Statistical Goodness and Utility

Lessons learned from multiband pulsar light curve fits

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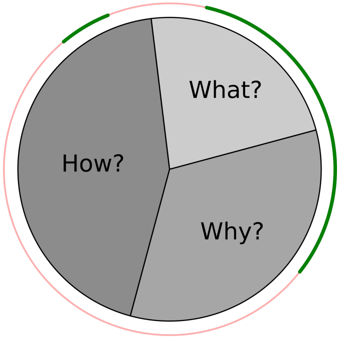
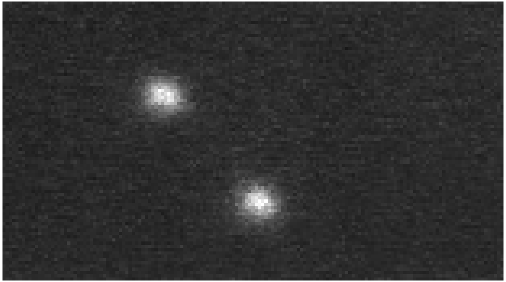
Tenth International Fermi Symposium, 9th–15th October 2022

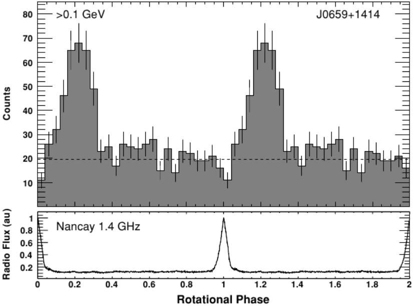
An invitation

This talk will focus on the "What?" (and some "Why?") of our work.
For more: Come and chat over coffee etc!

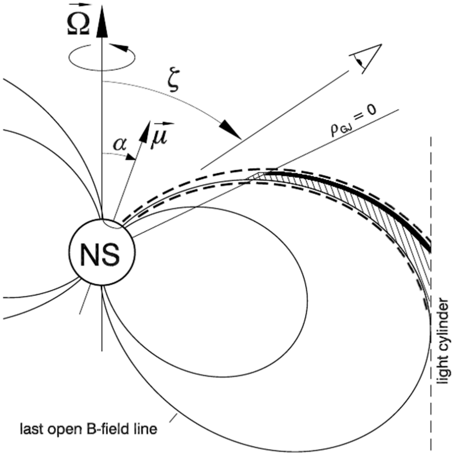


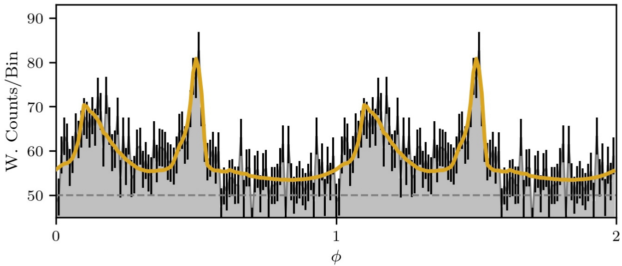
Pulsars and Light Curves (LCs)





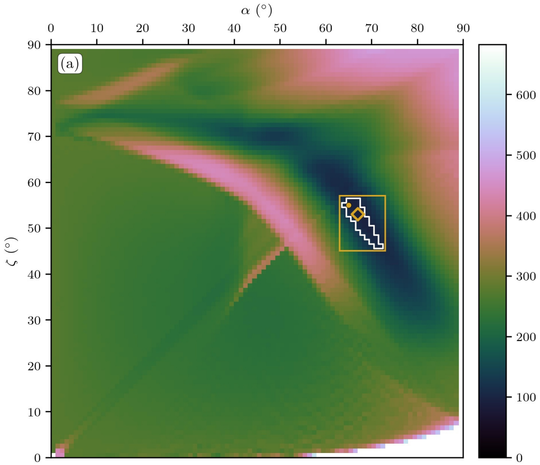
Models and Fits (single-band)





$$\chi^2(M) = \sum_{i=1}^{n} \left(\frac{D_i - M_i}{E_i} \right)^2$$

Models and Fits (single-band)



$\rightarrow$

Parameter estimates

$$(\alpha, \zeta) = (67^\circ, 53^\circ)$$

With constraints:

$$(\alpha, \zeta) = \left(67^{+5}_{-3},\ 53^{+3}_{-7}\right)^\circ$$

Models and Fits (dual-band) $\lhd\!\!-$ The Problem!

