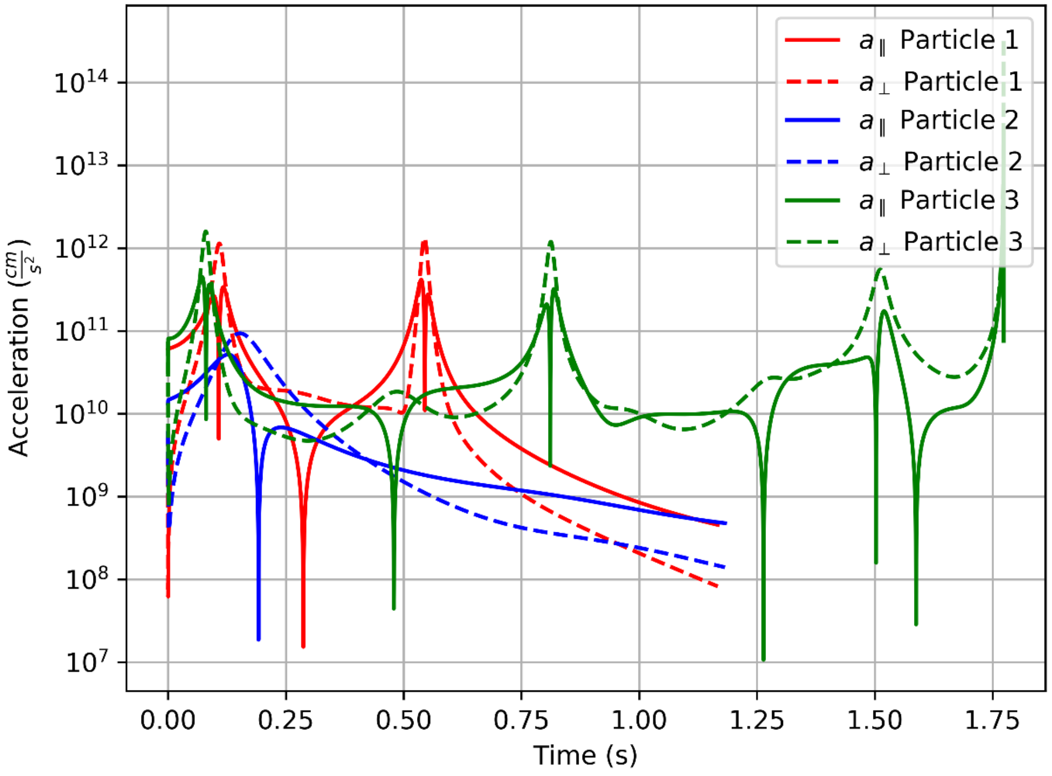
EXAMPLE TRAJECTORIES

- BBH separation: $10^8 cm$
- M_1: $30M_\odot$
- M_2: $35M_\odot$
- 50 Particles Randomly distributed



EXAMPLE SPEED AND ACCELERATION COMPONENTS





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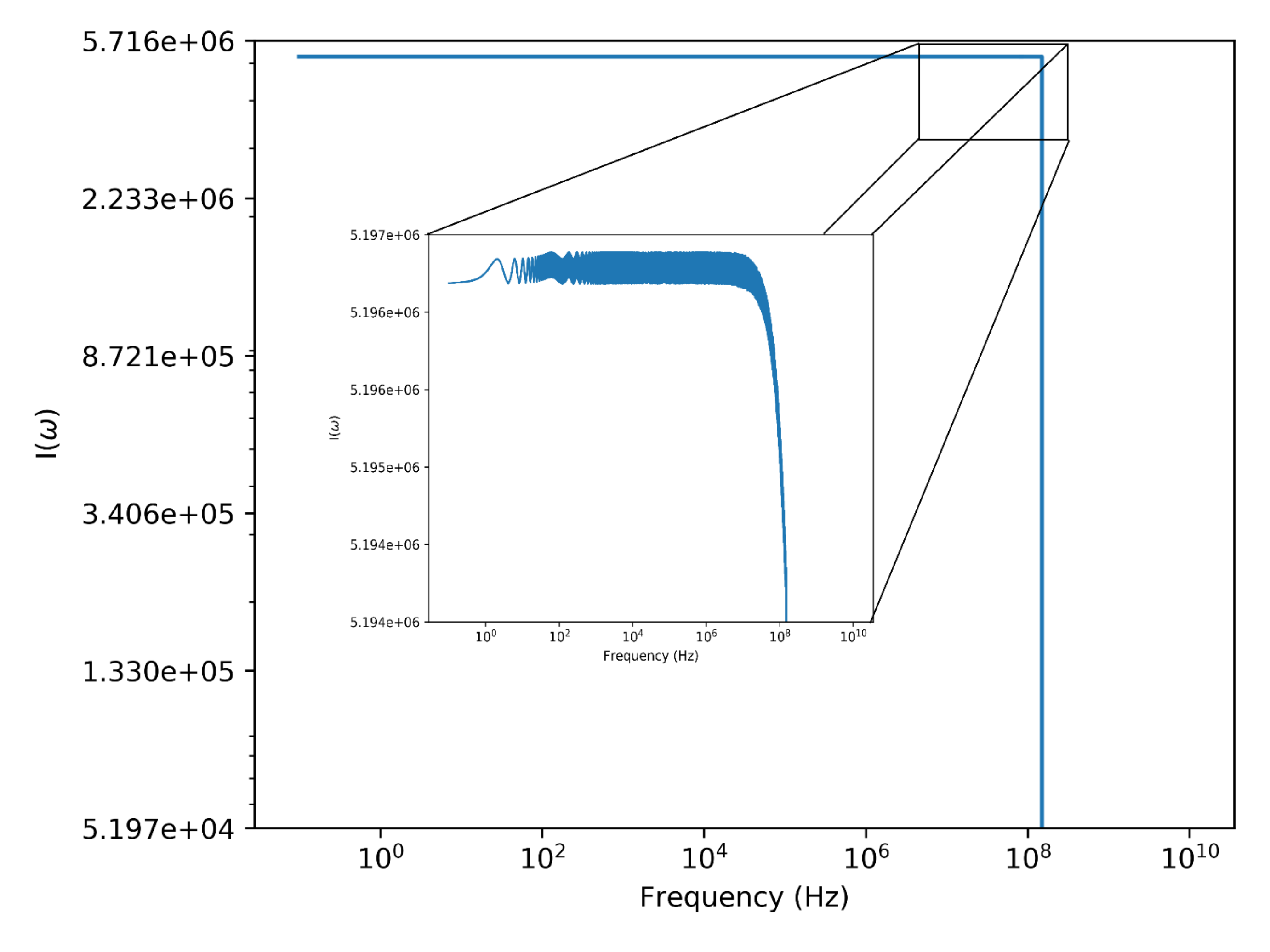
EXAMPLE TOTAL SED

- BBH separation: $10^8 cm$
- M_1: $30M_\odot$
- M_2: $35M_\odot$
- 50 Particles Randomly distributed
- Steep drop-off at $10^8 Hz$



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FINAL REMARKS AND CAVEATS

- This is a very naïve, single particle approach simulation.
- It is expected that EM fields will dominate over gravitational effects in these systems.
- More work still needs to be done to determine how much of the predicted spectrum could actually be detected.

Thank you for your attention!

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