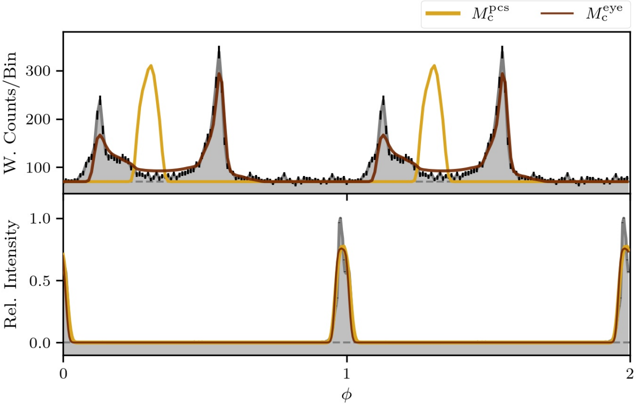
$$\chi^2_{\mathrm{c}}(M_{\mathrm{c}}) = \chi^2_{\mathrm{r}} + \chi^2_{\gamma} \qquad \rightarrow$$

Example

Using χ^2_{c}:
  the gold fit is better

But if:
  better shape $\Rightarrow$ better estimates

Then:
  the brown fit is better



Caveat: This talk will focus on the χ^2 statistic; there are numerous others.

!!! SPOILER !!!

We succeeded in eliminating the radio dominance![1]

[1] Papers in prep.

!!! SPOILER !!!

We succeeded in eliminating the radio dominance!

Focus of this talk

What more generally applicable lessons did we learn in doing so?

Lesson #1

There are two types of statistical fit[2]

Fits aimed at
Establishing goodness of fit

which require a
deviation statistic

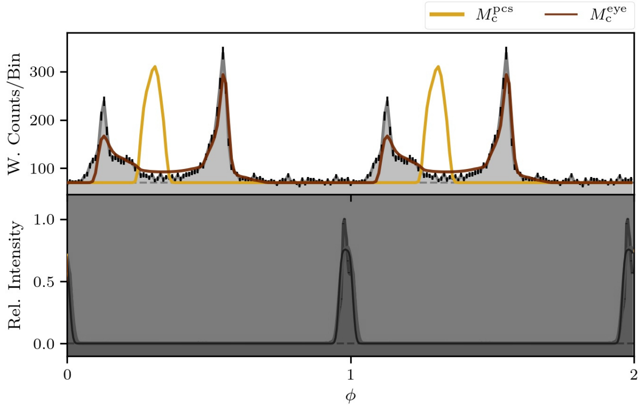
&

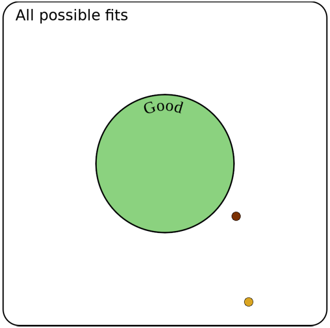
Fits aimed at
Parameter estimation

which require a
utility statistic

[2]Within the LC fitting context, at least.

Single-band fits — "Just use χ^2 and squint!"





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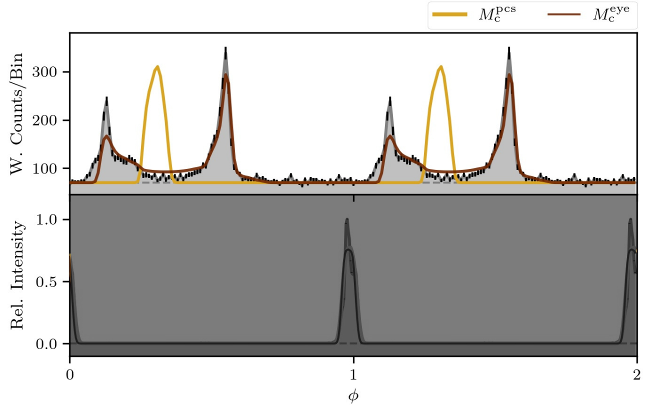
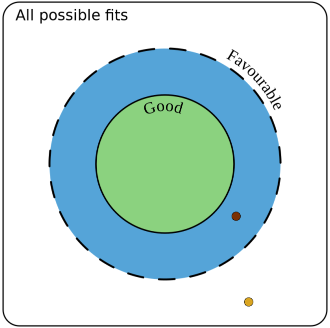
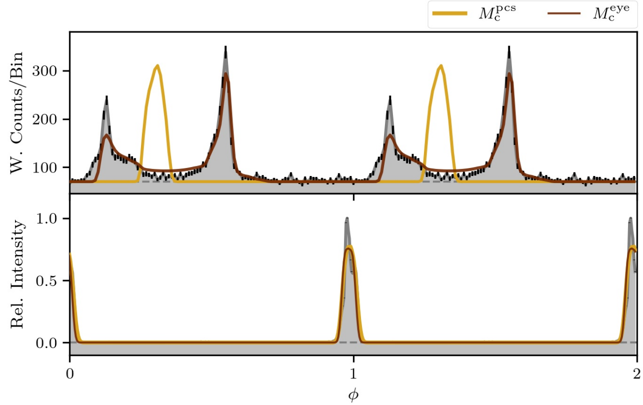
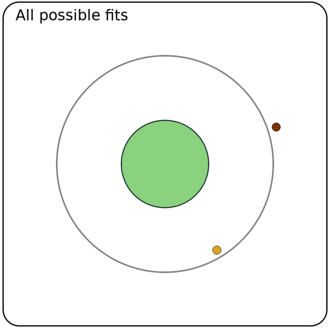


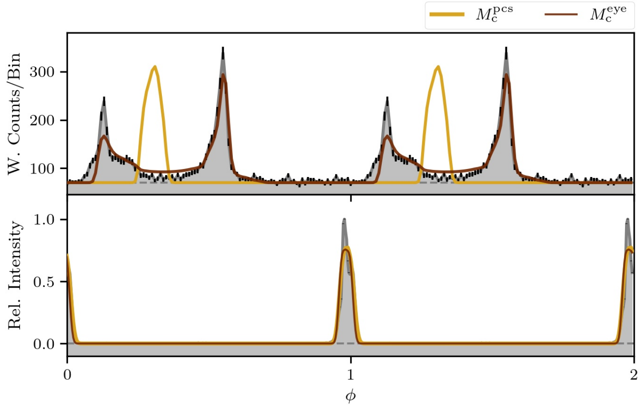


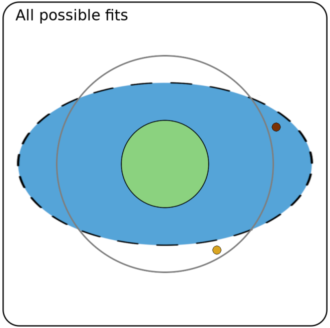
Dual-band fits — "Just use χ^2 and squint!"





Dual-band fits — "…Oh."





Lesson #2

Parameter estimation requires its own statistic
(in some contexts)

What's going on here??

$$\chi^2_{\mathrm{c}}(M_{\mathrm{c}}) = \chi^2_{\mathrm{r}} + \chi^2_{\gamma}$$

Most simply

- From the perspective of utility, χ^2_{r} and χ^2_{γ} carry different units
- This renders the addition operation improper

Therefore

We need a single-band statistic that carries units of utility

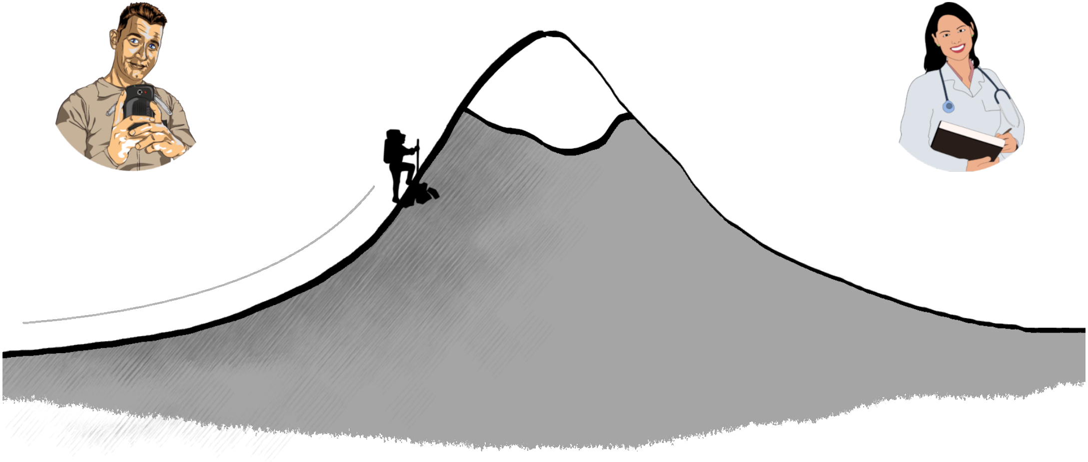
Bonus lesson: Dimensionless quantities (like χ^2) still carry units!

Lesson #3

Deviation is top-down, while Utility is bottom-up

A Silly Analogy

A Silly Analogy



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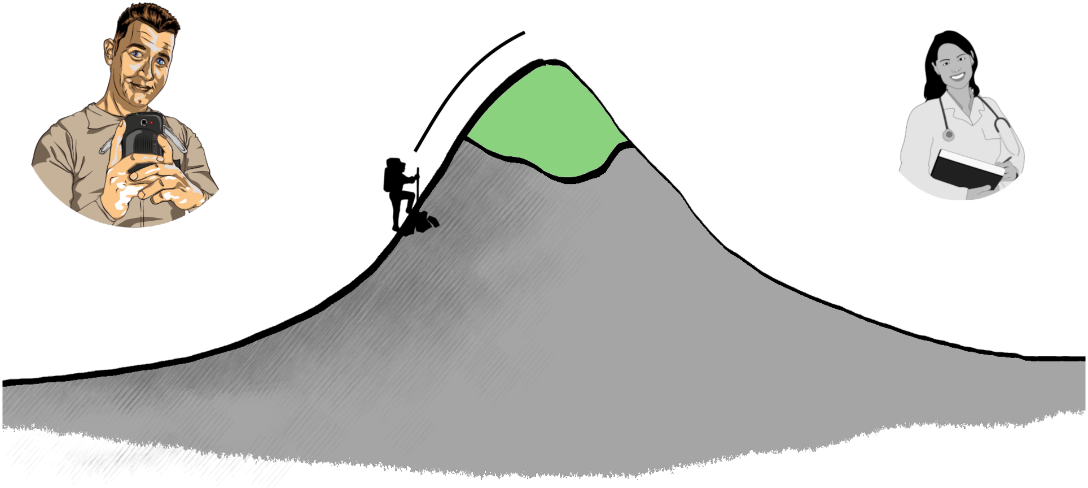

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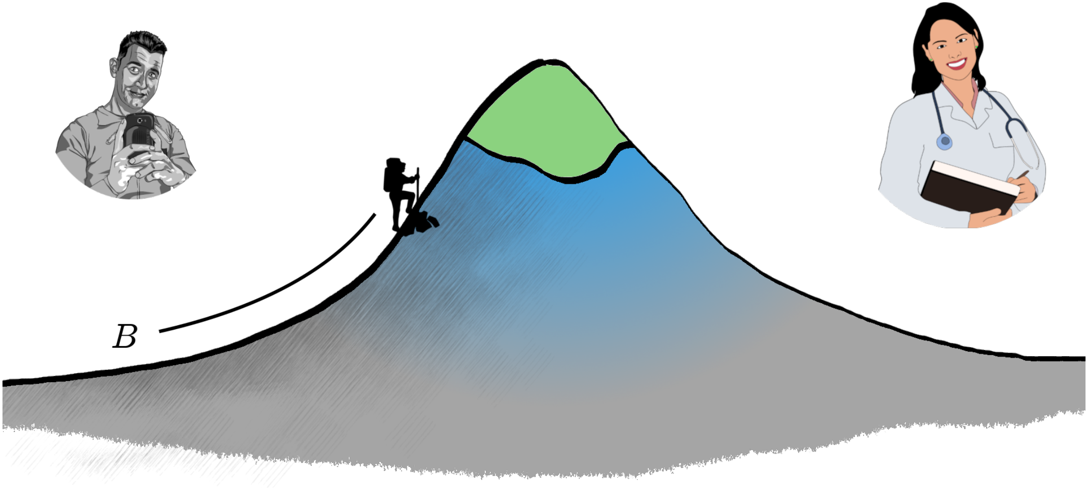

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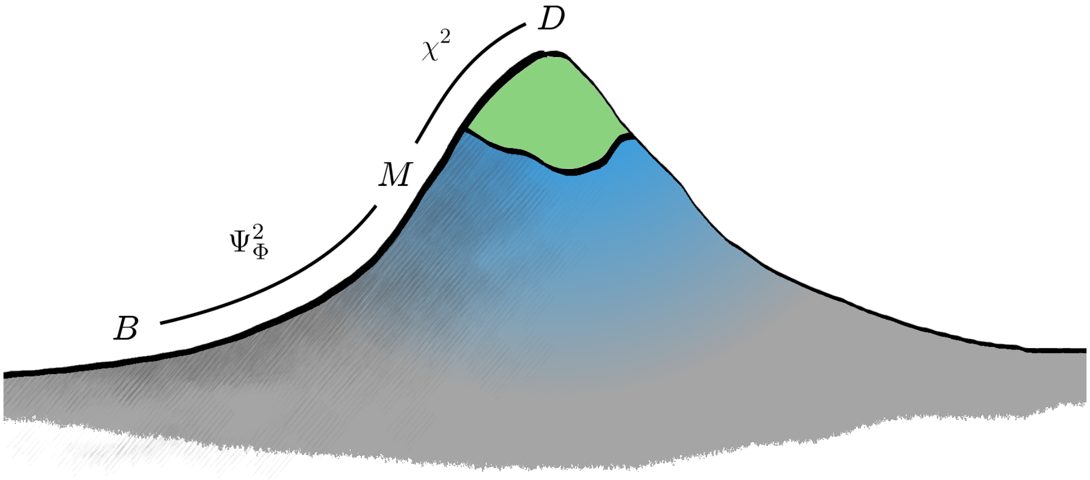
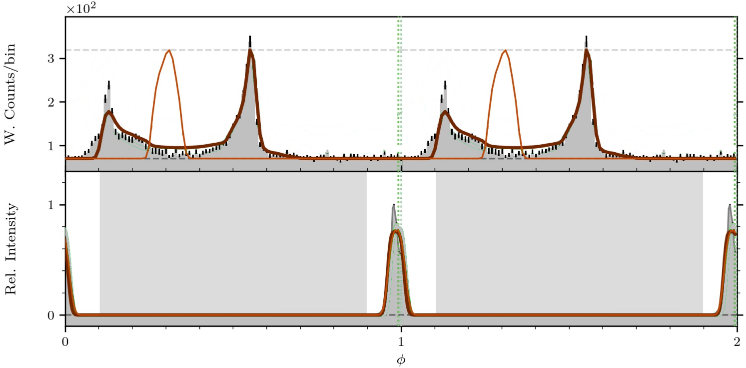


Dual-band fit: Revisited

$$\Psi^2_{\Phi,\mathrm{c}}(M_\mathrm{c}) = \frac{1}{2}\Psi^2_{\Phi,\mathrm{r}} + \frac{1}{2}\Psi^2_{\Phi,\gamma}$$



PSR J1048−5832
$\log(\lambda_{\mathrm{r}\gamma}) = 1.50$

$\Psi^2_{\Phi,\mathrm{r}}$: $0.95 \rightarrow 0.92$

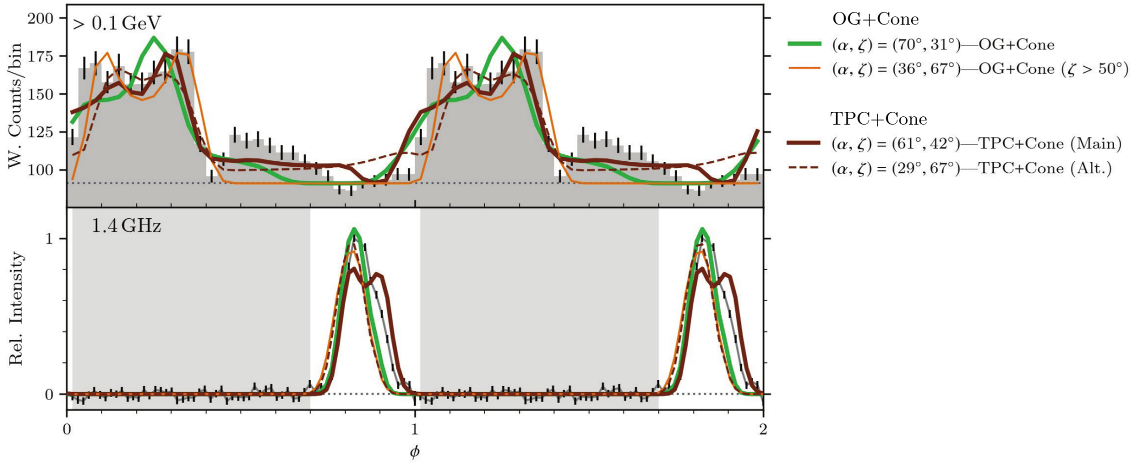
$\Psi^2_{\Phi,\gamma}$: $-4.12 \rightarrow 0.88$

Bonus lesson: Some observations are like perturbations; from B to D.

Lesson #4

$\chi^2_{\rm c}$ deviation and $\Psi^2_{\Phi,{\rm c}}$ utility are complementary

Example application for PSR J2039–5617[3]



[3] see Corongiu et al., 2020

Example application for PSR J2039–5617[3]

Fit	$\Psi^2_{\Phi,\mathrm{c}}$	$[\chi^2_{\mathrm{c}}]_\nu$	$\Psi^2_{\Phi,\mathrm{r}}$	$\chi^2_{\nu,\mathrm{r}}$	$\Psi^2_{\Phi,\gamma}$	$\chi^2_{\nu,\gamma}$
OG+Cone	0.883	6.02	0.967	5.01	0.799	9.02
OG+Cone ($\zeta > 50^\circ$)	0.834	11.12	0.924	11.50	0.743	11.52
TPC+Cone (Main)	0.846	16.50	0.859	21.21	0.833	7.53
TPC+Cone (Alt.)	0.841	10.46	0.929	10.69	0.753	11.12

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Example application for PSR J2039–5617[3]

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Summary & Conclusion

The big lessons that we learned are:

1. There are two types of statistical fit:
   - Establish goodness of fit, and
   - Parameter estimation
2. Parameter estimation requires its own statistic (in some contexts)
3. Deviation is top-down, while utility is bottom-up
4. χ^2_{c} deviation and $\Psi^2_{\Phi,\mathrm{c}}$ utility are complementary

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These lessons aren't confined to pulsar LC fitting, though!

In future we hope to apply these lessons to other fits where deviation-focussed statistics struggle; e.g., joint fitting to spectral data and surface brightness profiles for PWNe[4]

[4] see Van Rensburg, et al., 2020 for some preliminary results!

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—Thank you!—